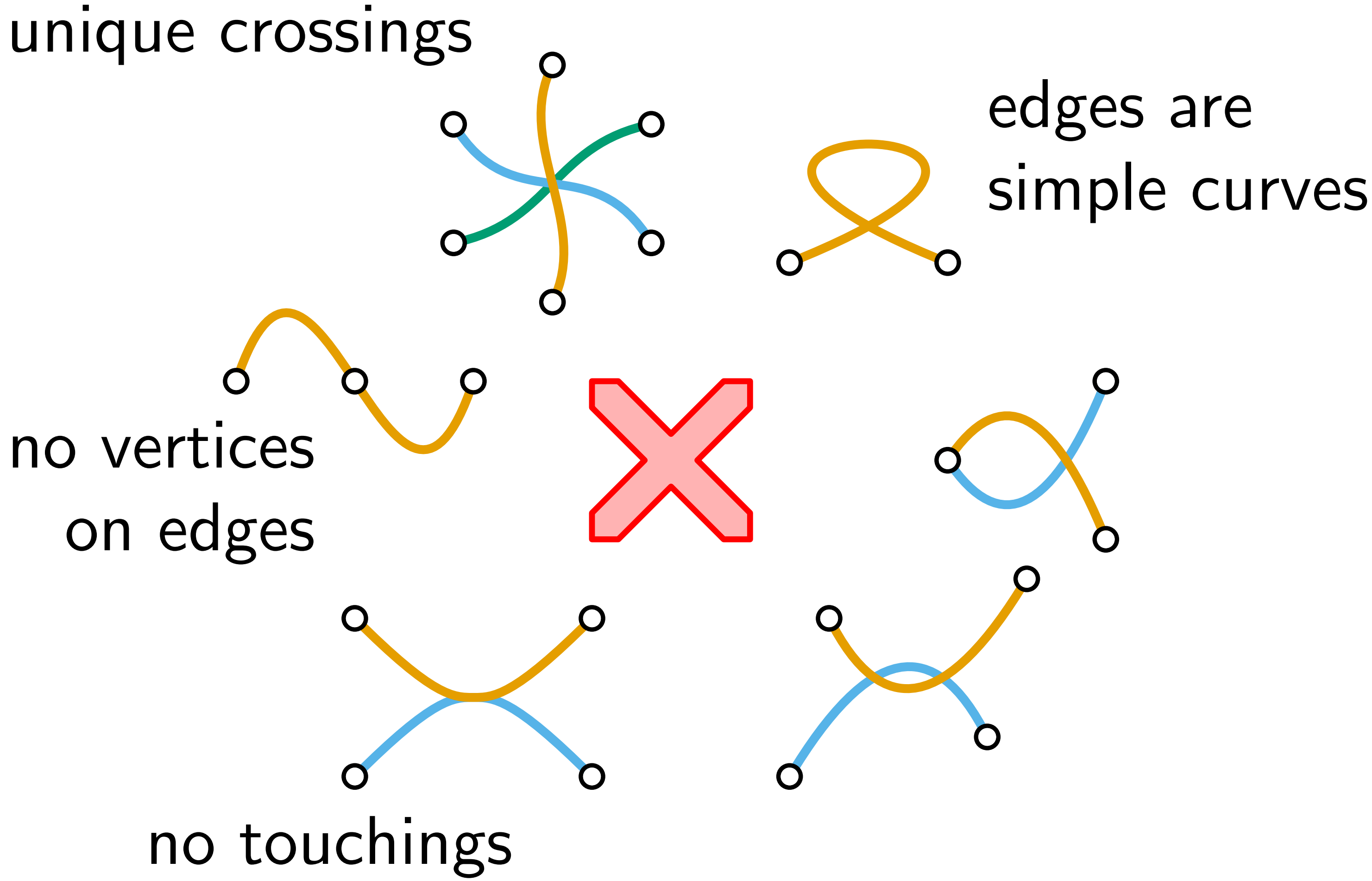
Separable Drawings:

Extendability and Crossing-Free Hamiltonian Cycles

Oswin Aichholzer, **Joachim Orthaber**, and **Birgit Vogtenhuber**

GD 2024: September 20

Introduction: Simple Drawings



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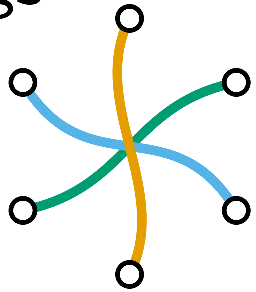
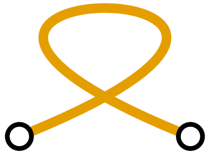
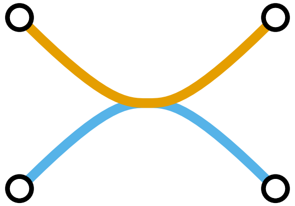
unique crossings



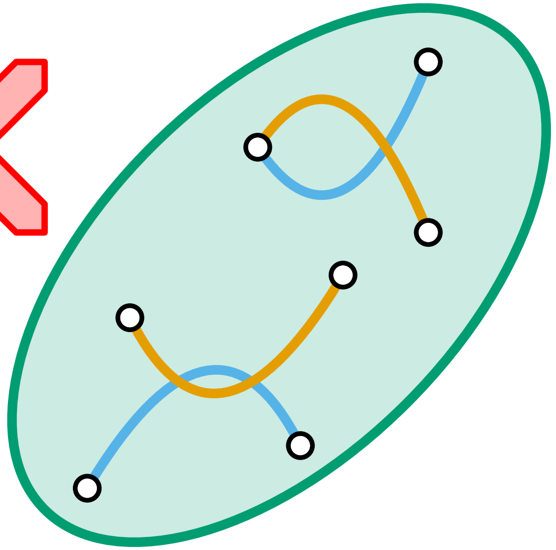
edges are simple curves



no vertices on edges



no touchings



two edges have at most one point in common

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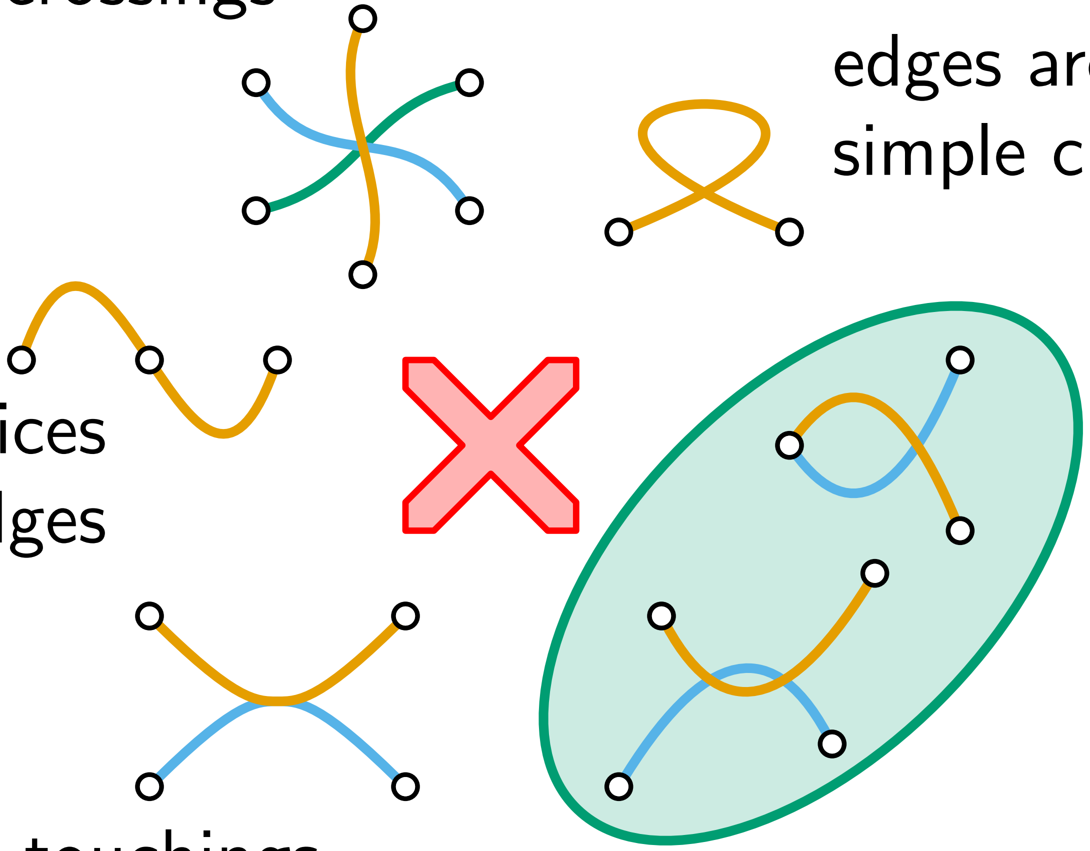

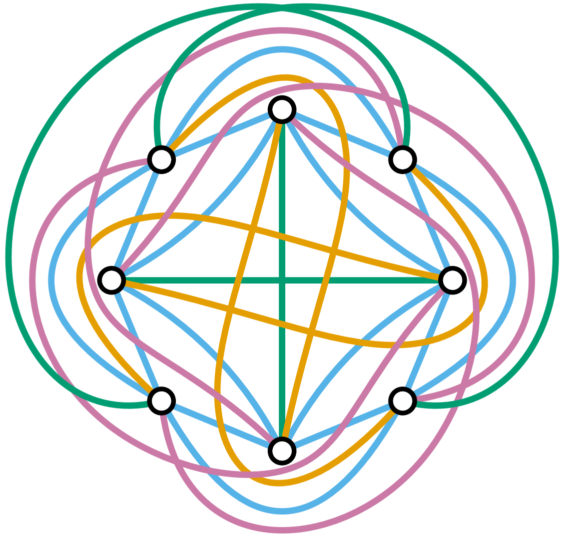

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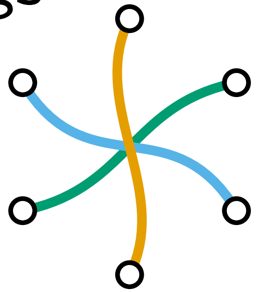

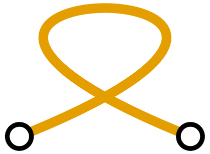

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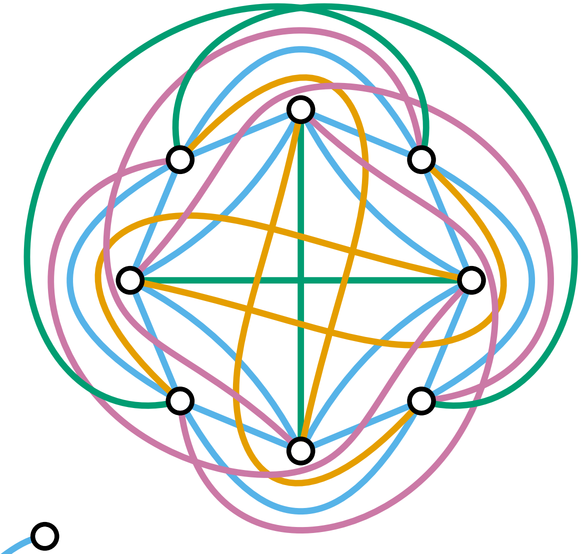

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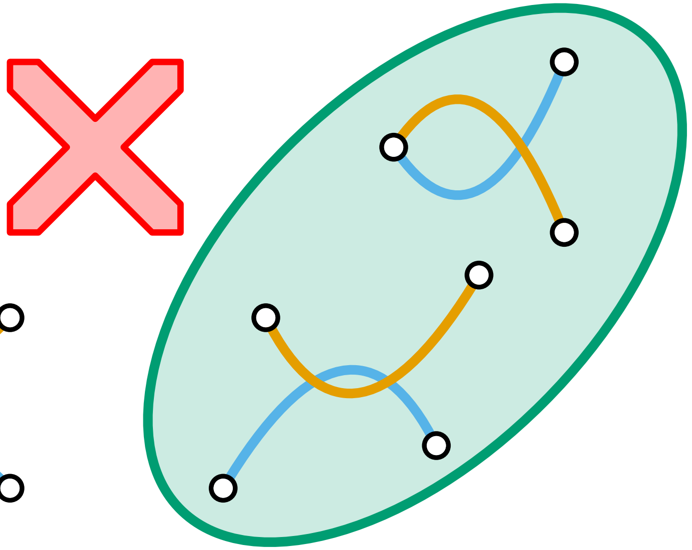

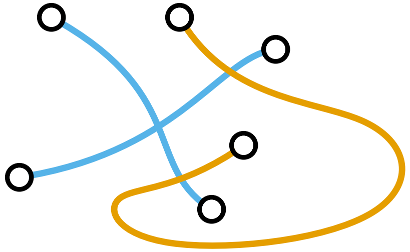

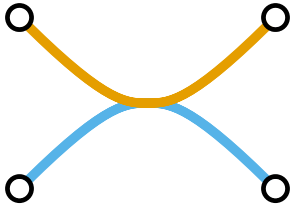

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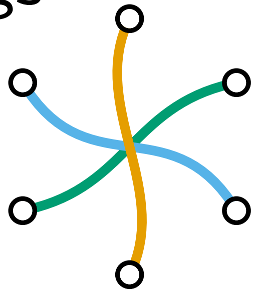

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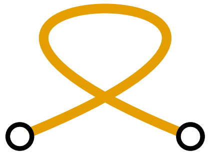

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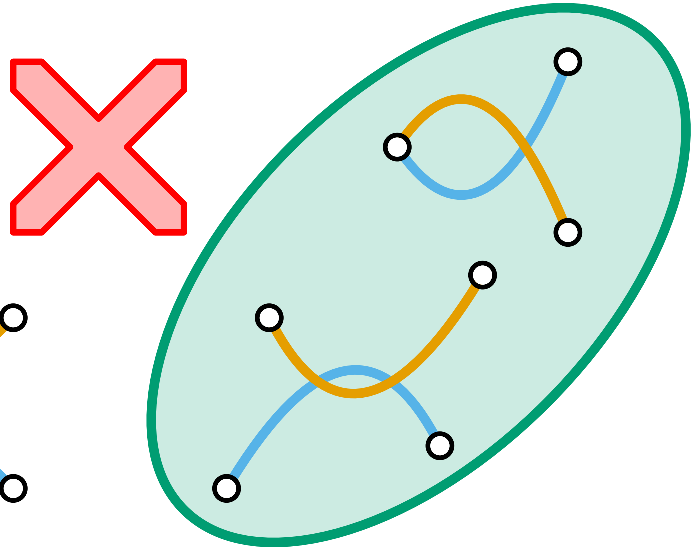

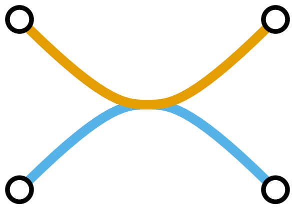

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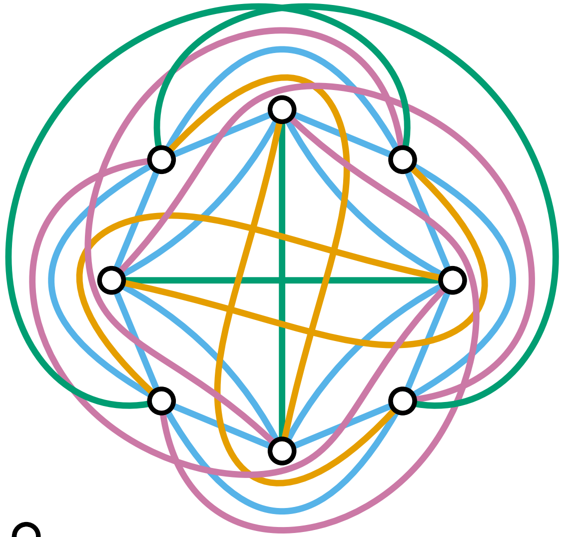

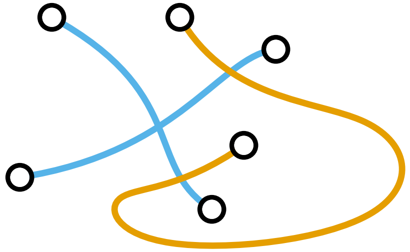

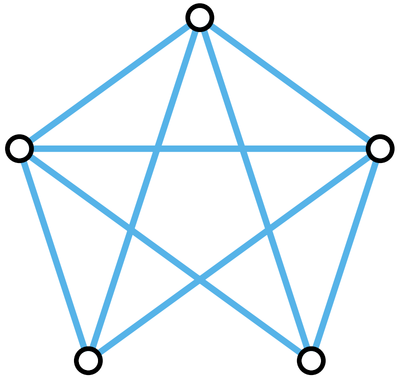

Introduction: Classic Questions

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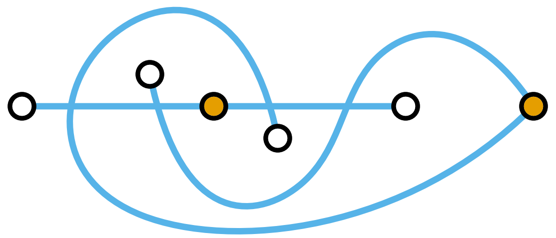

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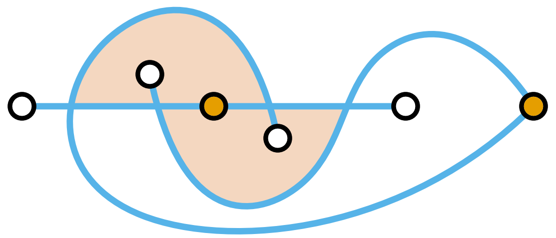

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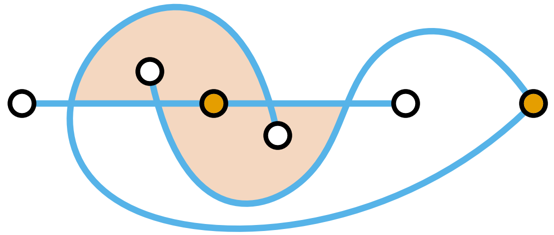

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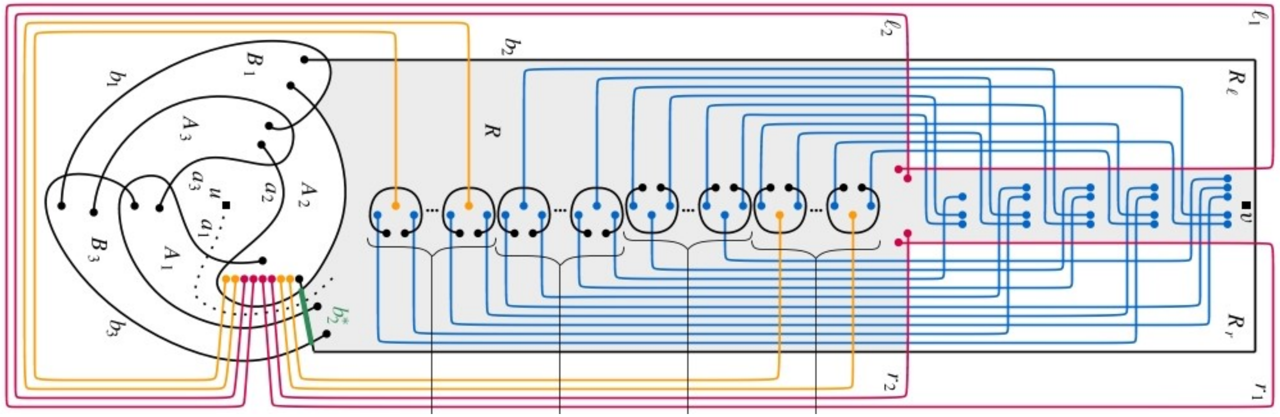

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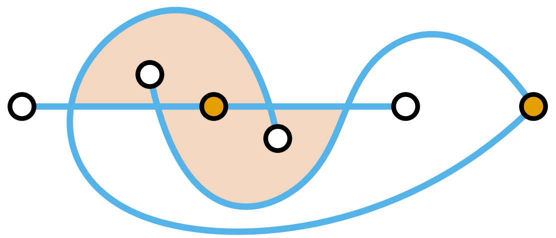

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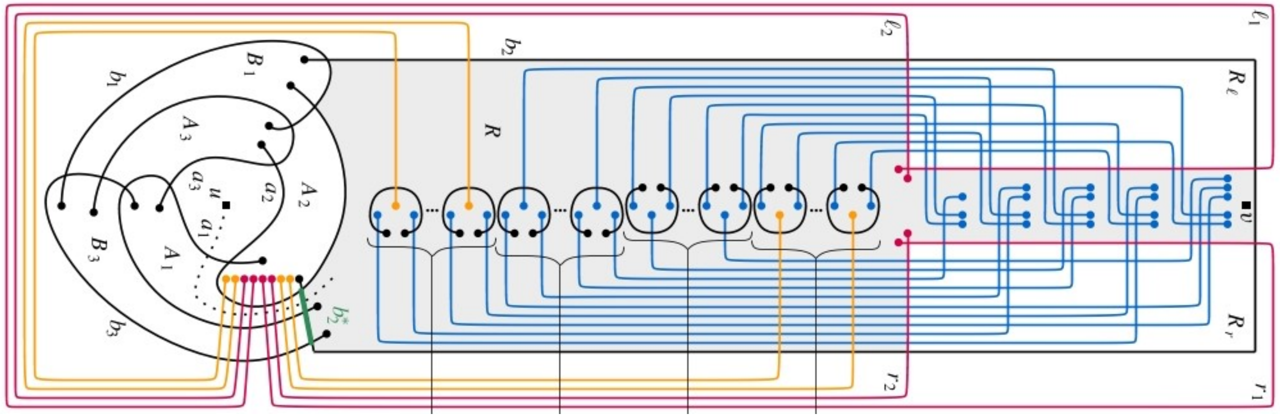

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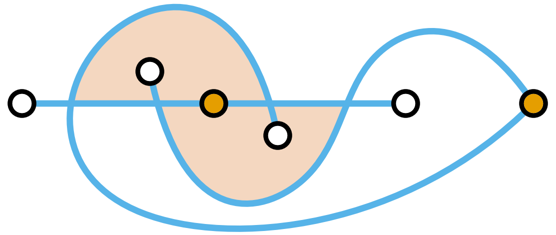

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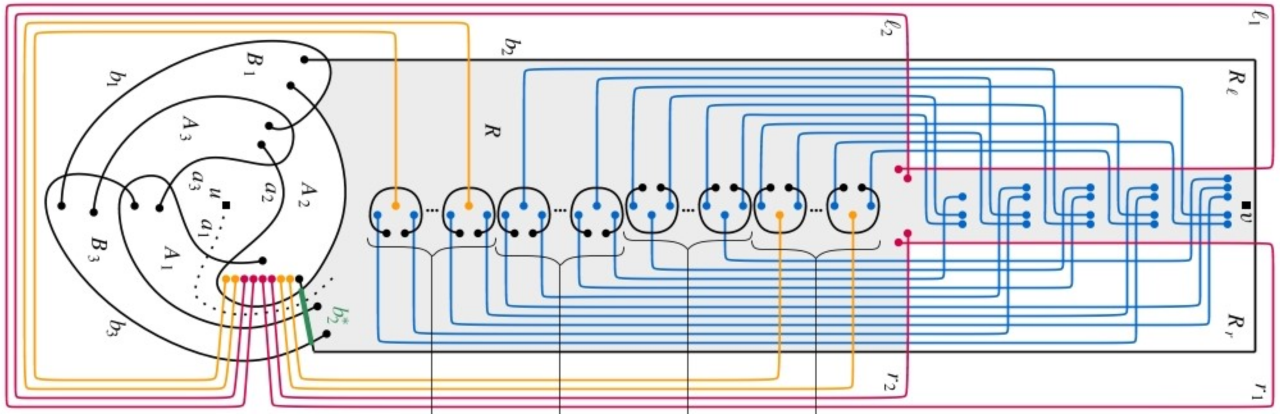

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x-monotone drawings always extendable

[Kynčl, Soukup, 2024]

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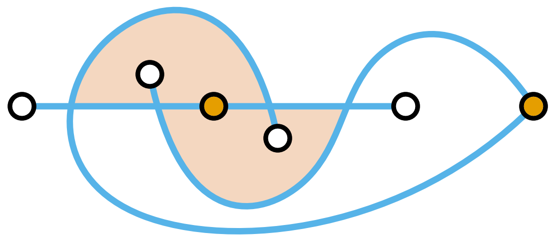

