

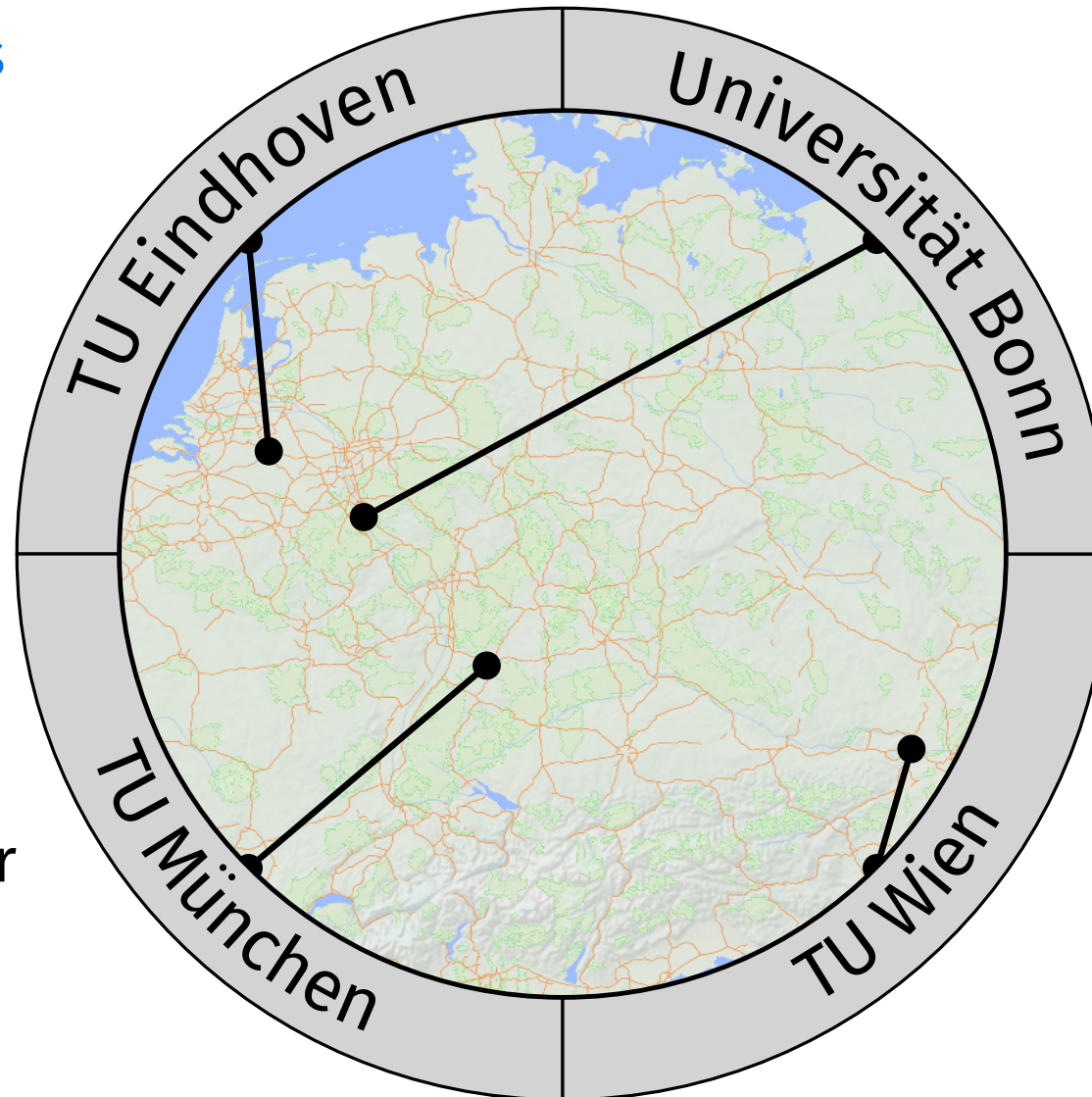
Boundary Labeling in a Circular Orbit

GD '24 ♦ 19.09.2024 ♦ Session 5.4

Soeren Terziadis

Jules Wulms
TU Eindhoven

Annika Bonerath
Universität Bonn



Markus Wallinger
TU München

Martin Nöllenburg
TU Wien

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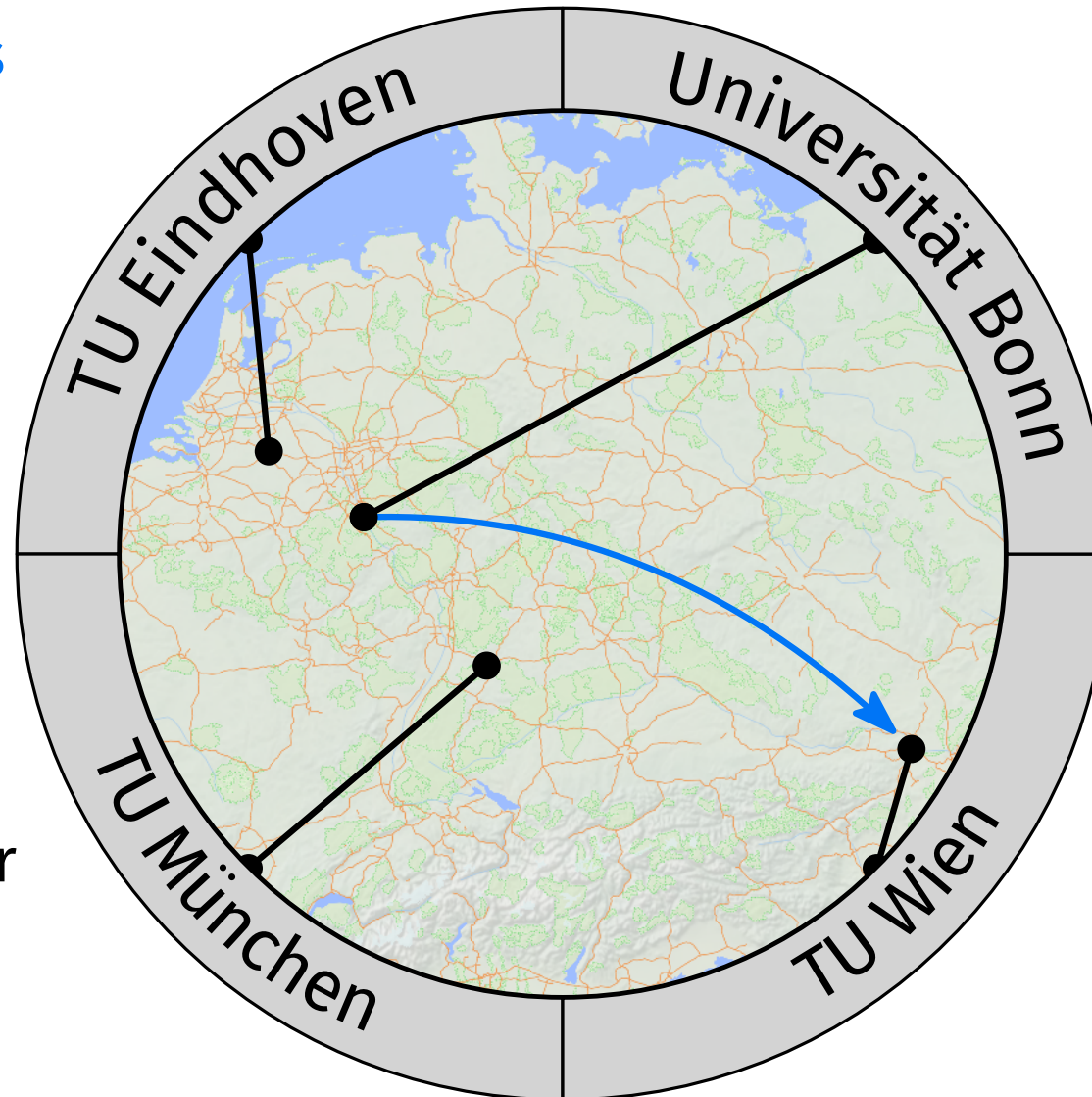
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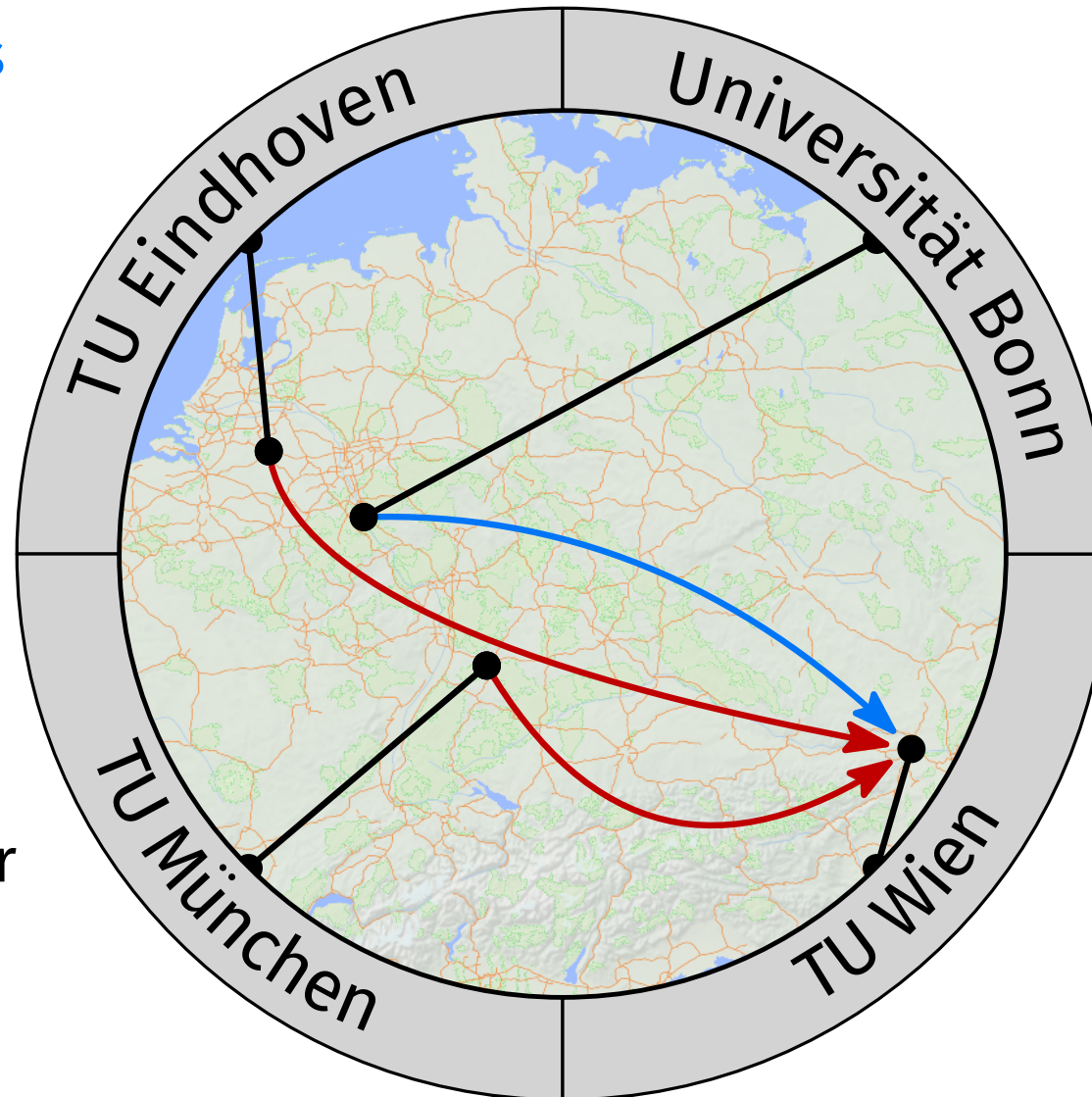
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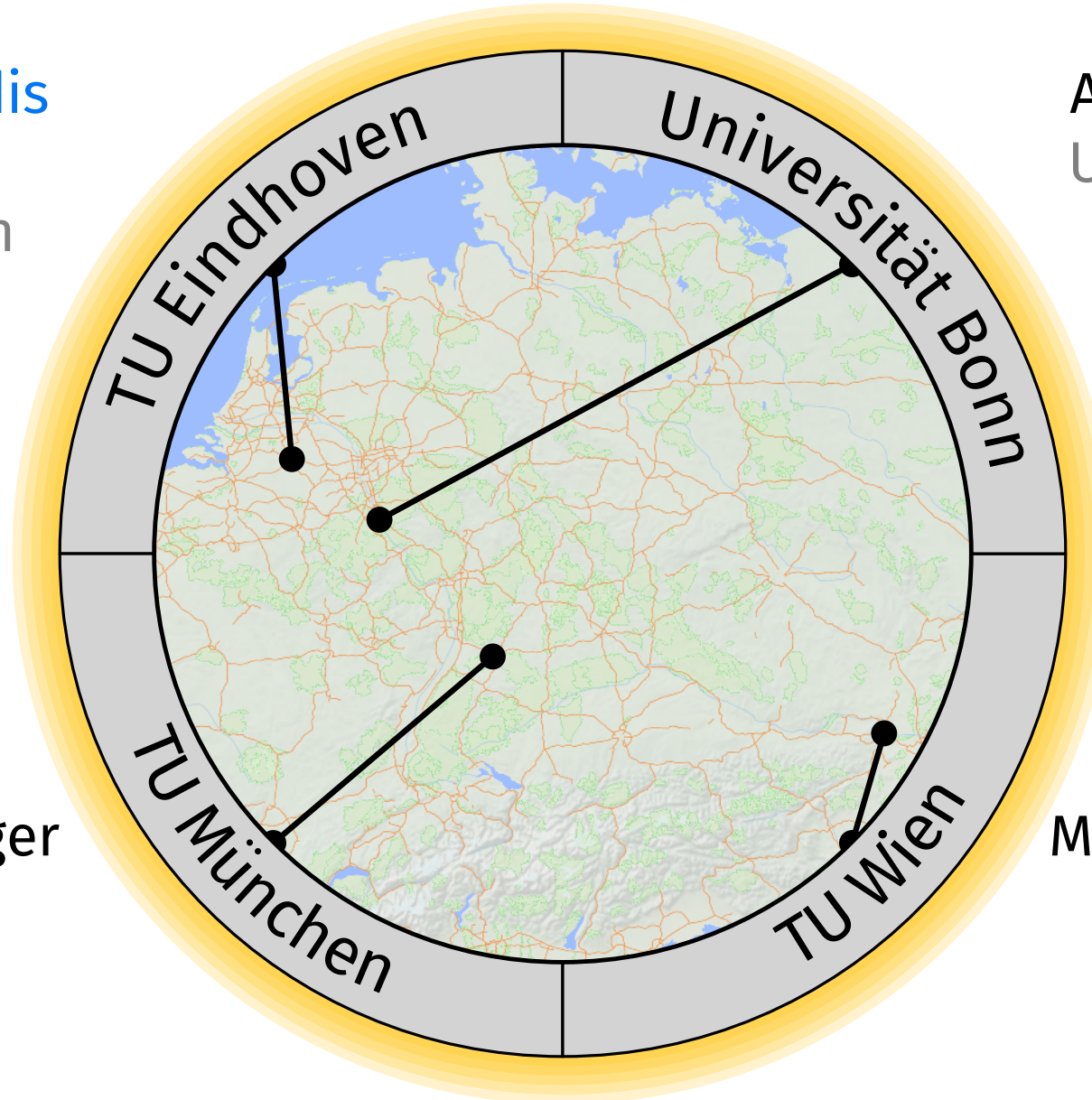
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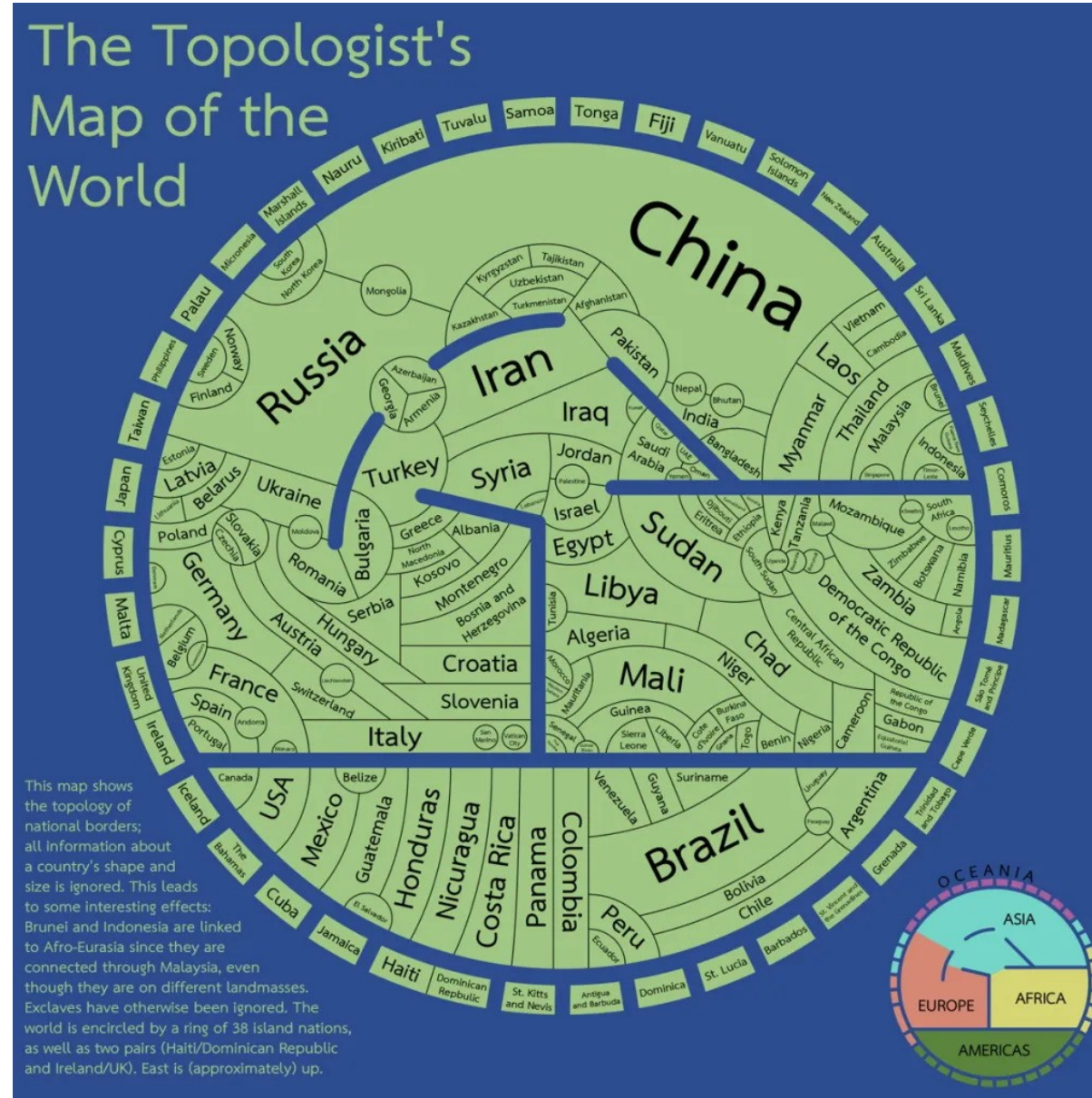
Circular Maps



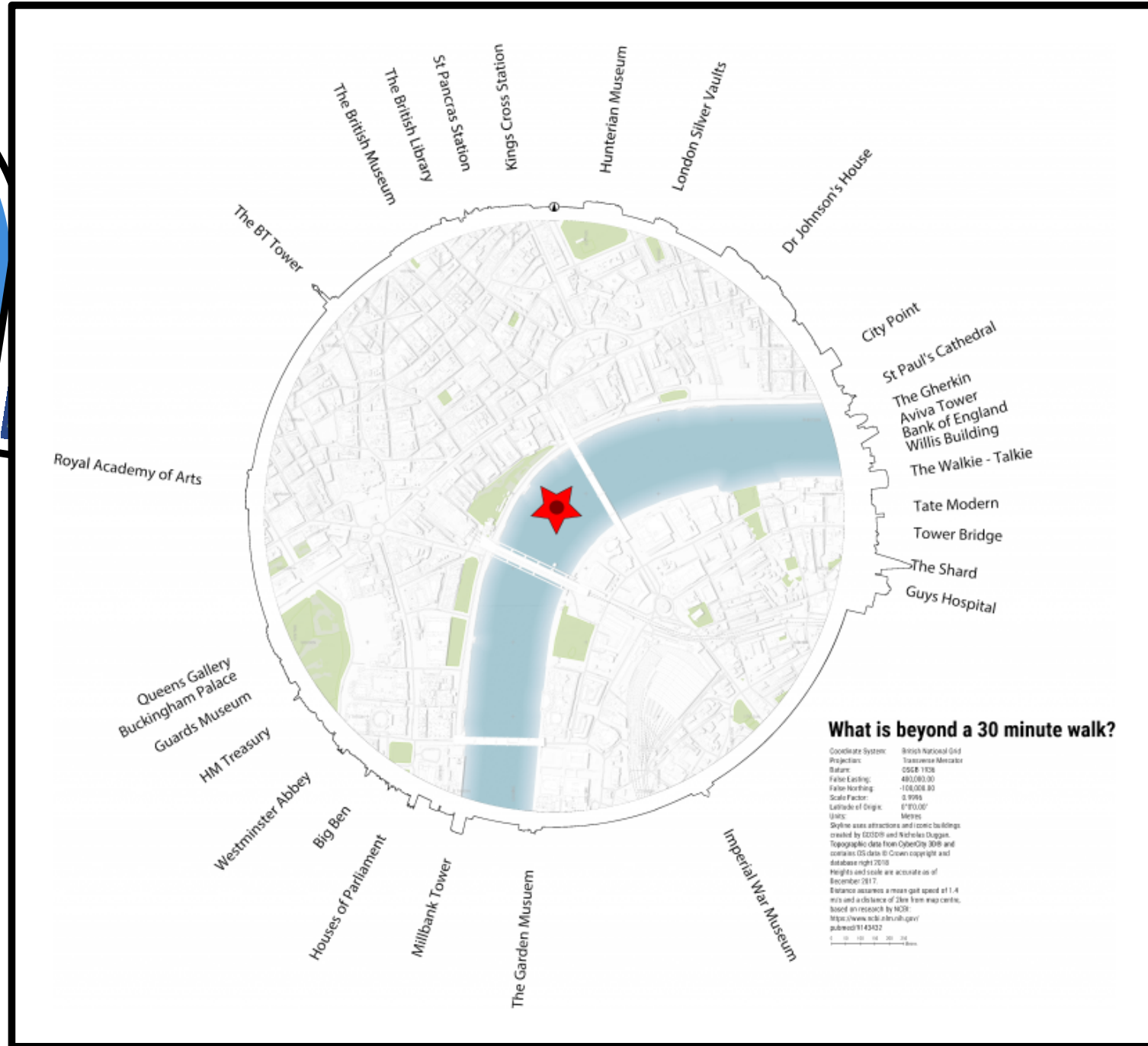
Circular Maps



The Topologist's Map of the World



Circular Maps



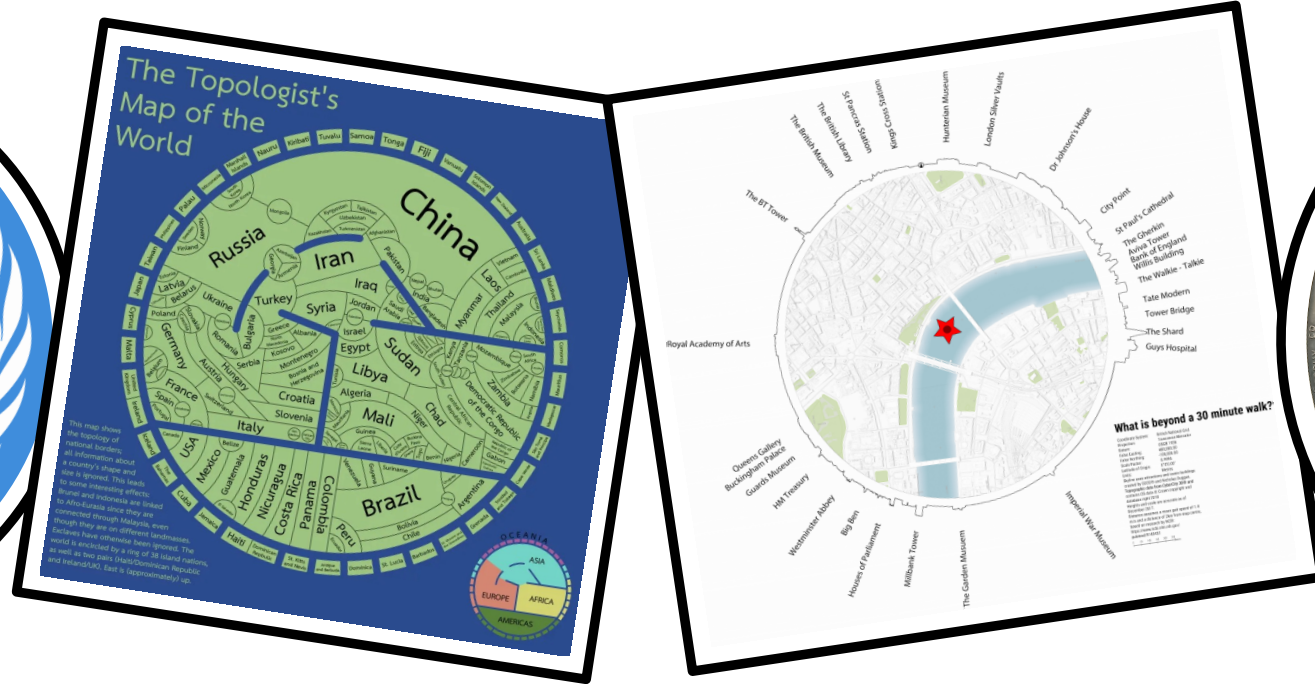
Circular Maps



The United Nations logo, featuring a white map of the world centered on the North Pole, surrounded by a wreath of olive branches, all in white on a blue circular background.



