Drawing Planar Graphs and 1-Planar Graphs Using Cubic Bézier Curves with Bounded Curvature

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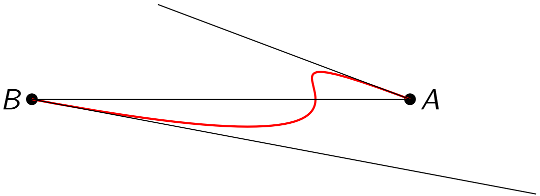
Bézier curves

$$At^3 + 3Ct^2(1-t) + 3Dt(1-t)^2 + B(1-t)^3$$



Curvature

The curvature of a twice-differentiable parameterized curve, $\mathbf{c}(t) = (x(t), y(t))$, can be defined as follows:

$$\kappa(t) = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}},$$

1-Bend

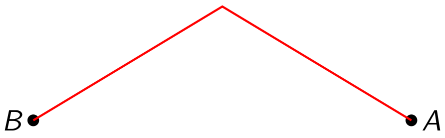
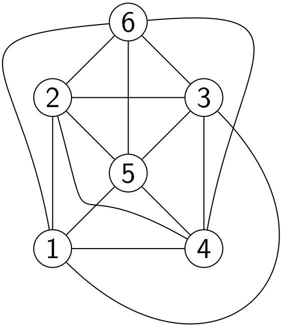
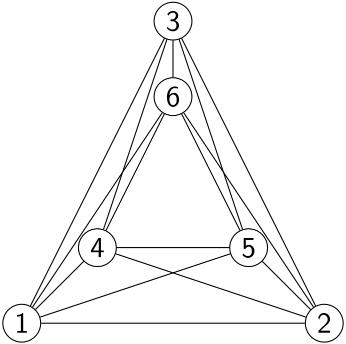


Figure 1: Infinite Curvature

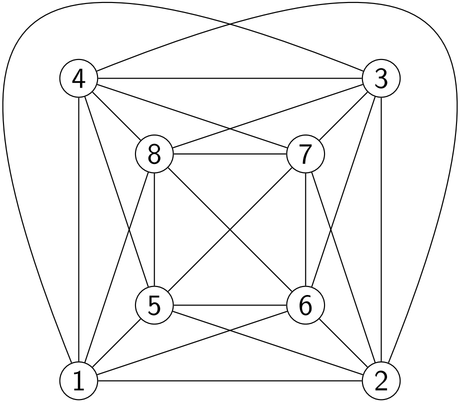
1-Planar Graphs



Straight Line Drawings



Always possible?



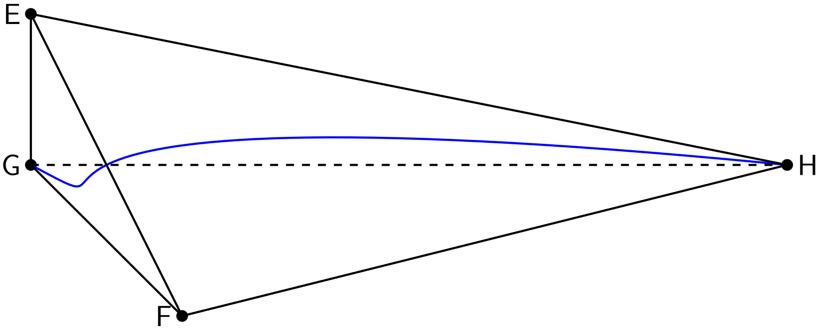
1-bend RAC Drawings

Theorem (Bekos, Didimo, Liotta, Mehrabi, and Montecchiani [1])

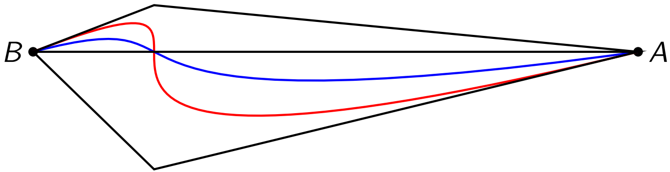
Let G be an n-vertex 1-planar graph. Then G admits a 1-planar 1-bend RAC drawing.

We adapt the proof to use Bézier curves

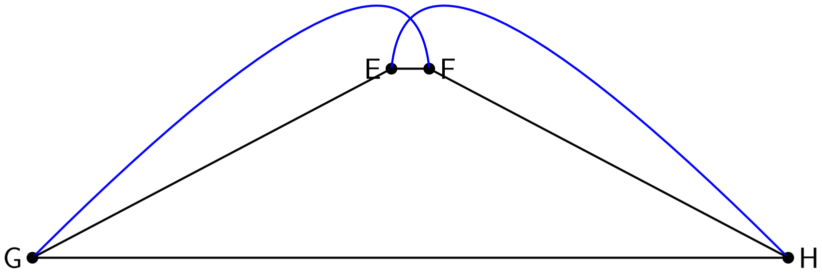
Using Bézier Curves inside



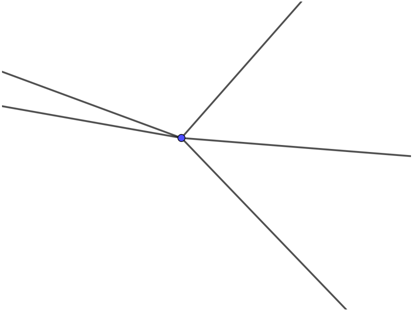
Bézier curves as a convex combination



Using Bézier curves outside

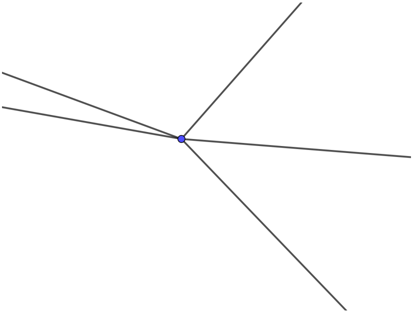


Angular Resolution in Planar Graphs



The smallest angle around a vertex

Angular Resolution in Planar Graphs



The smallest angle around a vertex

Theorem (Miyata [2])