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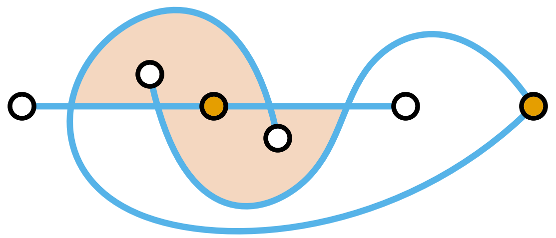

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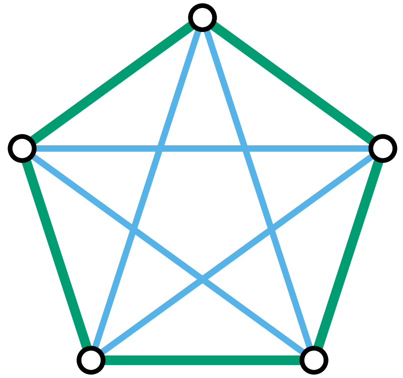

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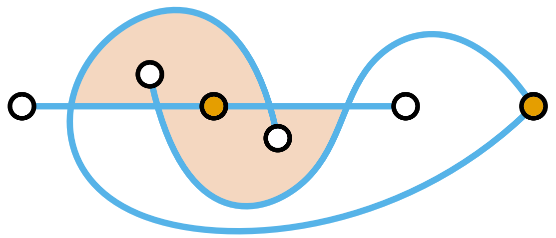

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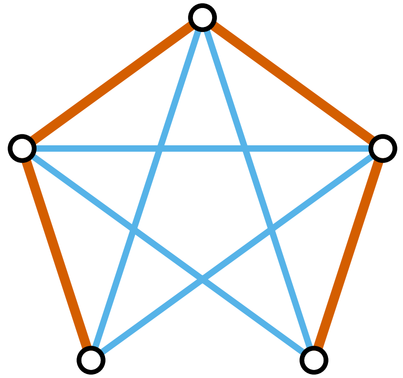

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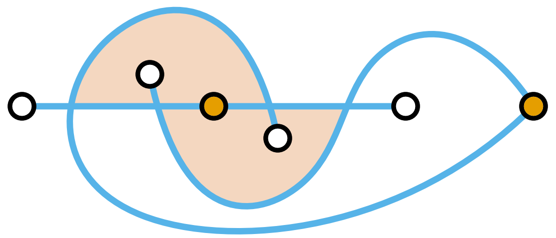

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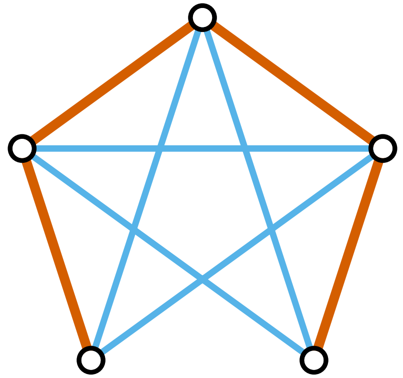

cylindrical, strongly c-monotone ✓

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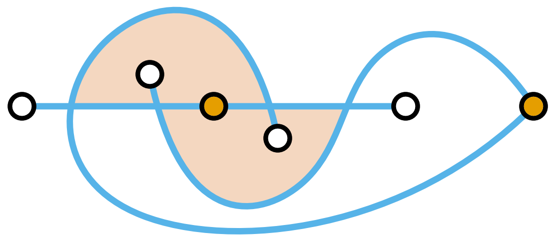

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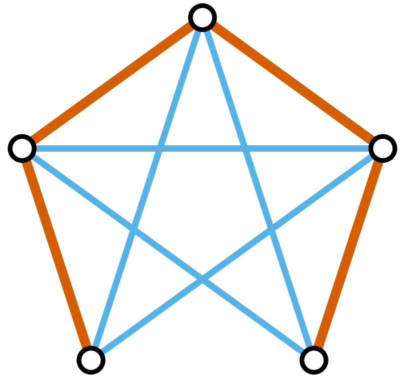

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g-convex drawings ✓
[Bergold, Felsner, M. Reddy, O, Scheucher, 2024]

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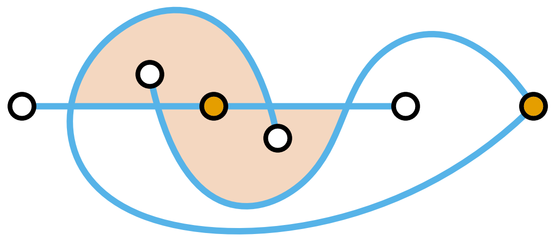

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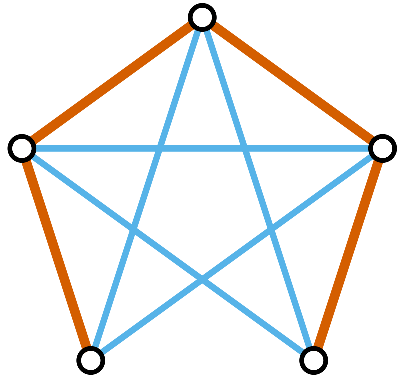

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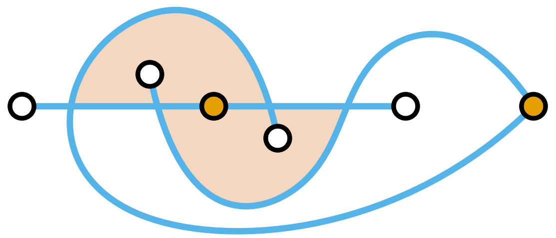

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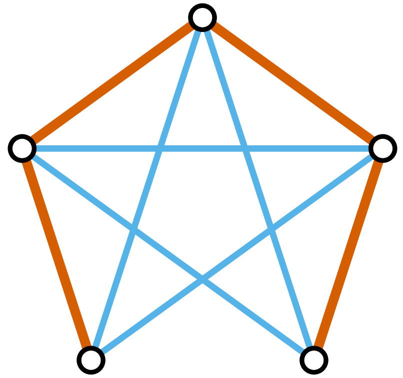

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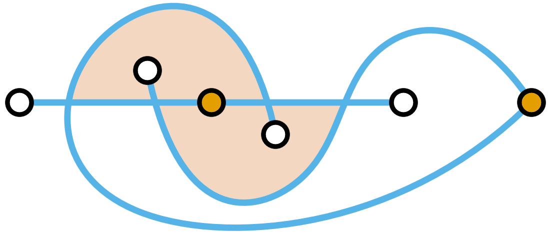

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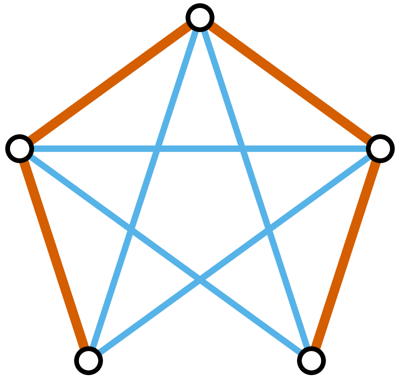

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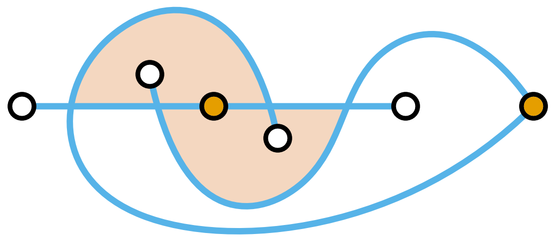

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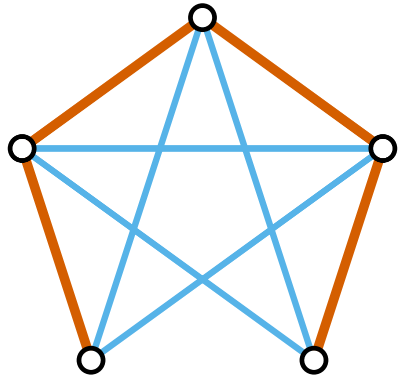

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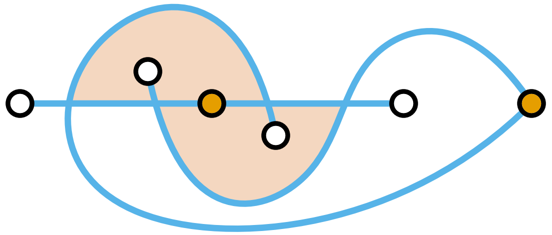

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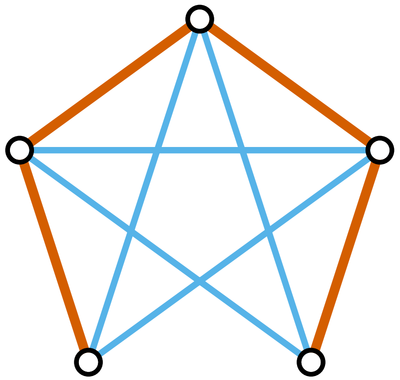

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[Arroyo, Bensmail, Richter, 2021]

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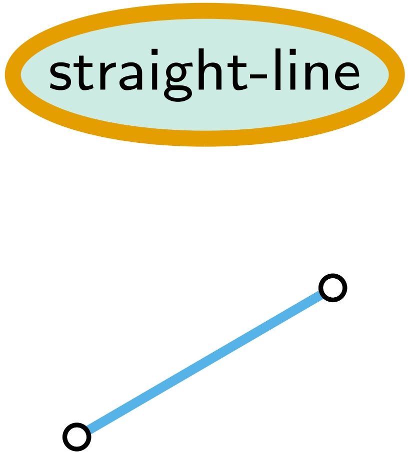

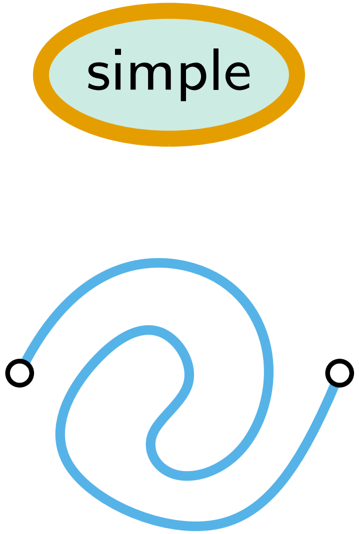

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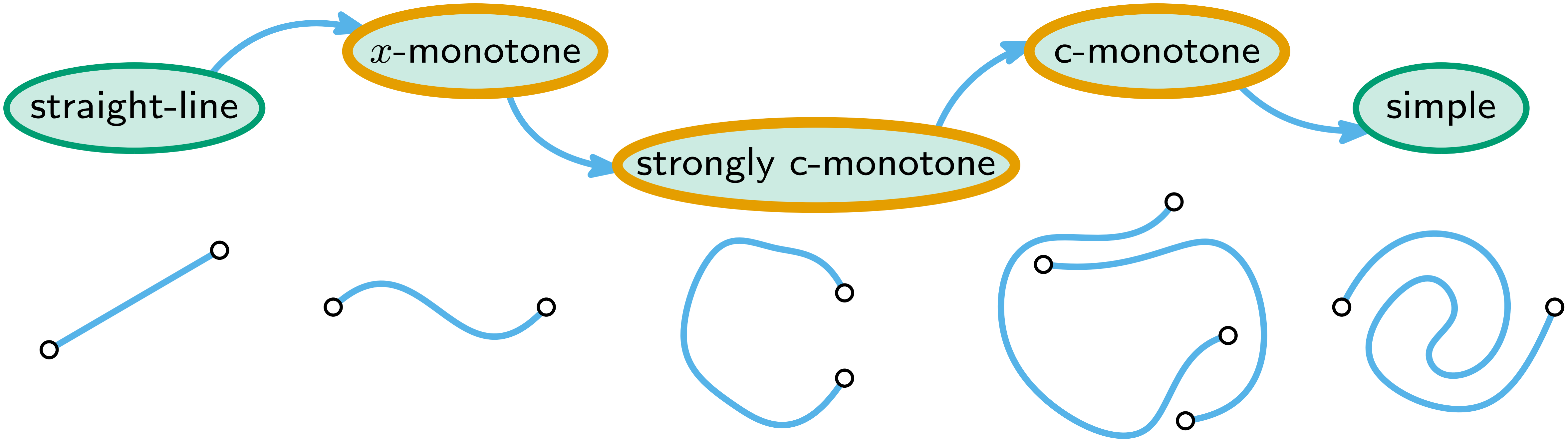

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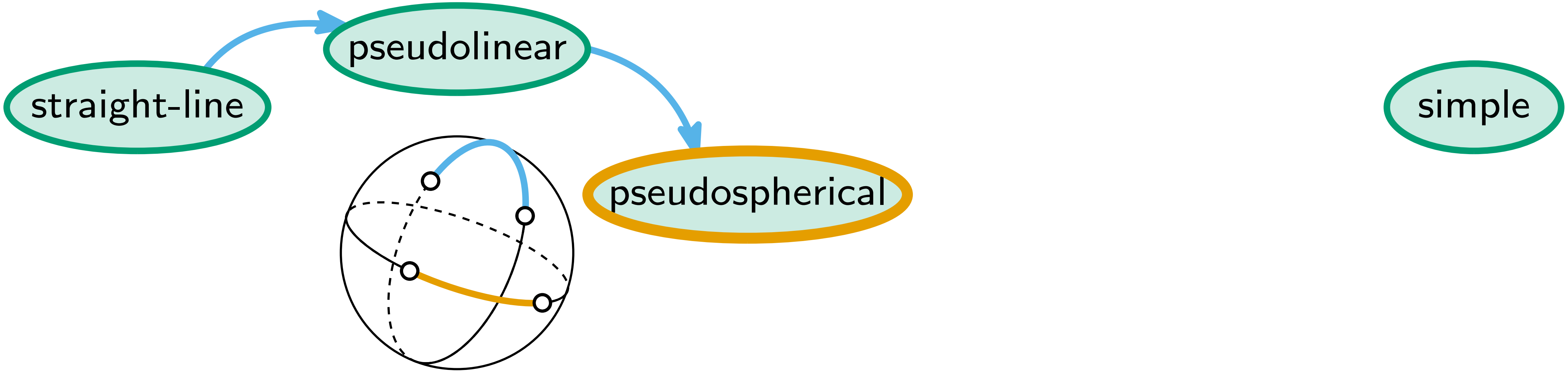

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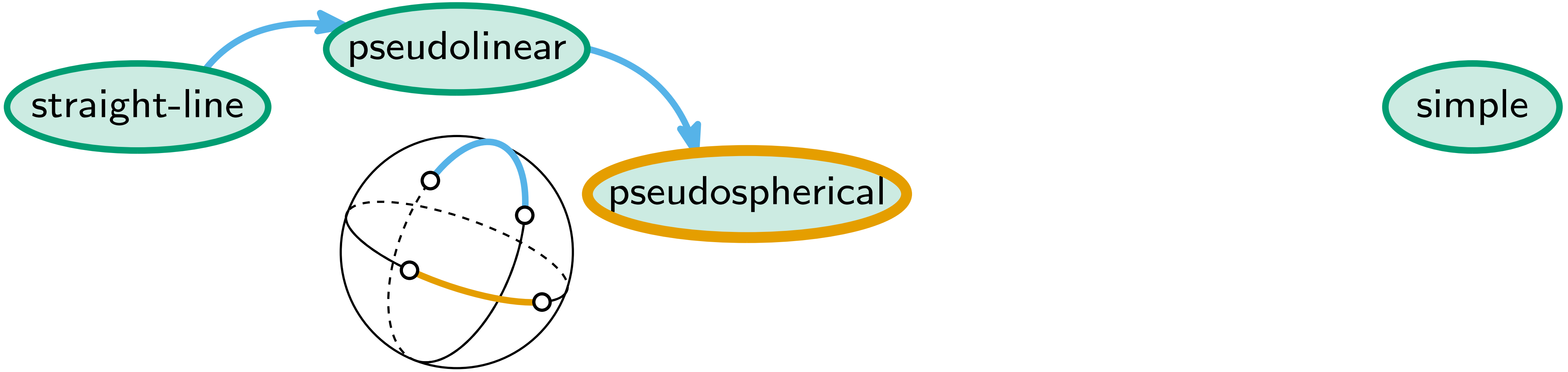

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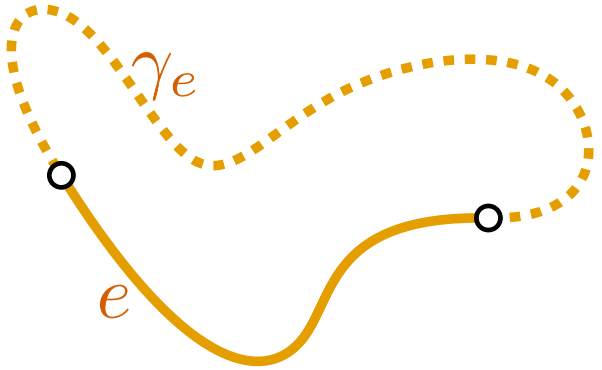

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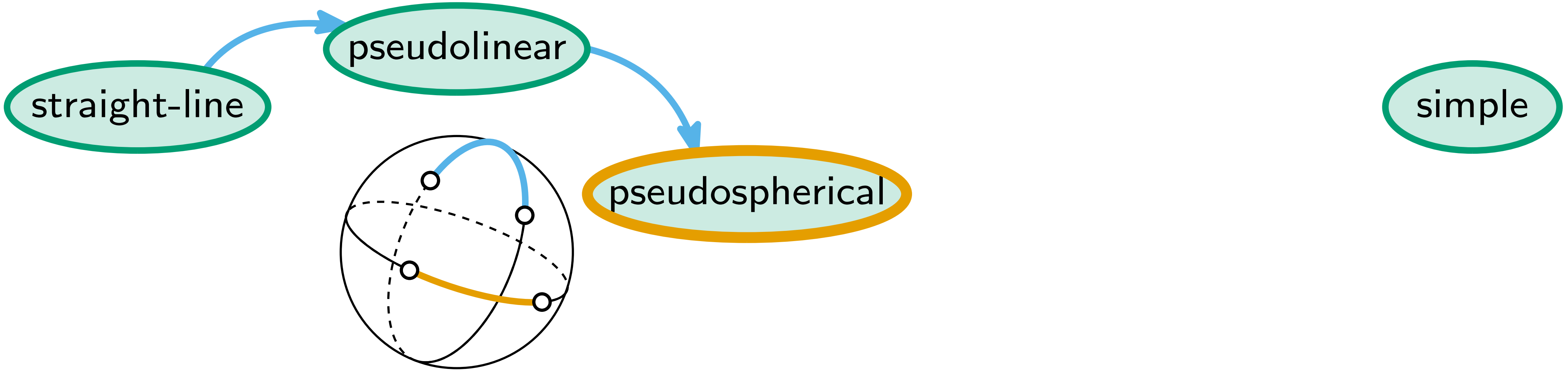

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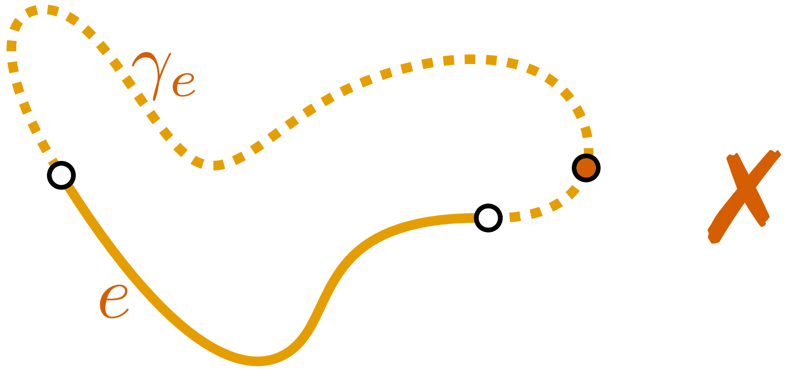
- only the end-vertices of e lie on γ_e



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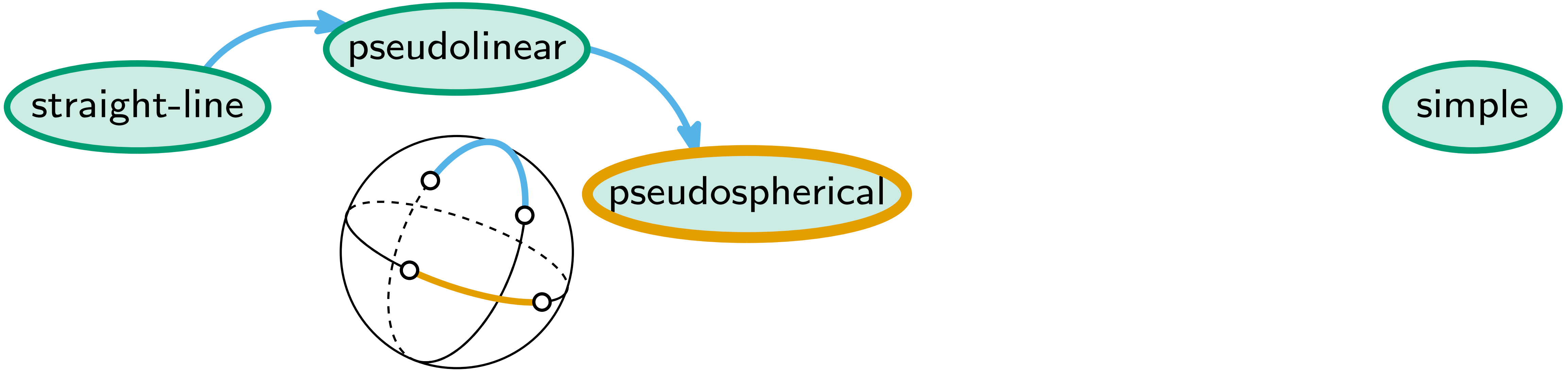

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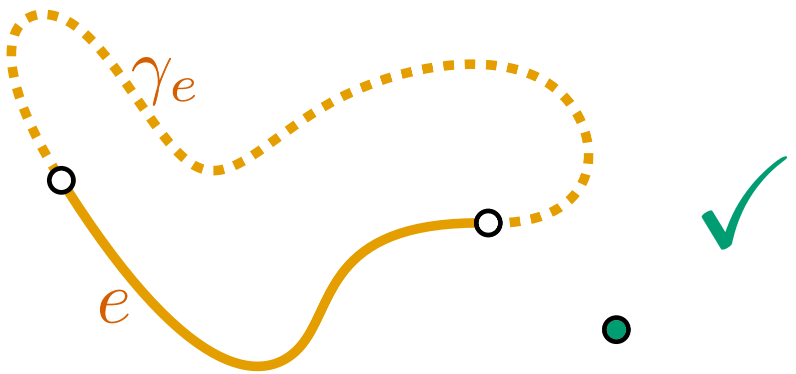

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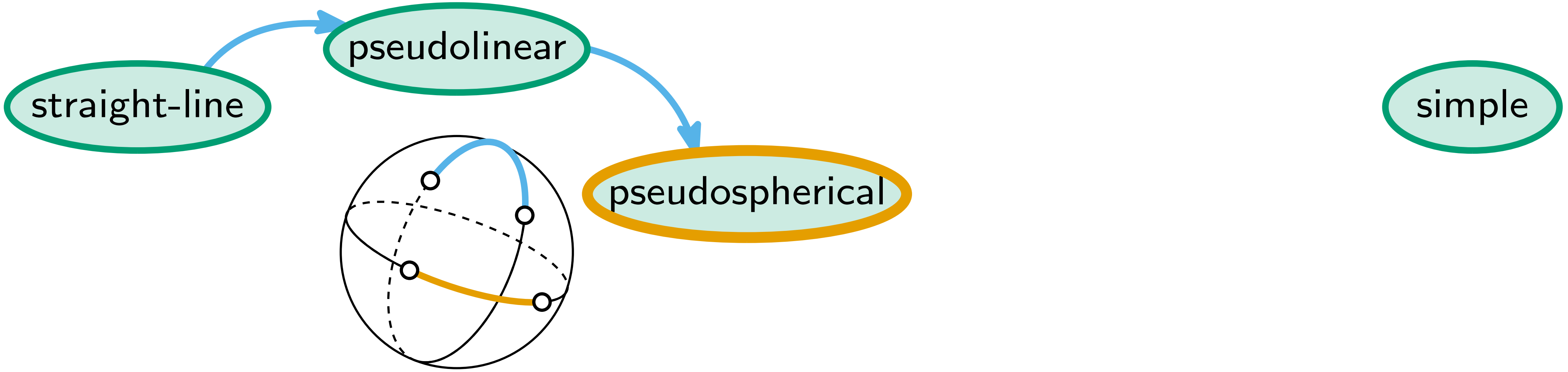