Circular Maps



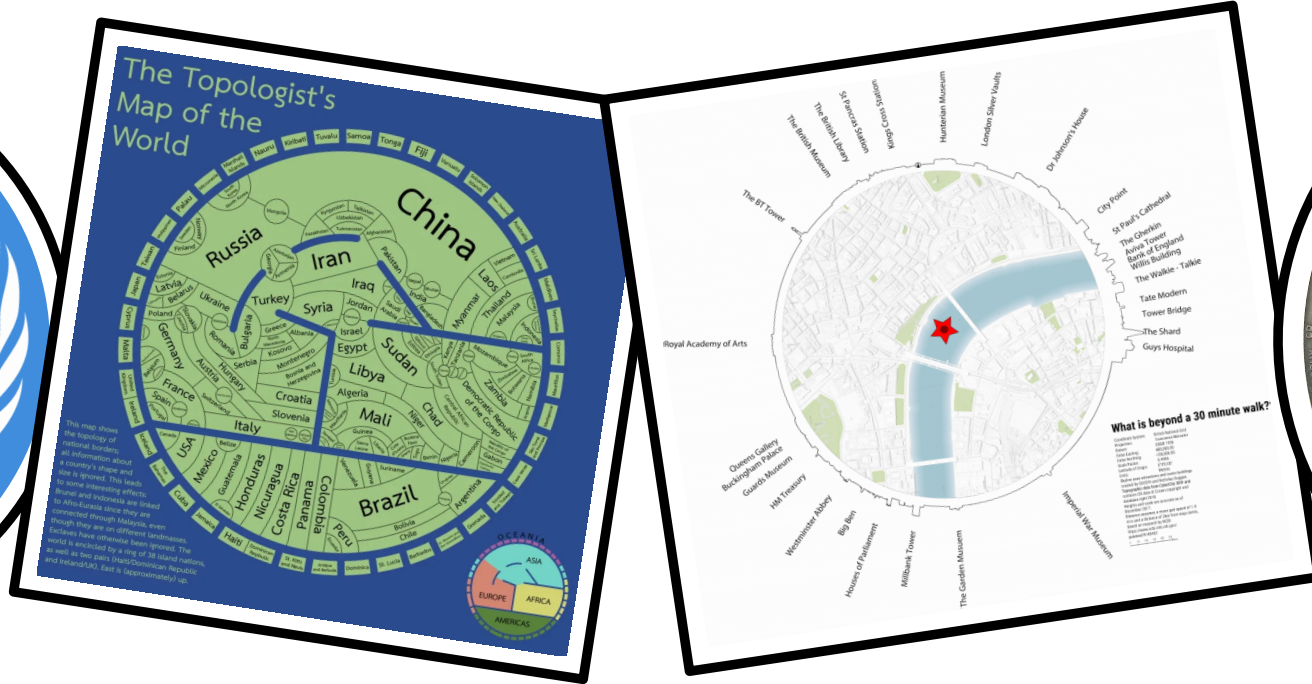
maps with **point features** on small displays like **smart watches**

The United Nations logo, featuring a white map of the world centered on the North Pole, surrounded by a laurel wreath, all within a blue circle.

Apple Watch Series 6



Circular Maps



maps with **point features** on small displays like **smart watches**

Apple Watch Series 6



Google Pixel Watch



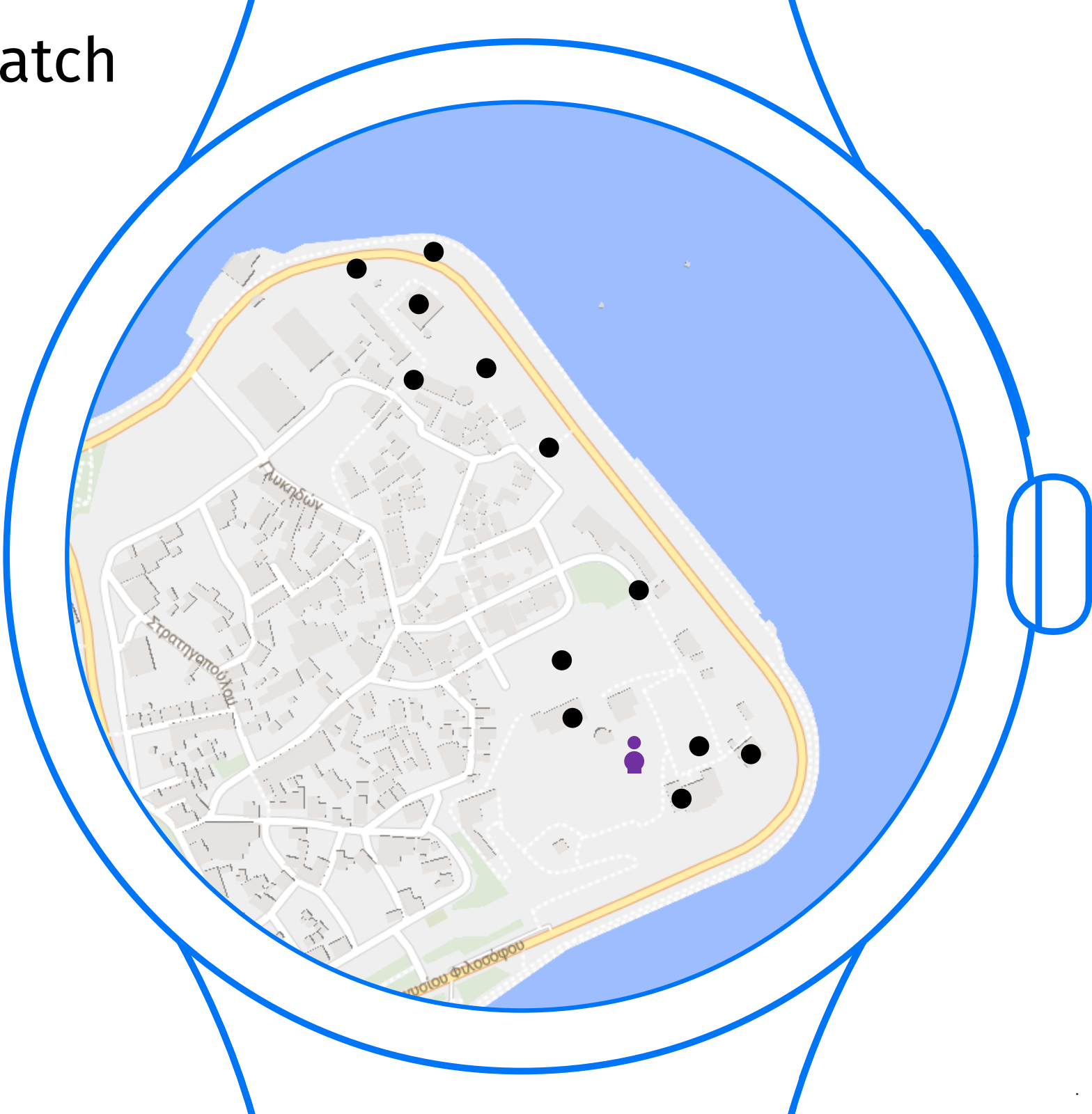
Huawei Watch GT 2



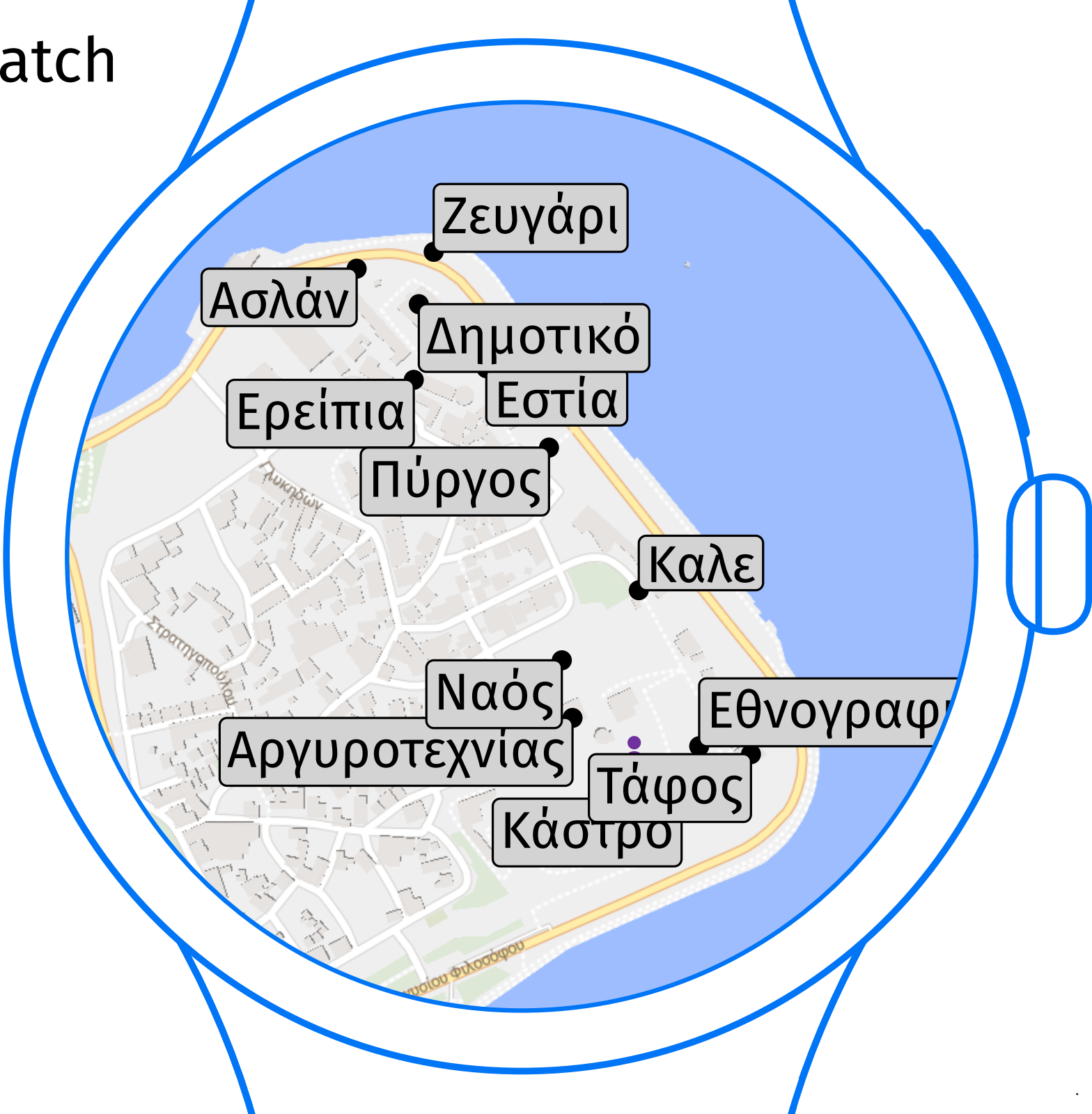
Samsung Galaxy Watch



Labeling on a Smart Watch

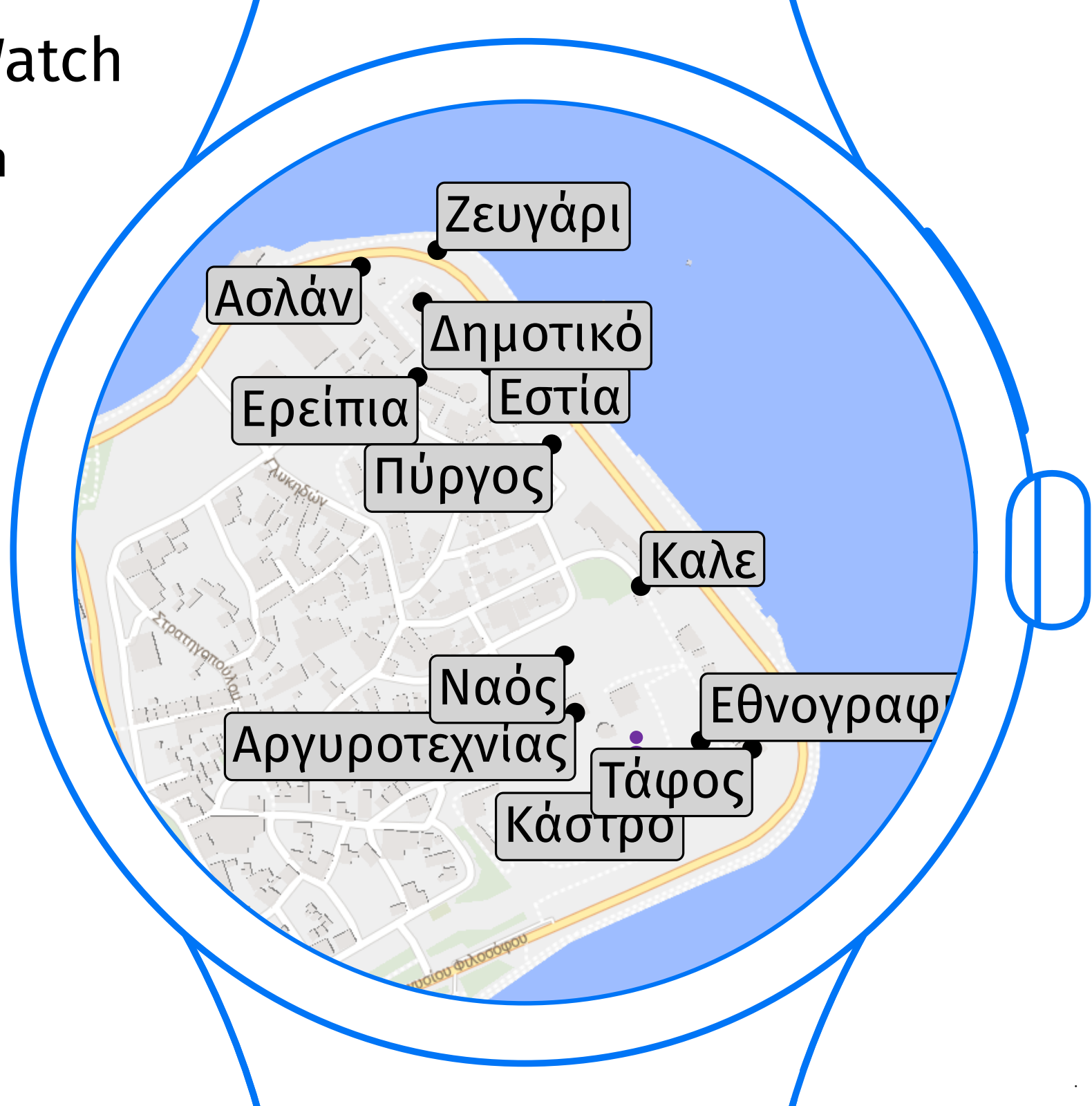


Labeling on a Smart Watch



Labeling on a Smart Watch

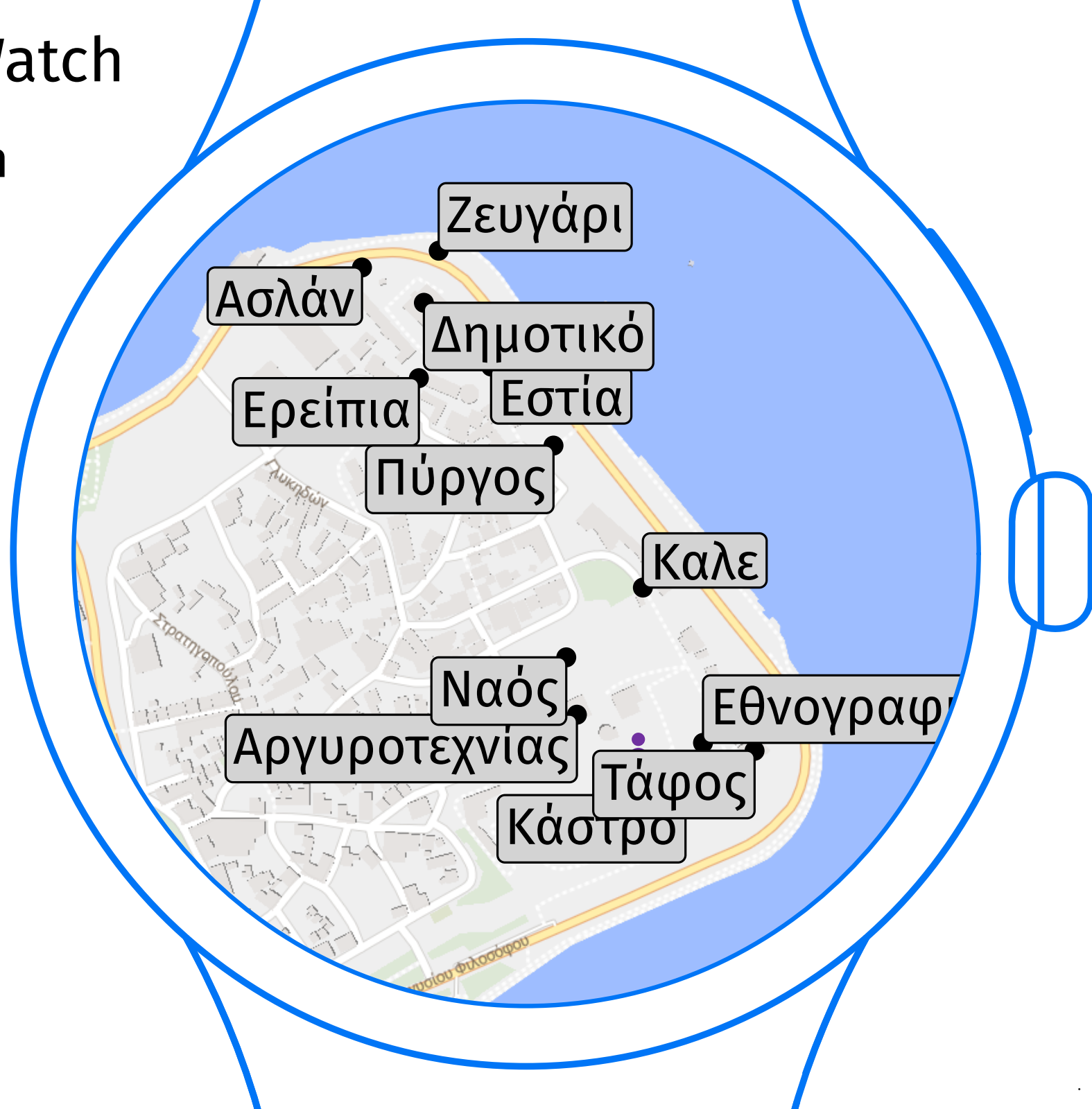
- reduce map obstruction
- avoid label overlap



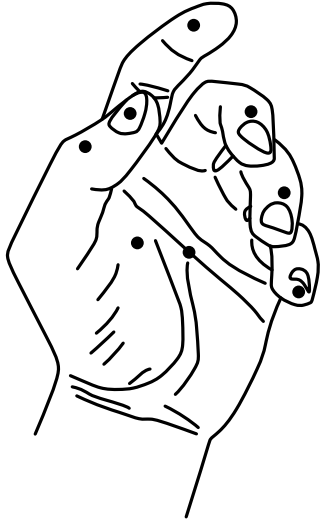
Labeling on a Smart Watch

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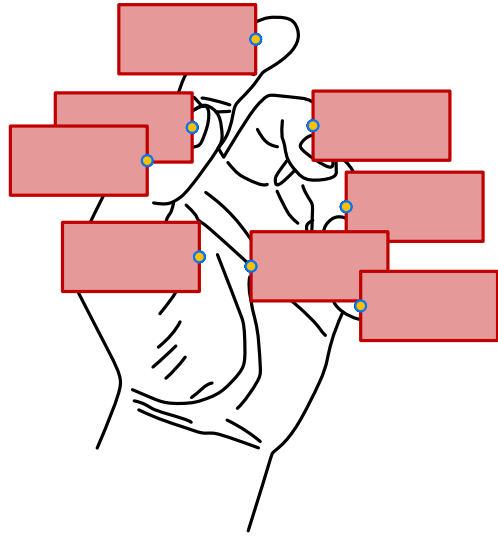
Labels **outside** the map?



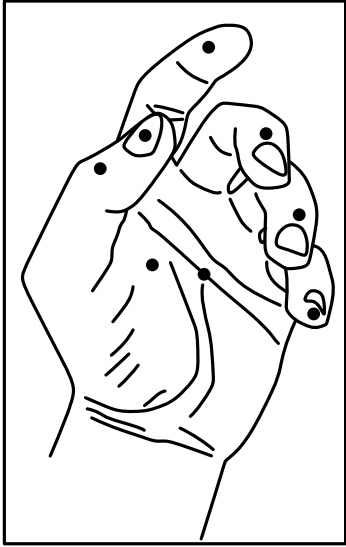
External Labeling [Bekos, Niedermann & Nöllenburg, '21]



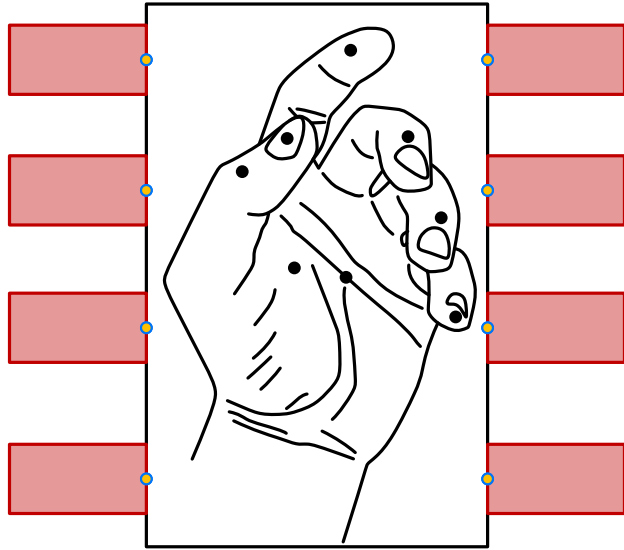
External Labeling [Bekos, Niedermann & Nöllenburg, '21]



External Labeling [Bekos, Niedermann & Nöllenburg, '21]

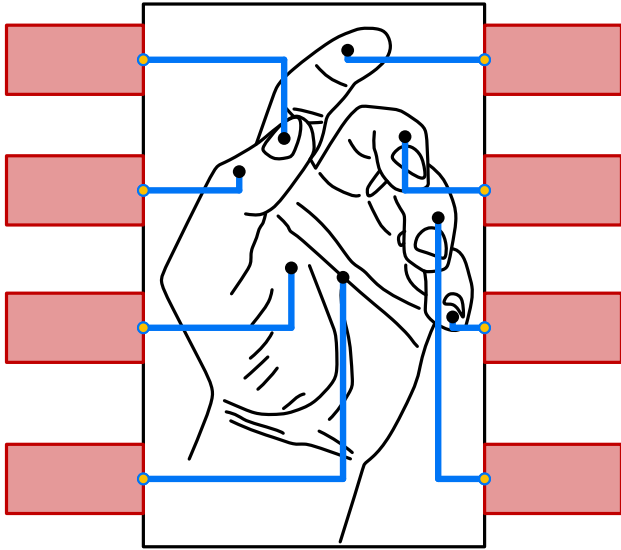


External Labeling [Bekos, Niedermann & Nöllenburg, '21]



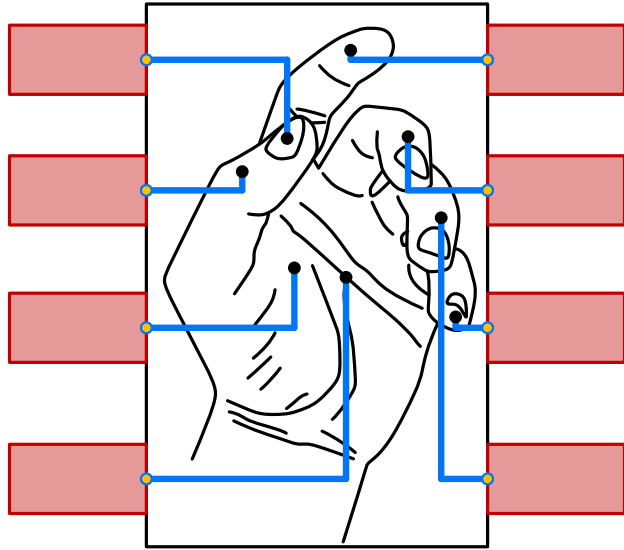
External Labeling [Bekos, Niedermann & Nöllenburg, '21]

