Noncrossing Longest Paths and Cycles

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shortest subgraphs

Observation

For every point set in the plane, the shortest perfect matching (spanning path, spanning cycle) is plane.

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Question

Are there ARBITRARILY LARGE point sets in the plane s.t. the longest perfect matching (spanning path, spanning cycle) is plane?

shortest subgraphs

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Theorem (Álvarez-Rebollar, Cravioto-Lagos, Marín, Solé-Pi, and Urrutia (Graphs and Combinatorics, 2024))

YES for the longest perfect matching

shortest subgraphs

Question (Álvarez-Rebollar, Cravioto-Lagos, Marín, Solé-Pi, and Urrutia 2024)

For every sufficiently large planar point set, must the longest spanning path have two edges that cross each other?

Conjecture (Álvarez-Rebollar, Cravioto-Lagos, Marín, Solé-Pi, and Urrutia 2024)

For every sufficiently large planar point set, the longest spanning cycle has two edges that cross each other.

Main Results

Theorem

For every integer $n \geq 1$ there exists a set of n points in the plane for which the longest spanning path is unique and plane.

Theorem

For every integer $n \geq 3$ there exists a set of n points in the plane for which the longest spanning cycle is unique and plane.

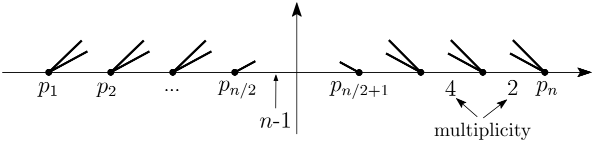
Longest path

Theorem

For every integer $n \geq 1$ there exists a set of n points in the plane for which the longest spanning path is unique and plane.

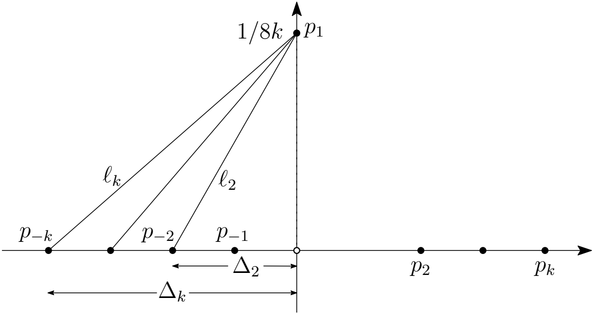
PROOF:

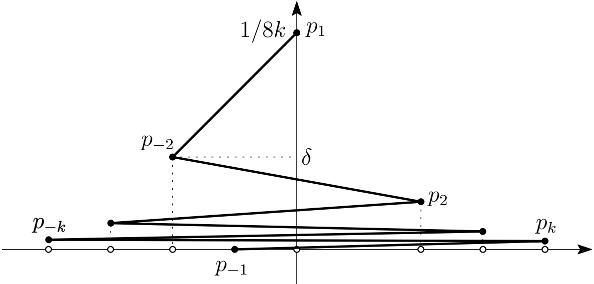
Step 1: n points on a line (n even):



Longest path

Step 2: n points almost on a line:

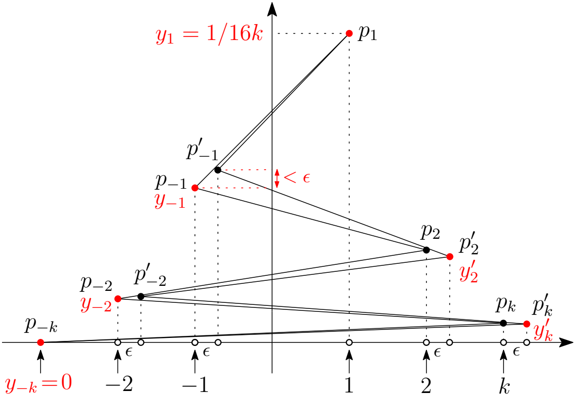




Longest cycle

Theorem

For every integer $n \geq 3$ there exists a set of n points in the plane for which the longest spanning cycle is unique and plane.



Longest cycle

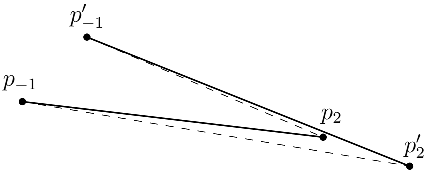


Figure: The longest cycle connects p_{-1} to p_2 and p'_{-1} to p'_2