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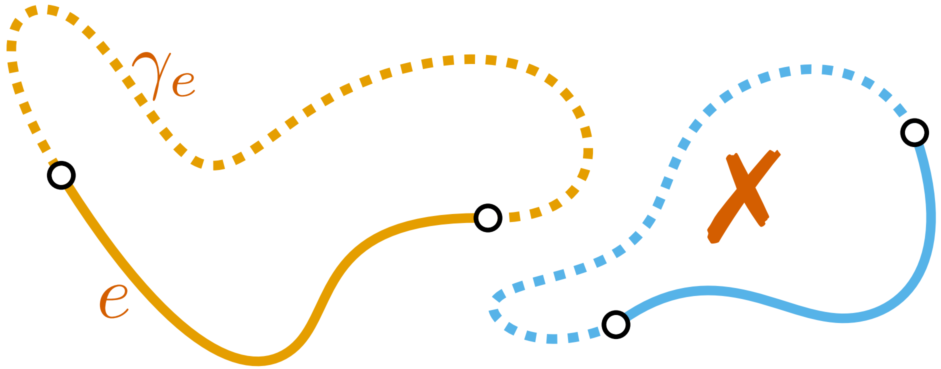

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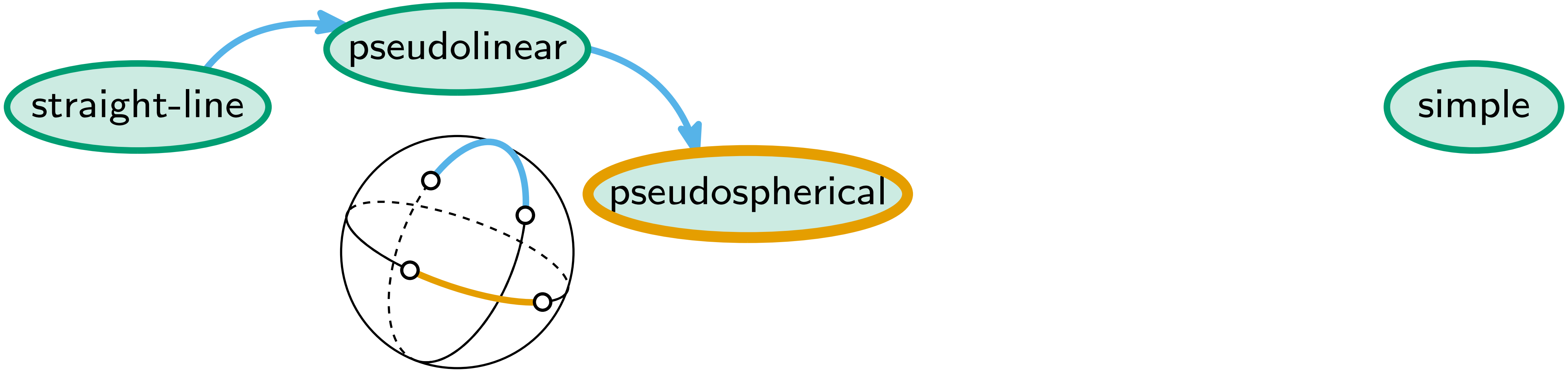

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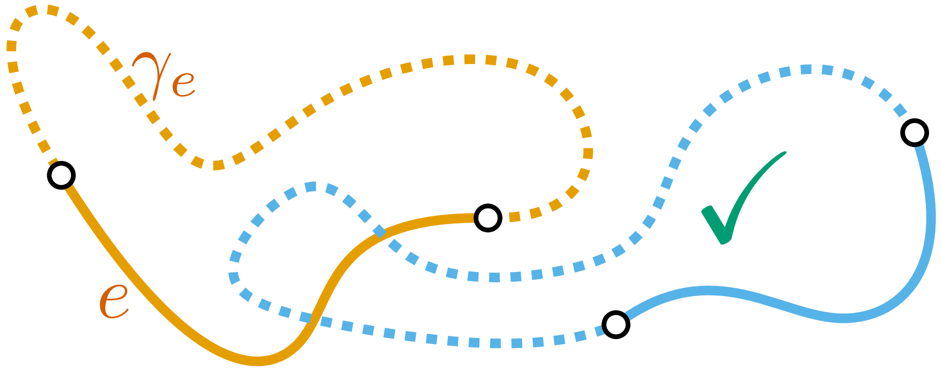

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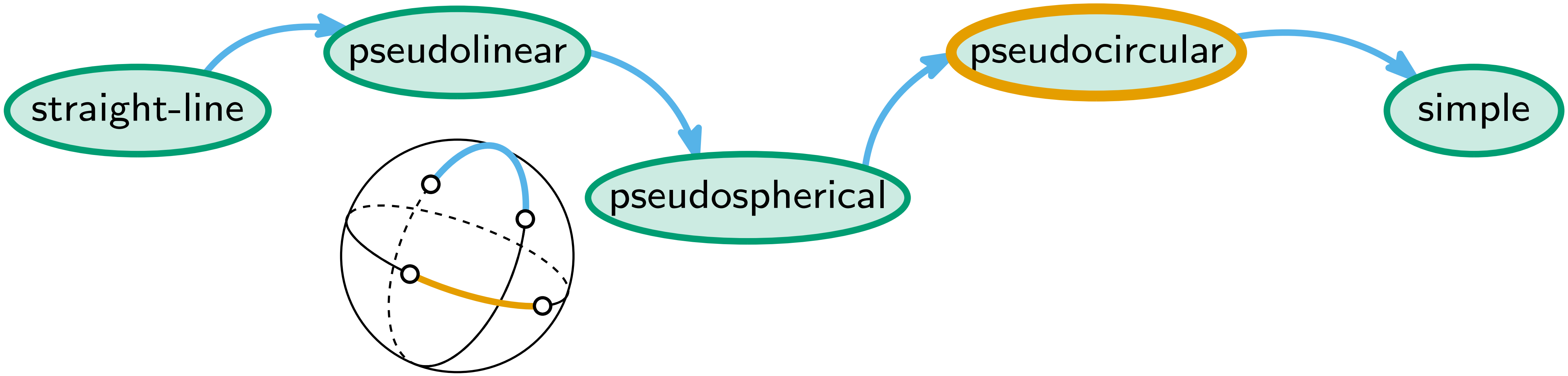

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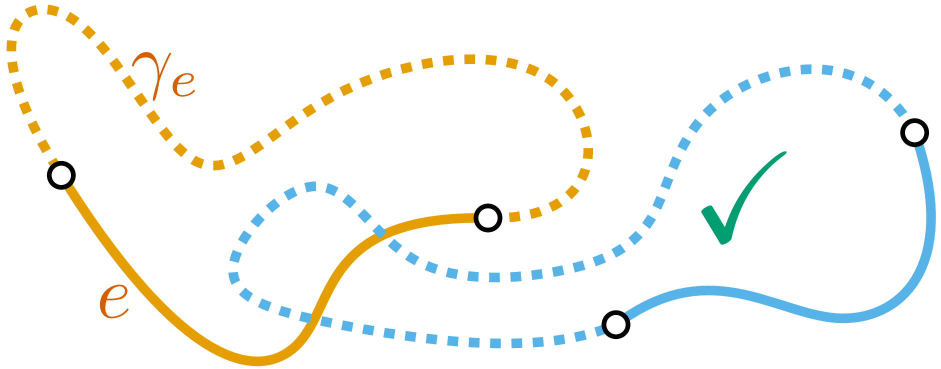

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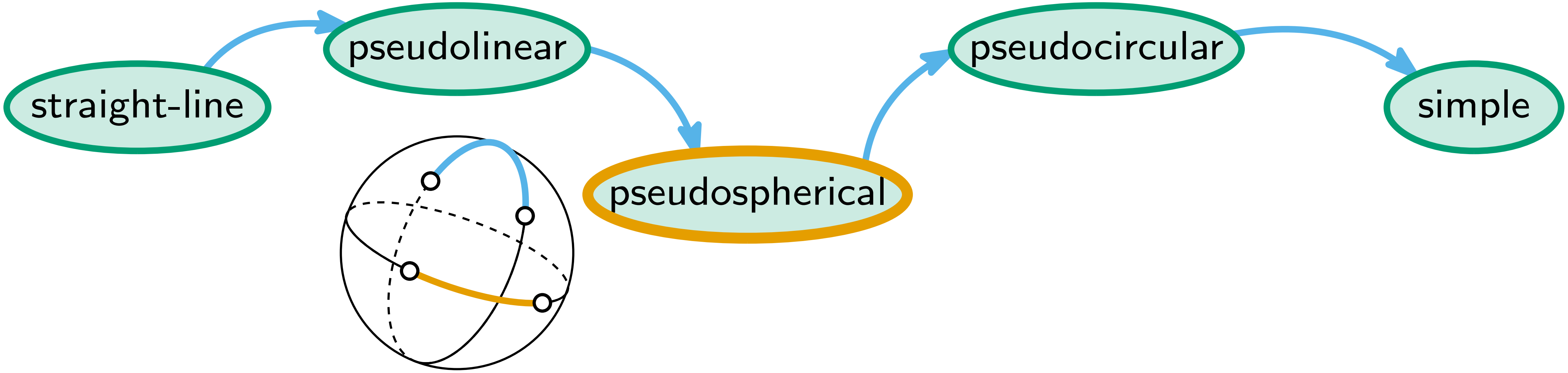

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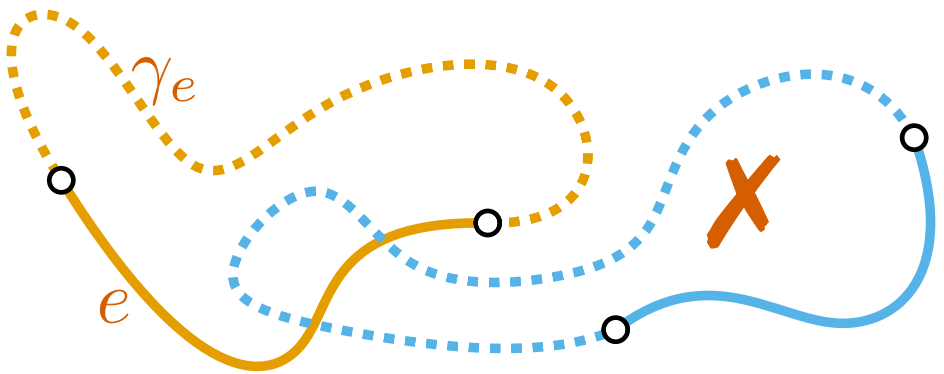

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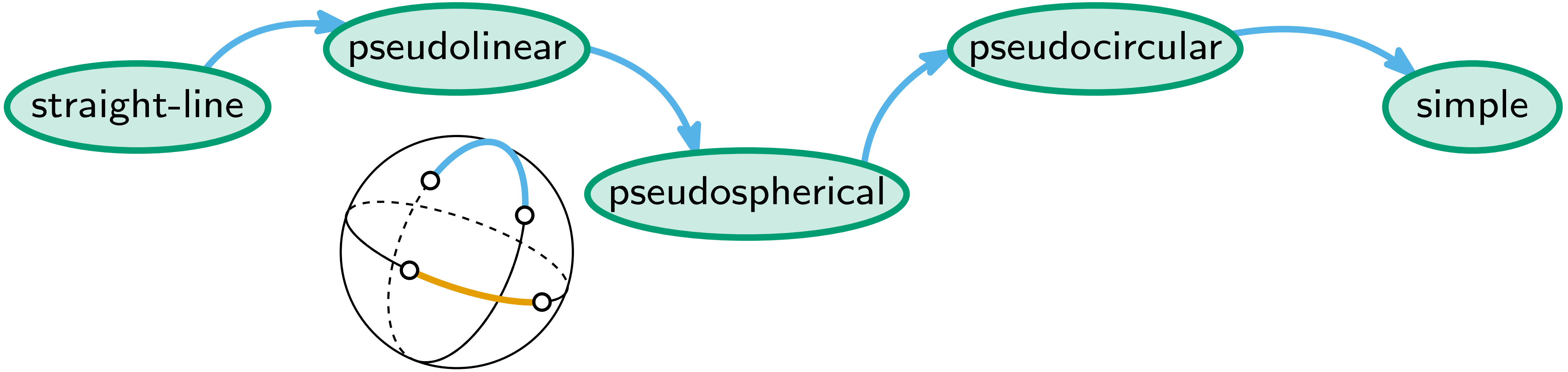

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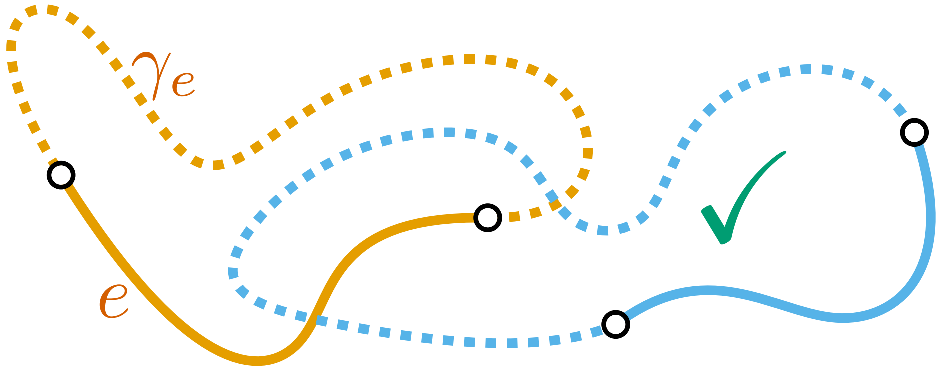

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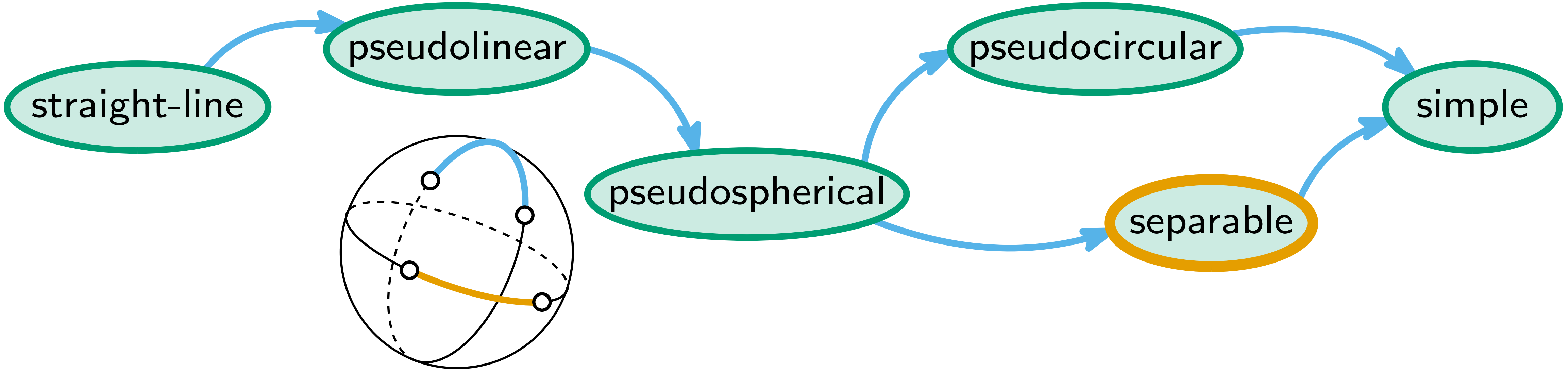

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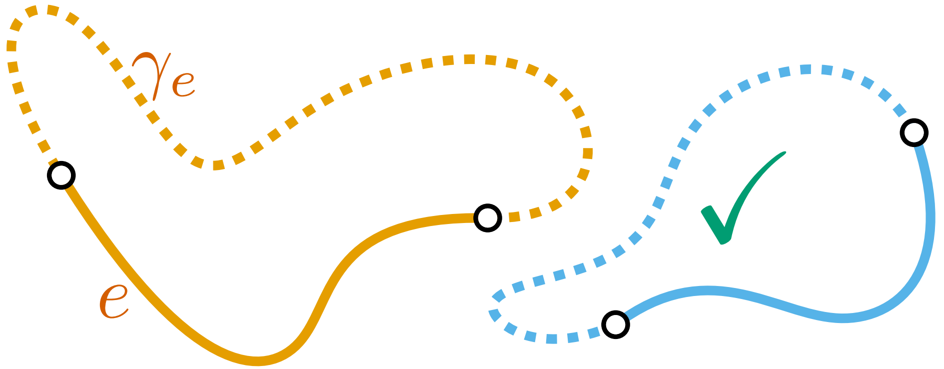

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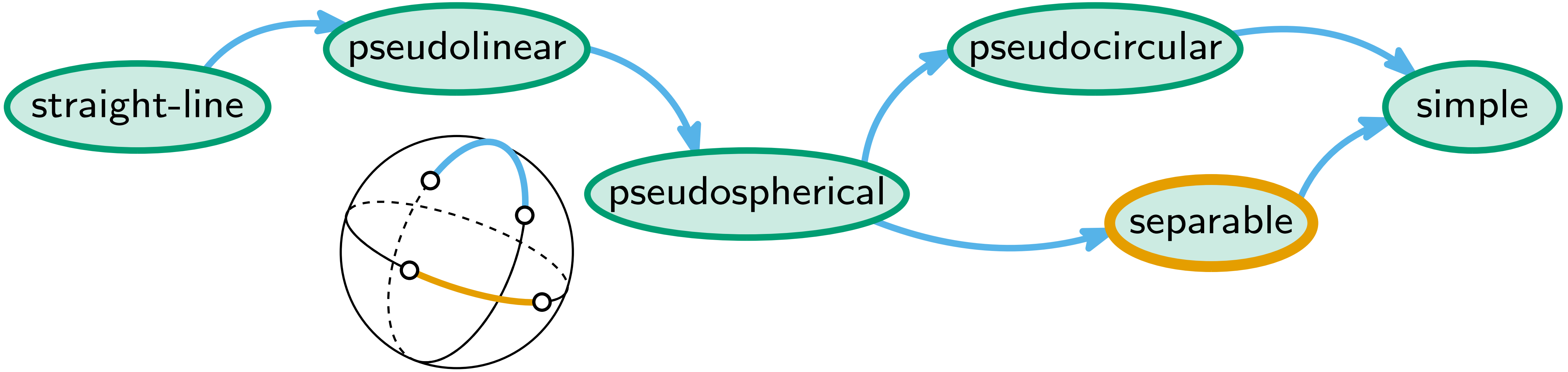

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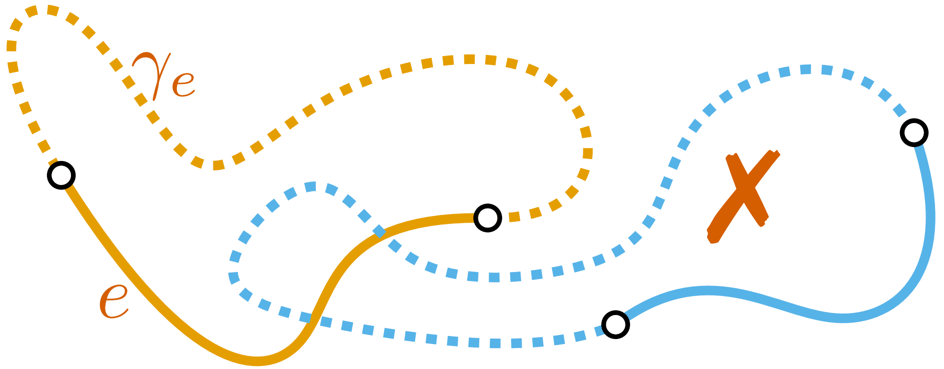

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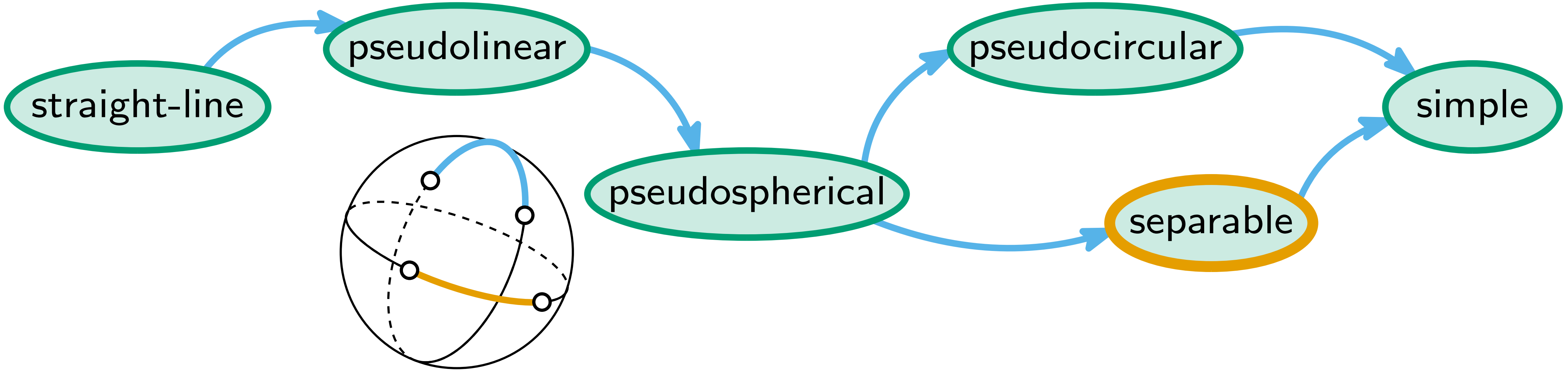

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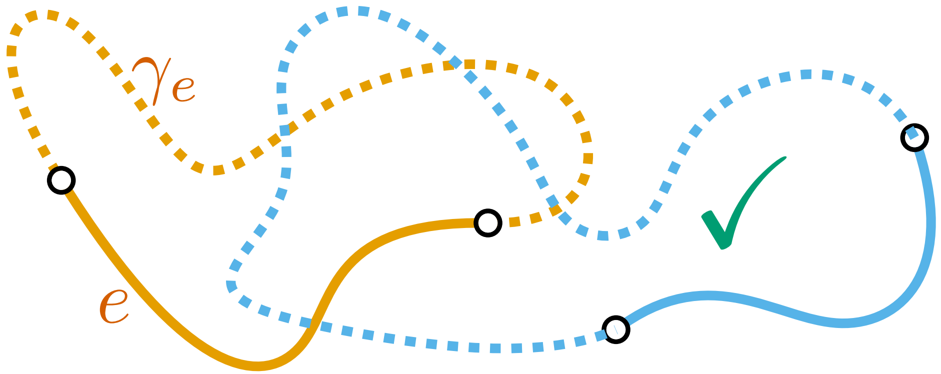