Boundary labeling

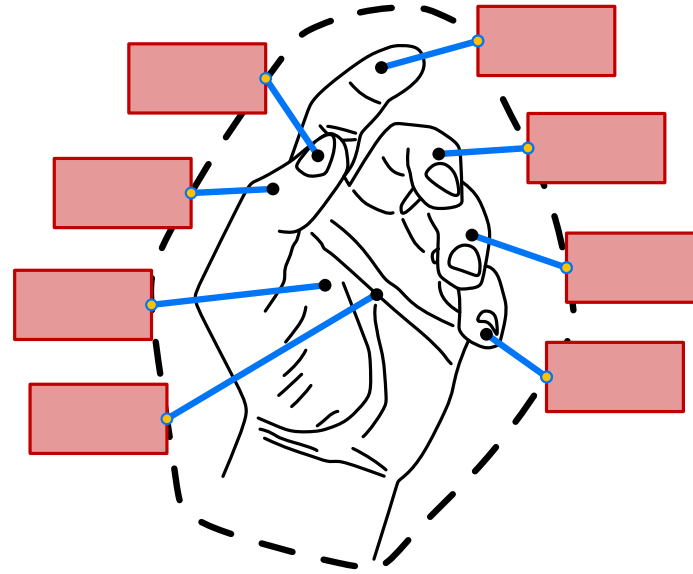


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

Boundary labeling

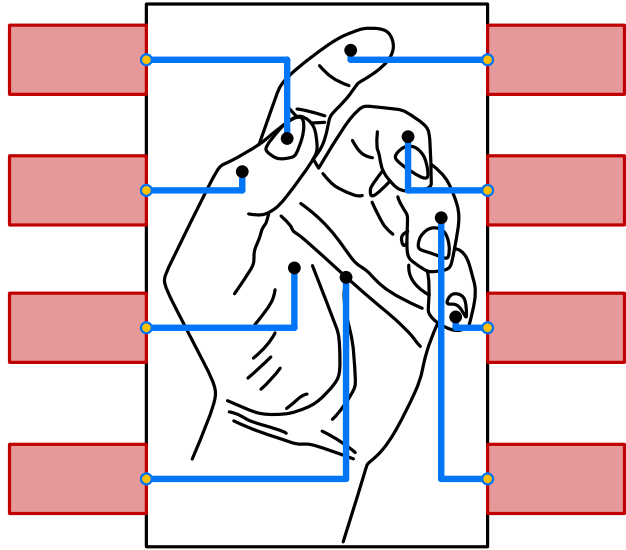


Contour labeling

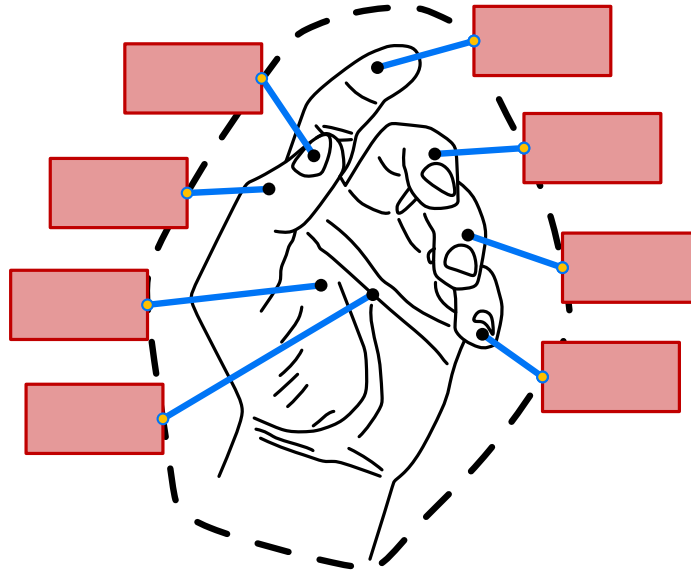


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

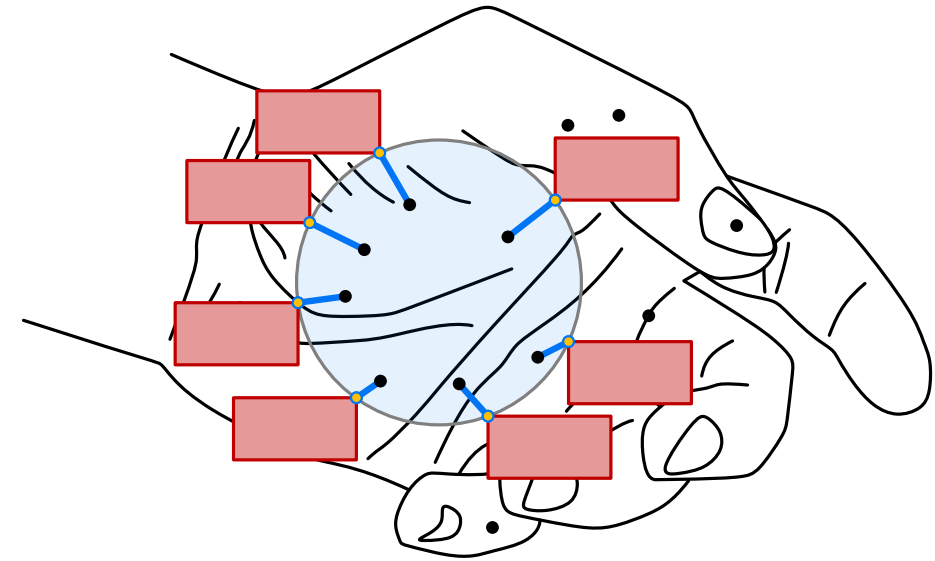
Boundary labeling



Contour labeling

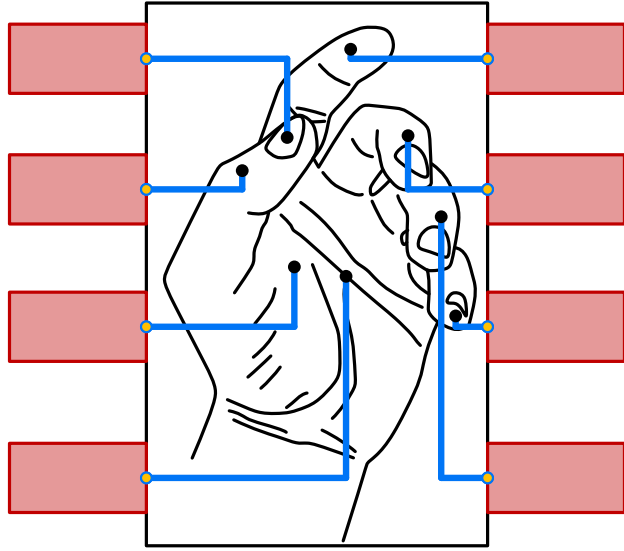


Eccentric labeling

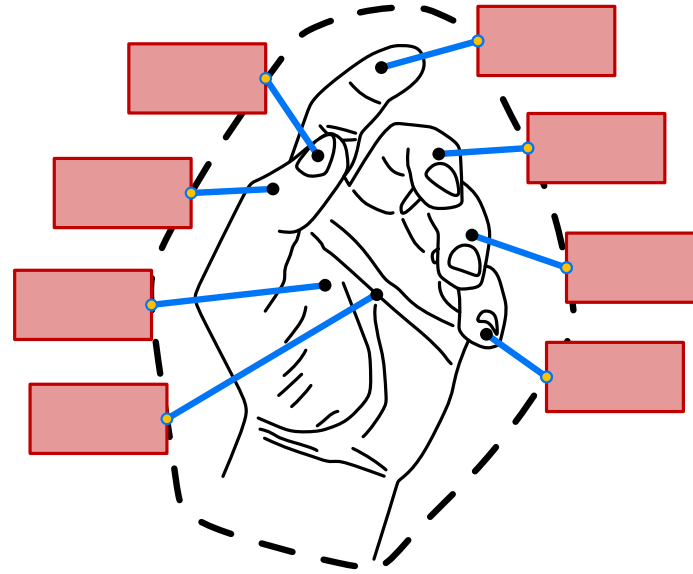


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

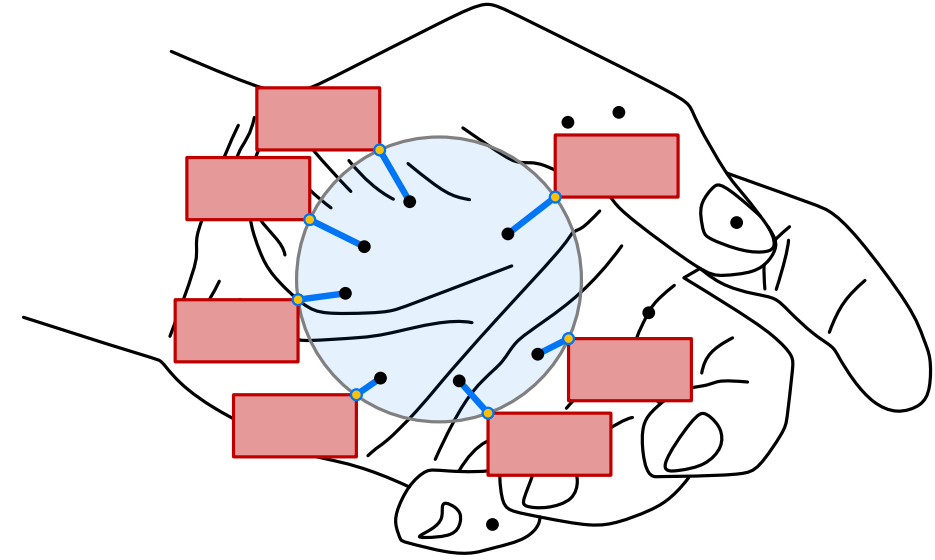
Boundary labeling



Contour labeling



Eccentric labeling

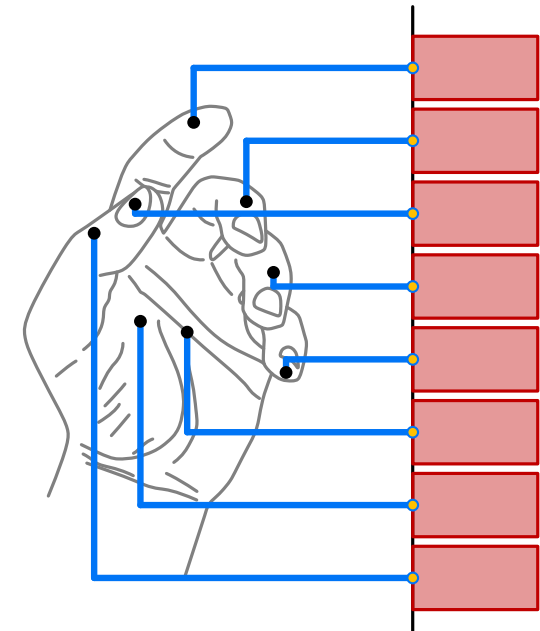


Boundary labeling (1-sided, po-leaders)

Generalized optimization function

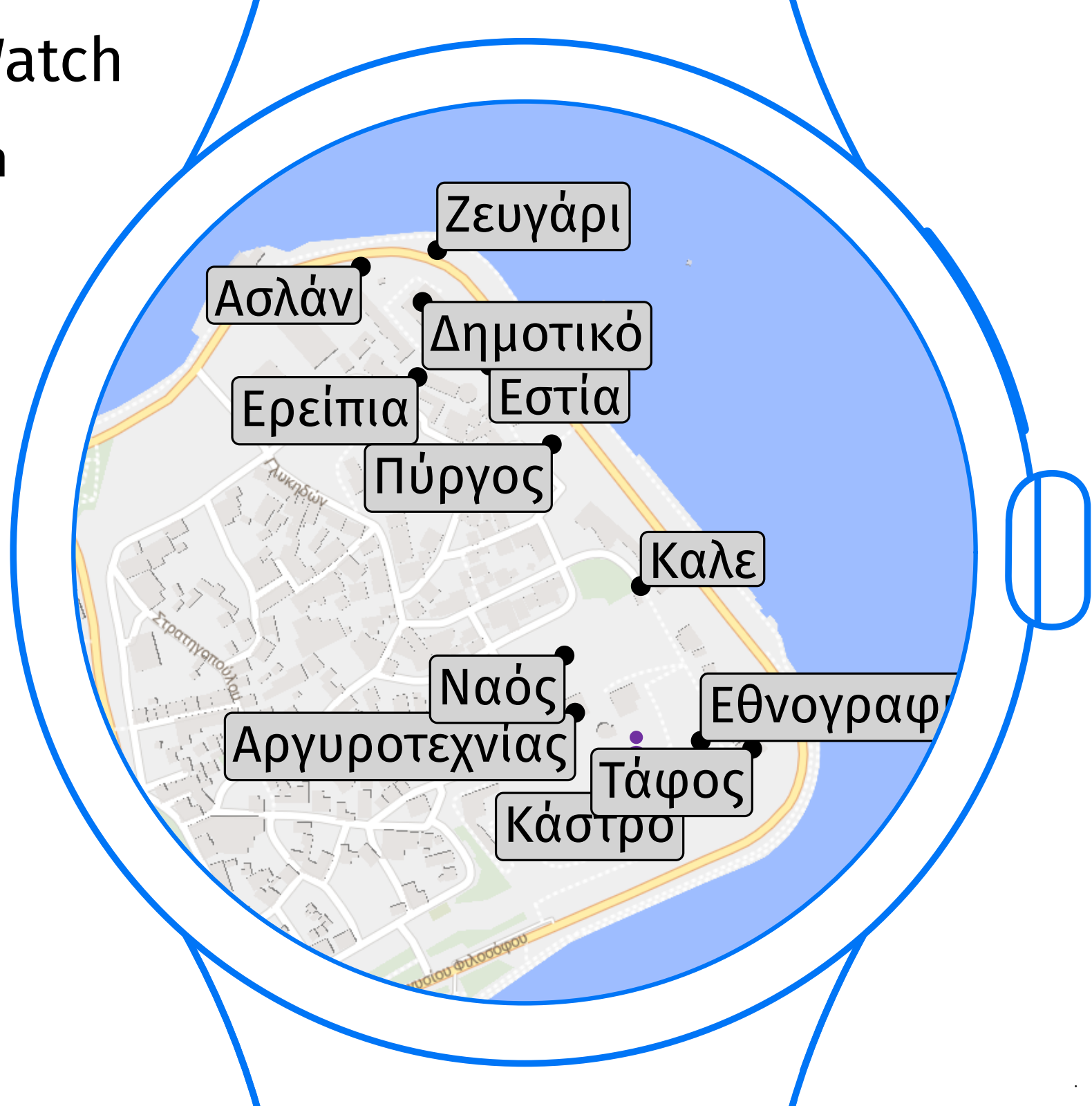
Dynamic program $O(n^3)$

[Benkert, Haverkort, Kroll, Nöllenburg, '09]



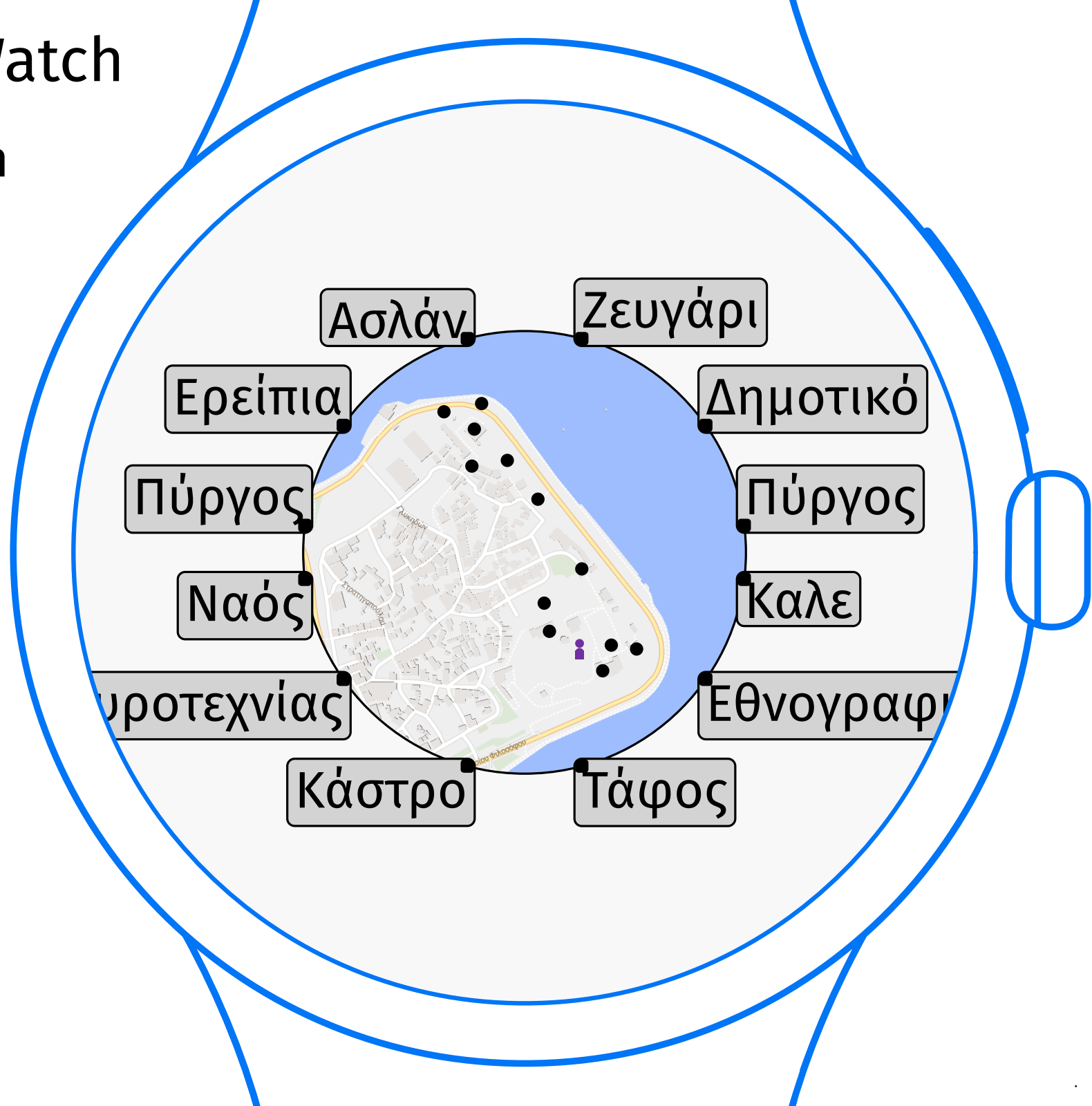
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap



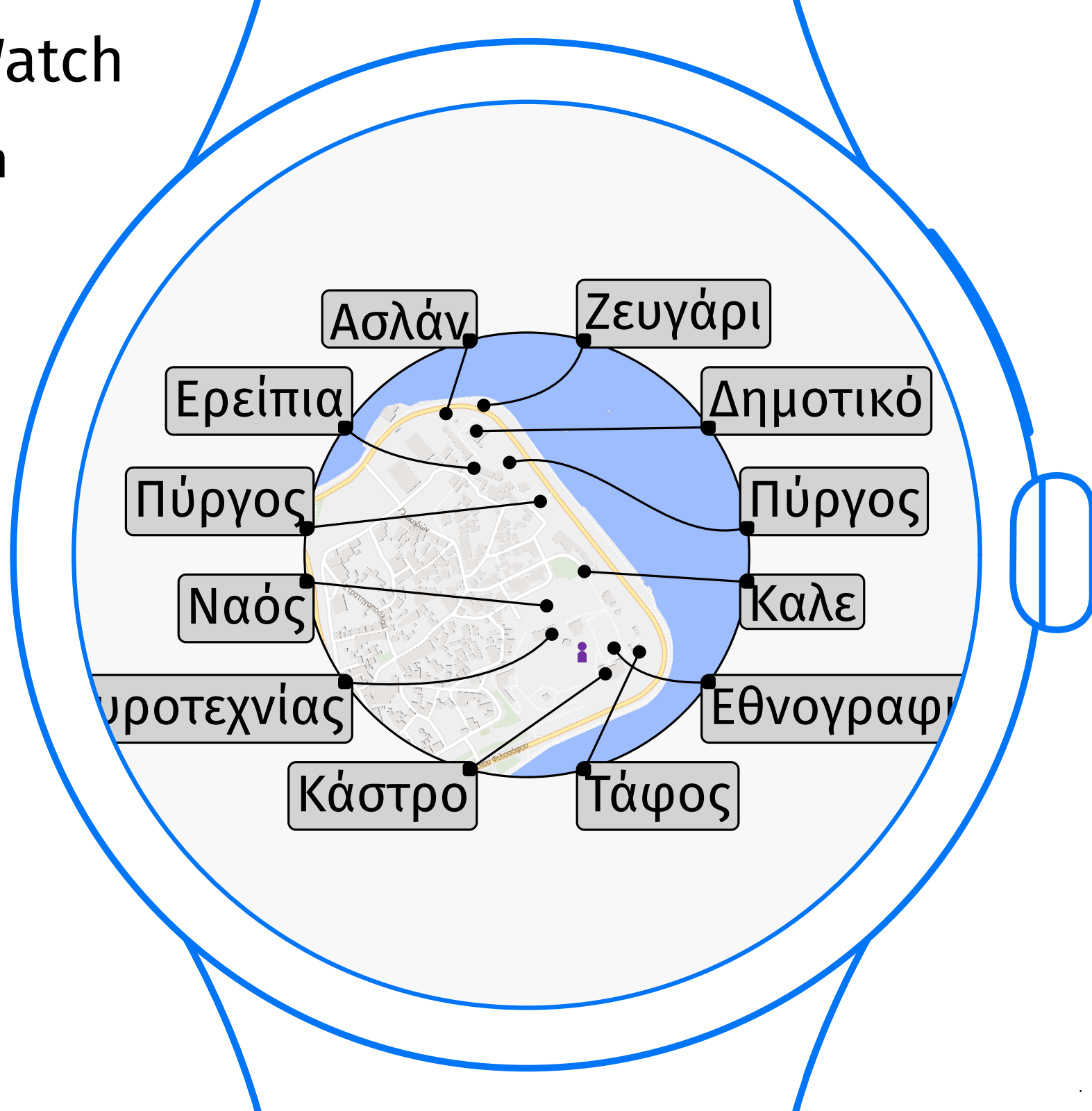
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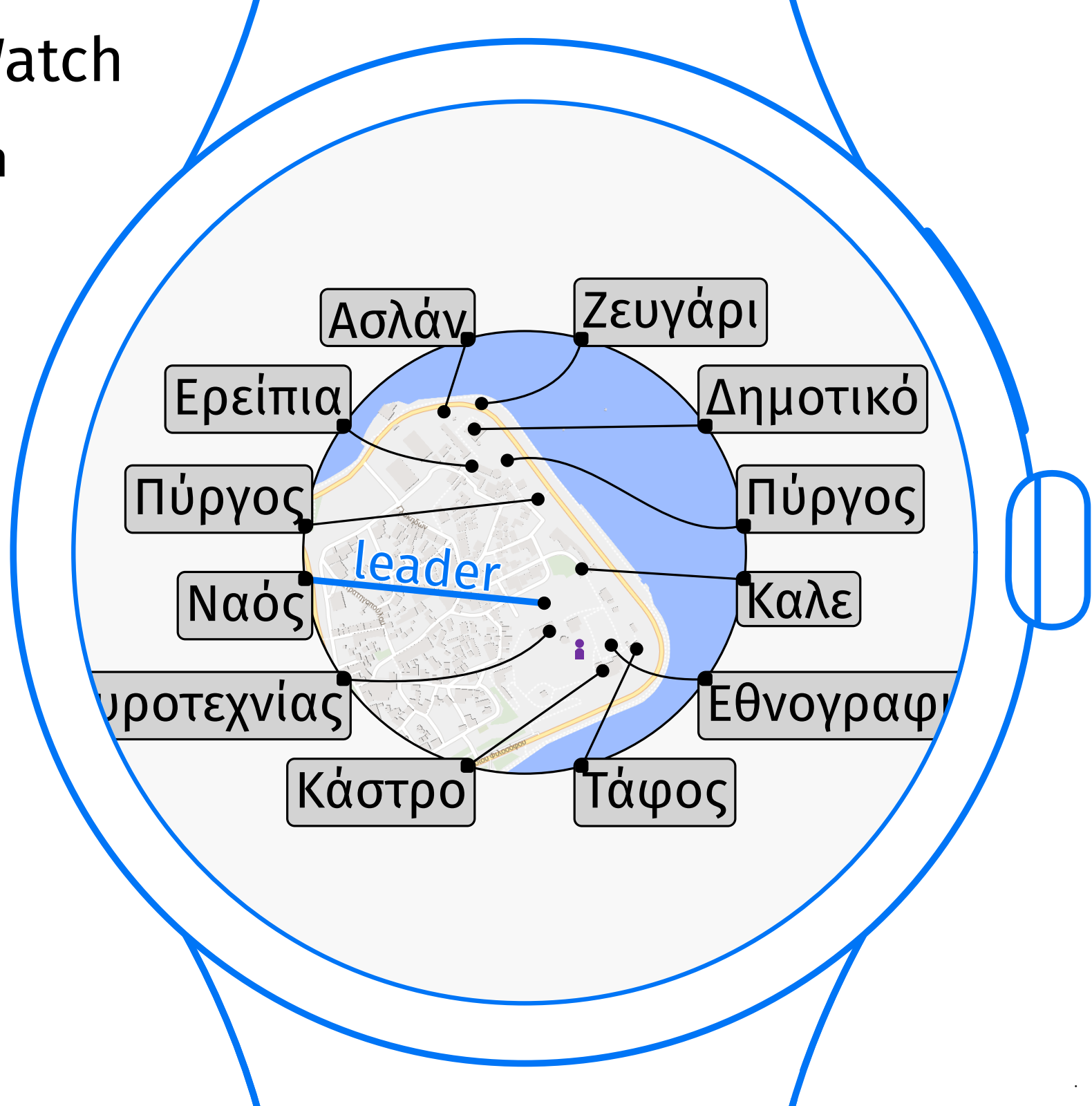
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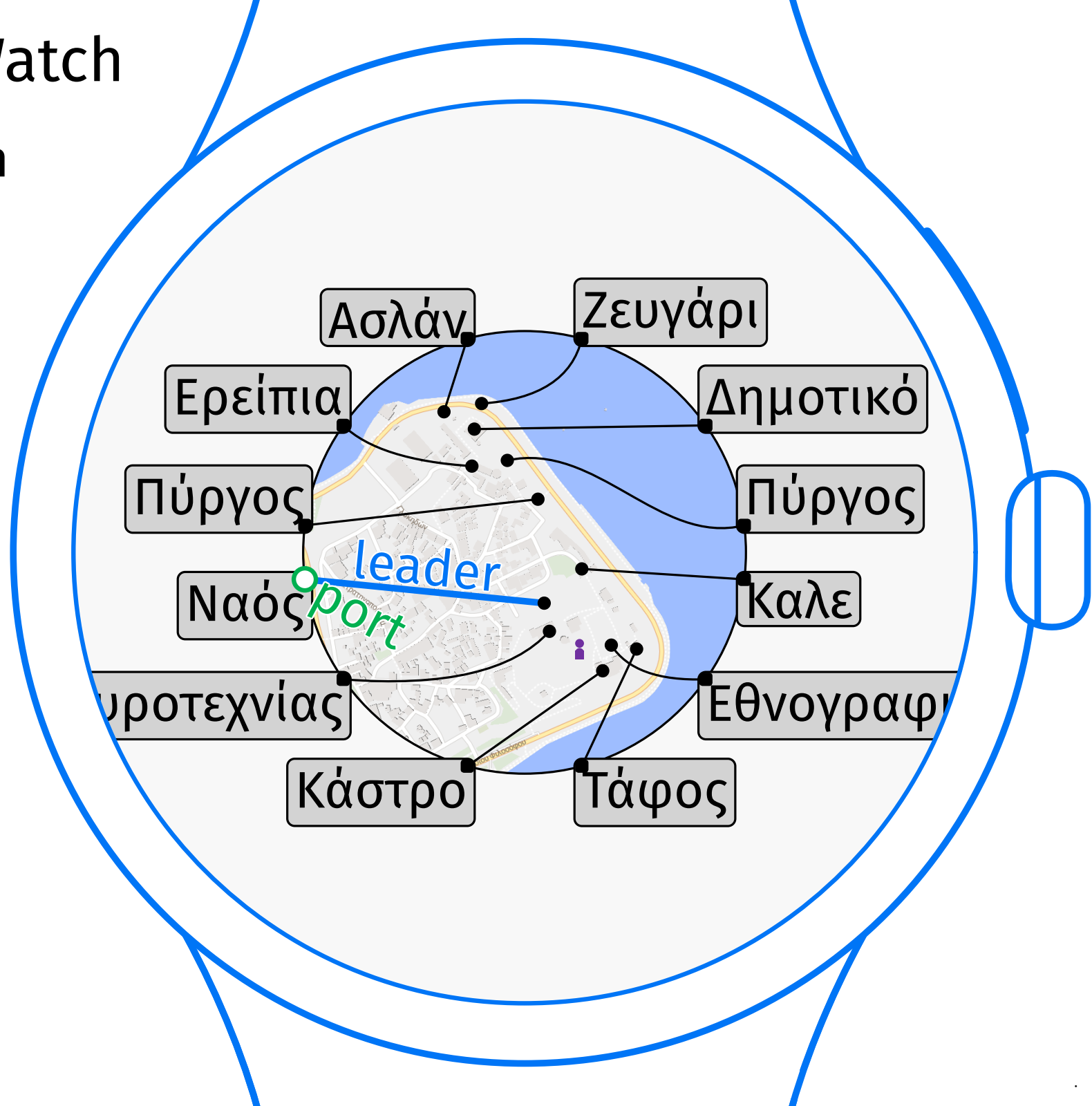
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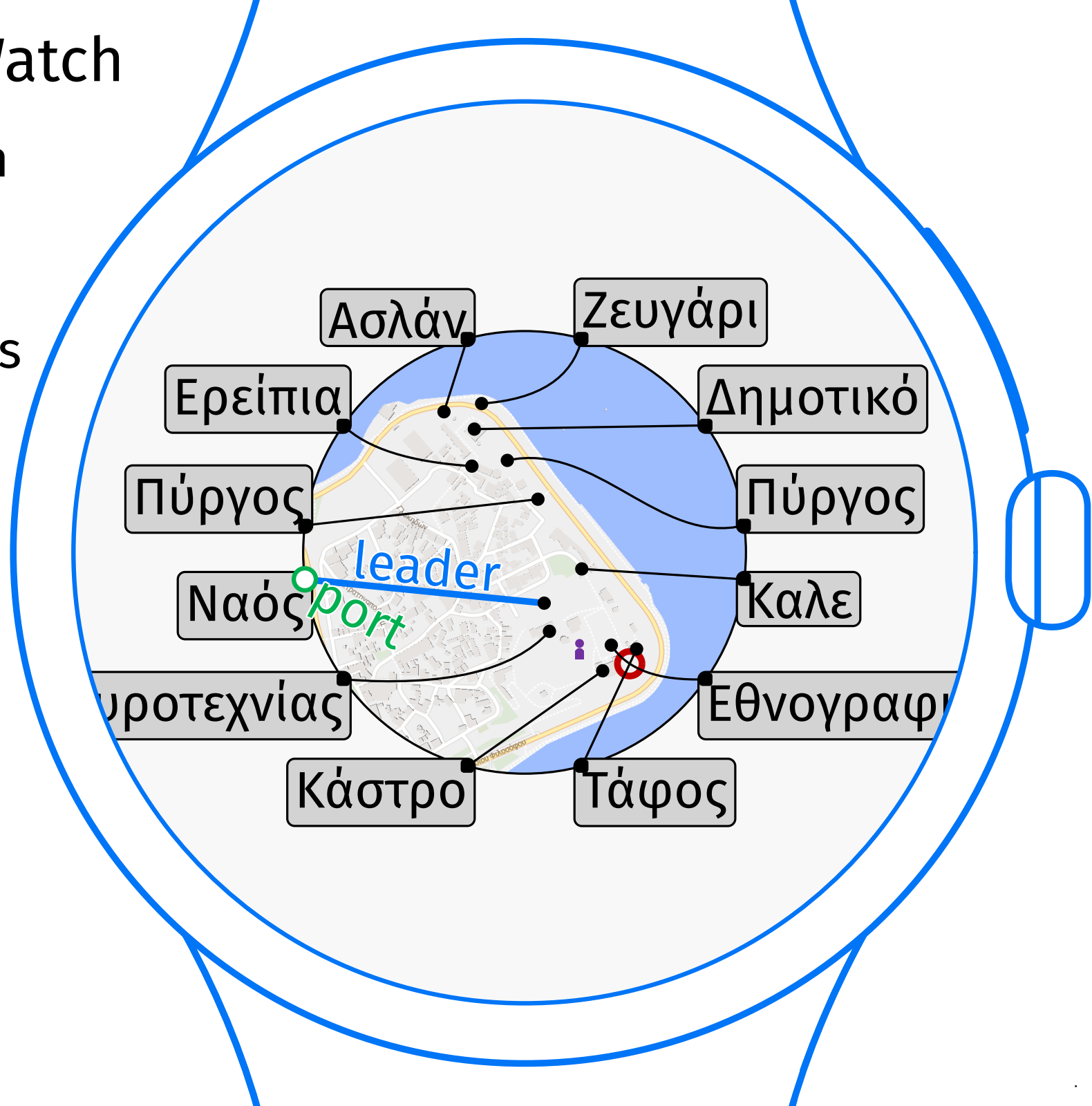
Labeling on a Smart Watch