Let ε be any positive constant. There is a family of planar graphs with maximum degree d that have angular resolution $O((\log d)^{\varepsilon}/d^{\frac{3}{2}})$

Using 2-Bend edges

Theorem (Goodrich and Wagner [3])

There is an $O(n)$ time algorithm to draw a planar graph on a grid with angular resolution $\Theta(1/d(v))$ using 2-Bend edges.

Theorem (Goodrich and Wagner [3])

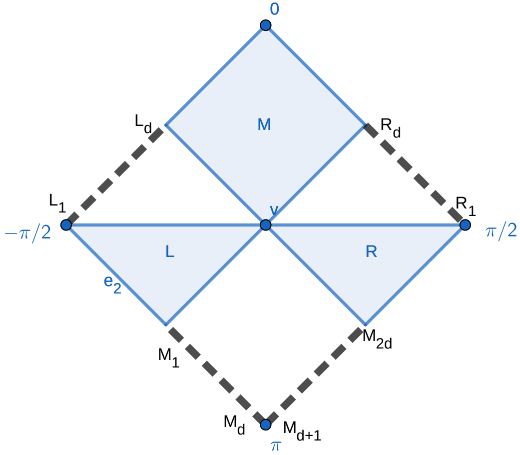
There is an $O(n)$ time algorithm to draw a planar graph on a grid with angular resolution $\Theta(1/d(v))$ using cubic Bézier curves.

Using 1-Bend edges

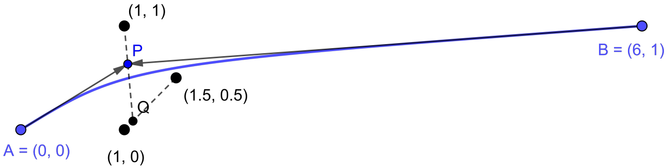
Theorem (Cheng, Duncan, Goodrich, and Kobourov [4])

Given a planar graph G, algorithm OneBend *produces in $O(n)$ time, a planar embedding on the $30n \times 15n$ grid with angular resolution $\Theta(1/d(v))$ using 1-Bend edges.*

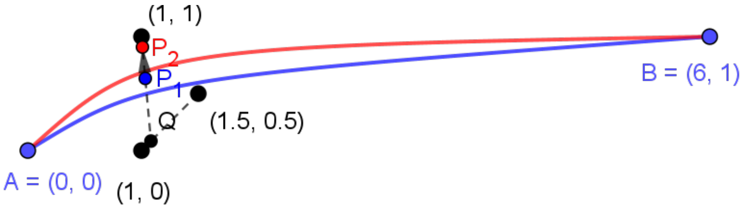
Vertex Joint Box



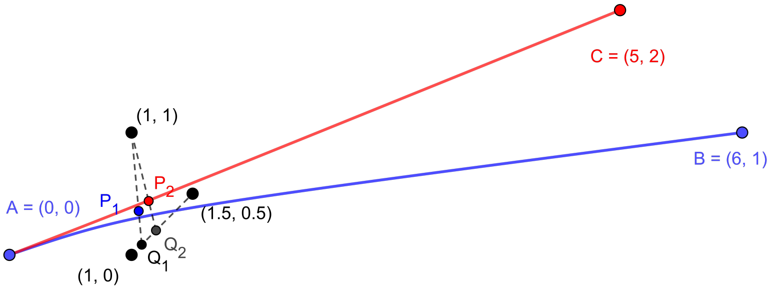
Drawing an edge



Non-intersection changing k



Non-intersection changing B



Curvature

Theorem

Given an n-vertex planar graph, G, we can draw G in an $O(n) \times O(n)$ grid and $\Omega(1/\mathrm{degree}(v))$ angular resolution, for each vertex $v \in G$, using a single cubic Bézier curve with curvature $O(\sqrt{n})$ per edge in $O(n)$ time.

Future Work

- Bounds on the Curvature for 1-Planar graphs
- Improve the Curvature for Planar graphs
- Use “C” shaped curves instead of “S” shaped curves

References

[1] M. A. Bekos, W. Didimo, G. Liotta, S. Mehrabi, and F. Montecchiani, "On rac drawings of 1-planar graphs," Theoretical Computer Science, vol. 689, pp. 48–57, 2017.

[2] H. Miyata, "A new upper bound for angular resolution," CoRR, vol. abs/2309.08401, 2023.

[3] M. T. Goodrich and C. G. Wagner, "A framework for drawing planar graphs with curves and polylines," Journal of Algorithms, vol. 37, no. 2, pp. 399–421, 2000.

[4] C. C. Cheng, C. A. Duncan, M. T. Goodrich, and S. G. Kobourov, "Drawing planar graphs with circular arcs," Discrete & Computational Geometry, vol. 25, pp. 405–418, 1999.

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