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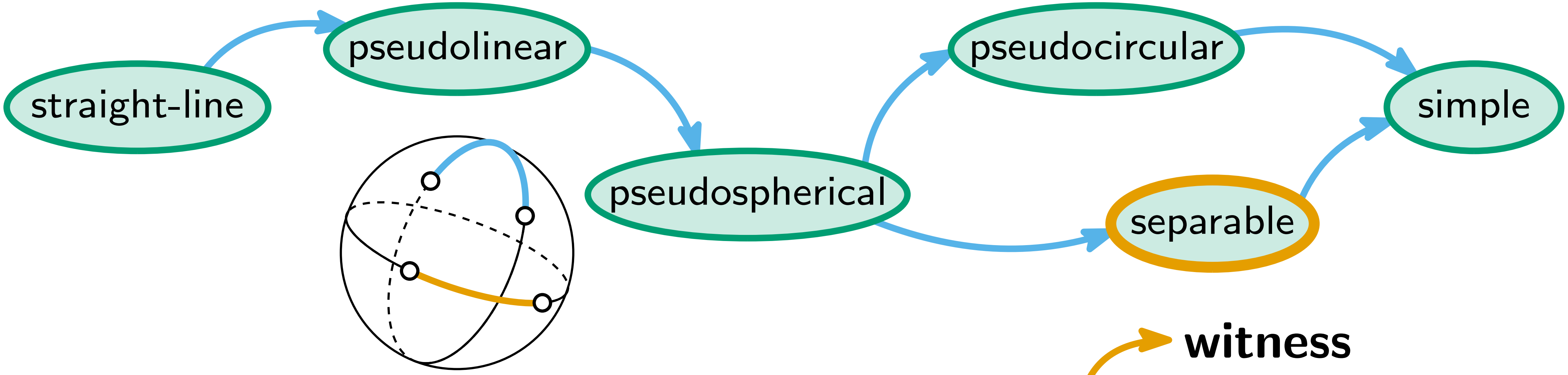

witness

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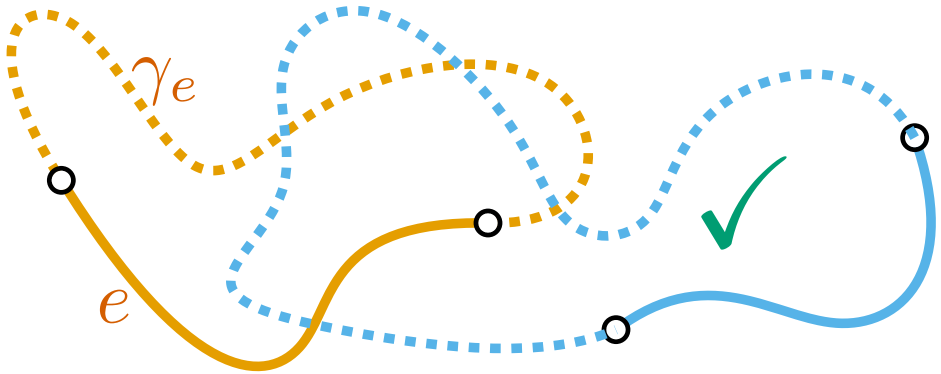
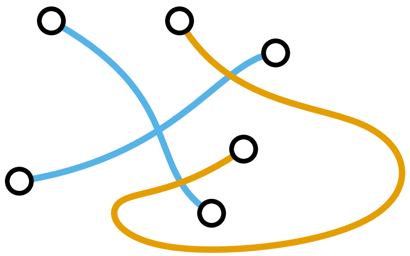


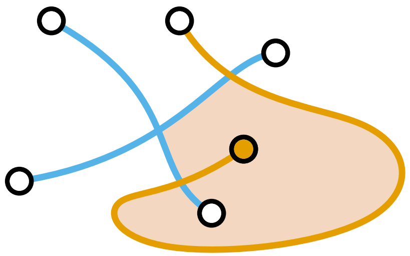
Separable Drawings

Not separable:



Separable Drawings

Not separable:



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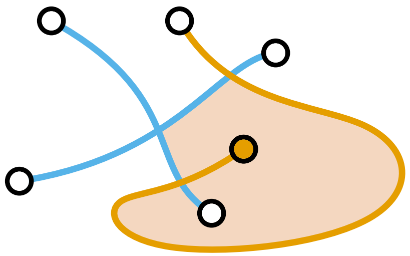
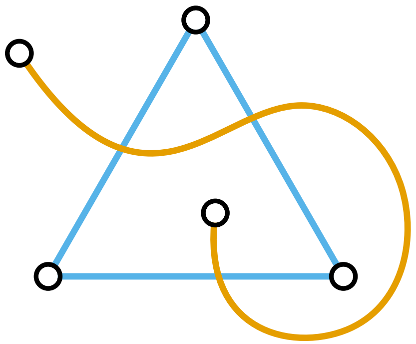
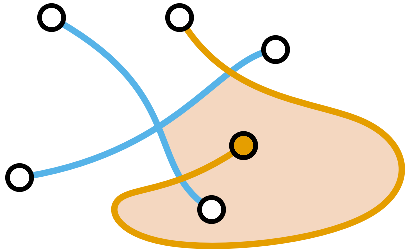
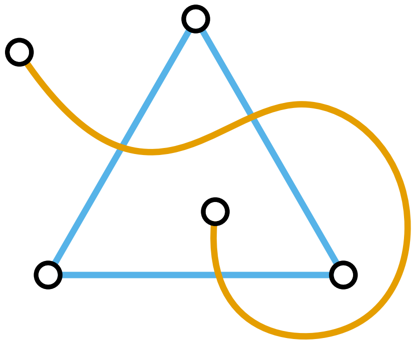




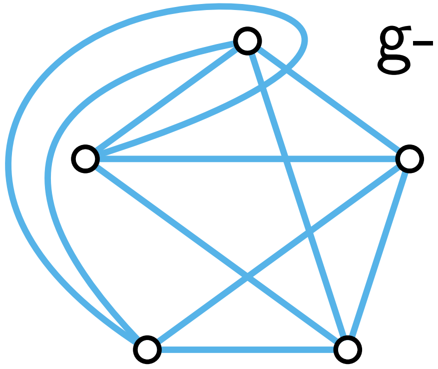
Separable Drawings

Not separable:





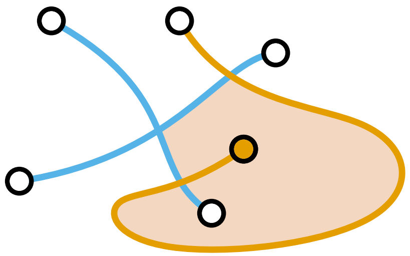
$\Rightarrow$

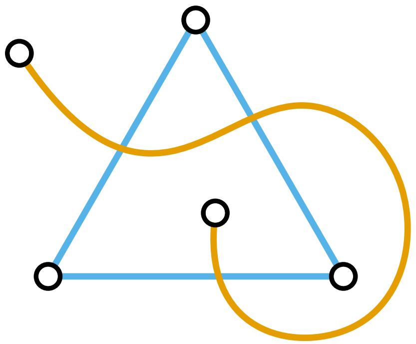


g-twisted

Separable Drawings

Not separable:





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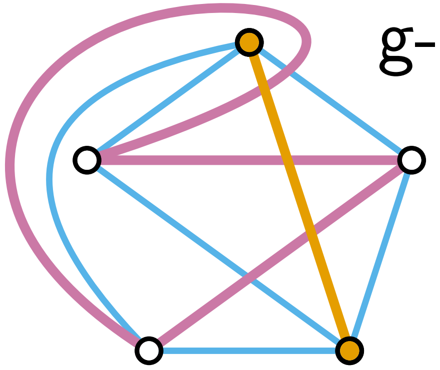

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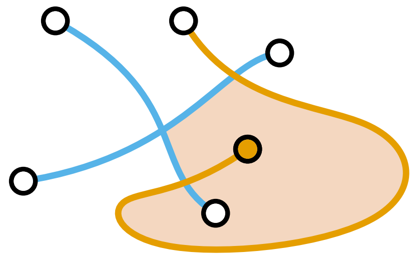

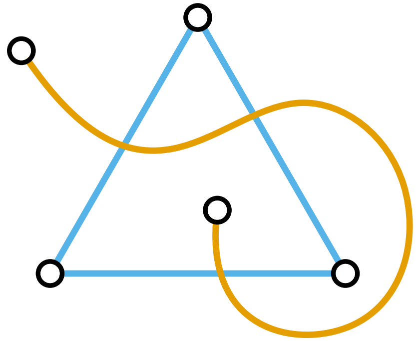

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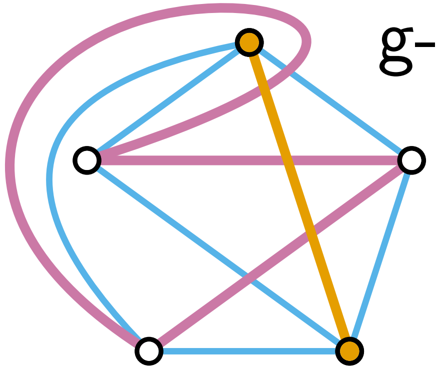
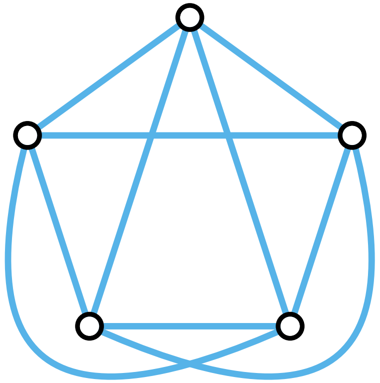


Separable:



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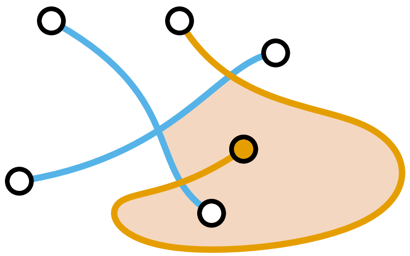

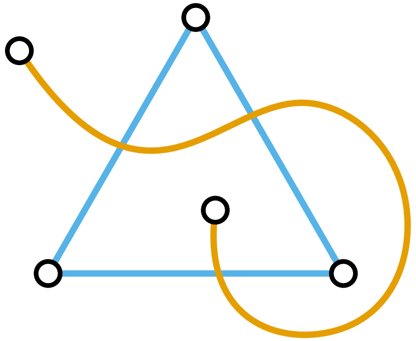

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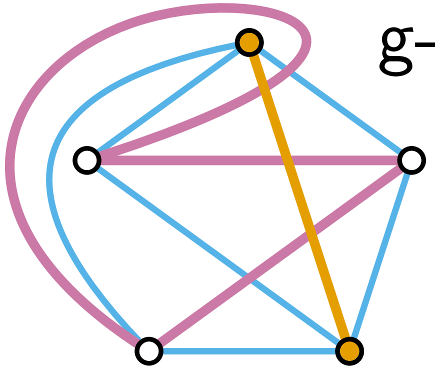

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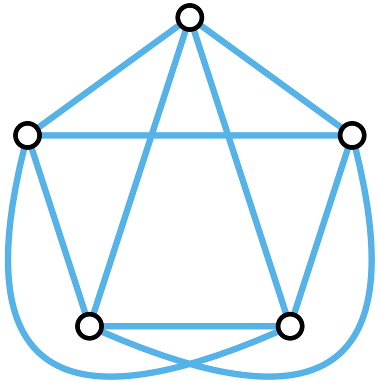
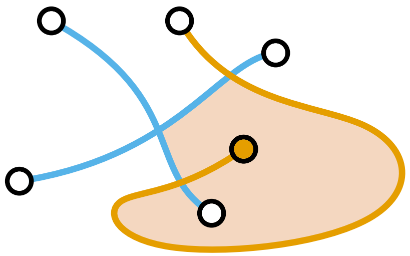
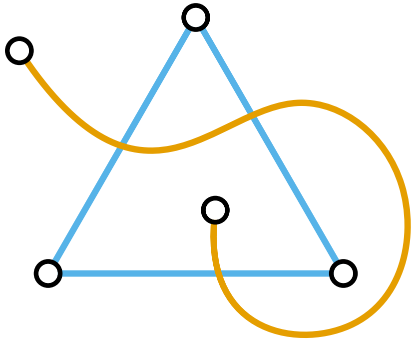


check each edge independently

Separable Drawings

Not separable:





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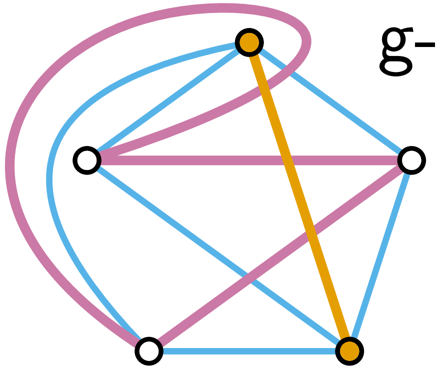

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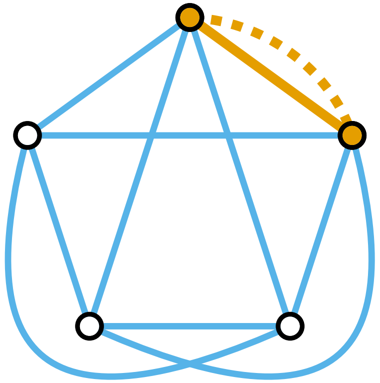

check each edge independently

uncrossed ✓

Separable Drawings

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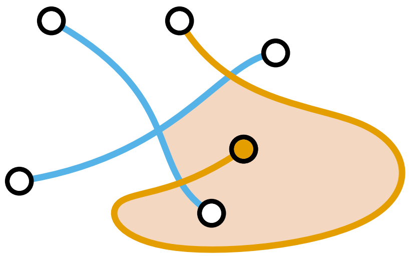

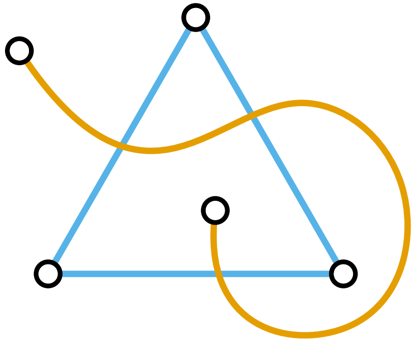

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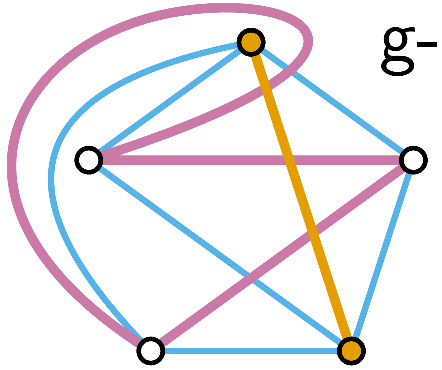

g-twisted

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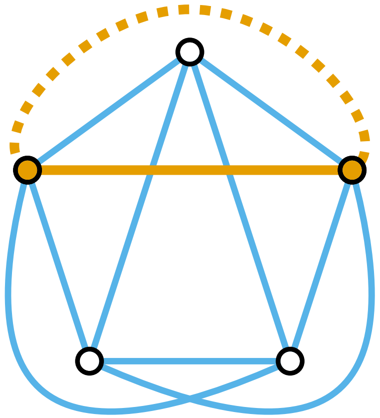

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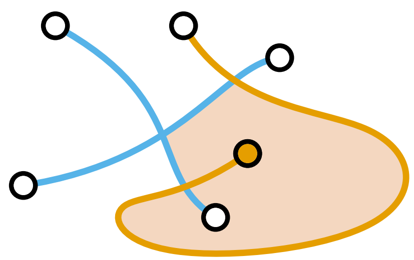
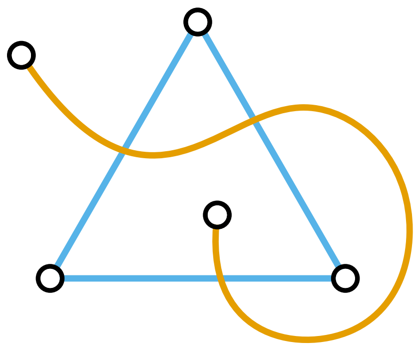
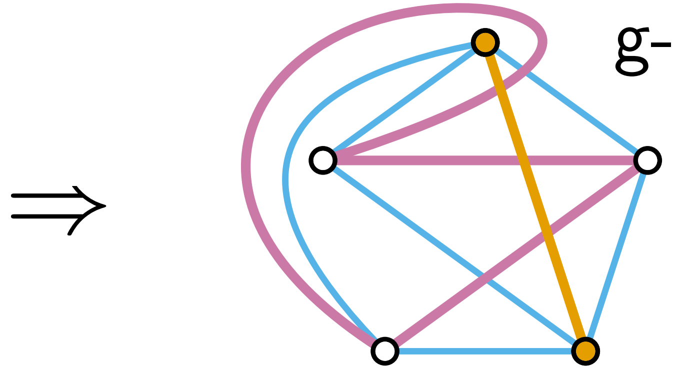
uncrossed ✓

end-points share face ✓

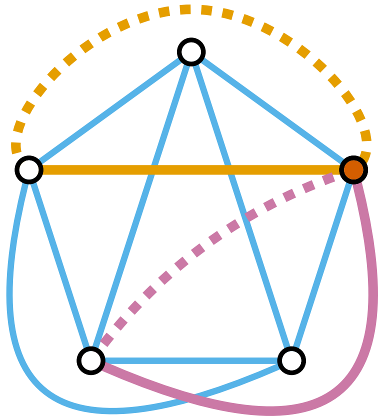
Not separable:







Separable:



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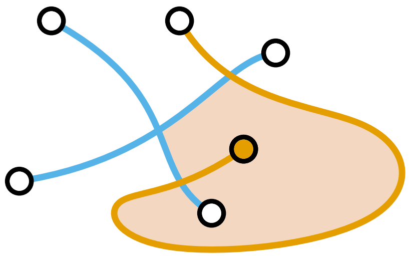
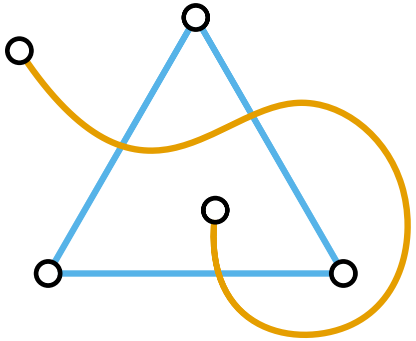
witnesses can just touch

Separable Drawings

Not separable:





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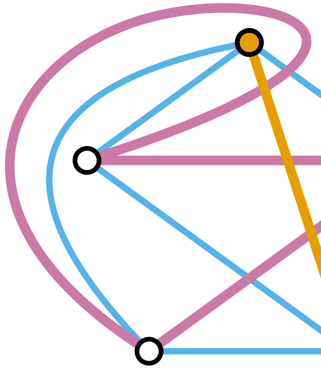
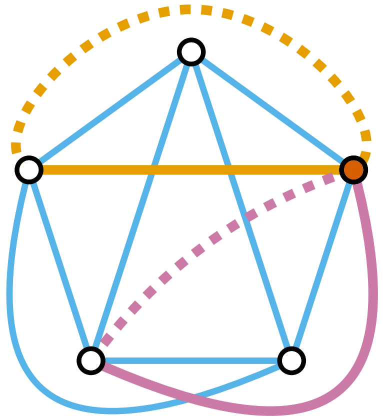


g-twisted

Separable:



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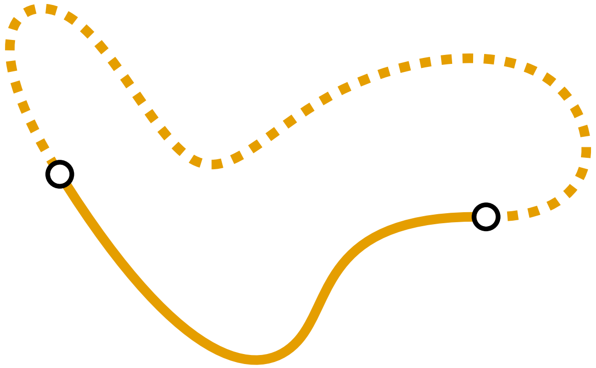

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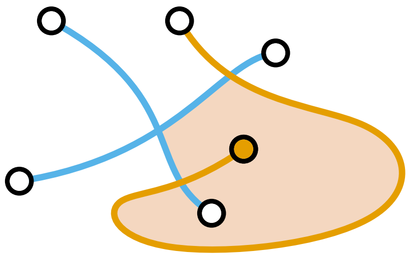

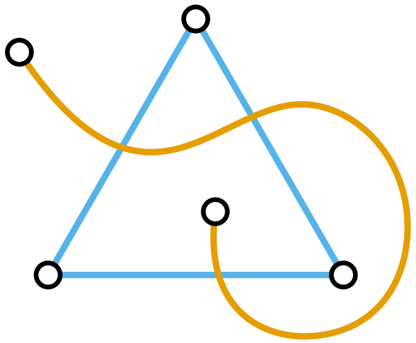

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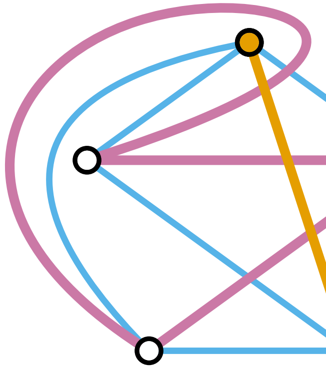

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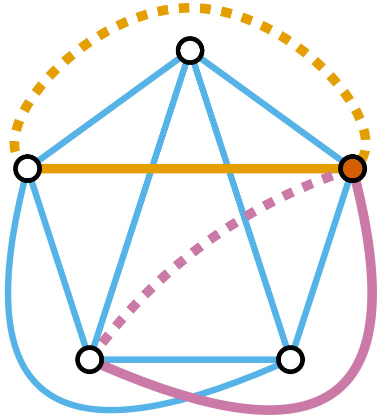

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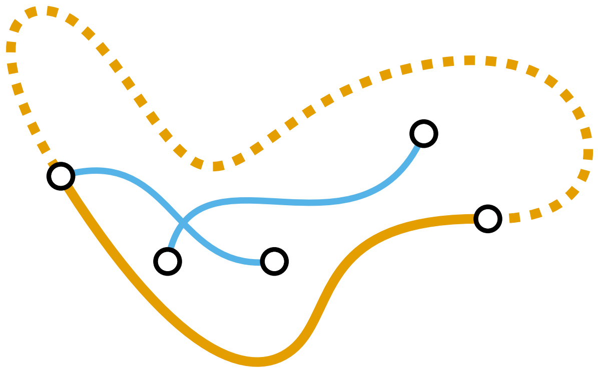

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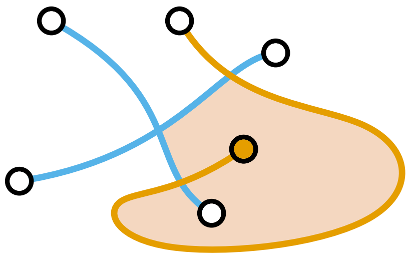

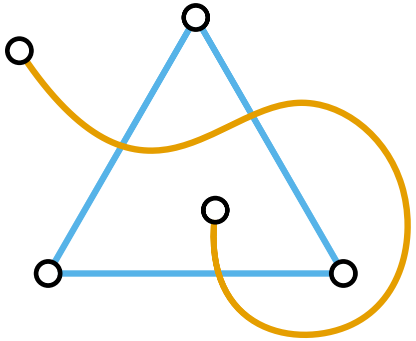

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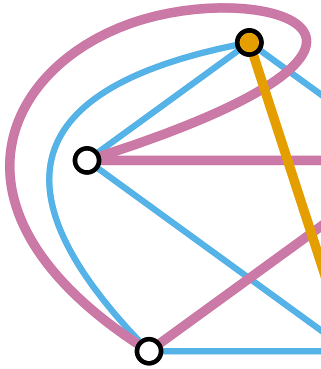
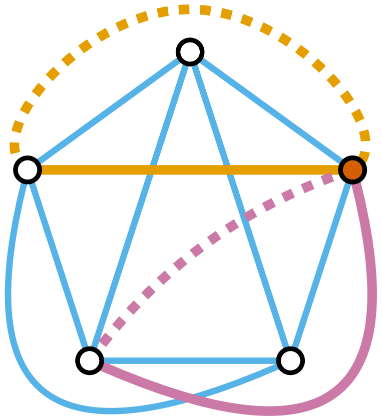


Separable:



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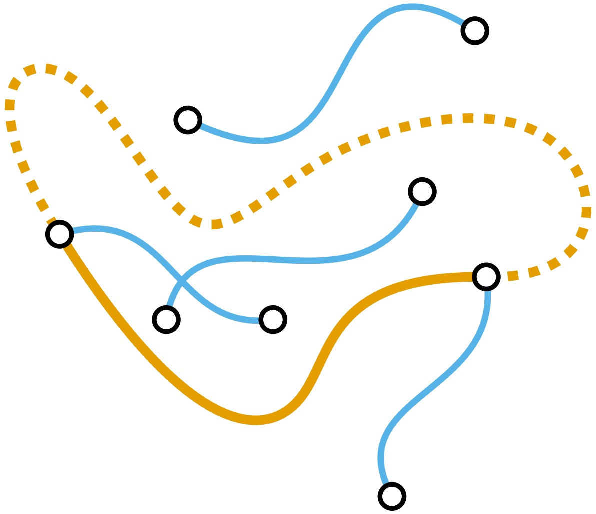

Result: Hamiltonicity (K_n)

Conjecture [Rafla 1988] Every simple drawing of K_n with $n \geq 3$ vertices contains a crossing-free Hamiltonian cycle.