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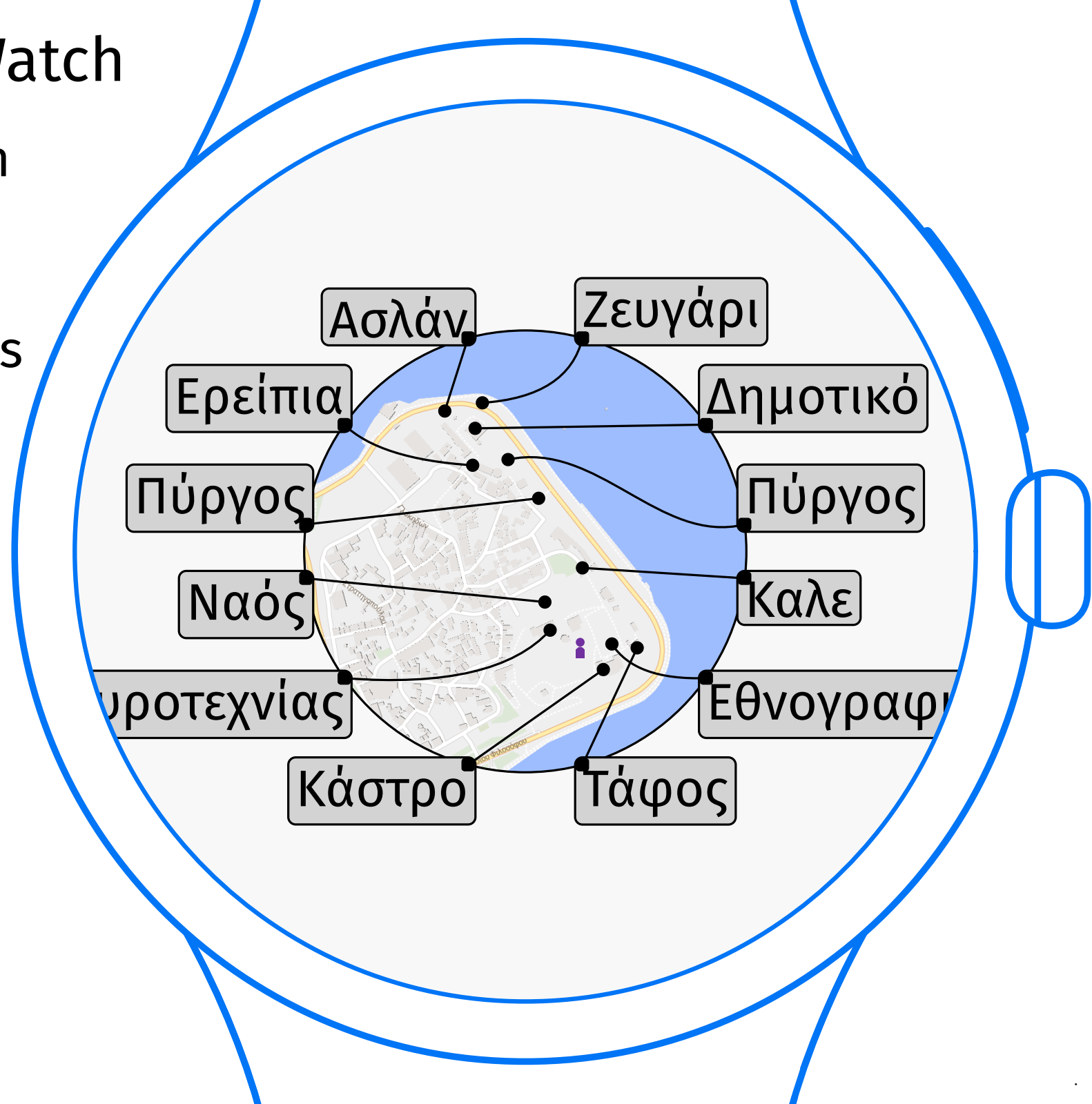
Labeling on a Smart Watch

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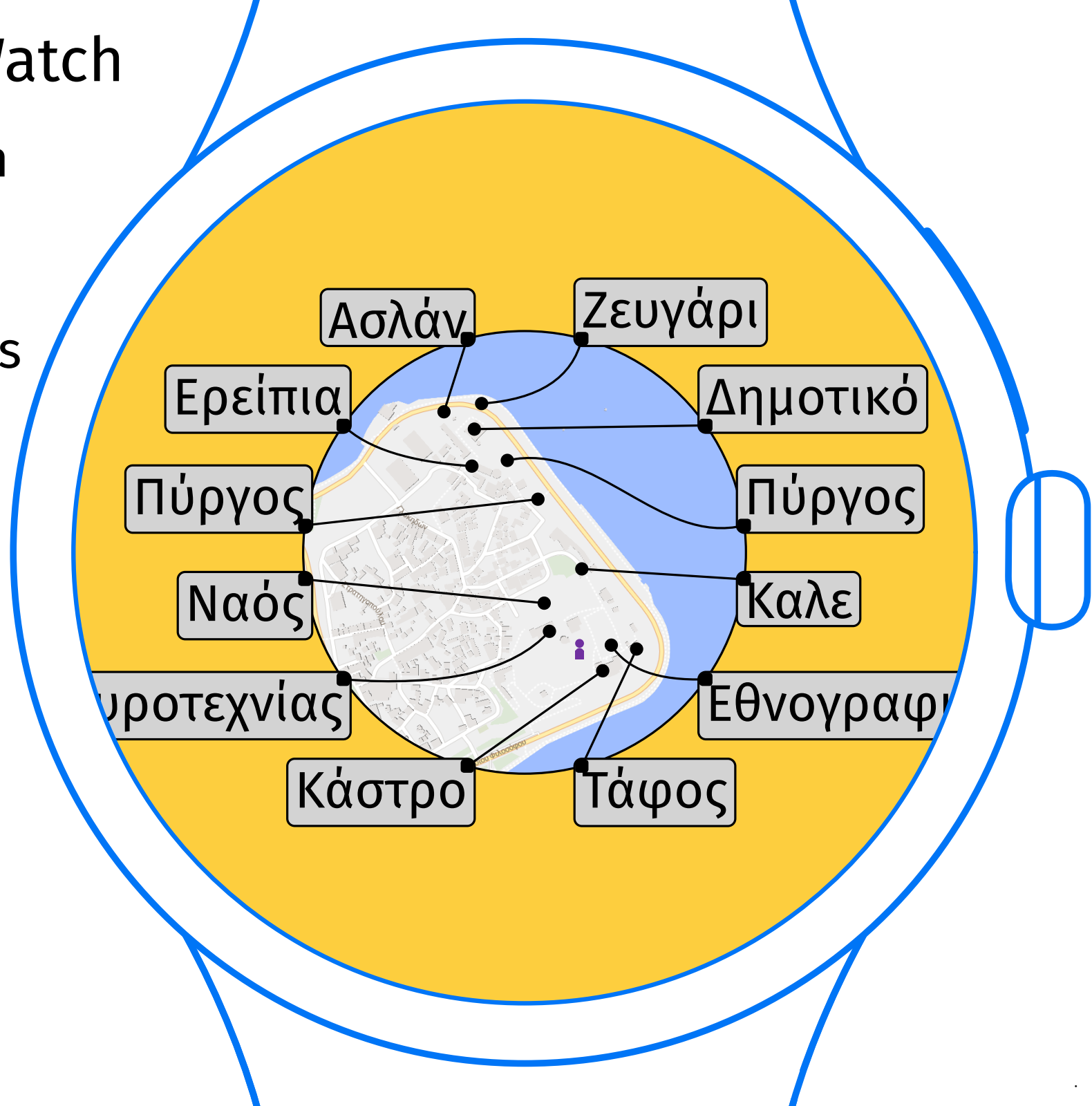
Labeling on a Smart Watch

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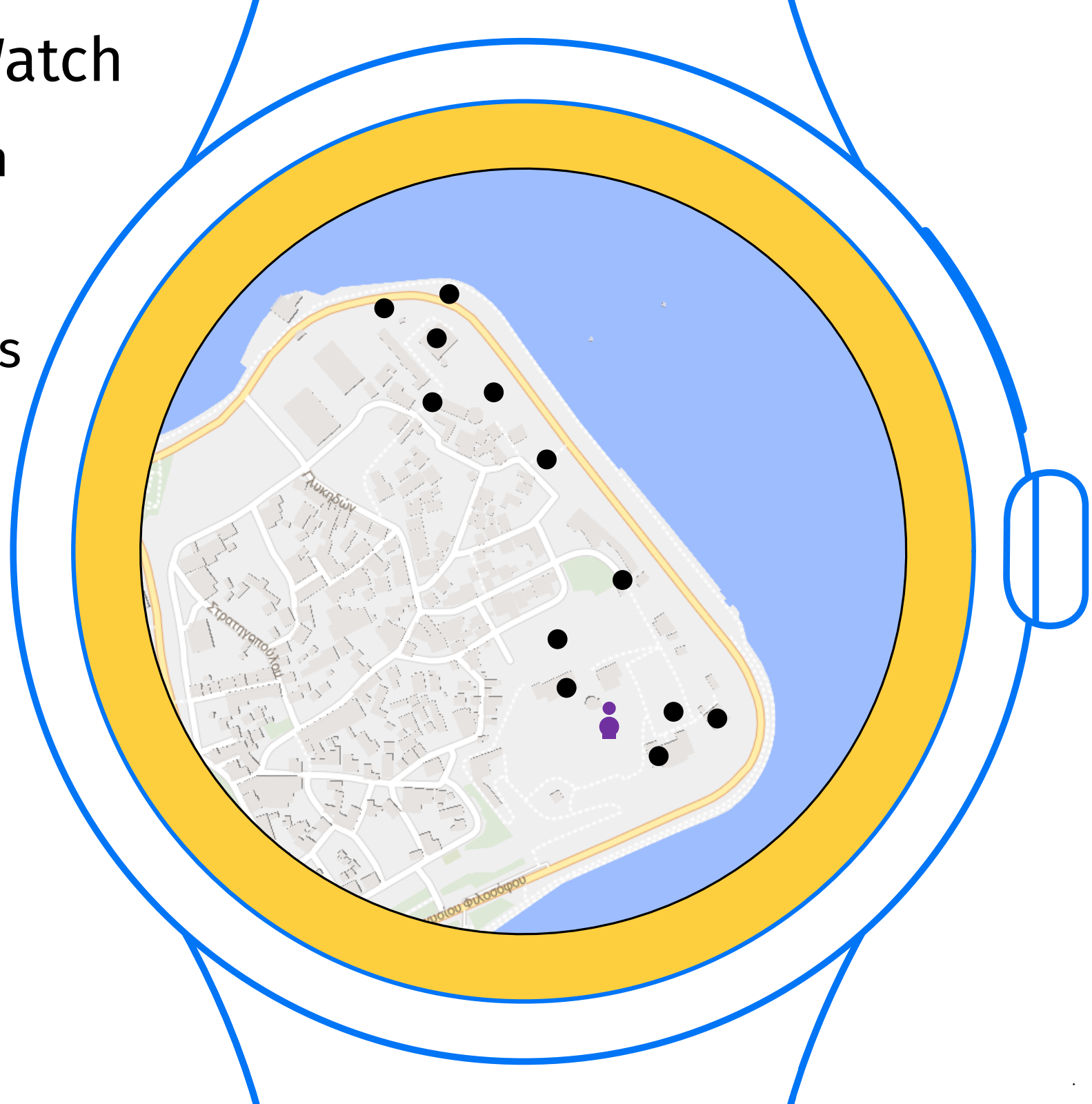
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage