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Conjecture [AOV 2023] Every simple drawing $\mathcal{D}$ of K_n with $n \geq 2$ vertices contains, for each pair of vertices $v \neq w$ in $\mathcal{D}$, a crossing-free Hamiltonian path with end-vertices v and w.

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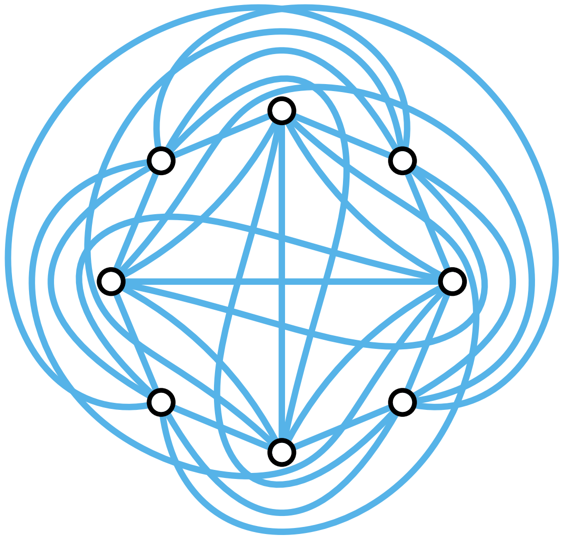

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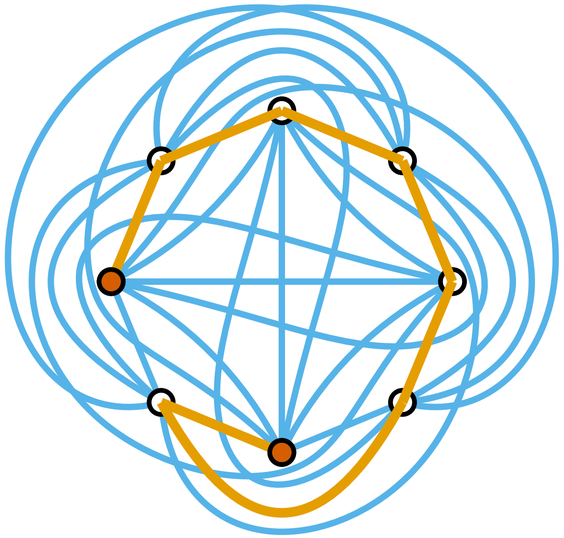

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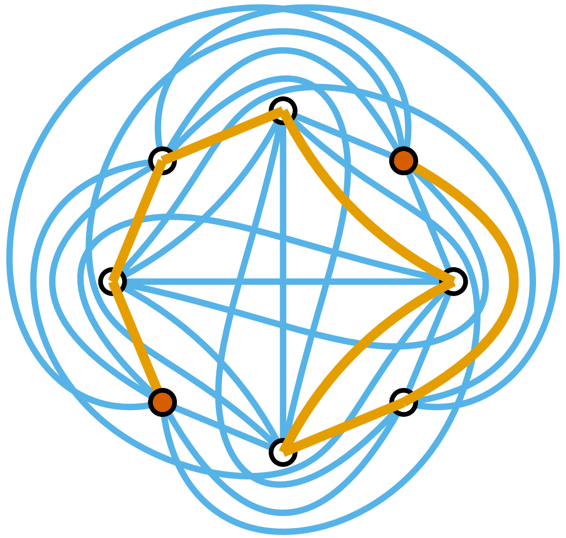

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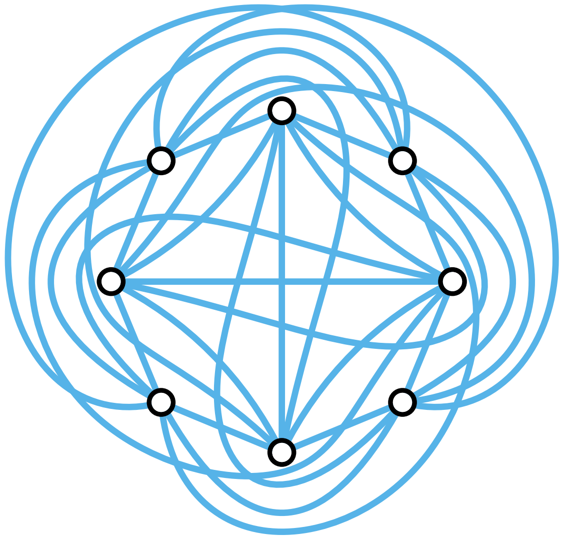

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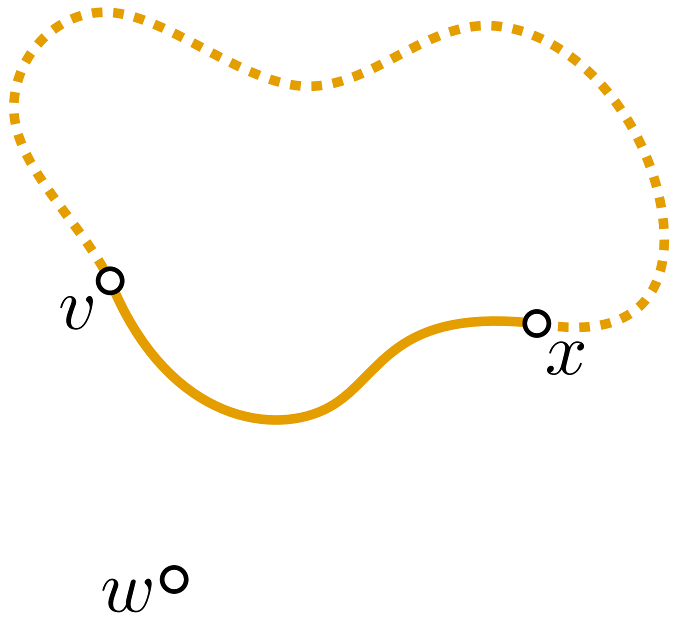

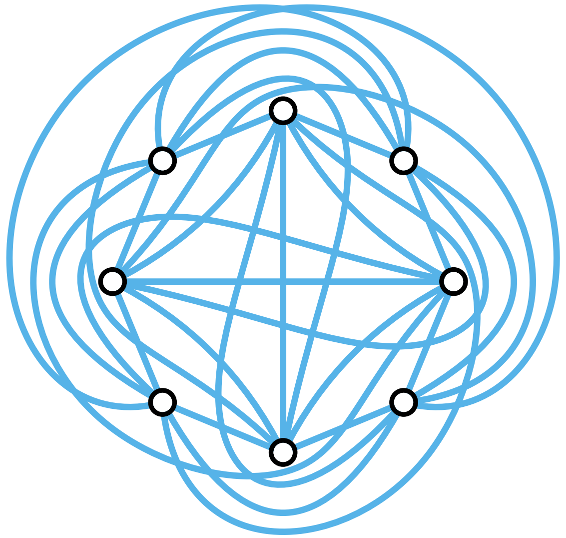

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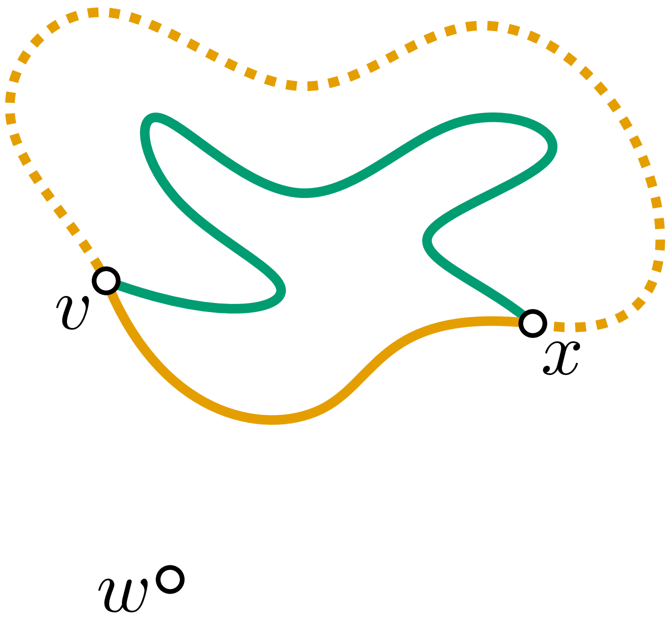

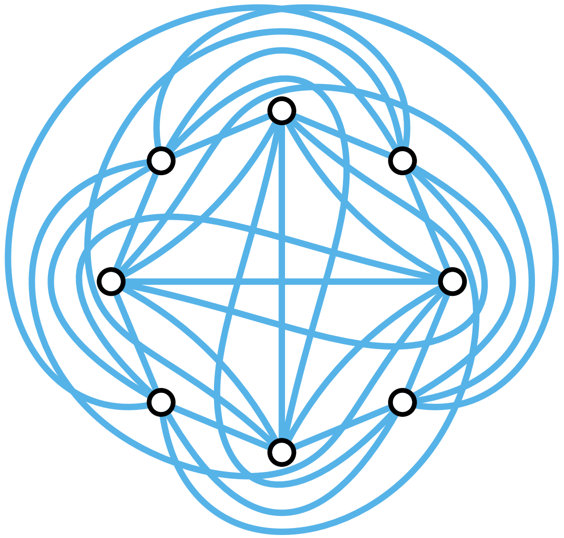

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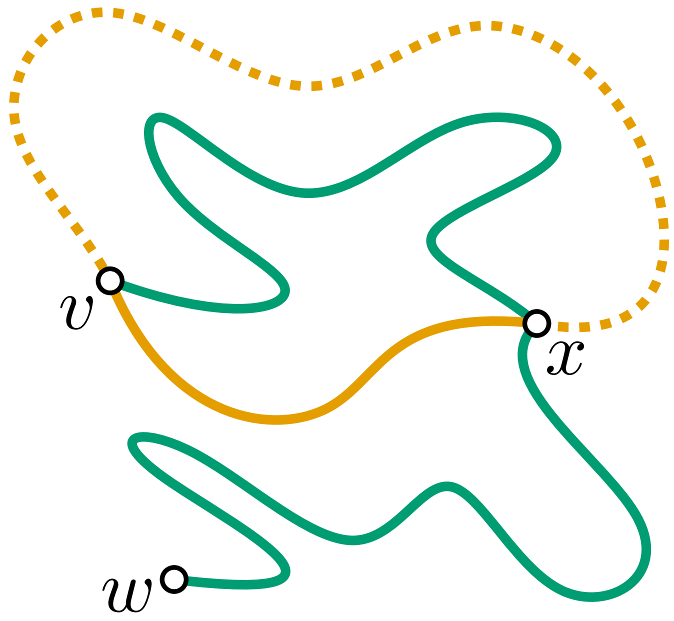

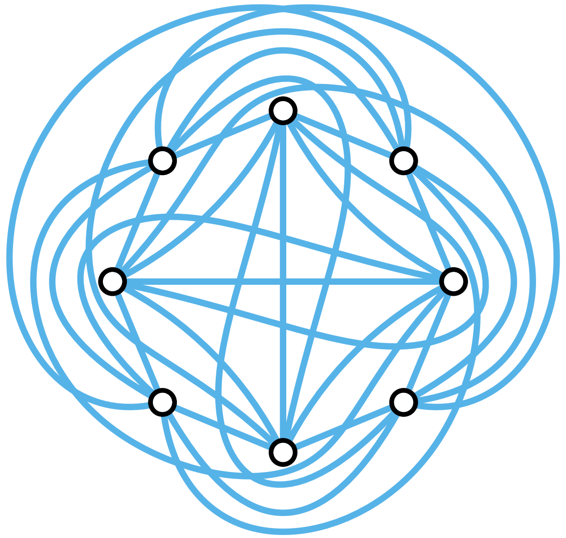

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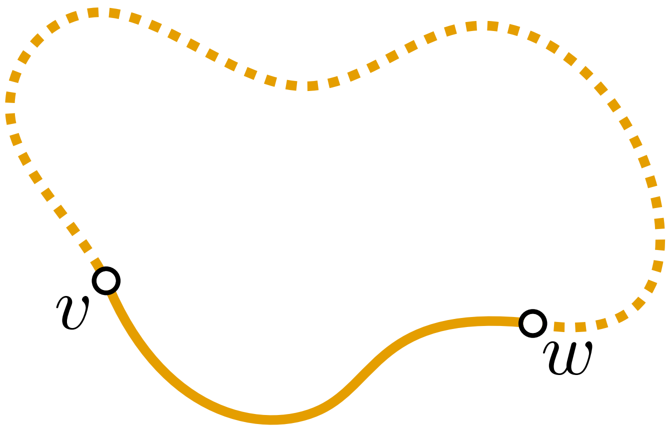

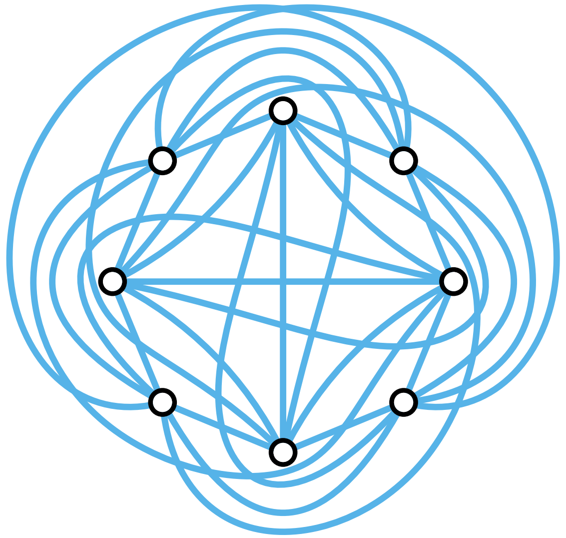

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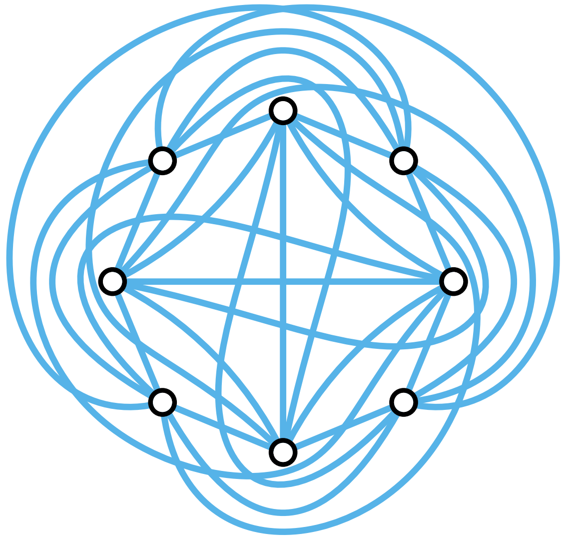

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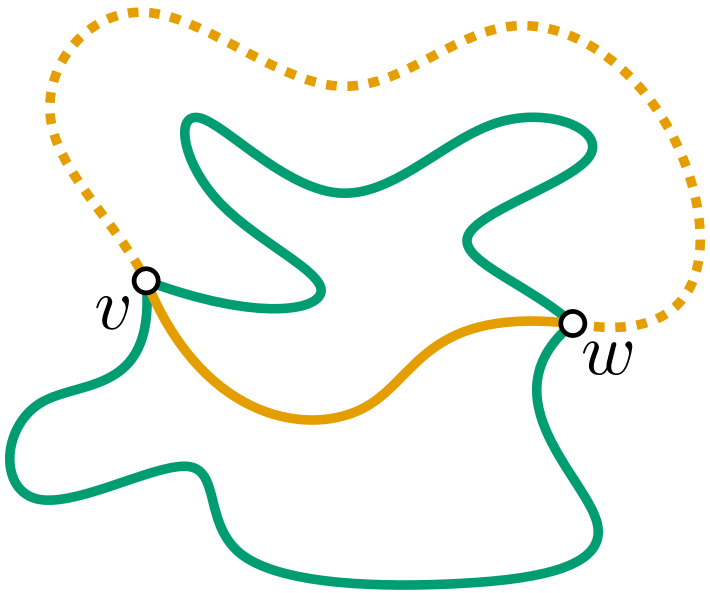

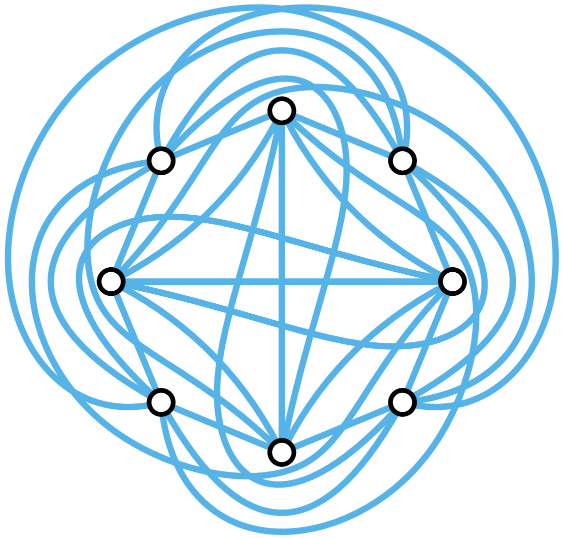

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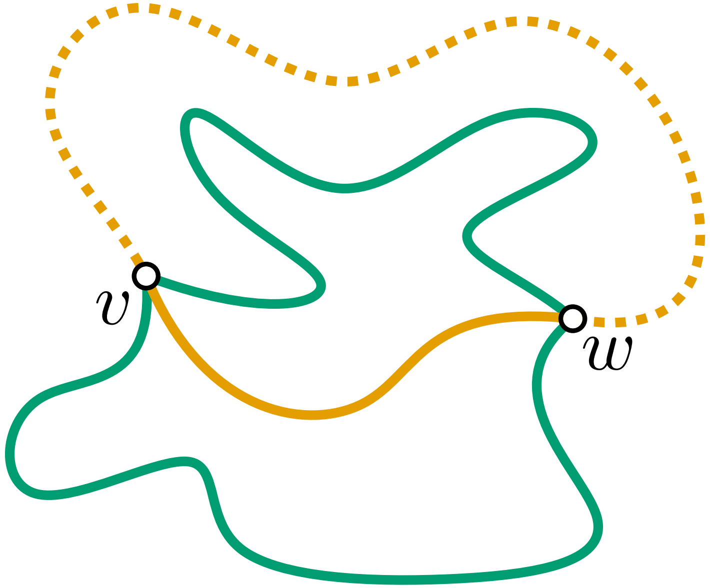

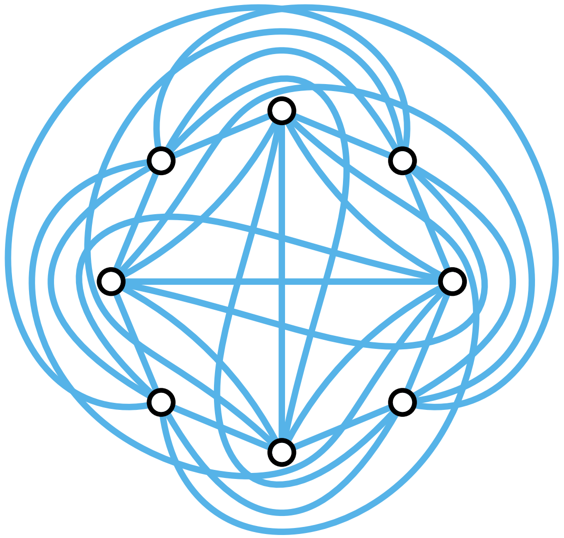

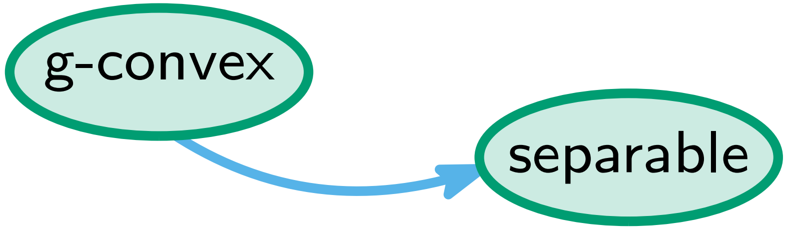

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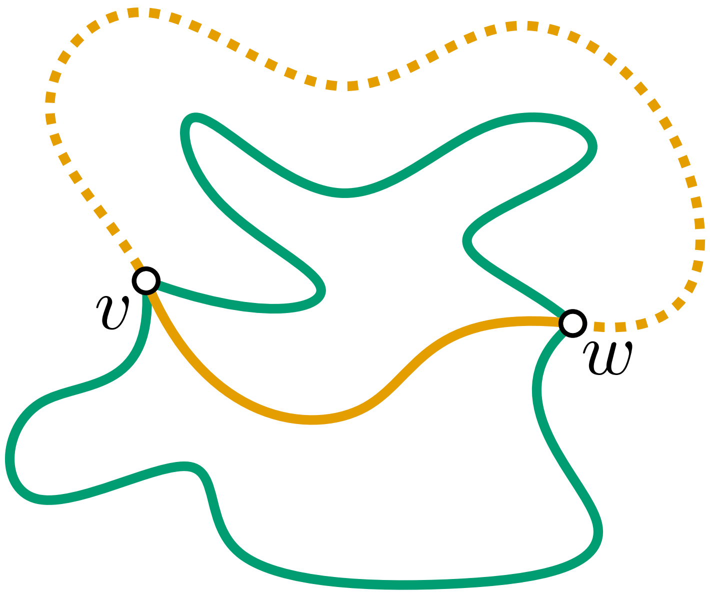
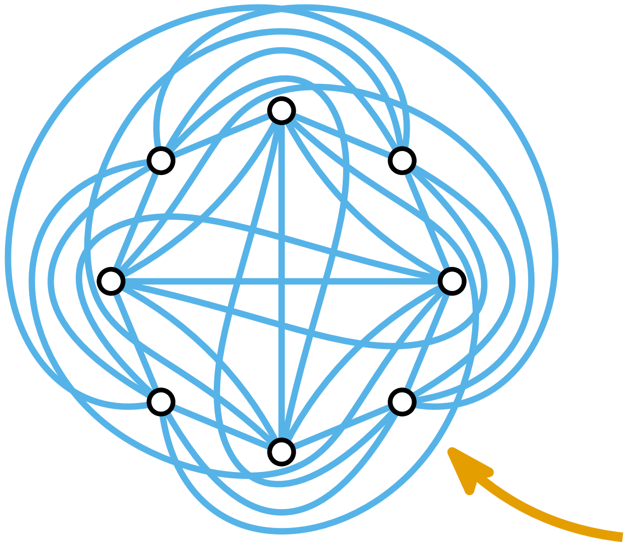
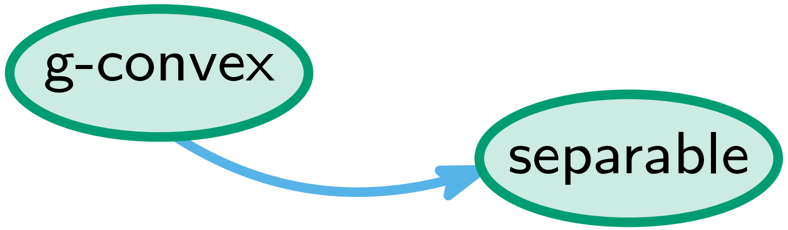
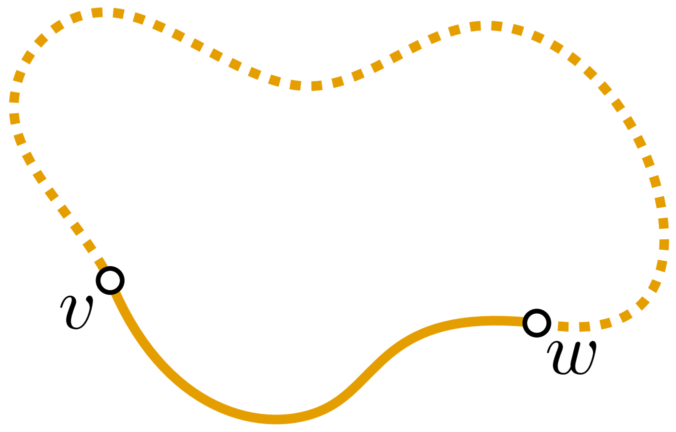






Result: Recognition (K_n) / Flips

for every witness:



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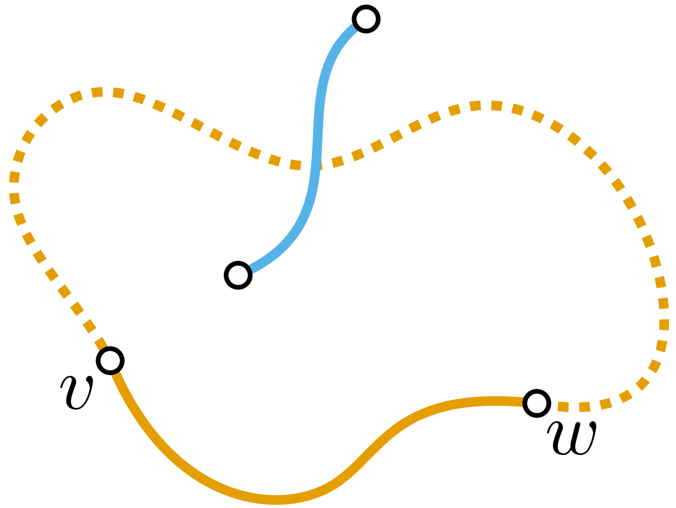

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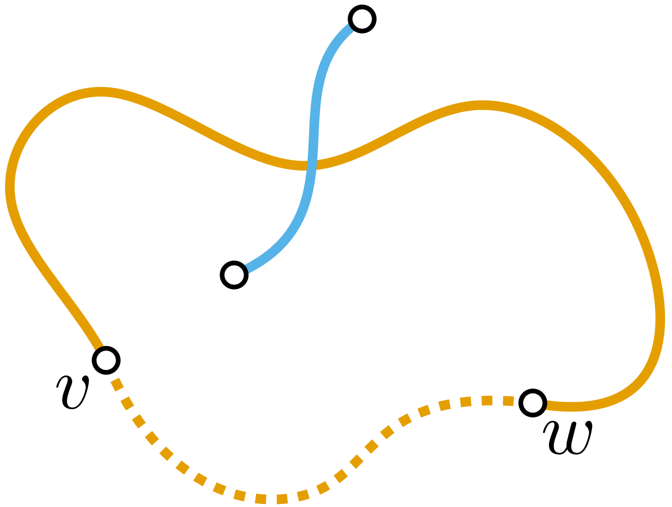

can exchange edge/non-edge part

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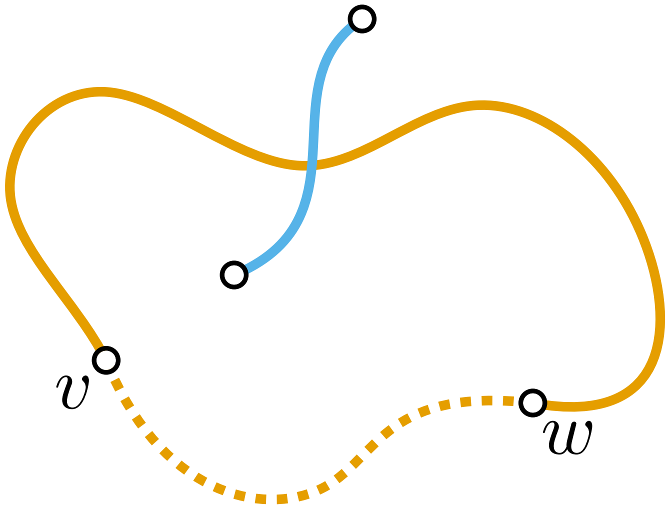

can exchange edge/non-edge part

$\rightarrow$ still simple

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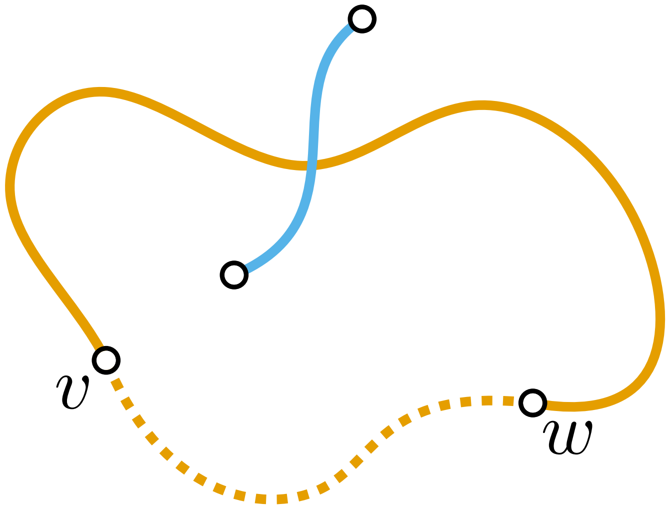

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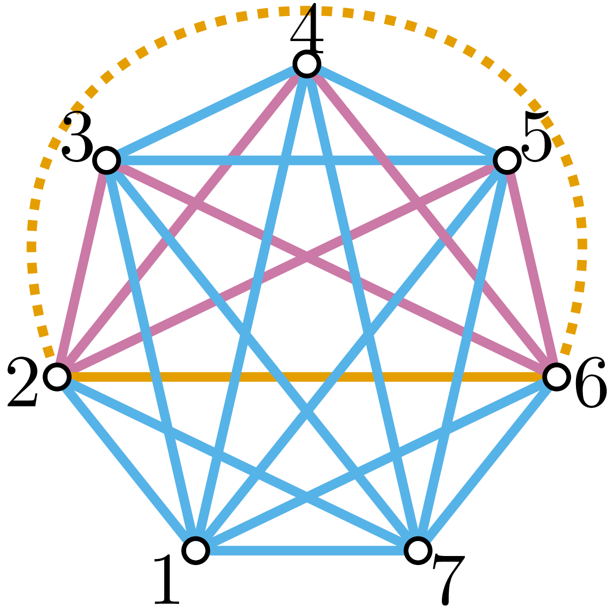
relation to **flips** in rotation systems

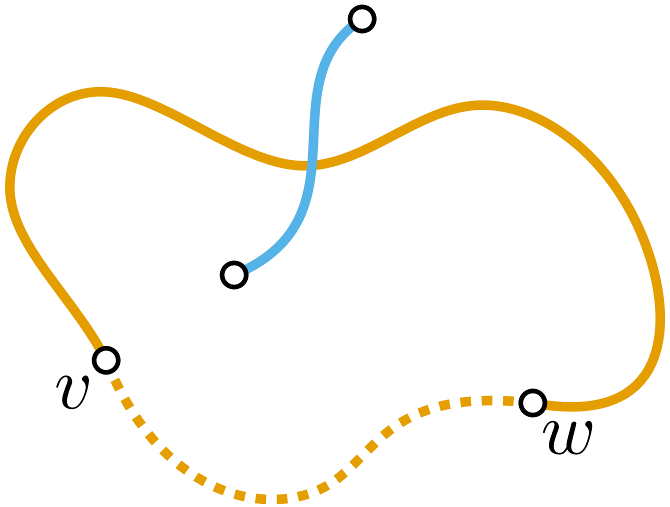
$1: 2\ 3\ 4\ 5\ 6\ 7$
$2: 1\ 3\ 4\ 5\ 6\ 7$
$3: 1\ 2\ 4\ 5\ 6\ 7$
$4: 1\ 2\ 3\ 5\ 6\ 7$
$5: 1\ 2\ 3\ 4\ 6\ 7$
$6: 1\ 2\ 3\ 4\ 5\ 7$
$7: 1\ 2\ 3\ 4\ 5\ 6$



Result: Recognition (K_n) / Flips

Theorem: It can be decided in $\mathcal{O}(n^6)$ time whether a given simple drawing of K_n is separable.

for every witness:



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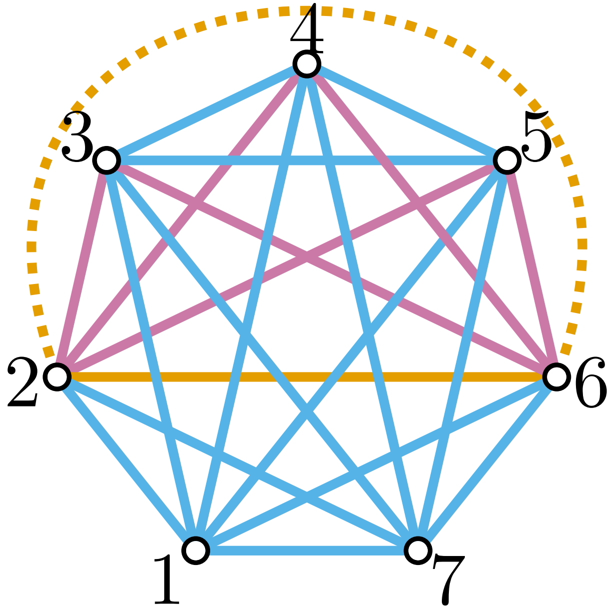

Result: Extendability

Theorem: Every separable drawing on n vertices can be extended to a simple drawing of K_n.

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1. add one edge: minimize crossings with the **witnesses**

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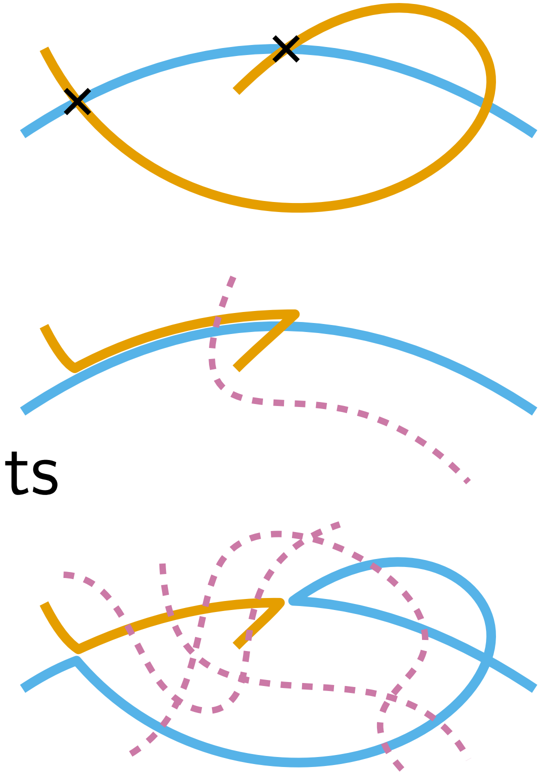

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standard rerouting/exchanging arguments

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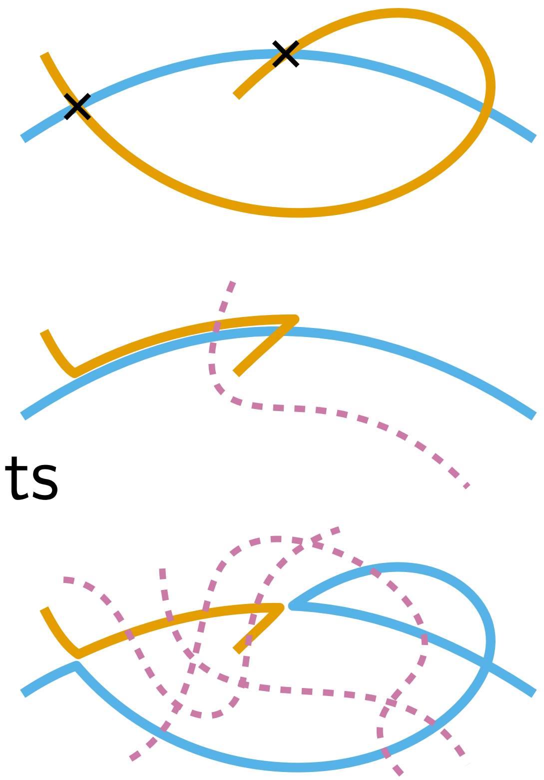

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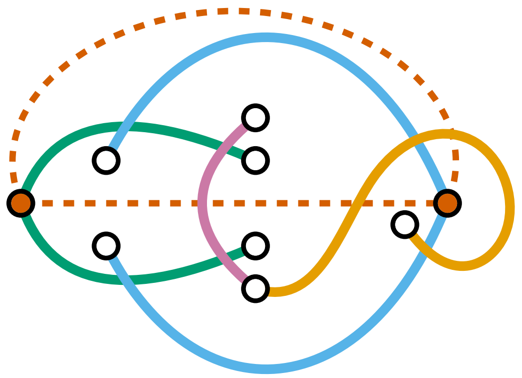
result need not be separable!



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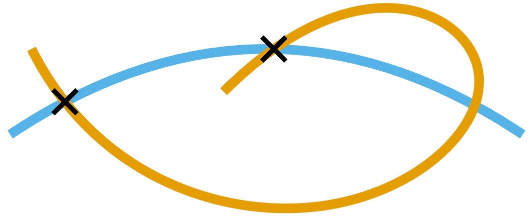

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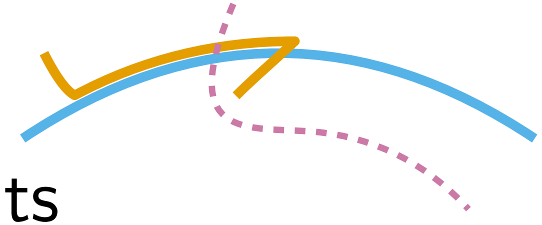

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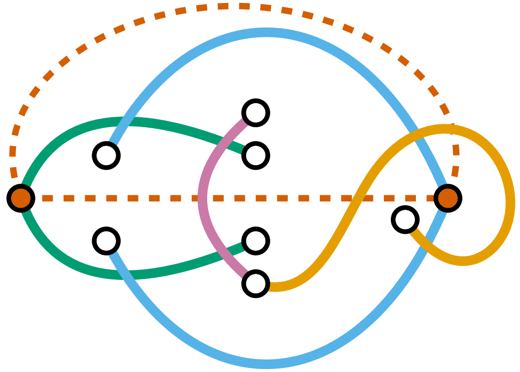
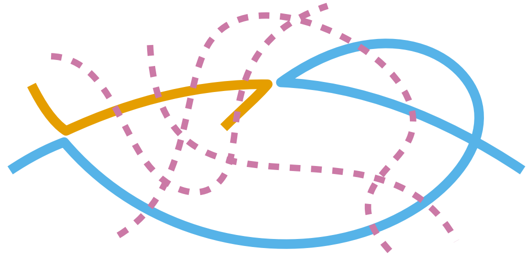


2 add all missing edges respecting 1
+ minimize crossings among them



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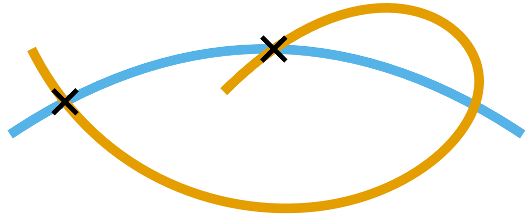

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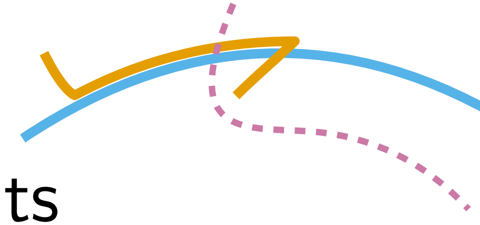

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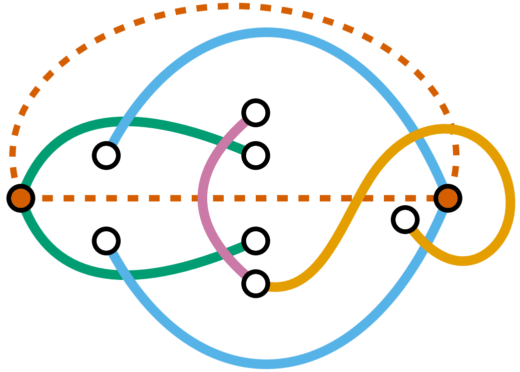
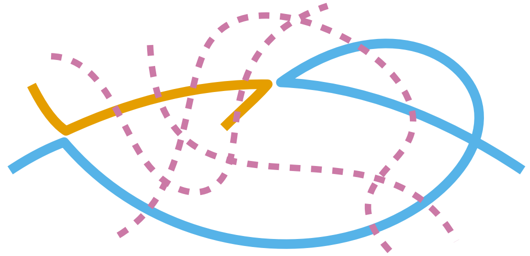
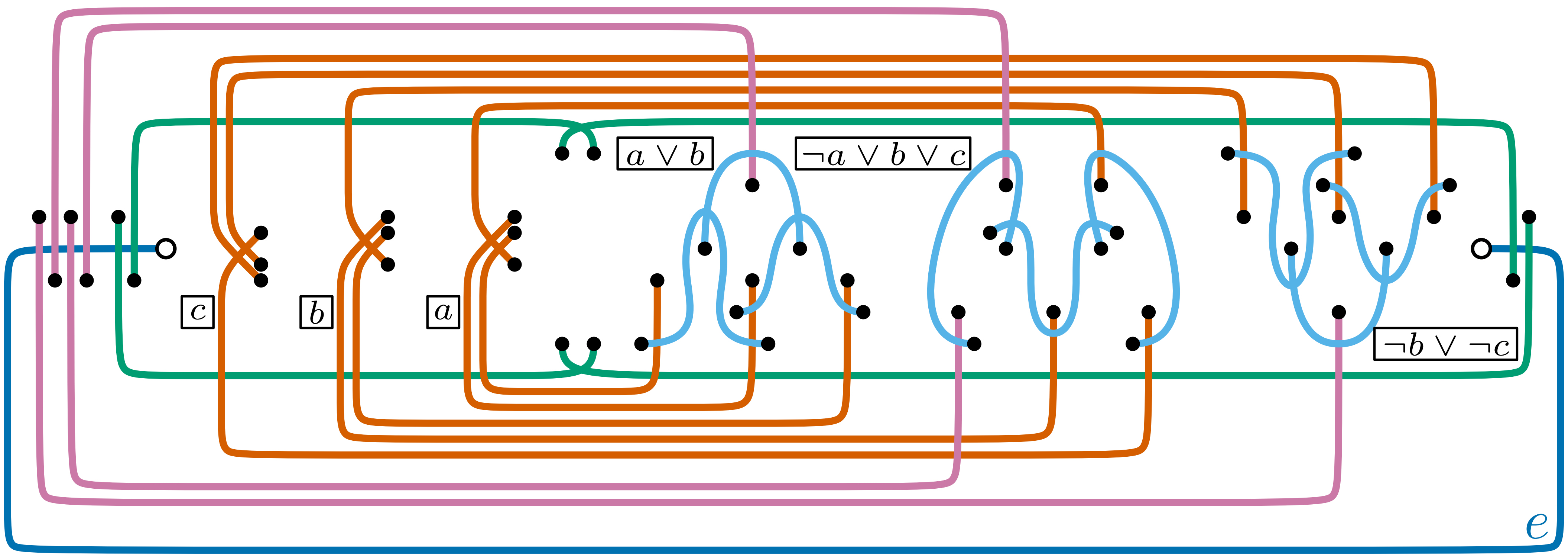


❷ add all missing edges respecting ❶ + minimize crossings among them



Theorem: Every crossing-minimizing drawing on n vertices can be extended to a simple drawing of K_n.

Result: Recognition / NP-Hardness



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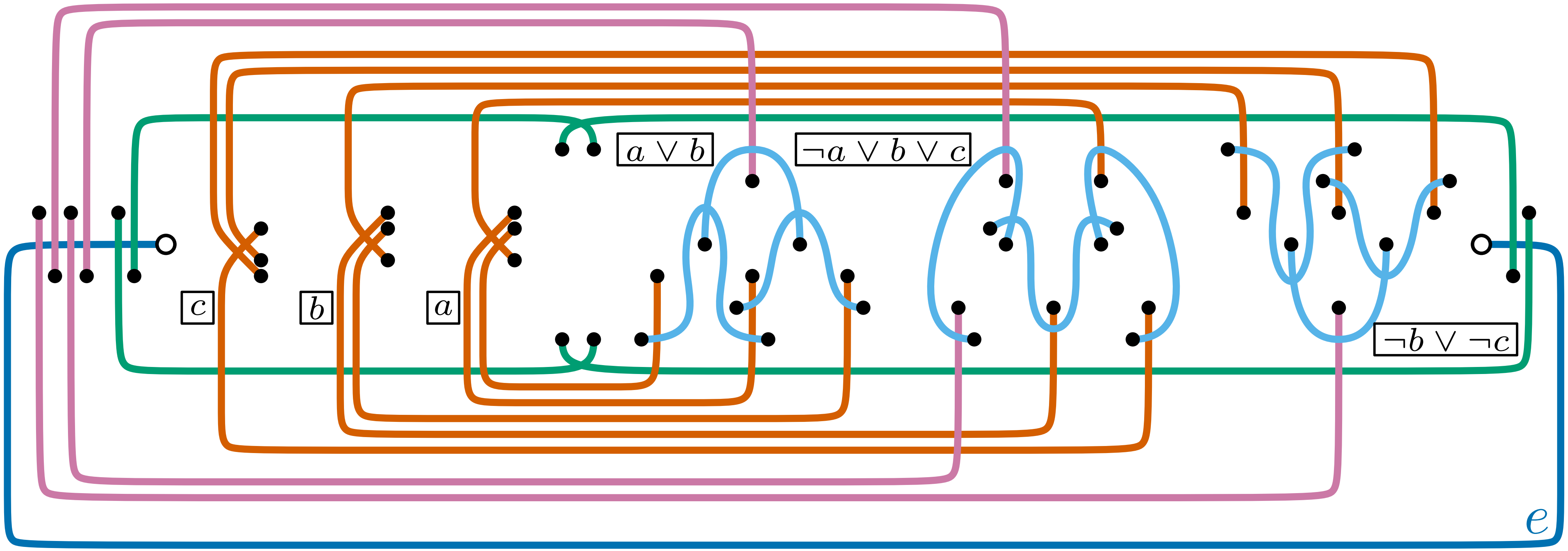
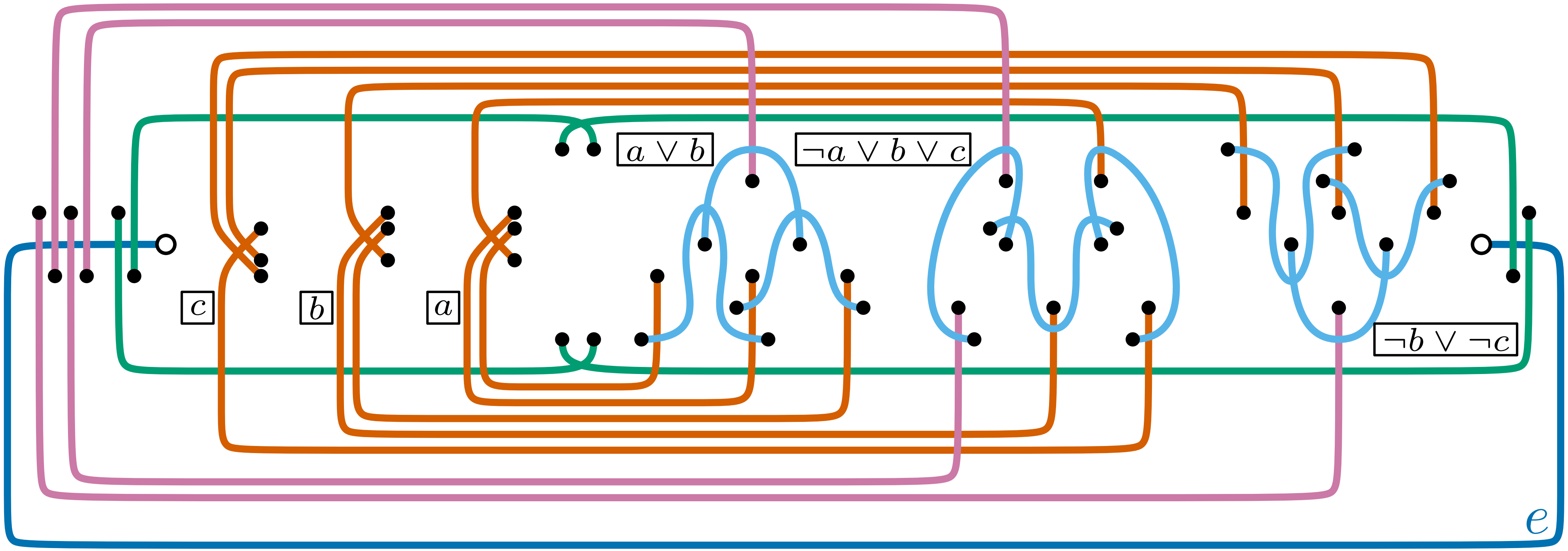