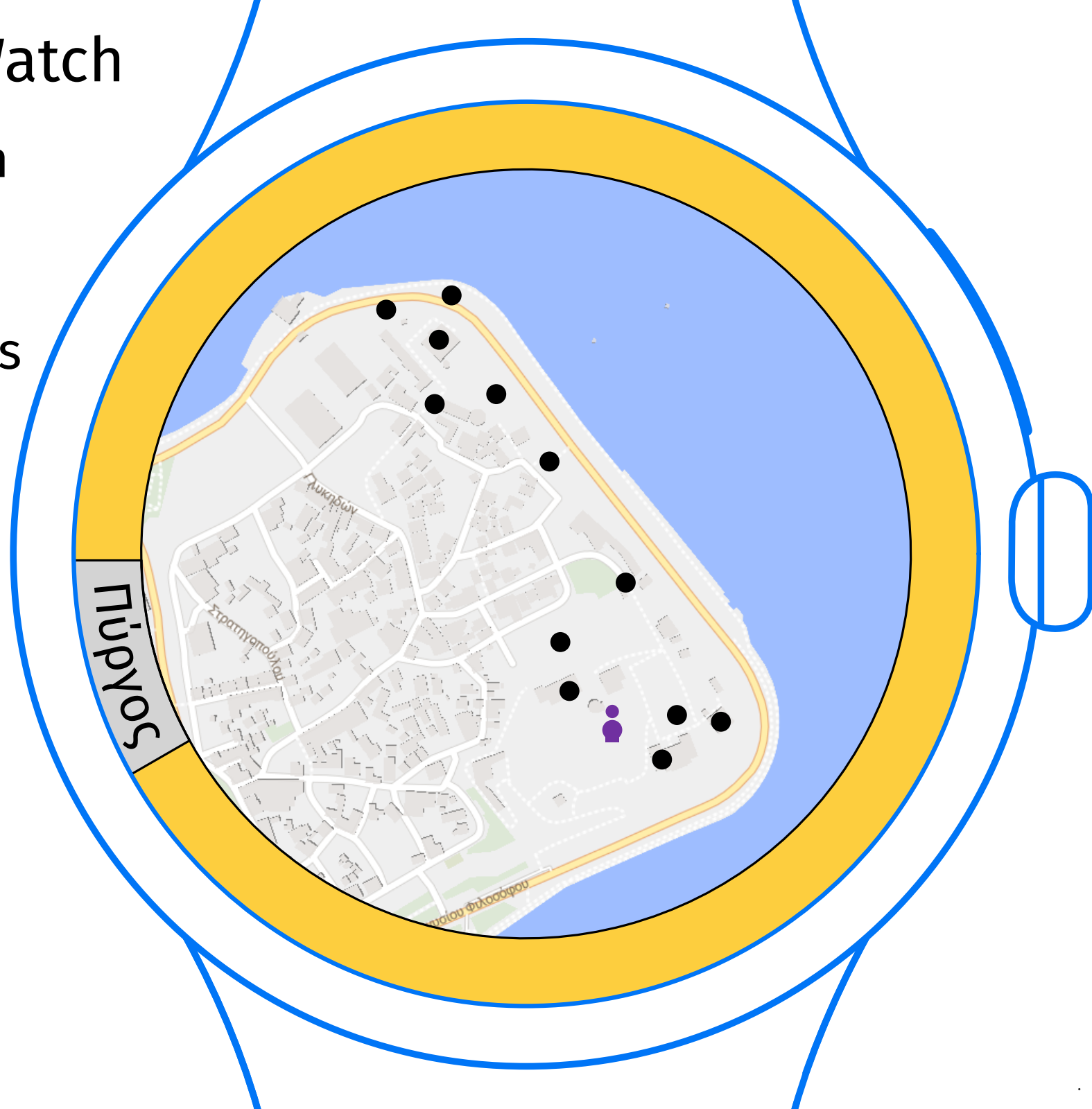
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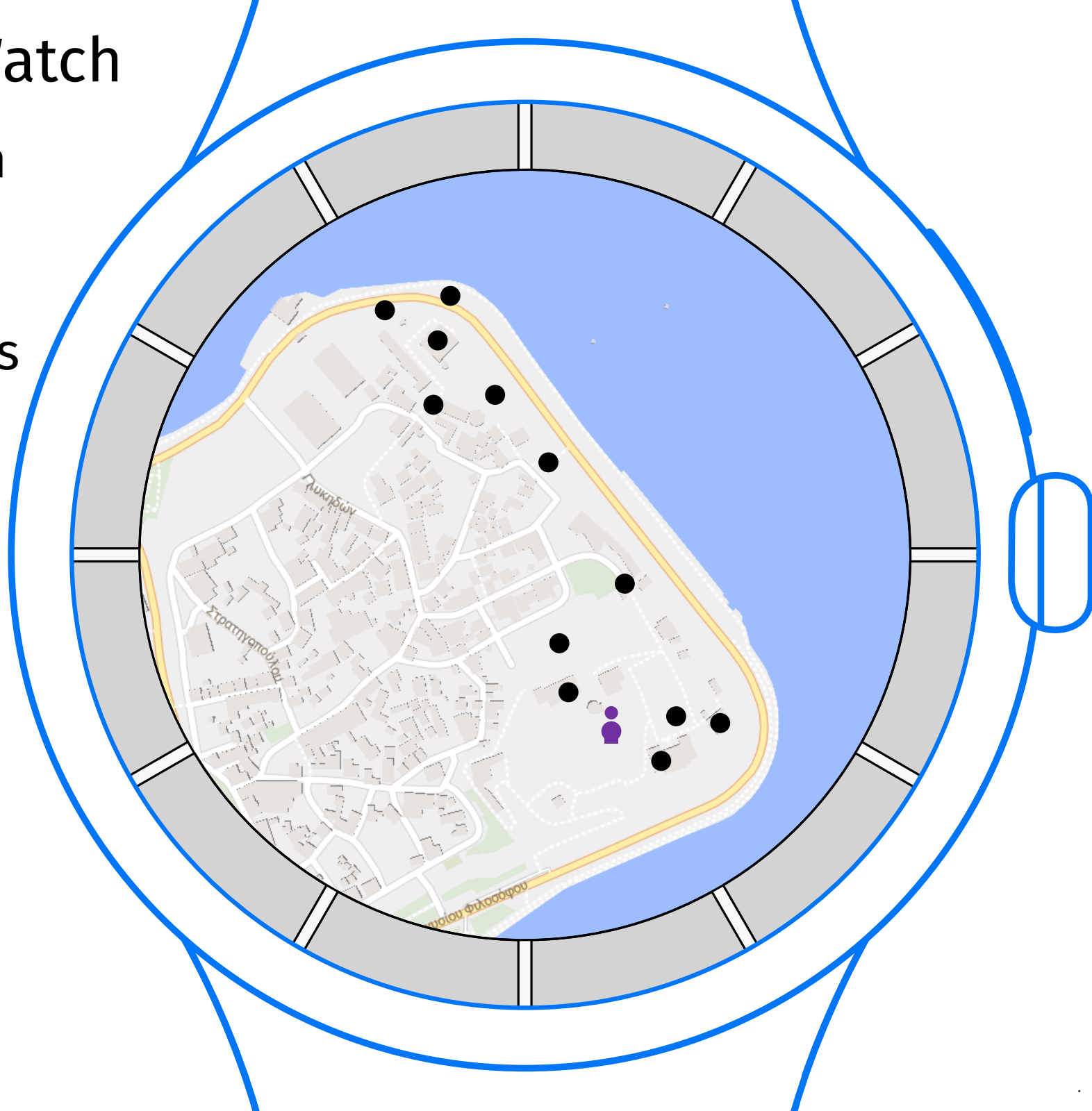
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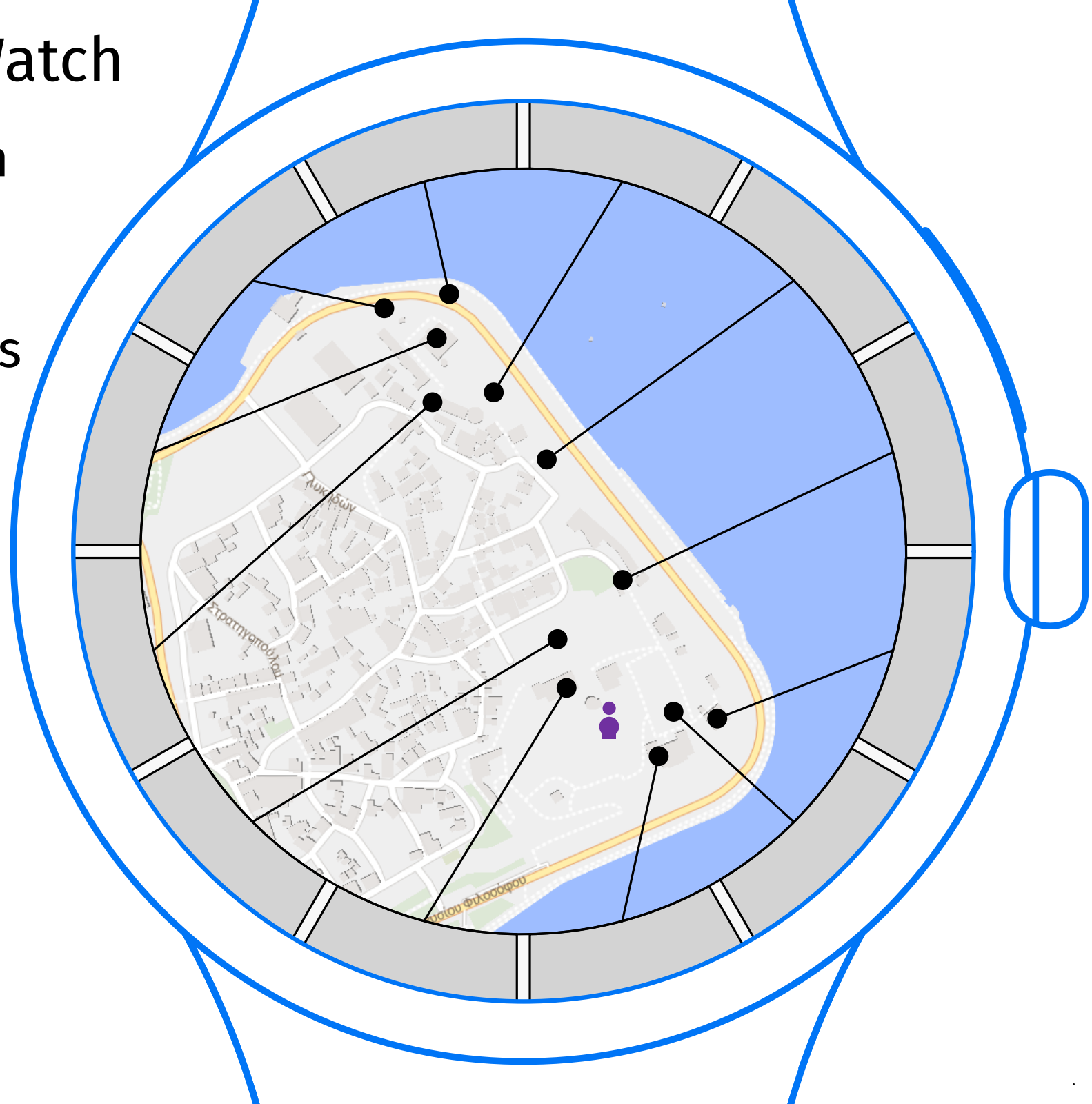
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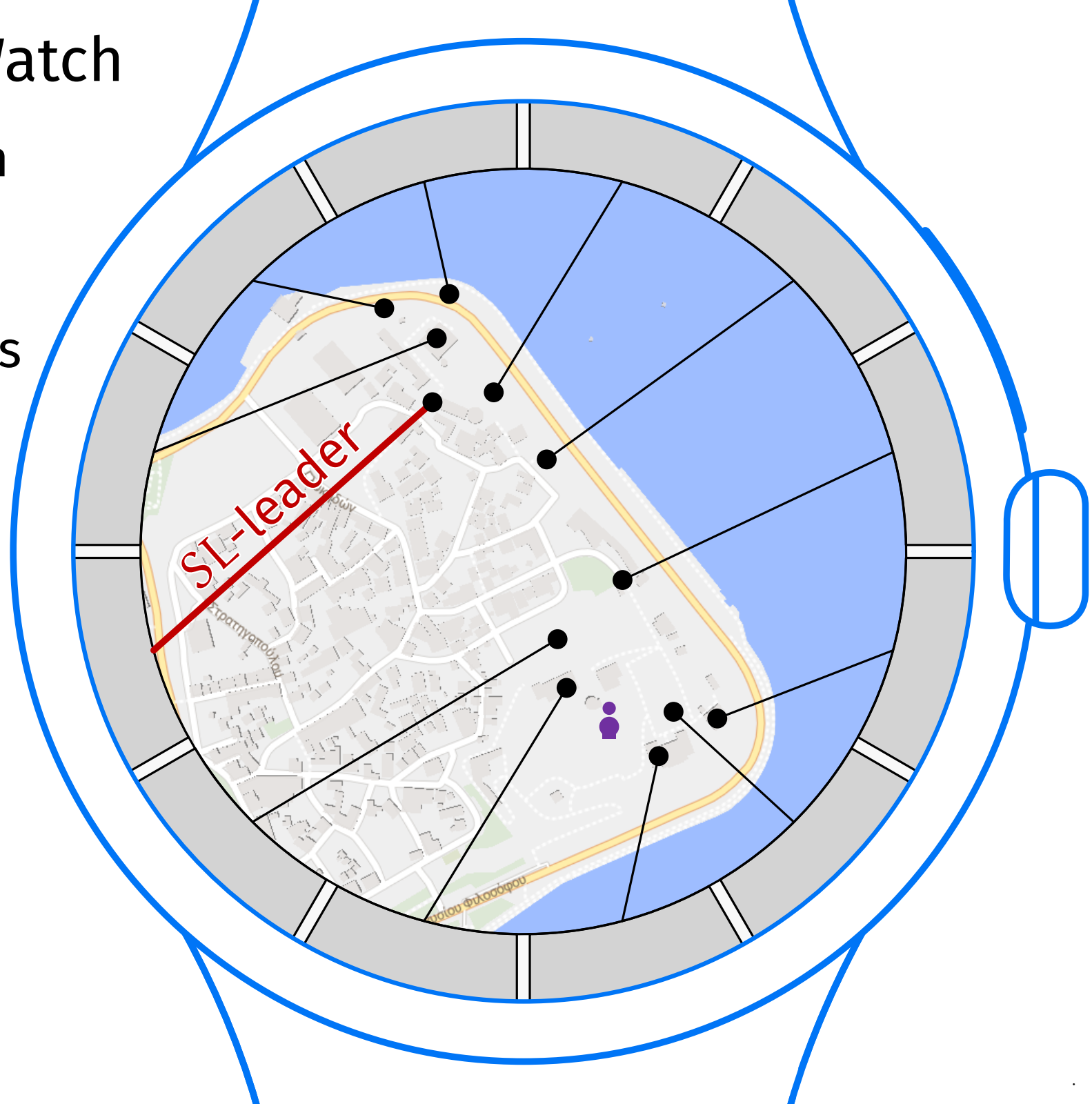
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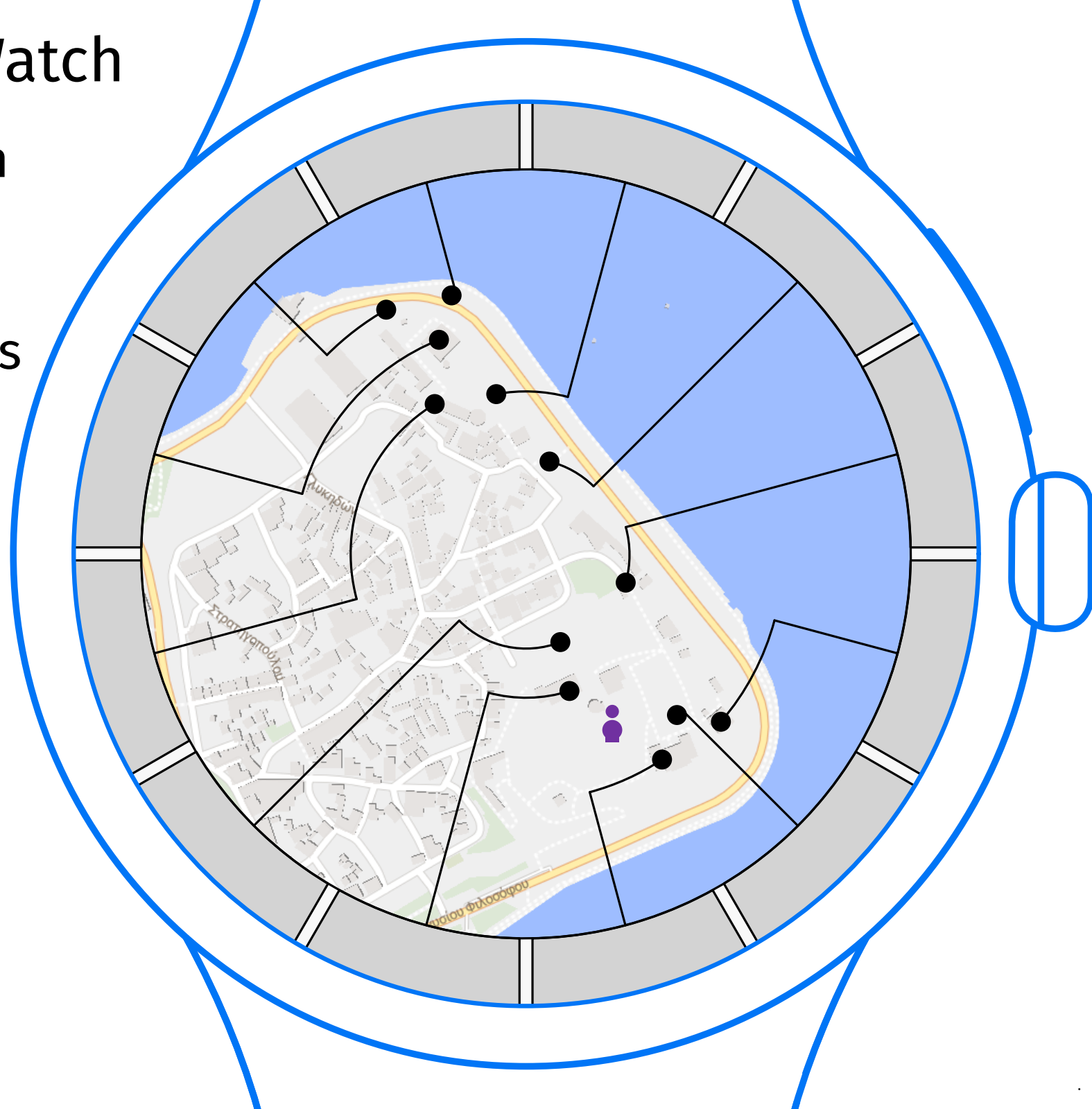
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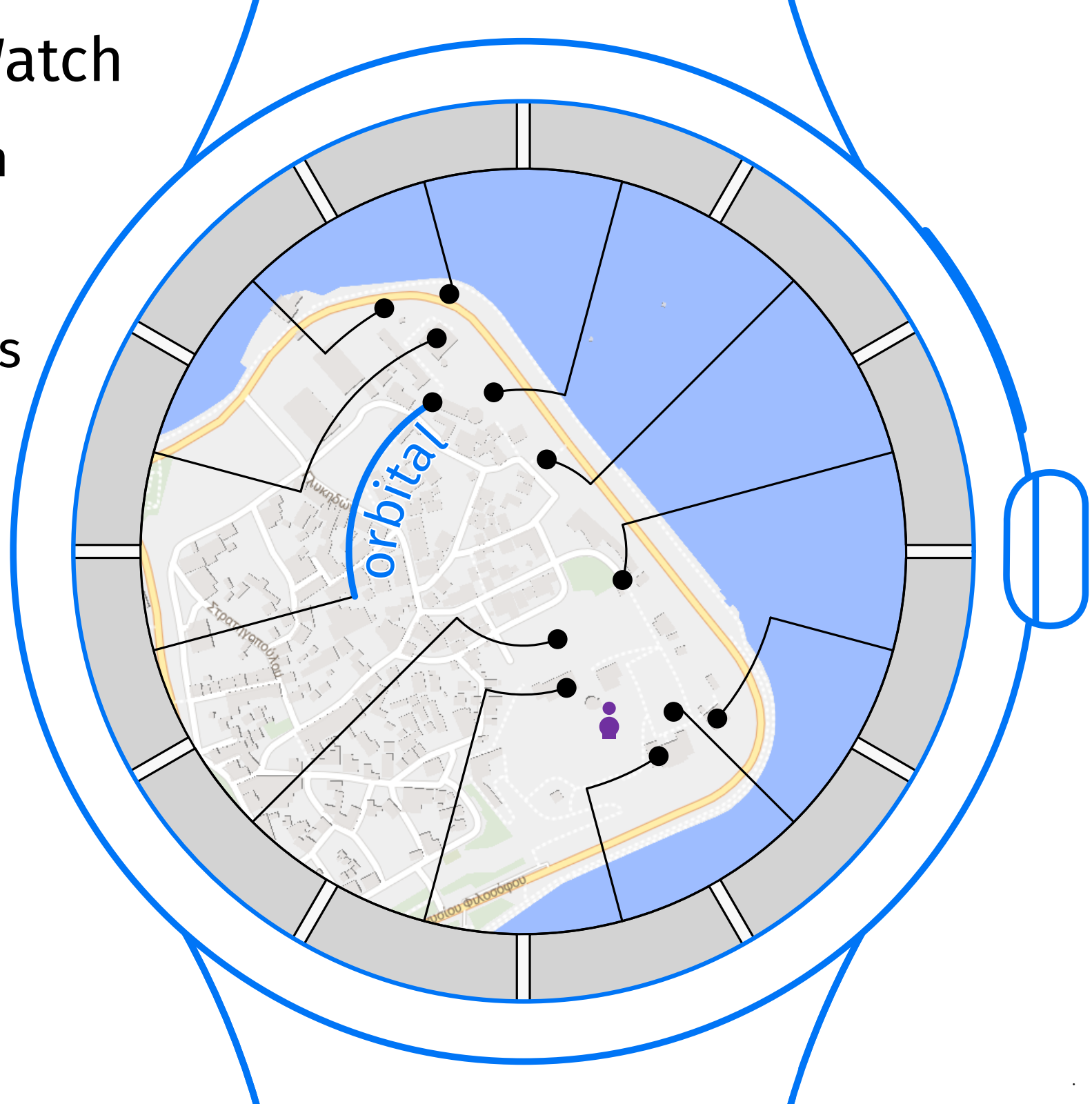
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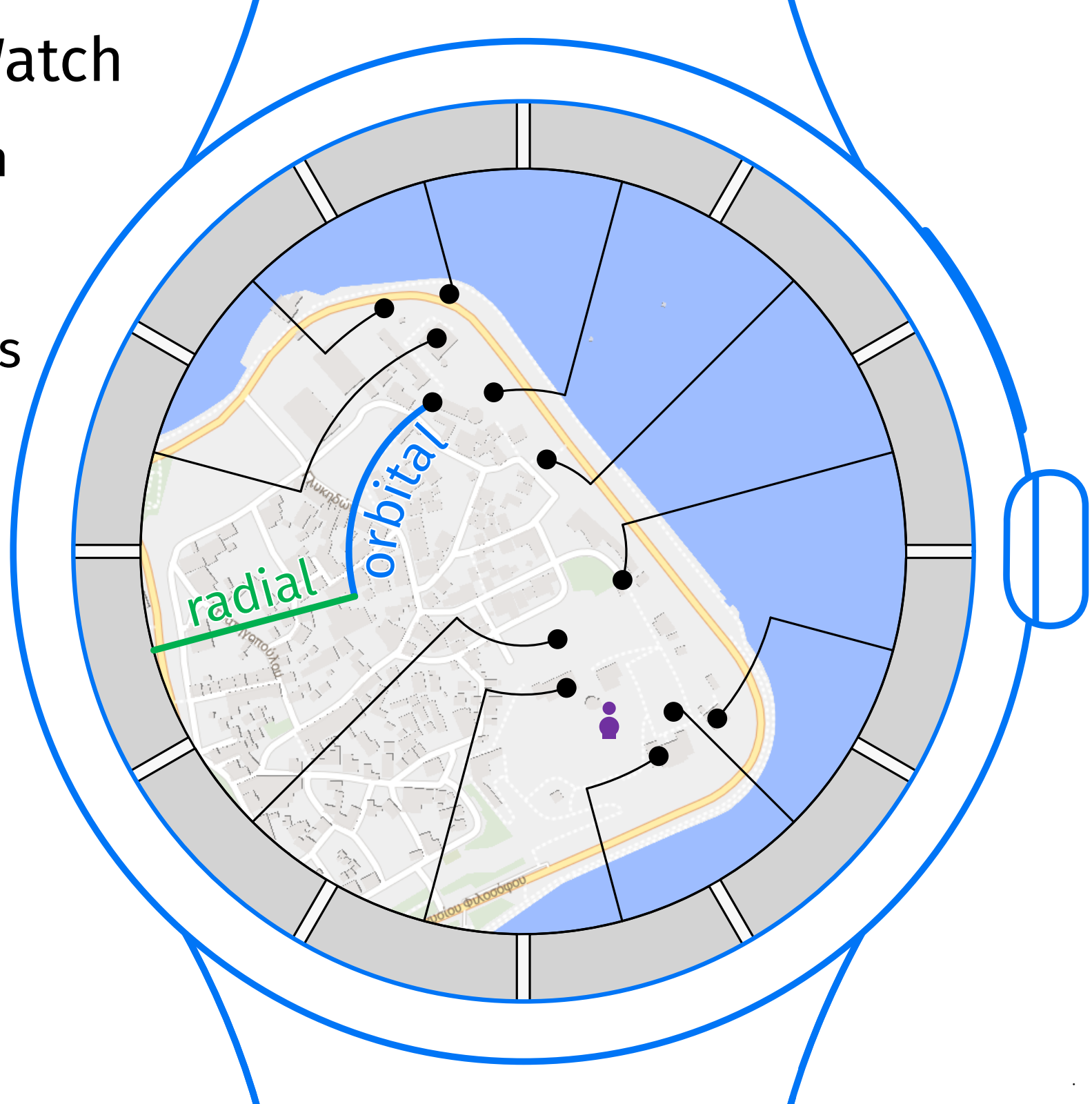
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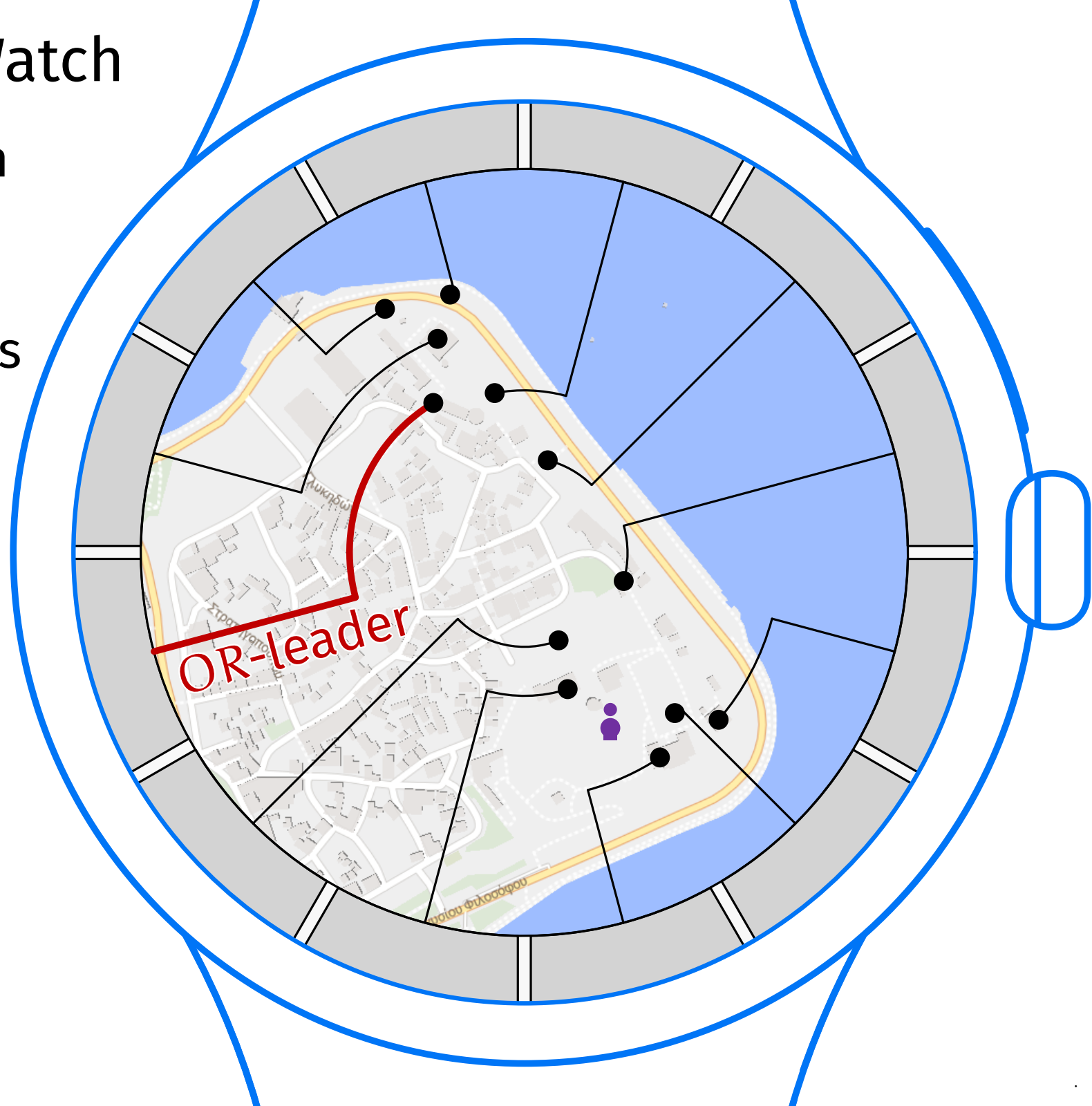
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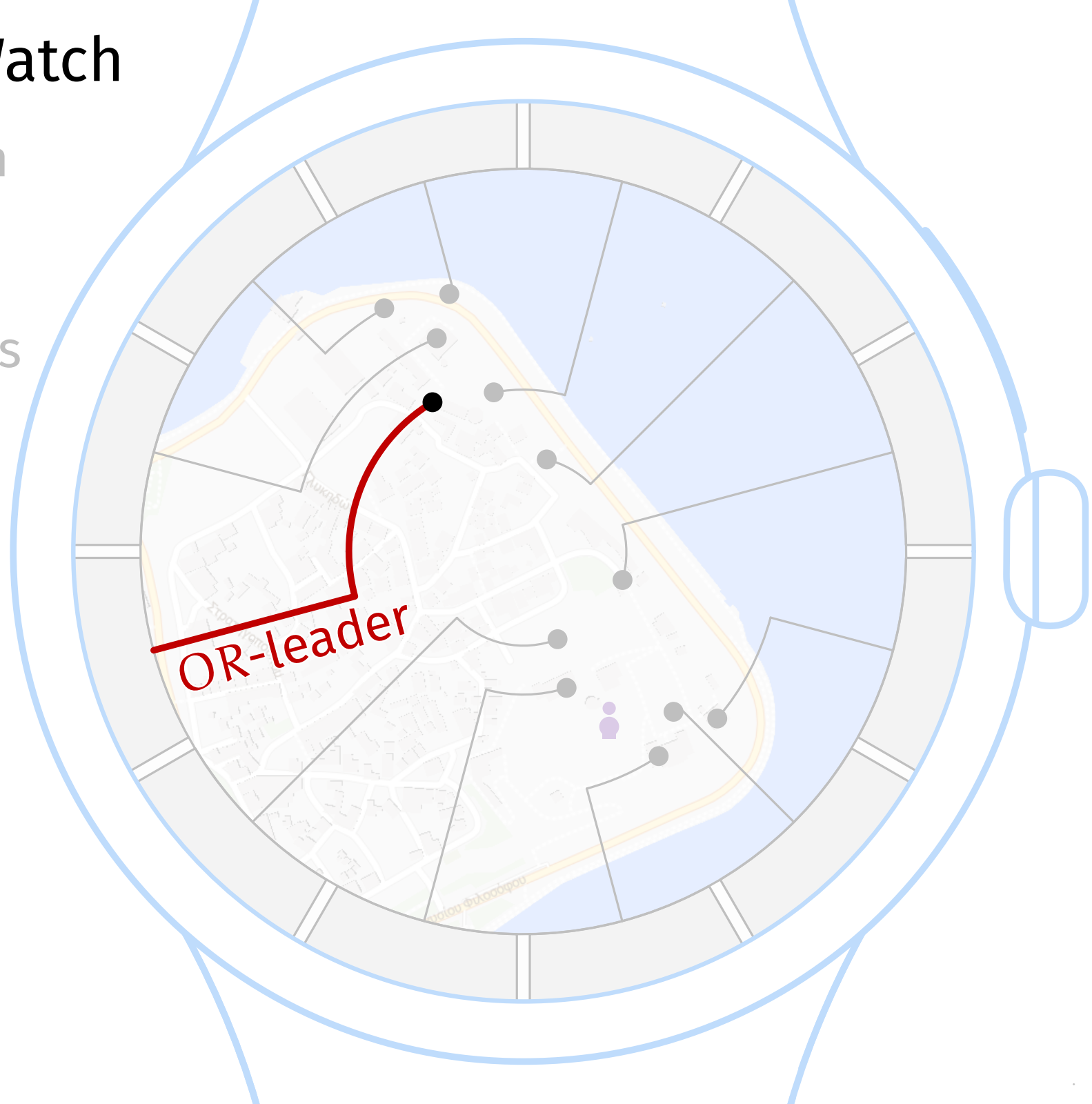
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Labeling on a Smart Watch

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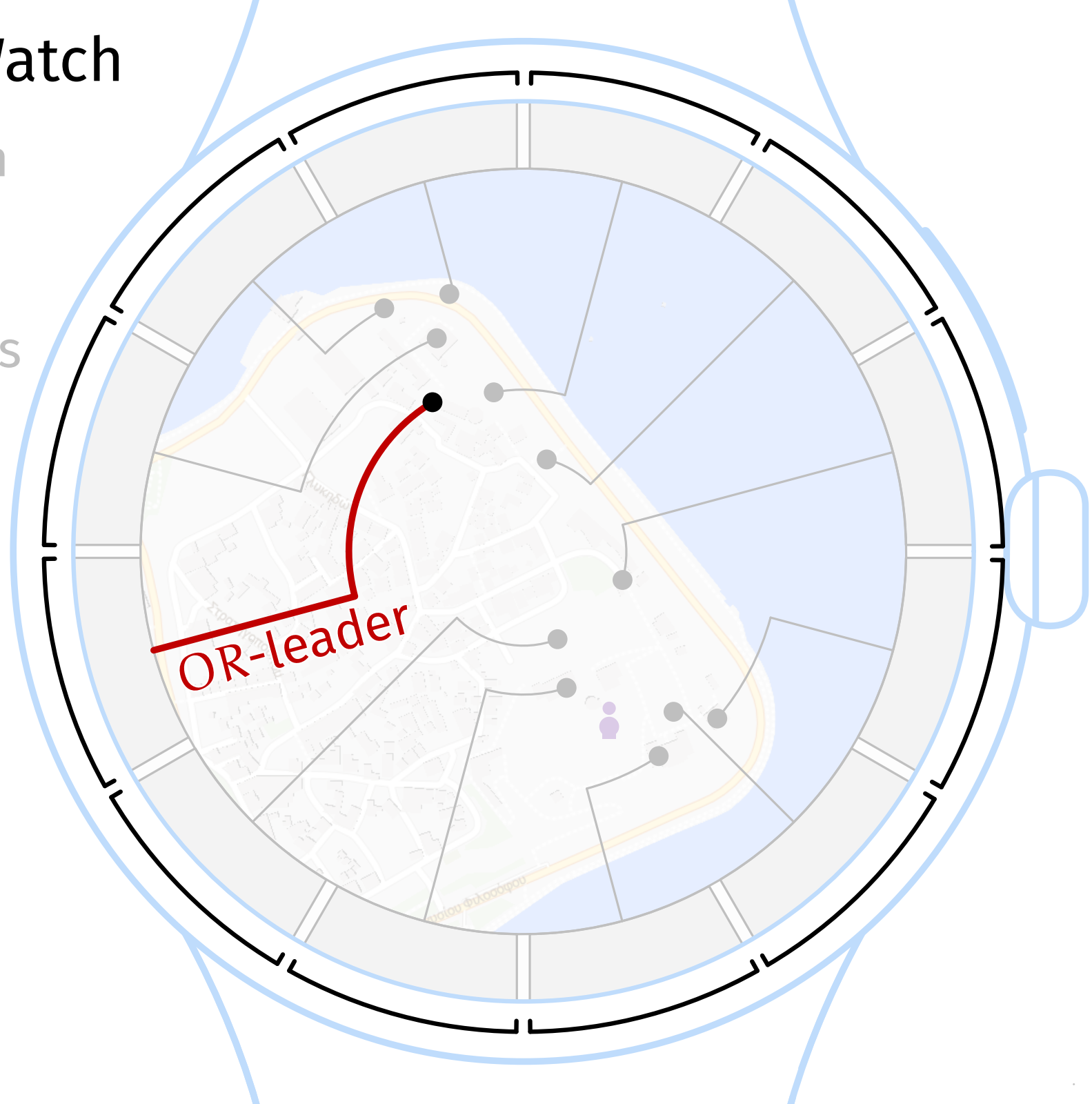
This presentation:
OR-leaders



Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

This presentation:
OR-leaders
Unit labels



Labeling on a Smart Watch

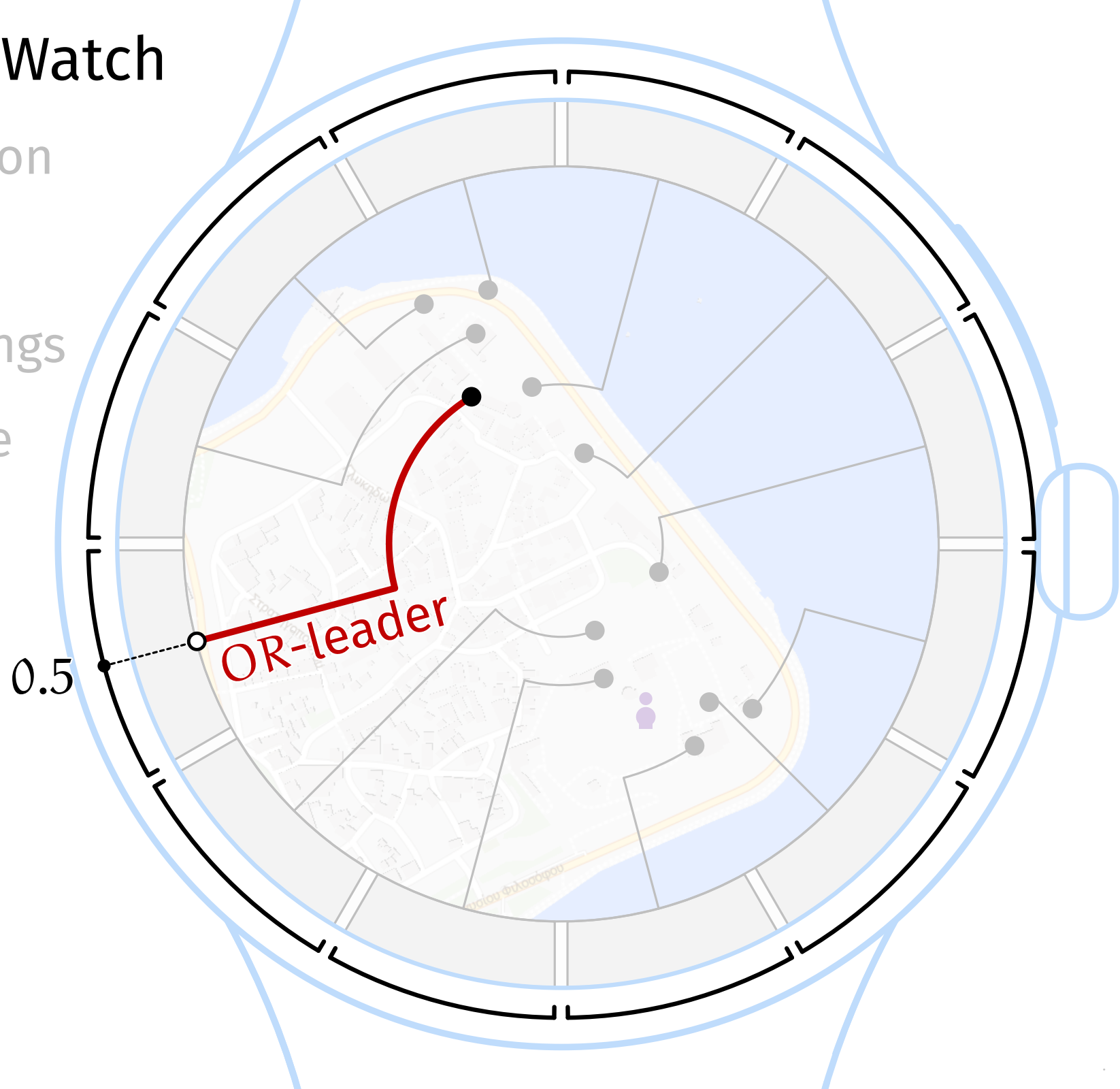
- reduce map obstruction
- avoid label overlap
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This presentation:

OR-leaders

Unit labels

Ports at center



Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
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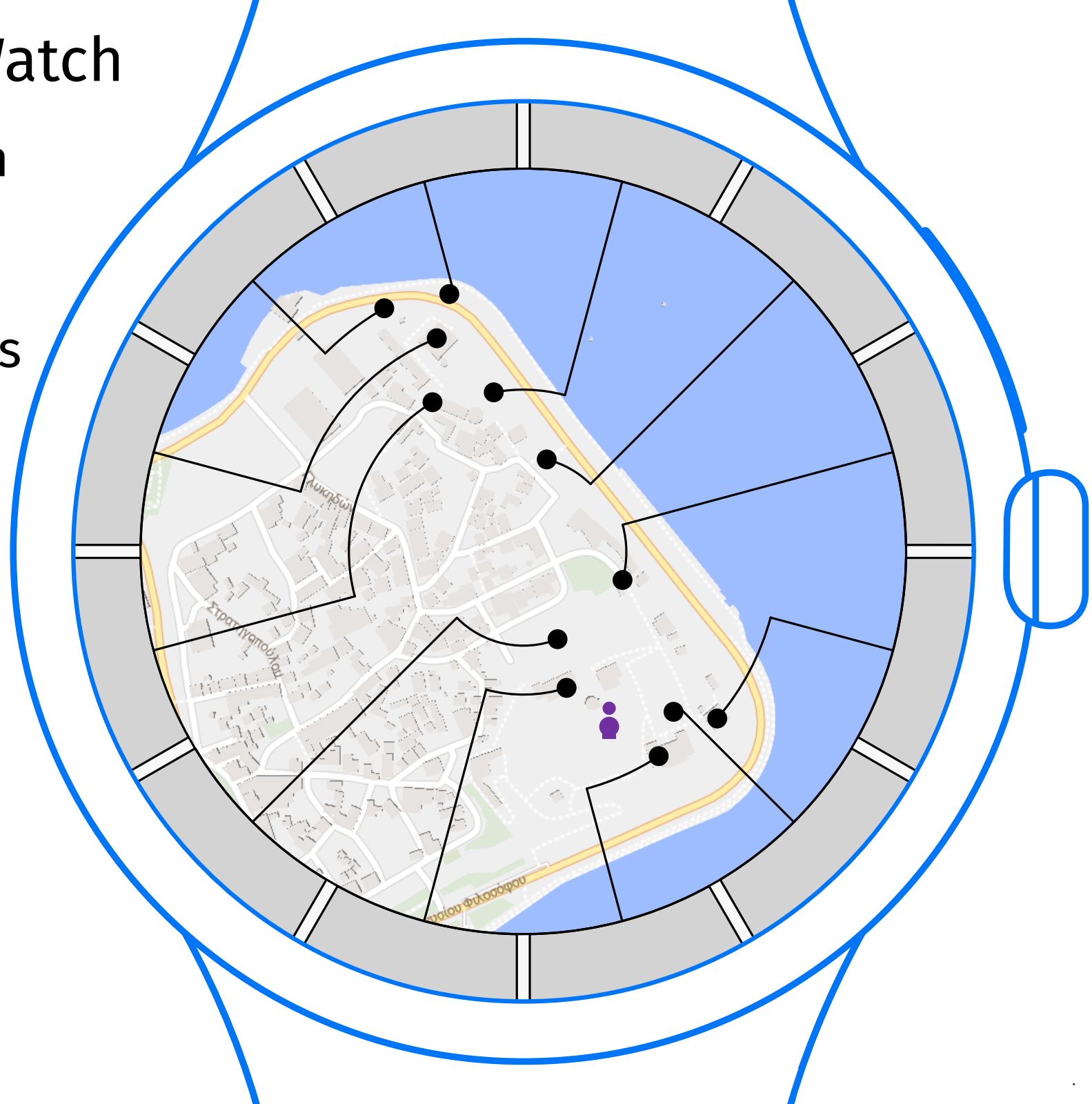
This presentation:

OR-leaders

Unit labels

Ports at center

goal: minimize sum of
leader lengths



Labeling on a Smart Watch

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- avoid label overlap
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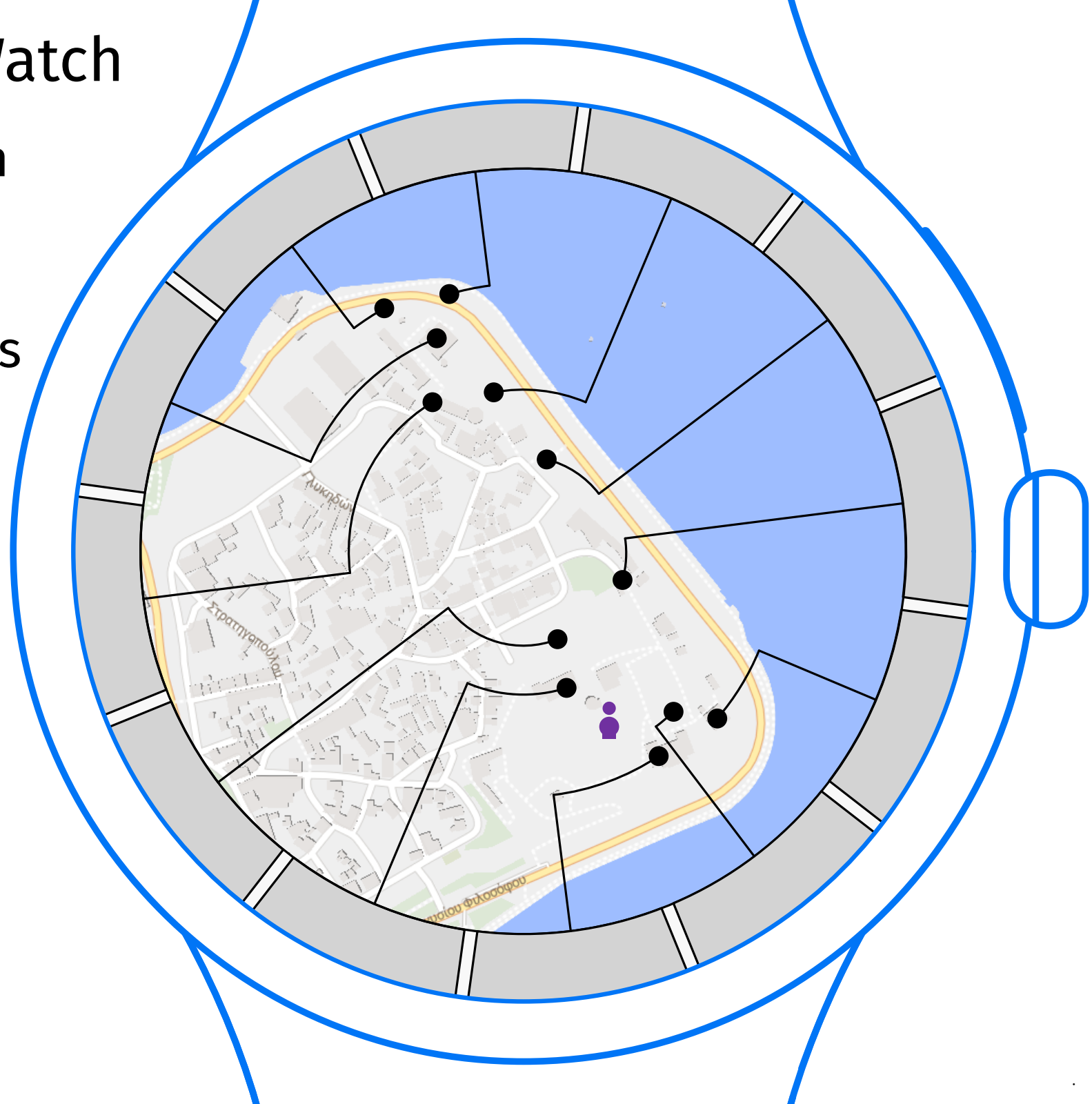
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Labeling on a Smart Watch

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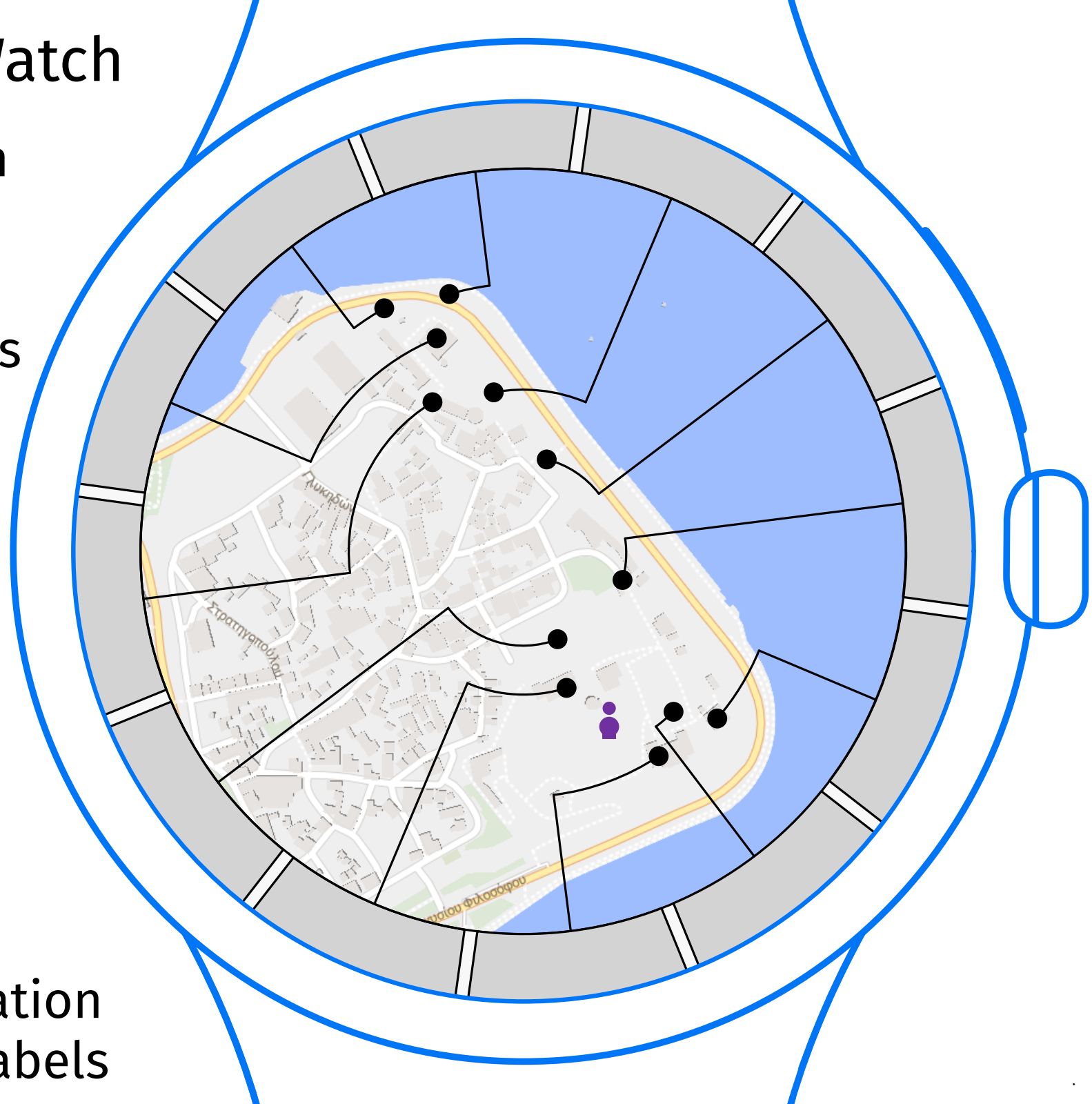
Unit labels

Ports at center

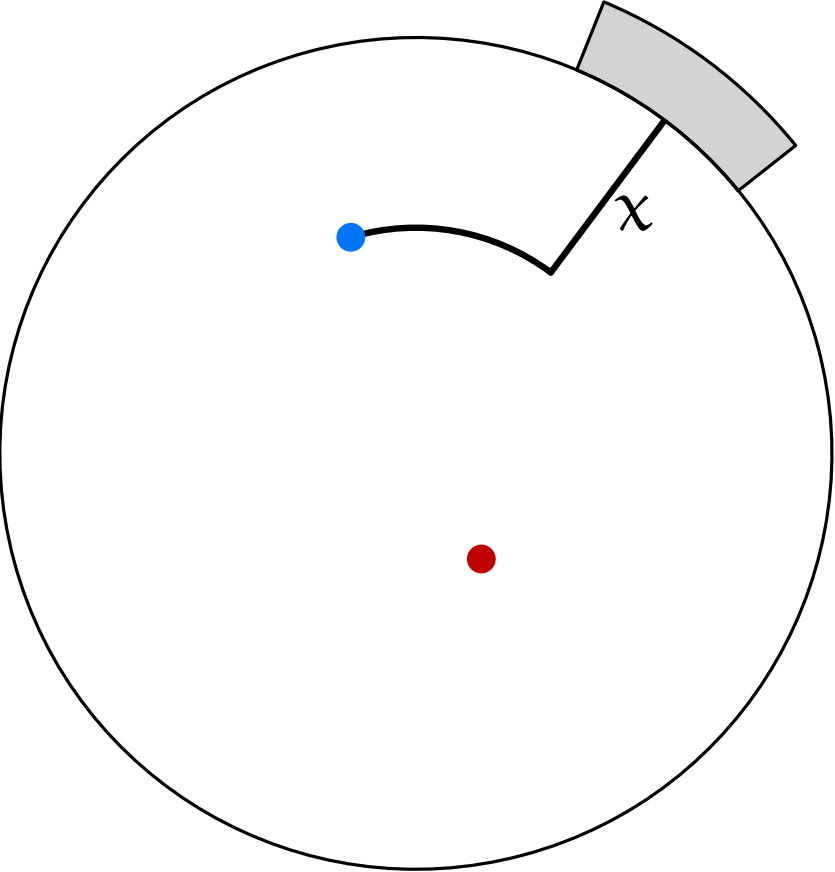
goal: minimize sum of
leader lengths

under:

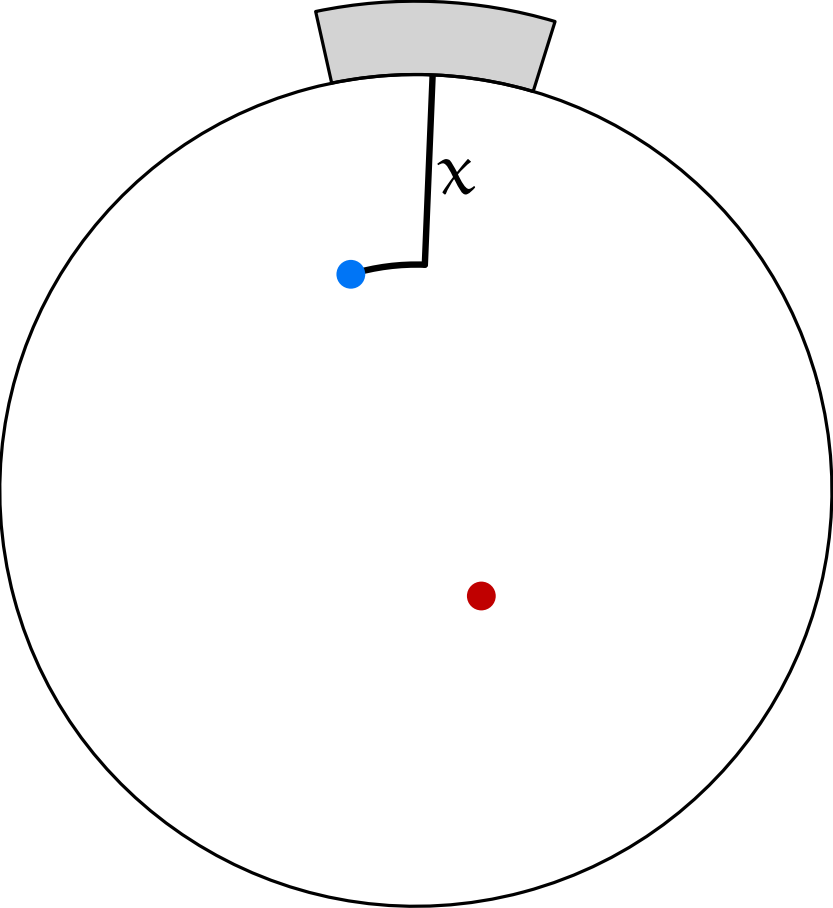
- continuous rotation
- fixed order of labels