reduction from **linked planar 3-SAT with negated edges on one side**

Result: Recognition / NP-Hardness



reduction from **linked planar 3-SAT with negated edges on one side**

satisfiable $\Leftrightarrow$ e has witness

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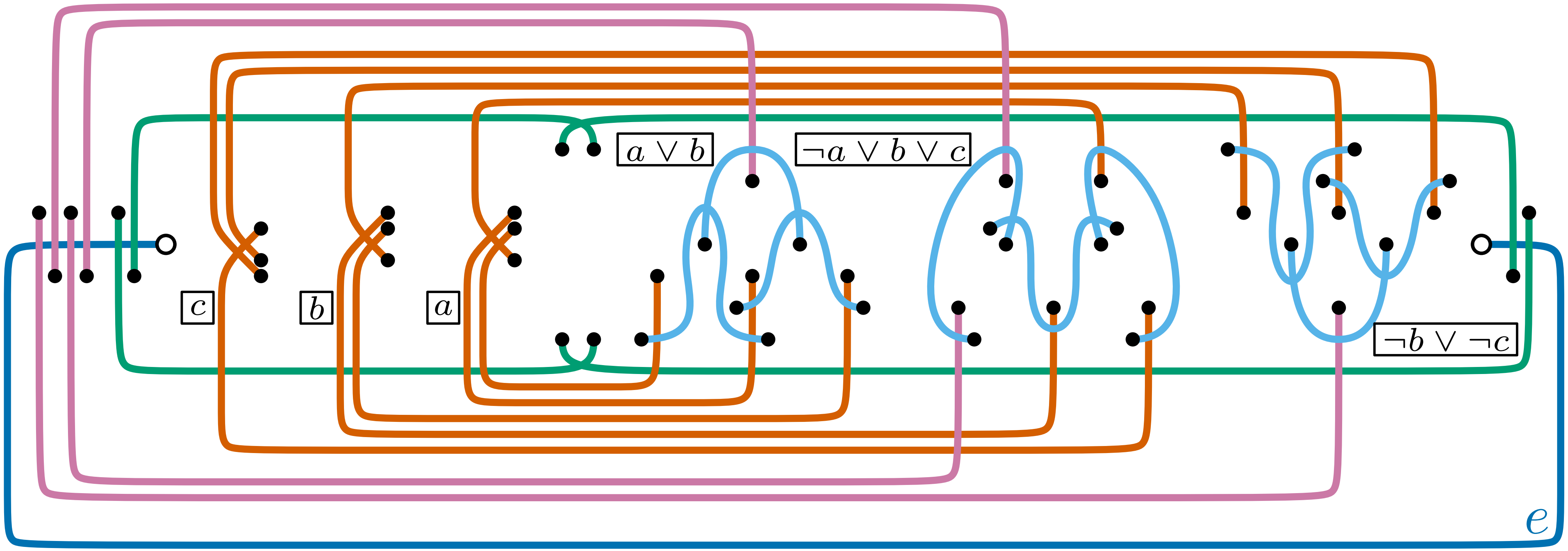
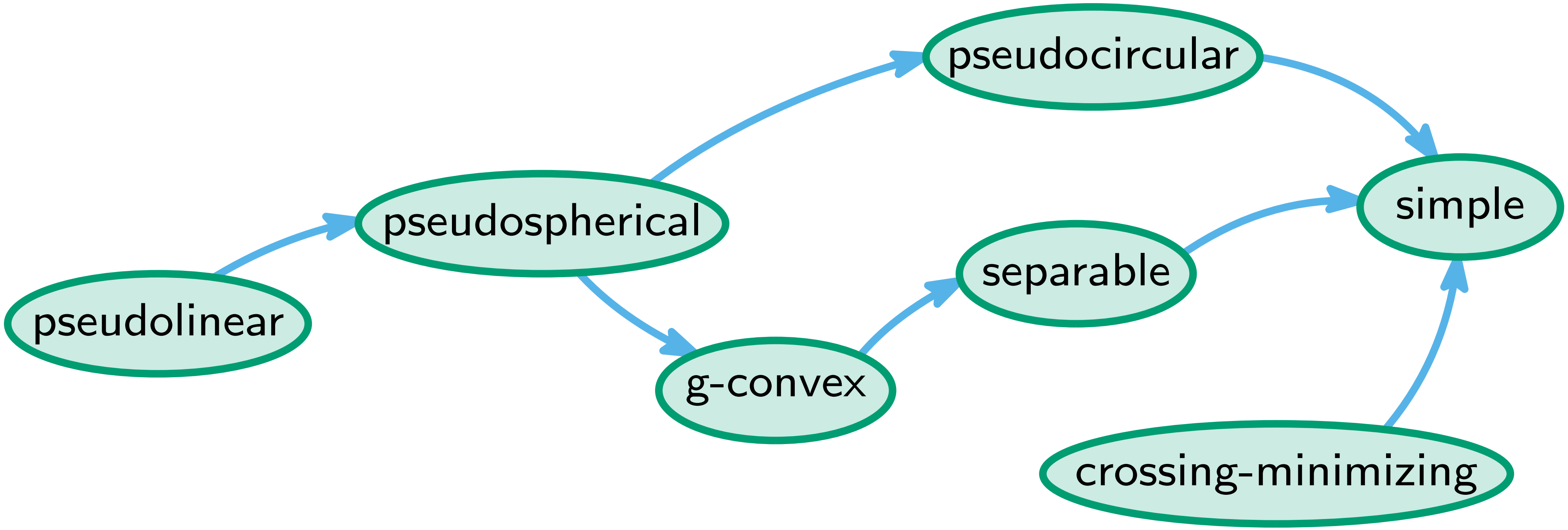


reduction from **linked planar 3-SAT with negated edges on one side**

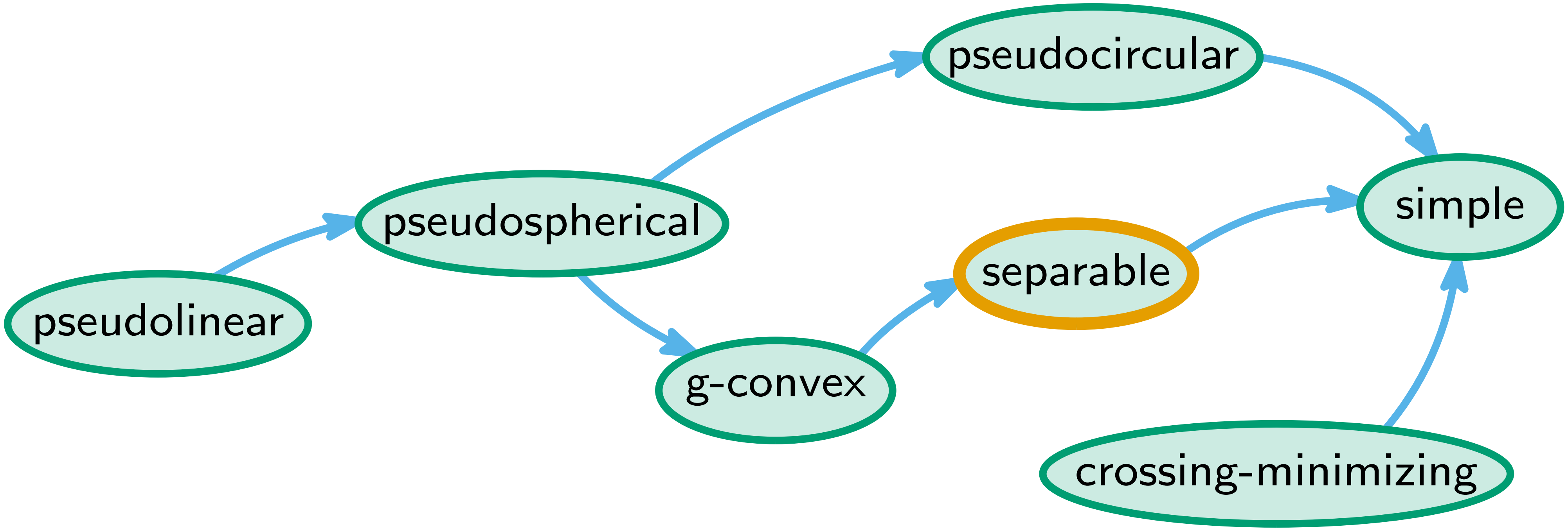
satisfiable $\Leftrightarrow$ e has witness

→ all other edges have witnesses

Summary


pseudocircular
pseudospherical
simple
separable
pseudolinear
g-convex
crossing-minimizing

Summary



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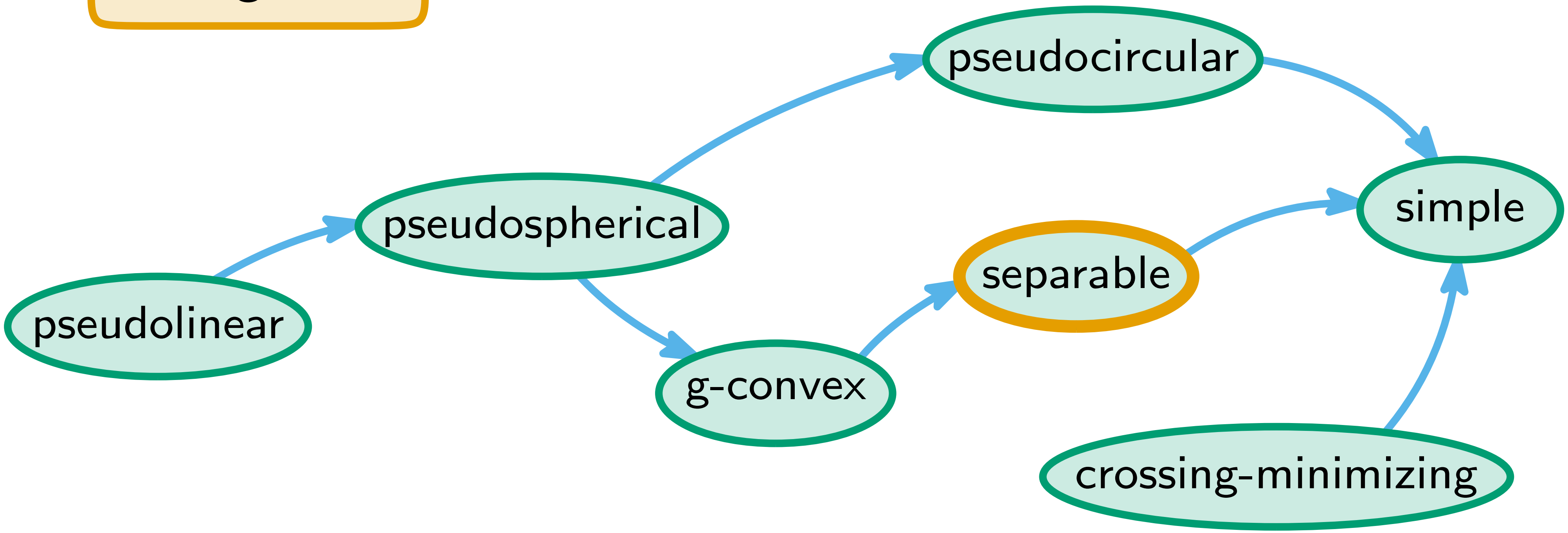
Recognition



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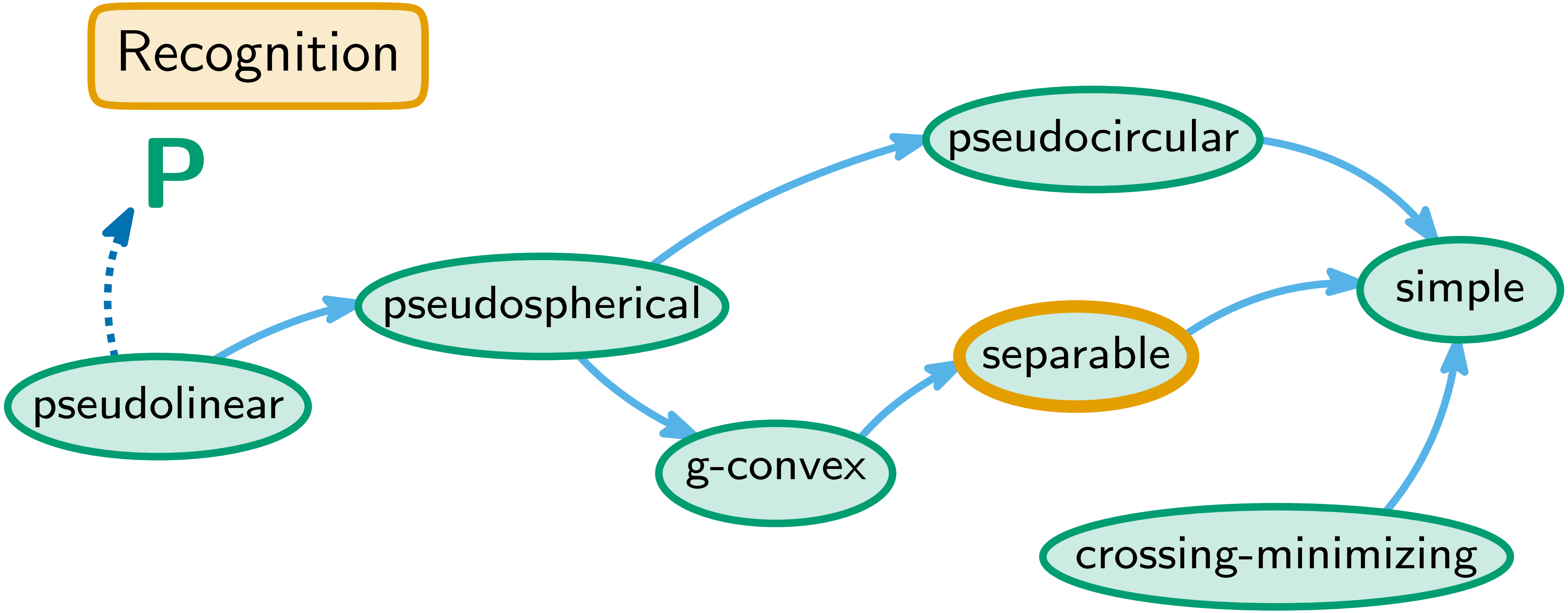
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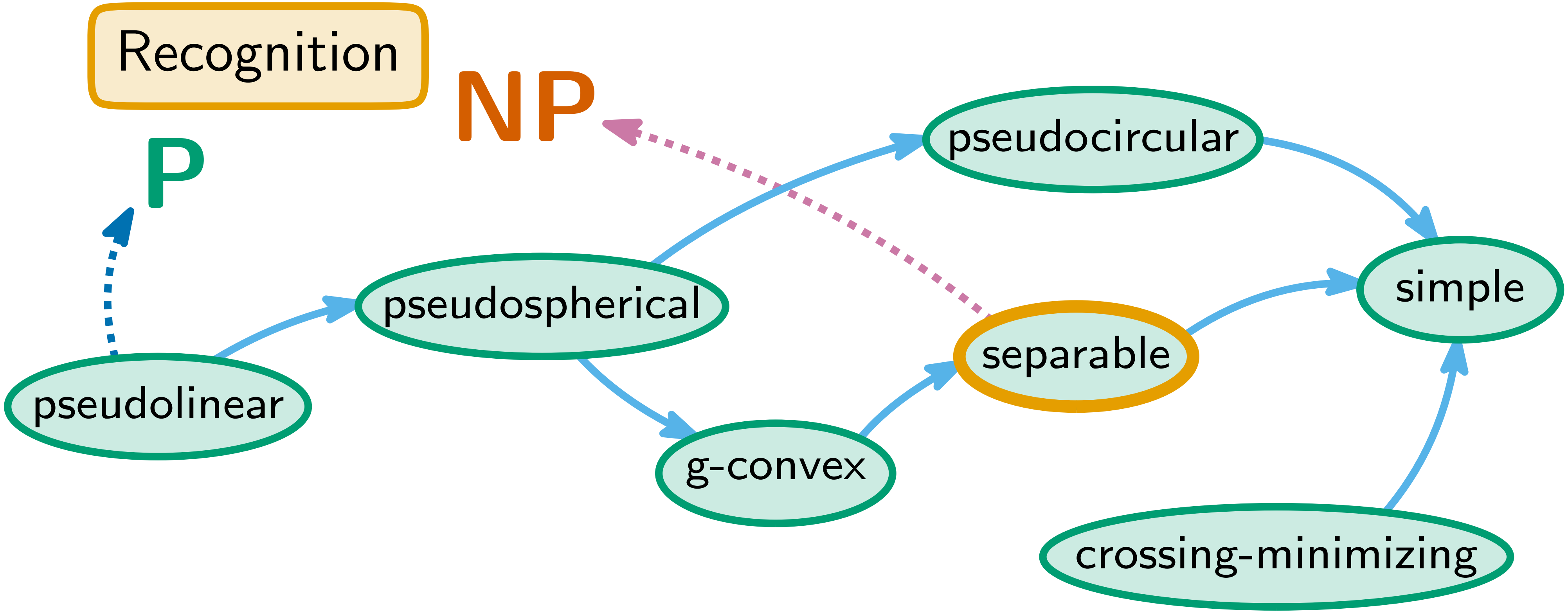

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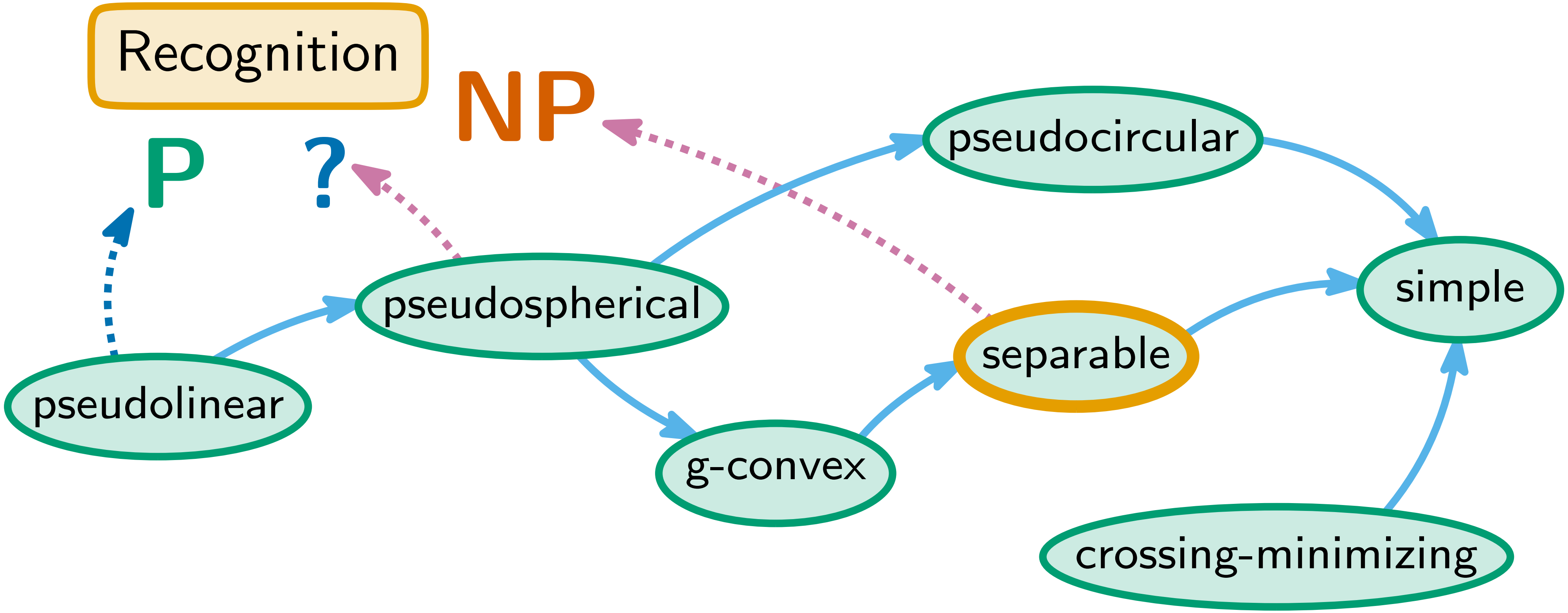
Recognition
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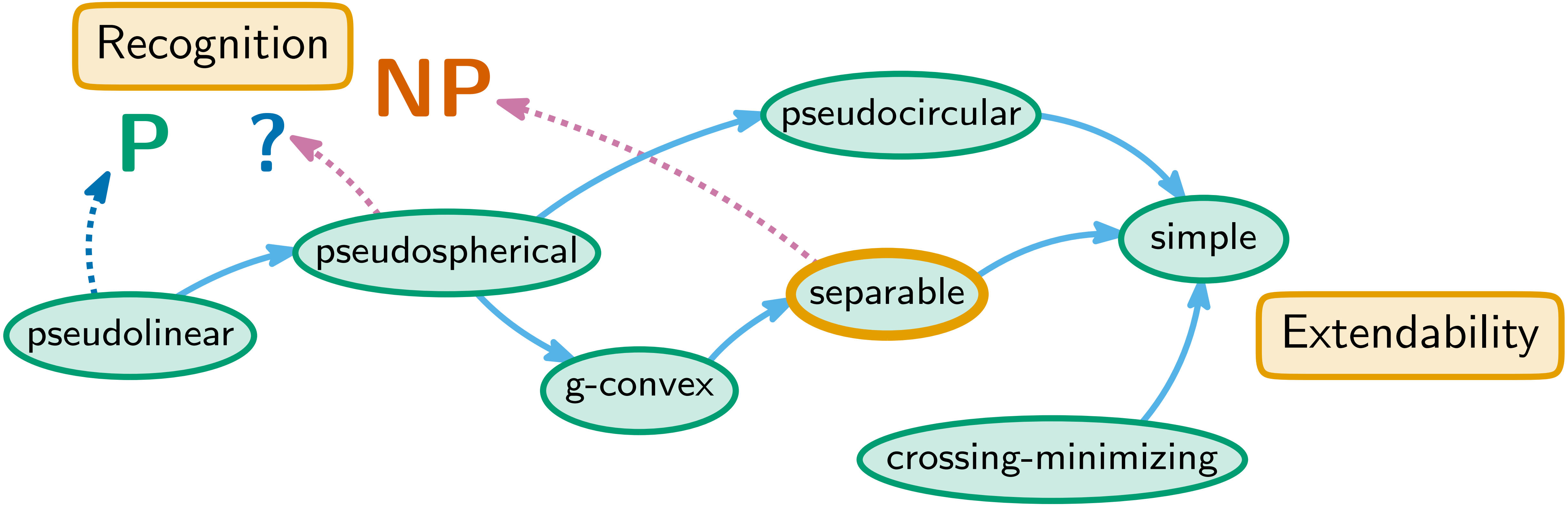
Summary