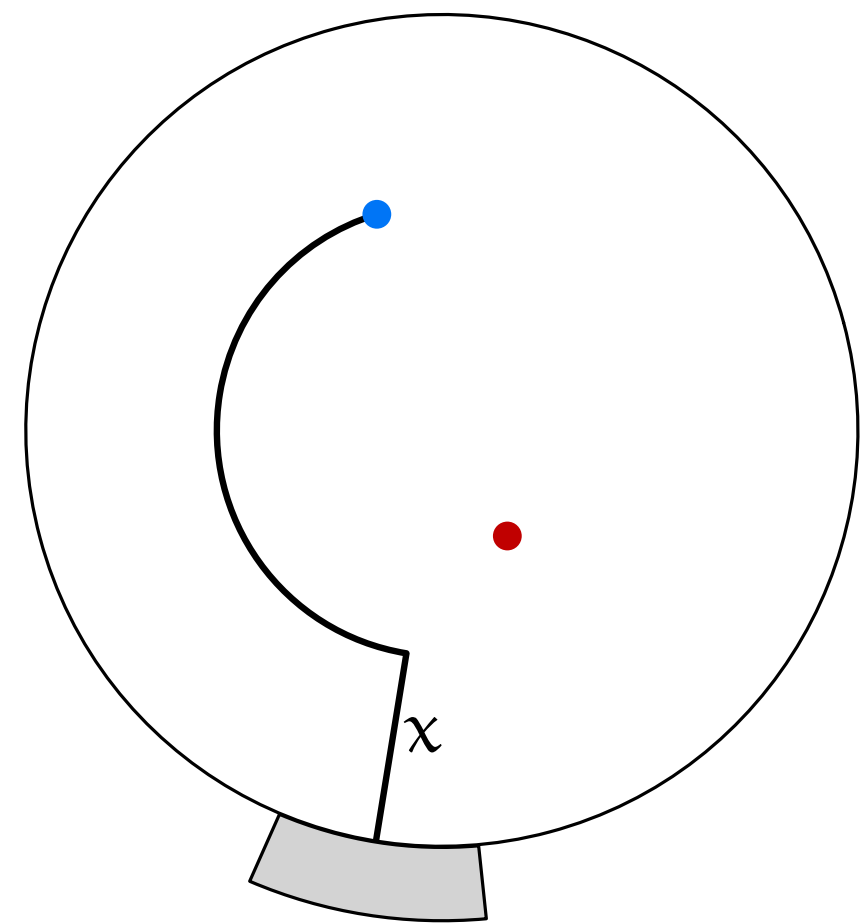
Rotating the Labels



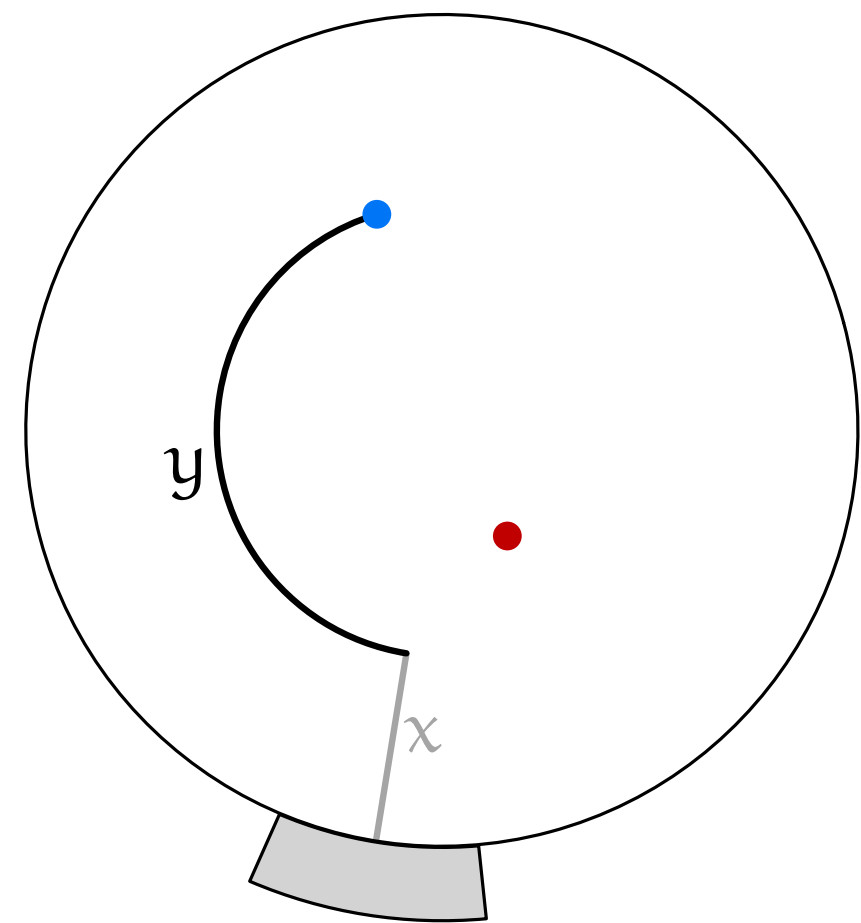
Rotating the Labels



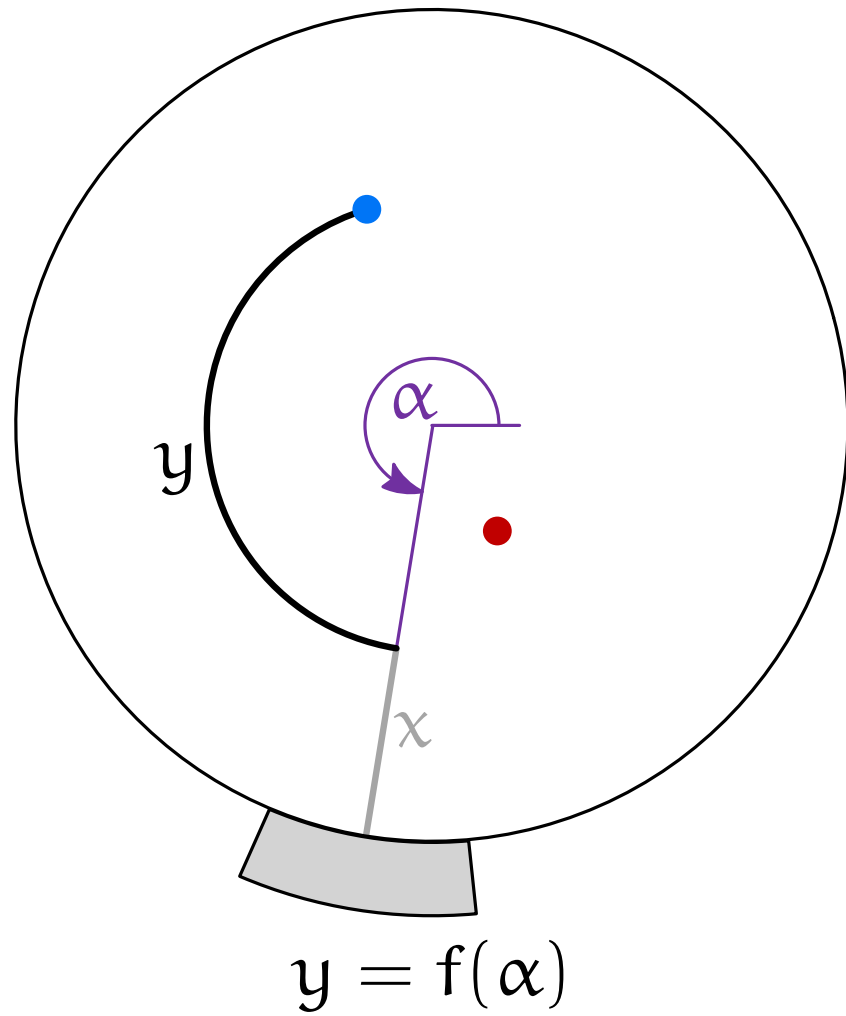
Rotating the Labels



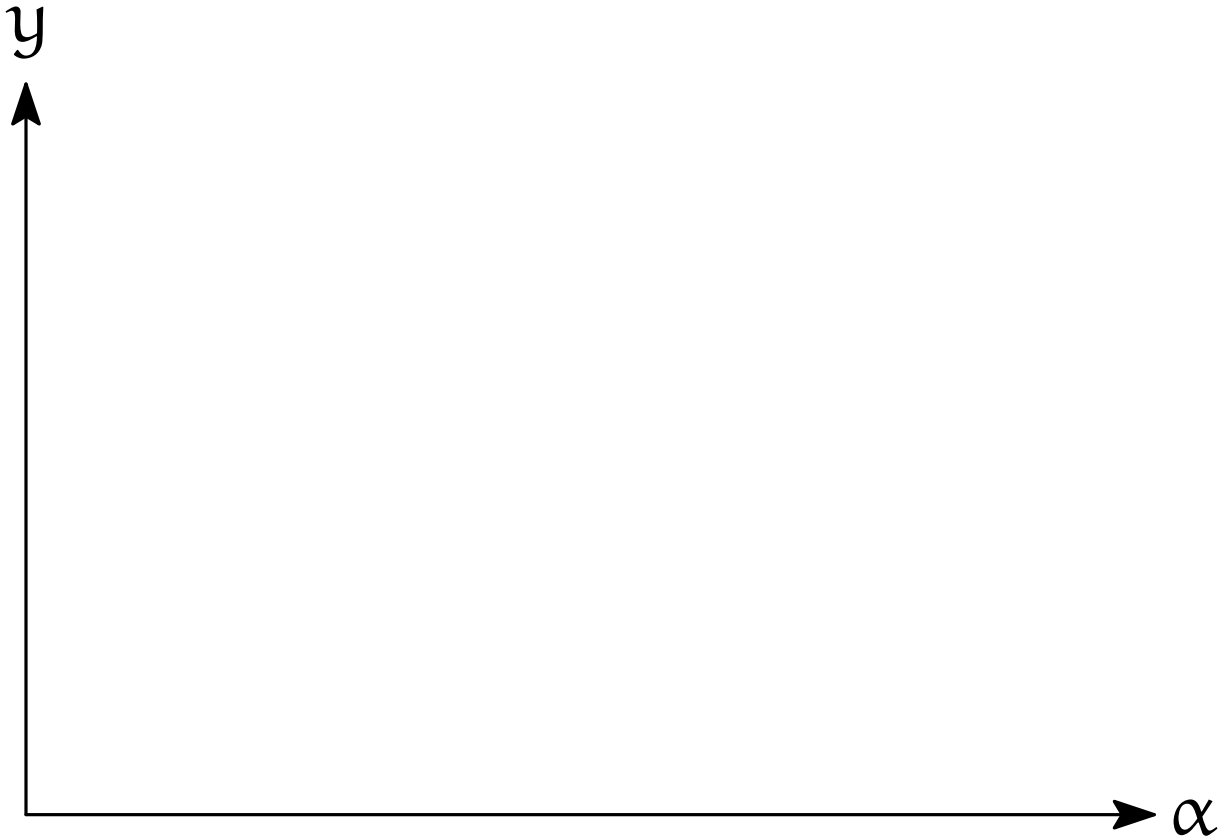
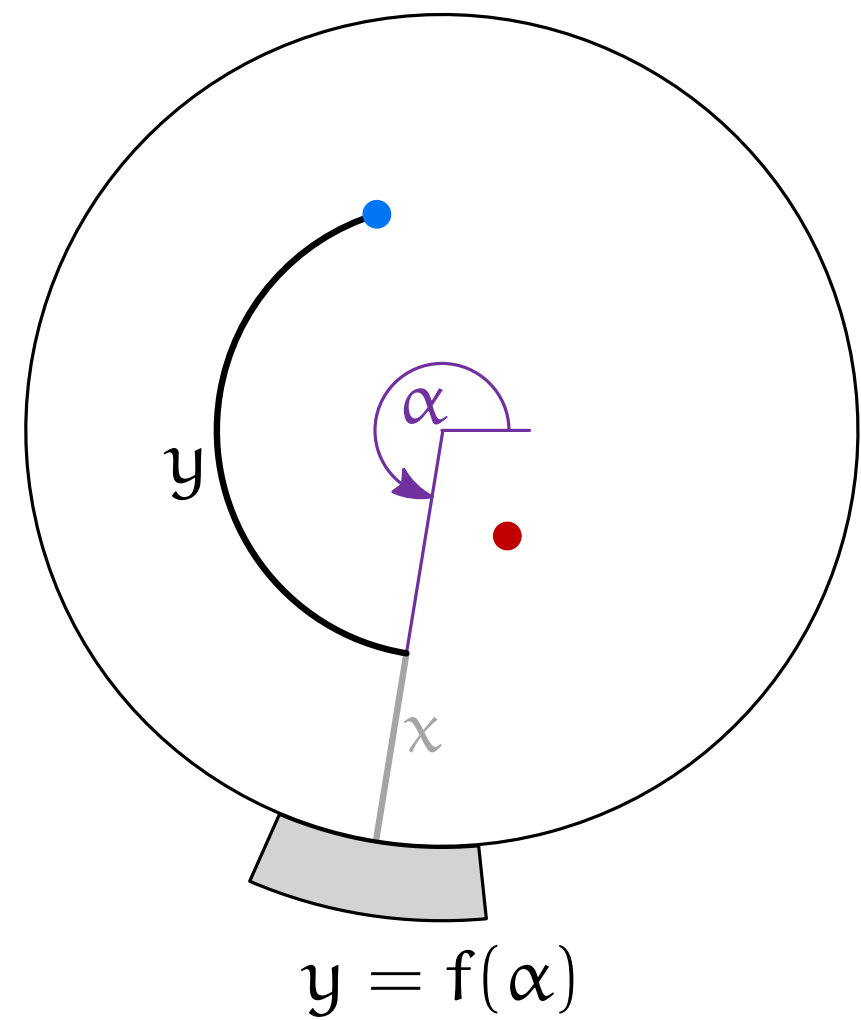
Rotating the Labels



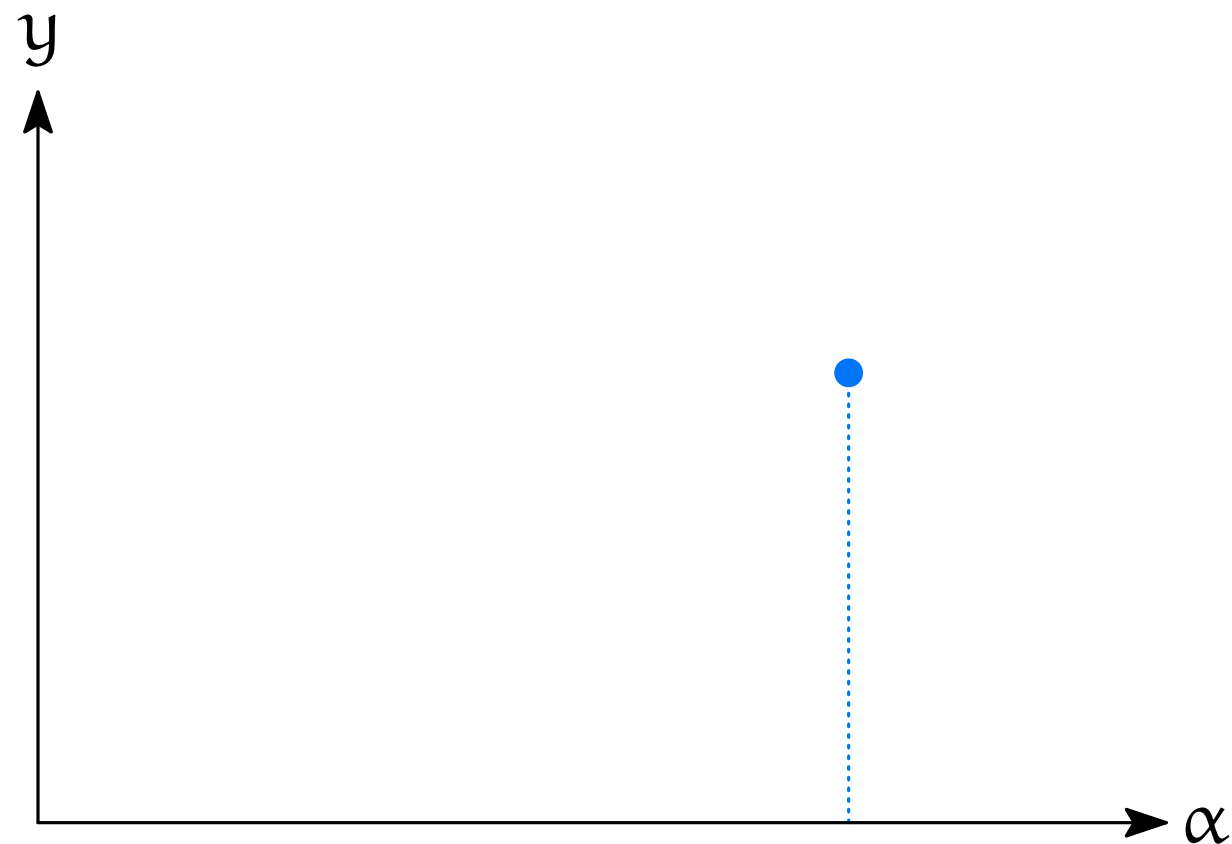
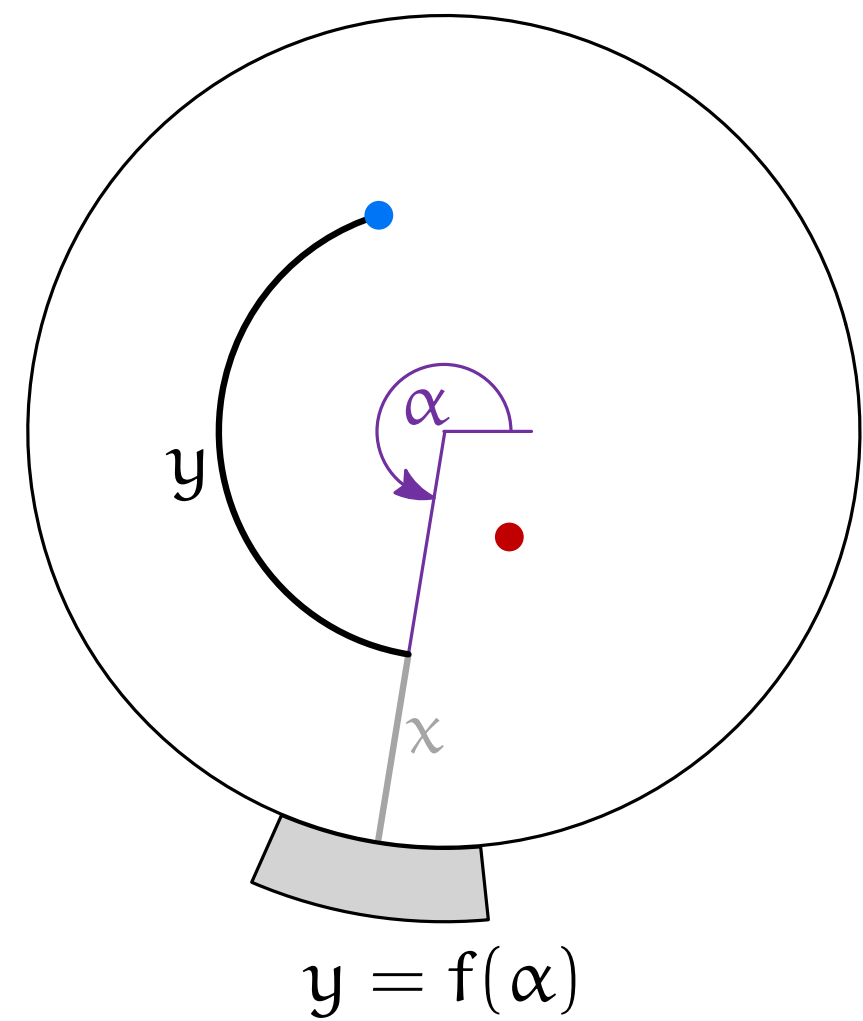
Rotating the Labels



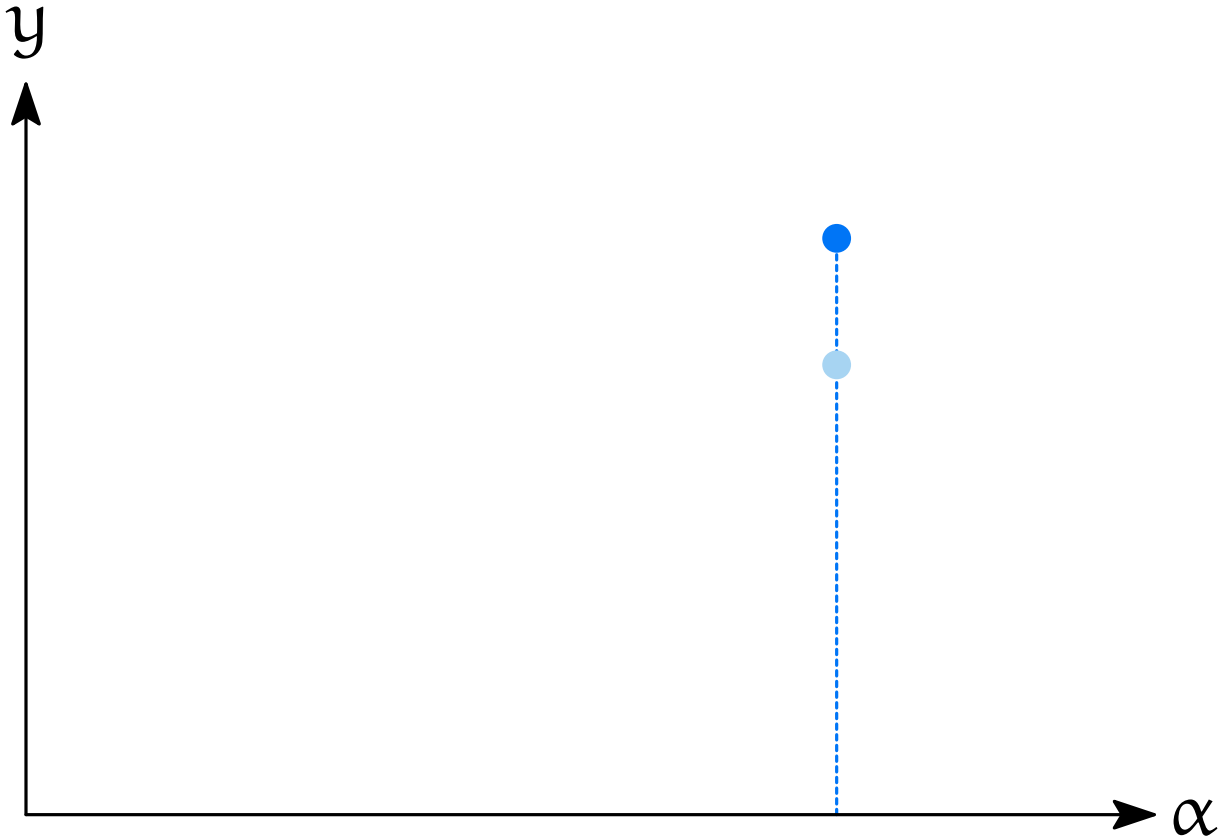
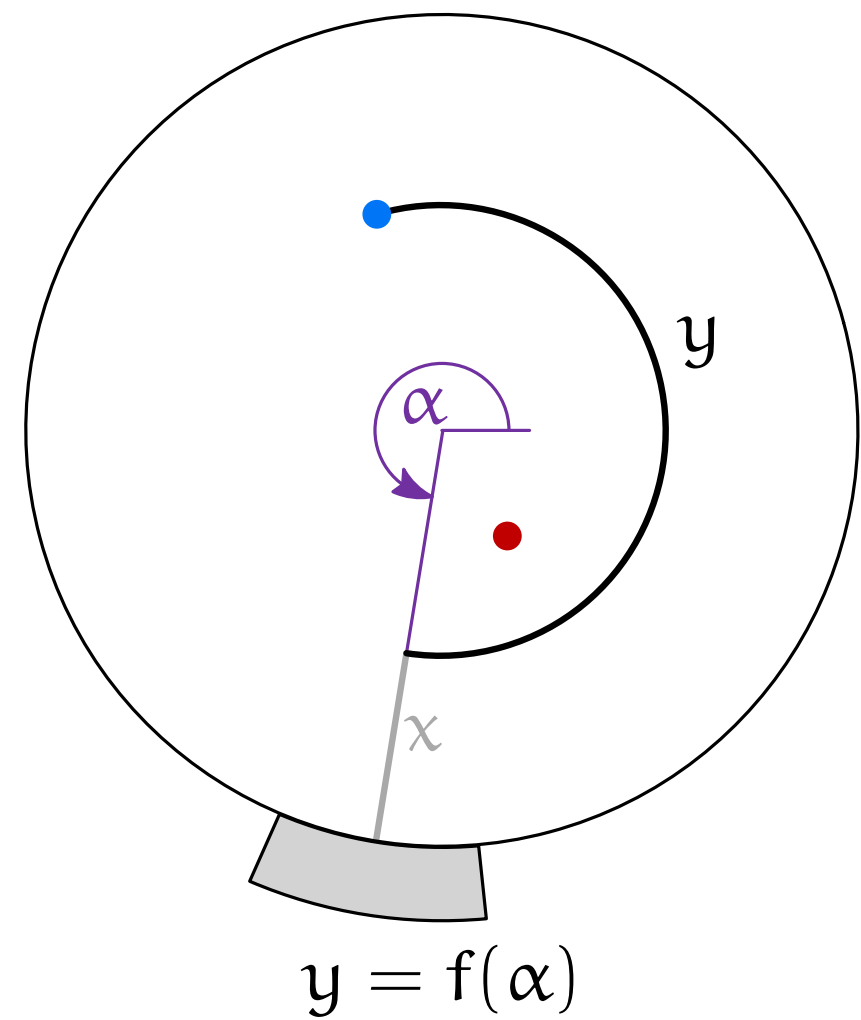
Rotating the Labels