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Extendability
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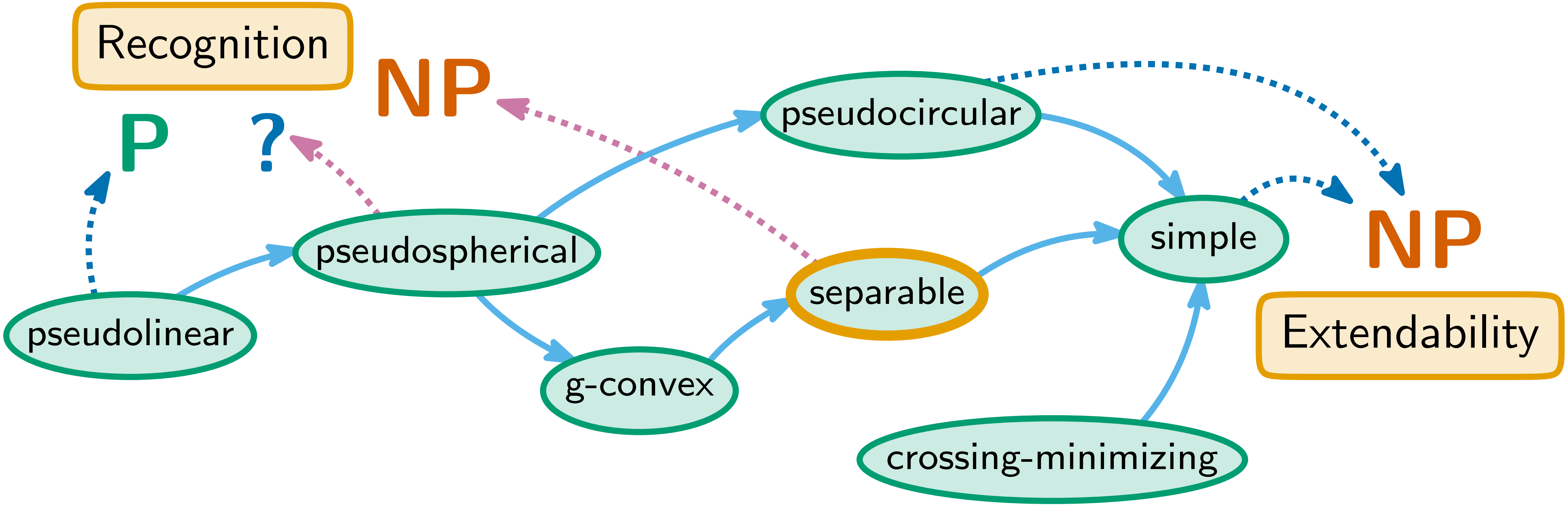
Summary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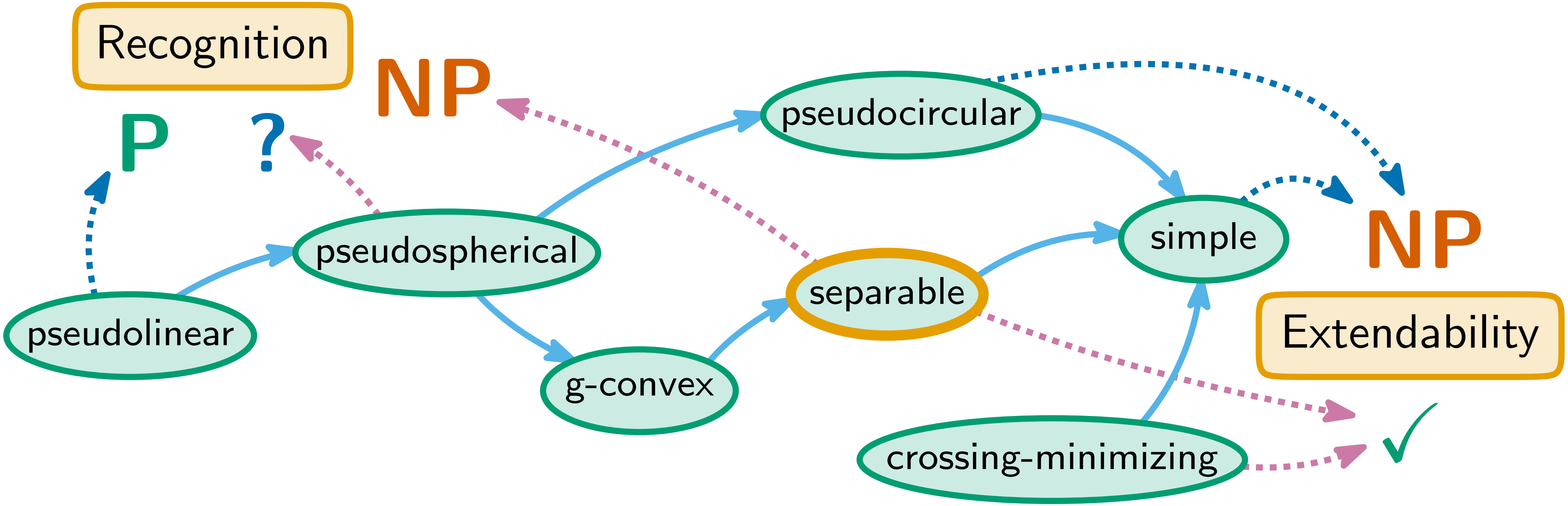
NP
Extendability

Summary


Recognition
NP
P
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separable
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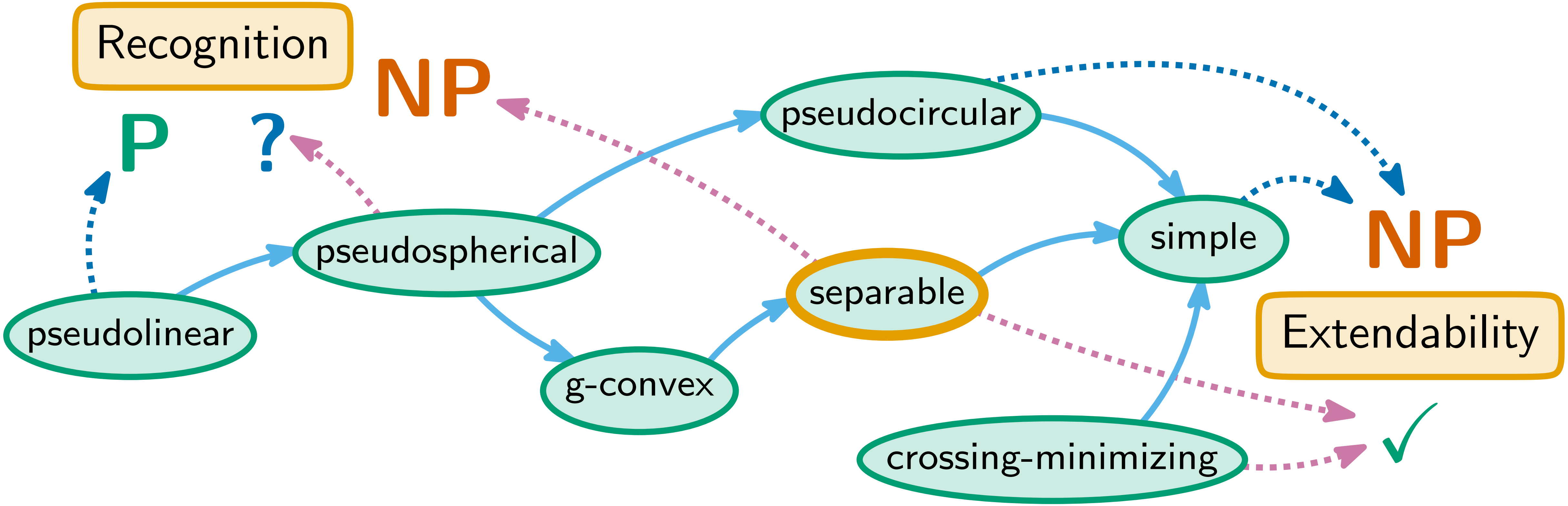
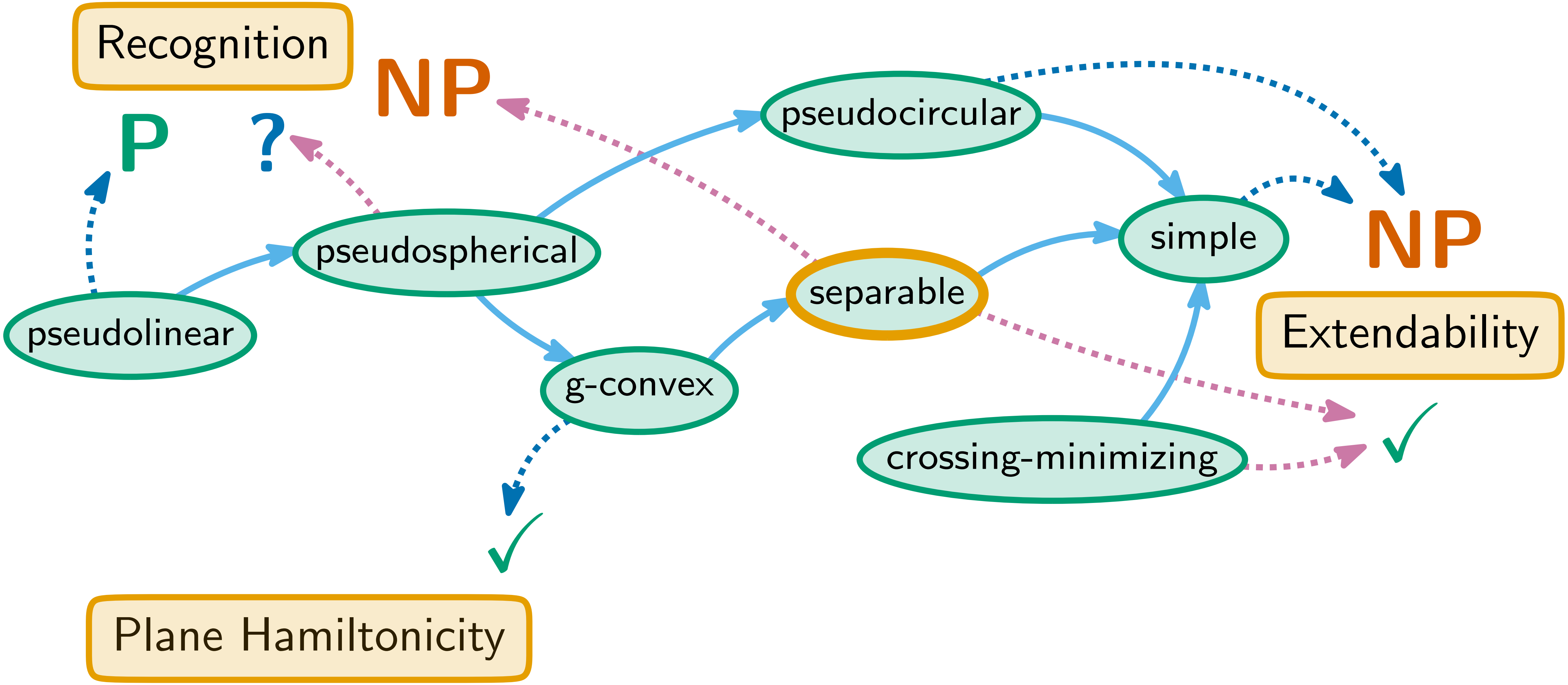
Extendability
g-convex
crossing-minimizing
✓

Summary



Plane Hamiltonicity

Summary


Recognition
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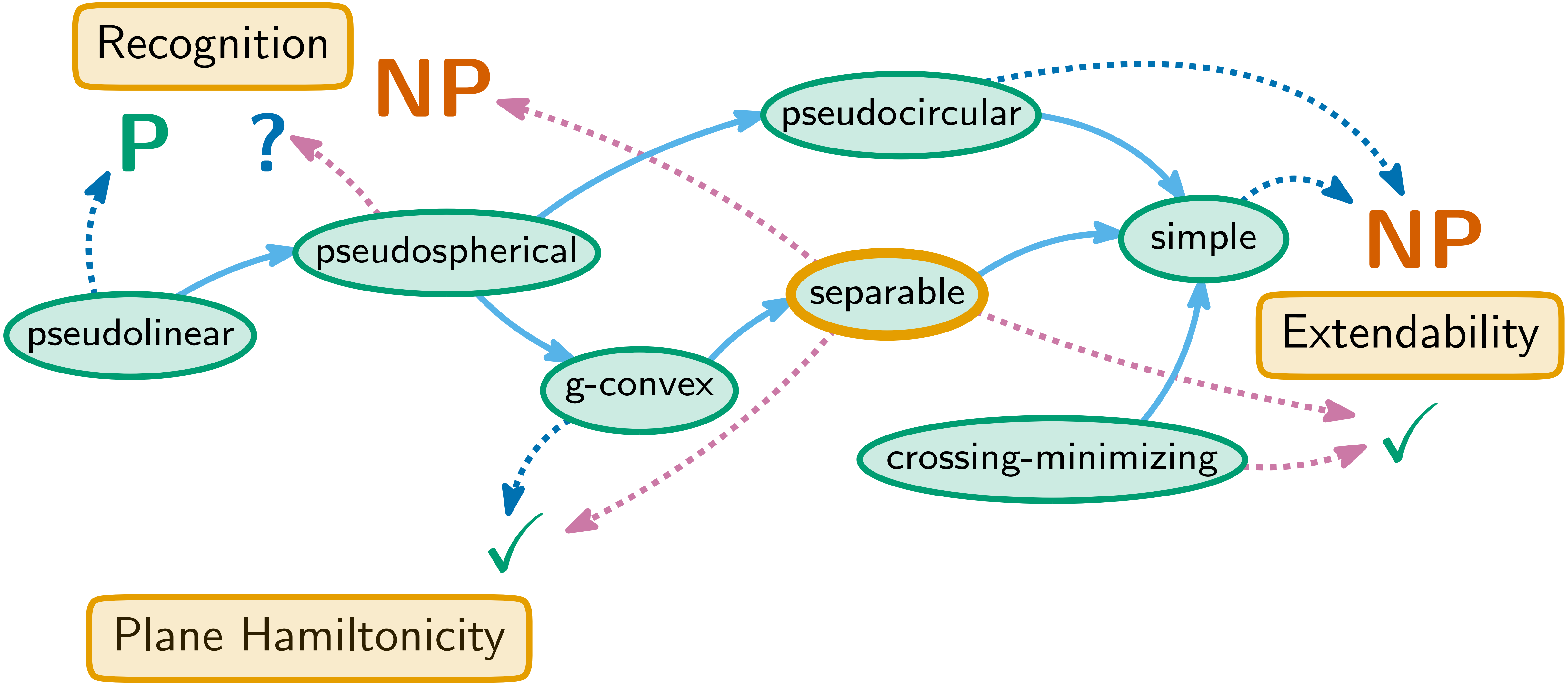
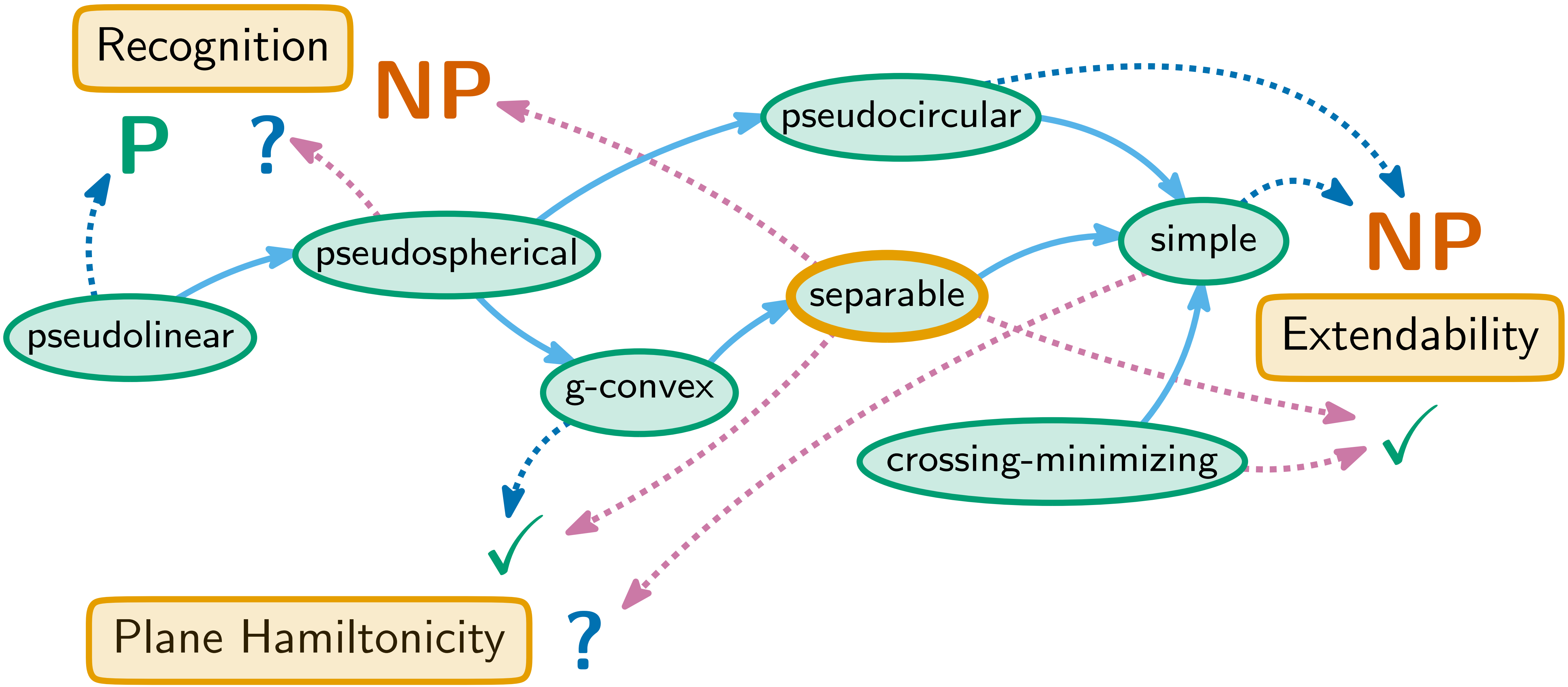
Plane Hamiltonicity

Summary


Recognition
P
?
NP
pseudocircular
pseudospherical
simple
NP
separable
pseudolinear
Extendability
g-convex
✓
crossing-minimizing
✓
Plane Hamiltonicity

Summary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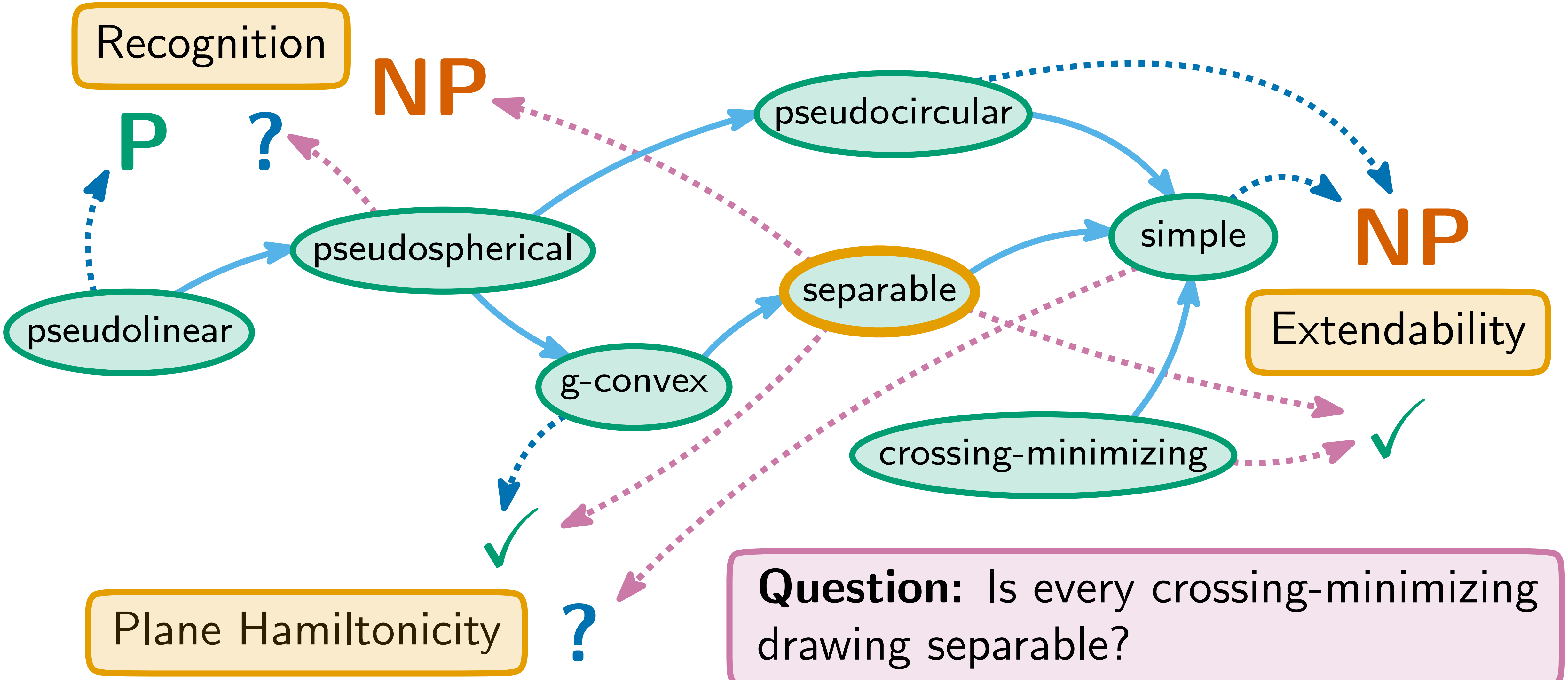
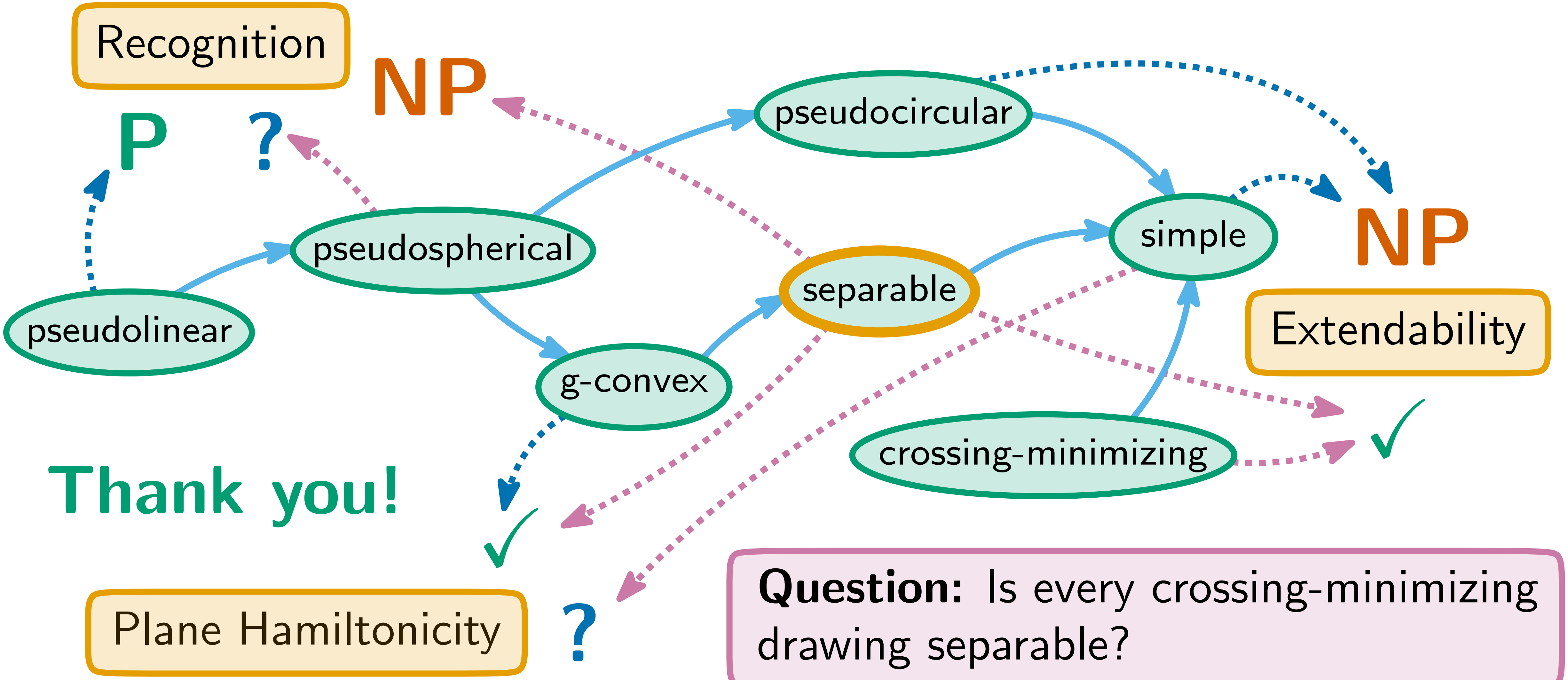
Summary



Question: Is every crossing-minimizing drawing separable?

Summary



Thank you!

Question: Is every crossing-minimizing drawing separable?