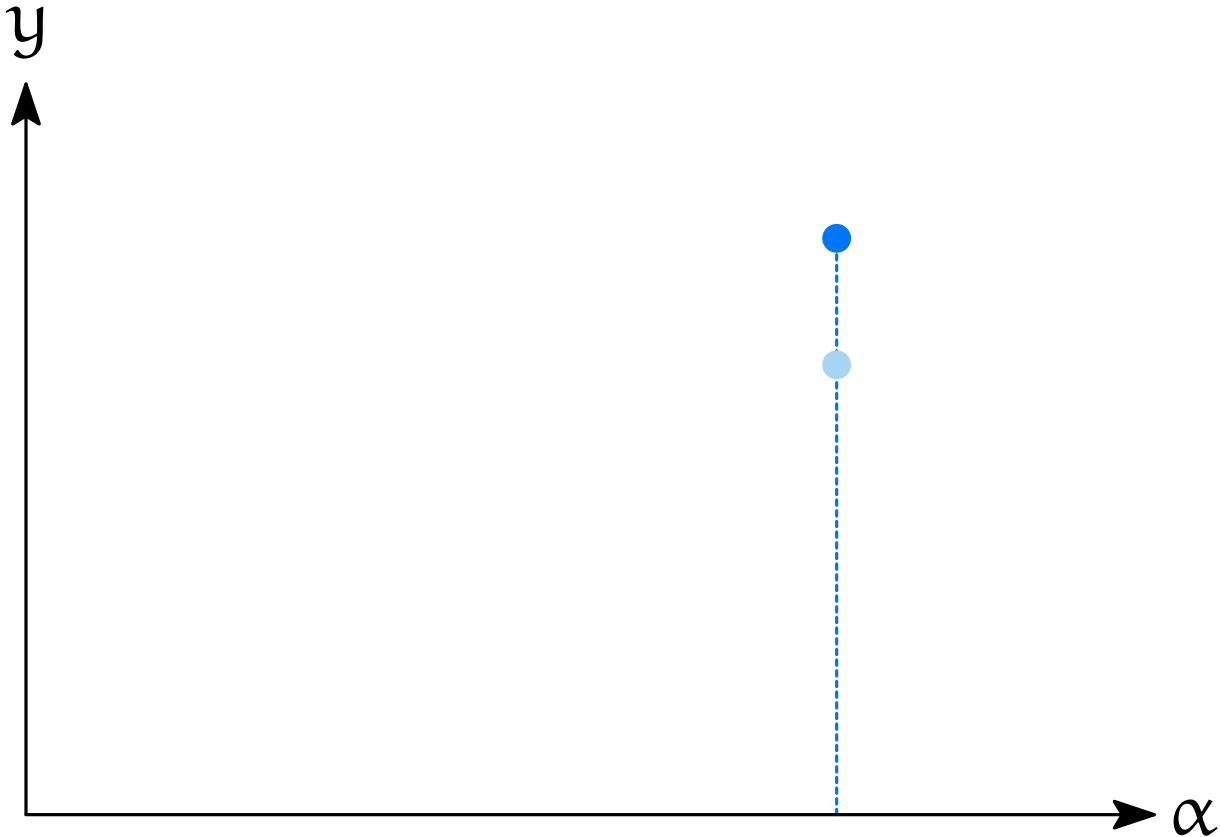
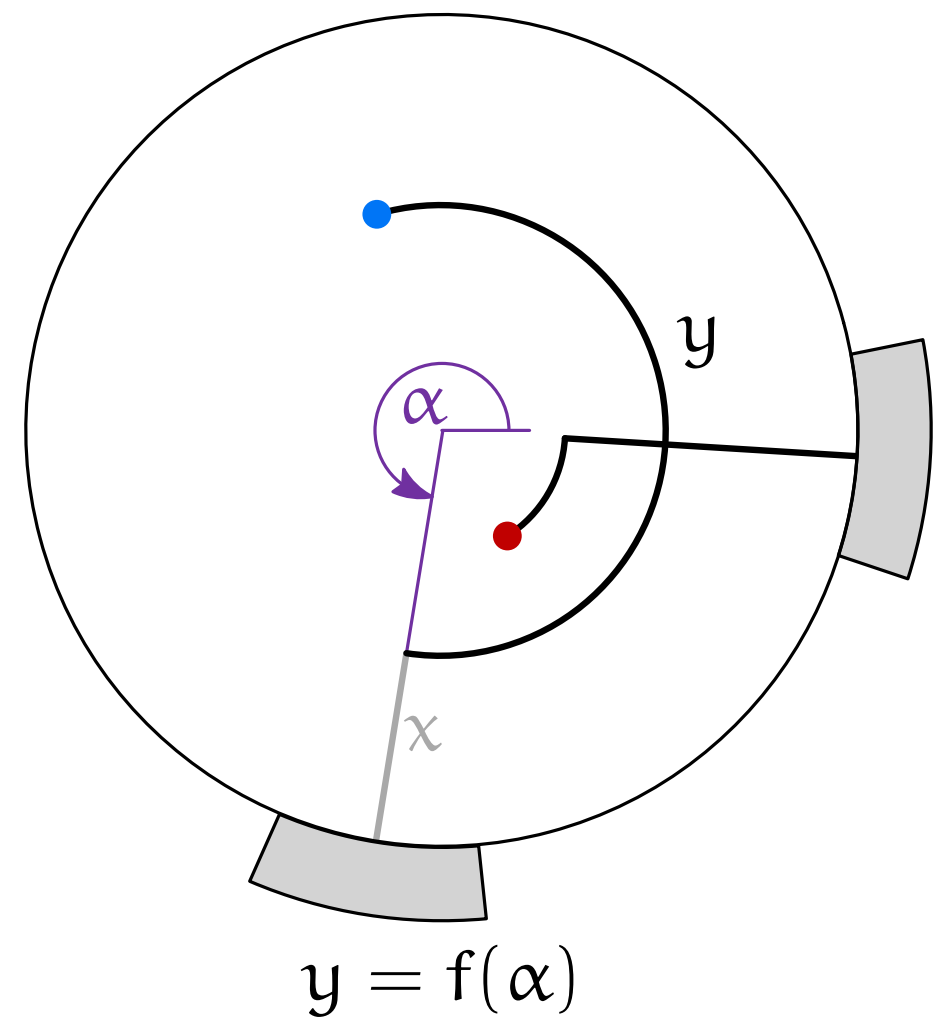
Rotating the Labels



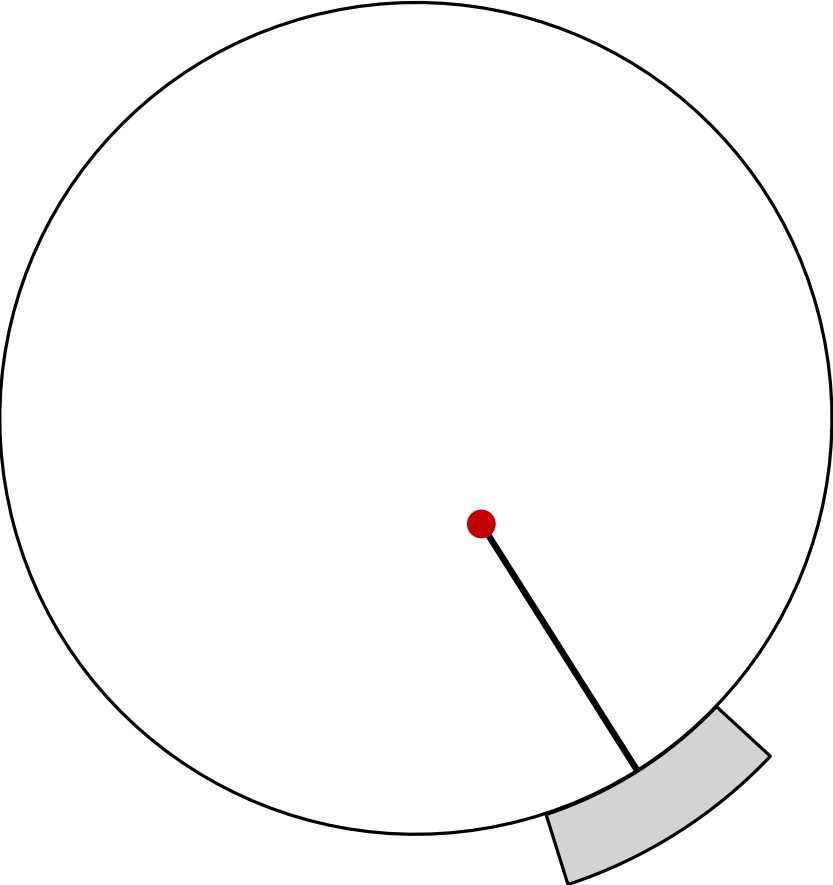
Rotating the Labels



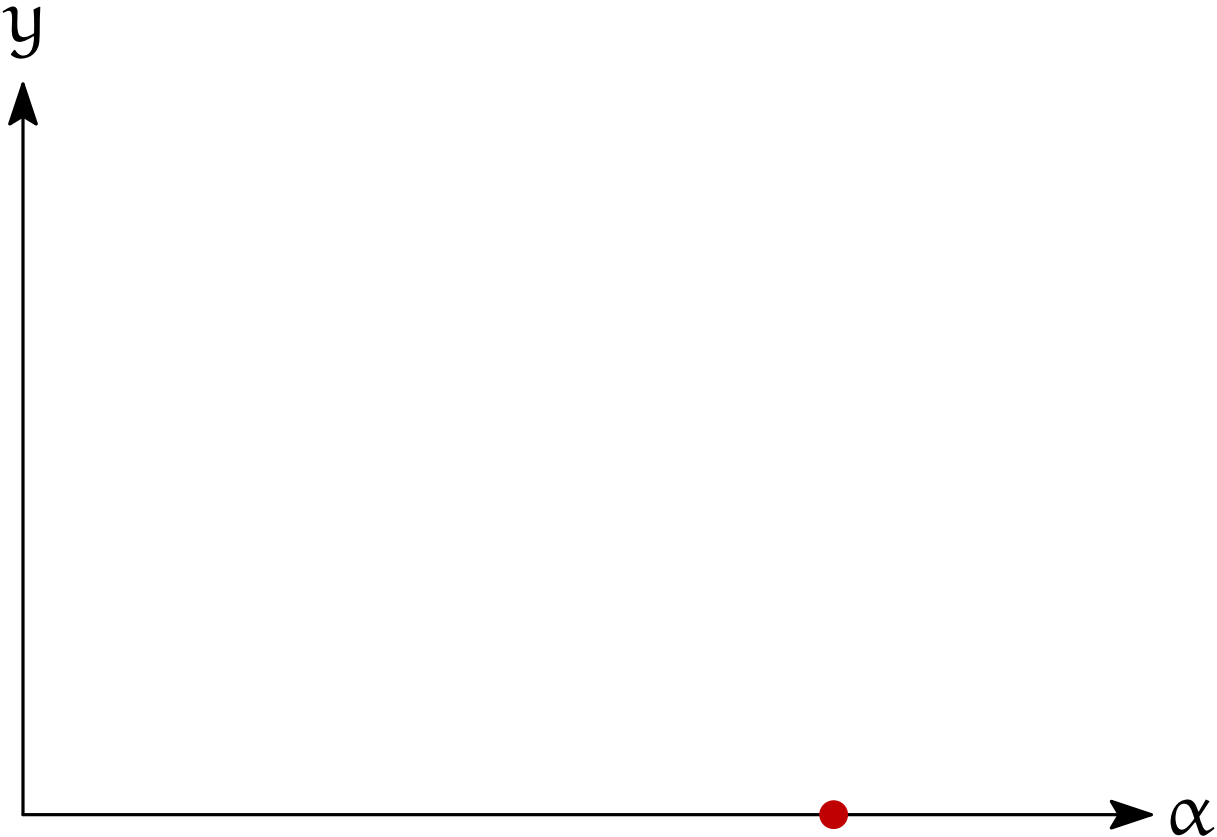
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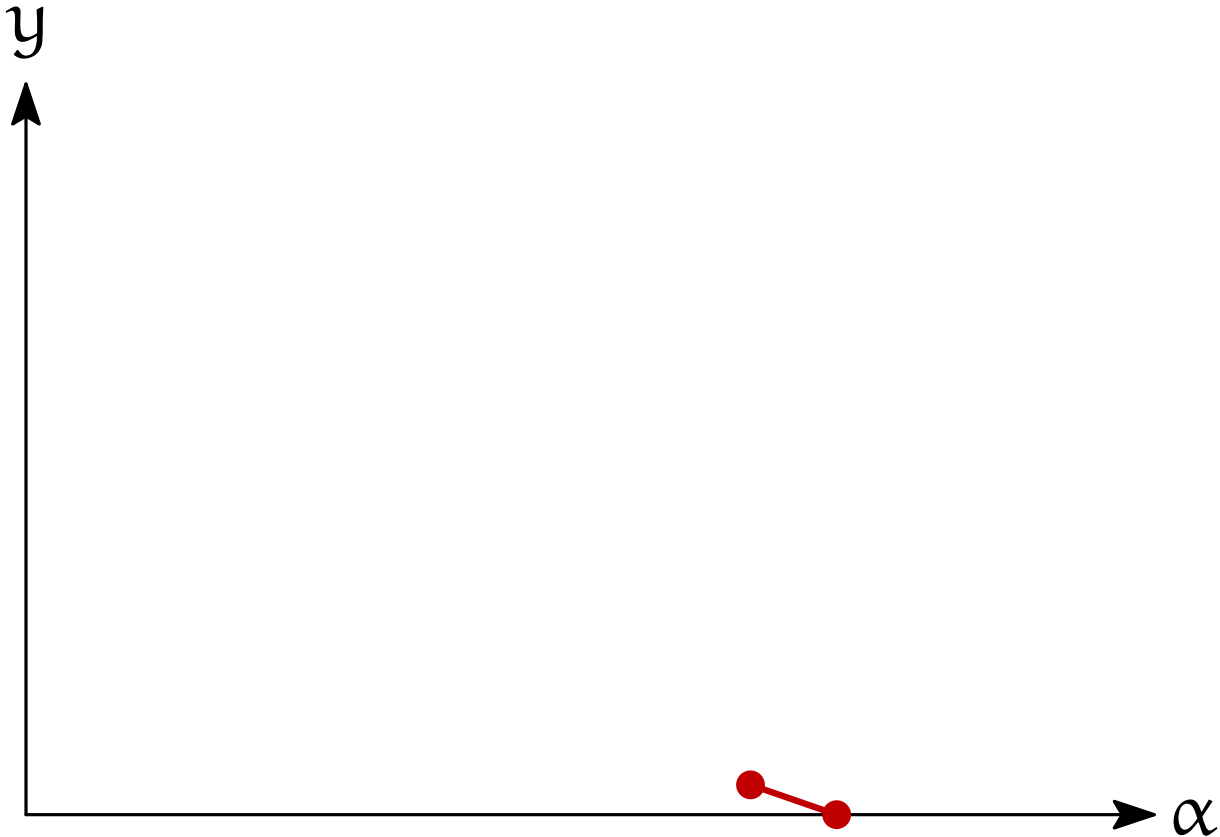
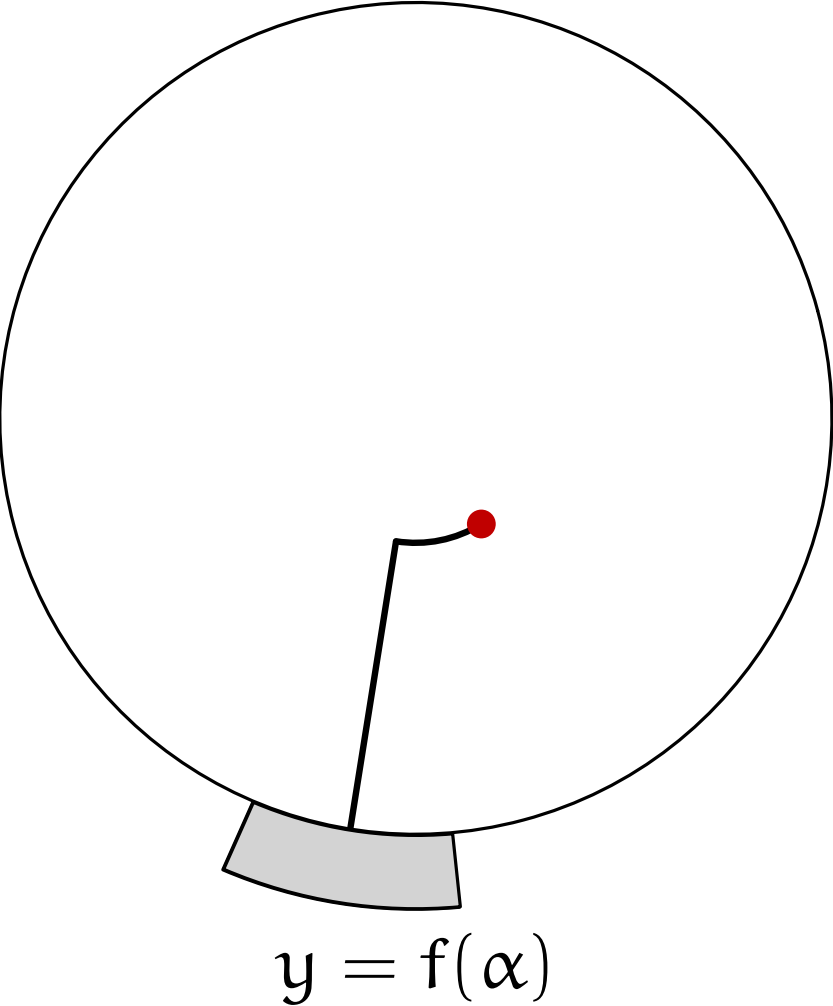
Rotating the Labels



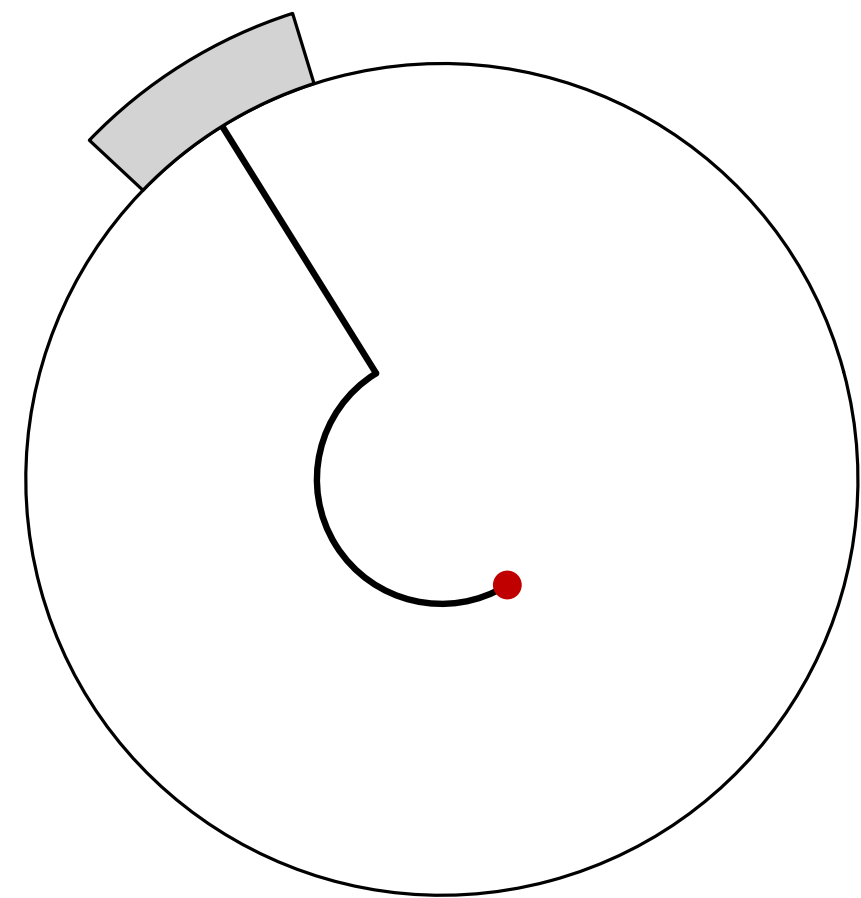
$y = f(\alpha)$



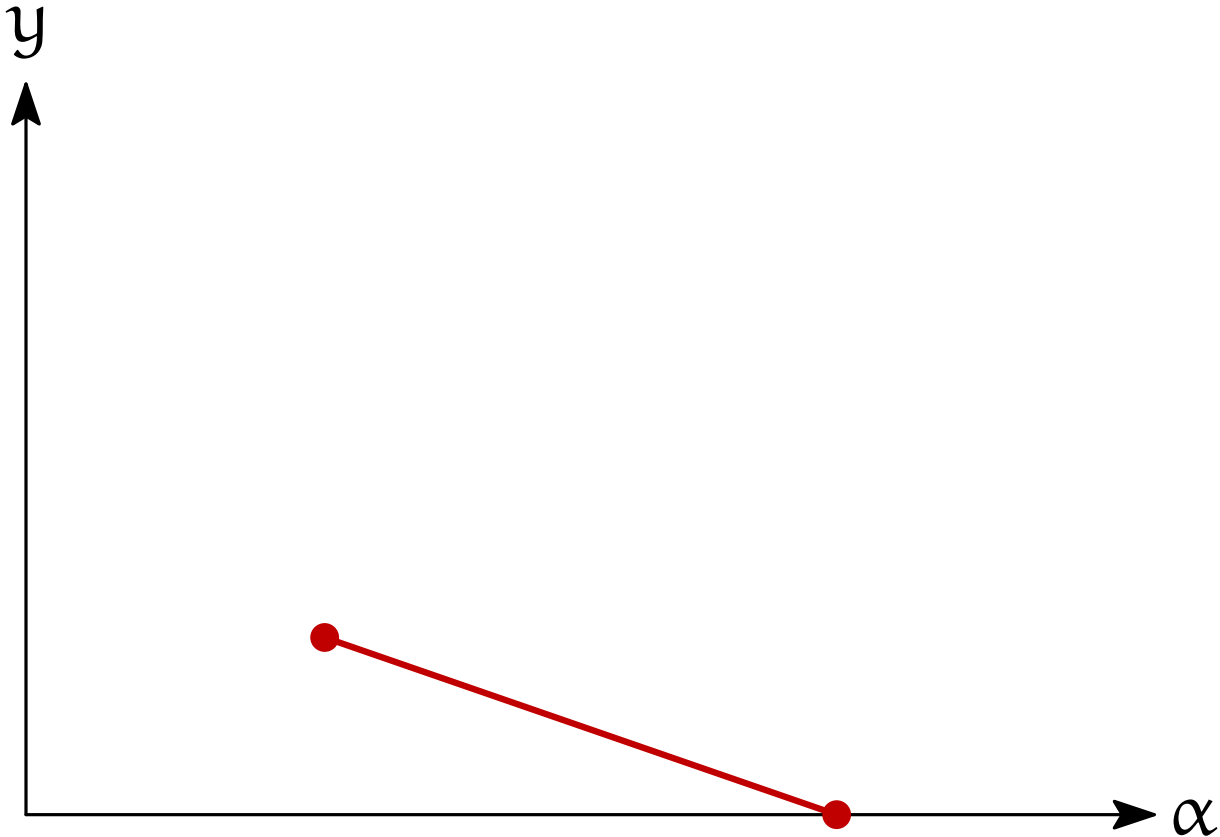
Rotating the Labels



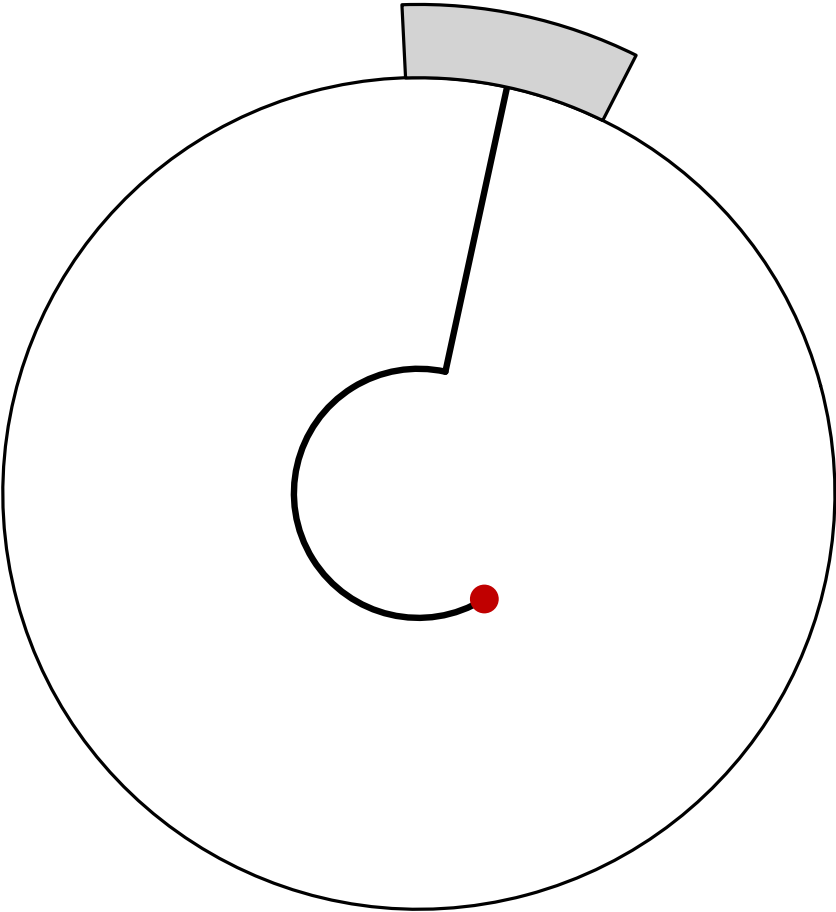
Rotating the Labels



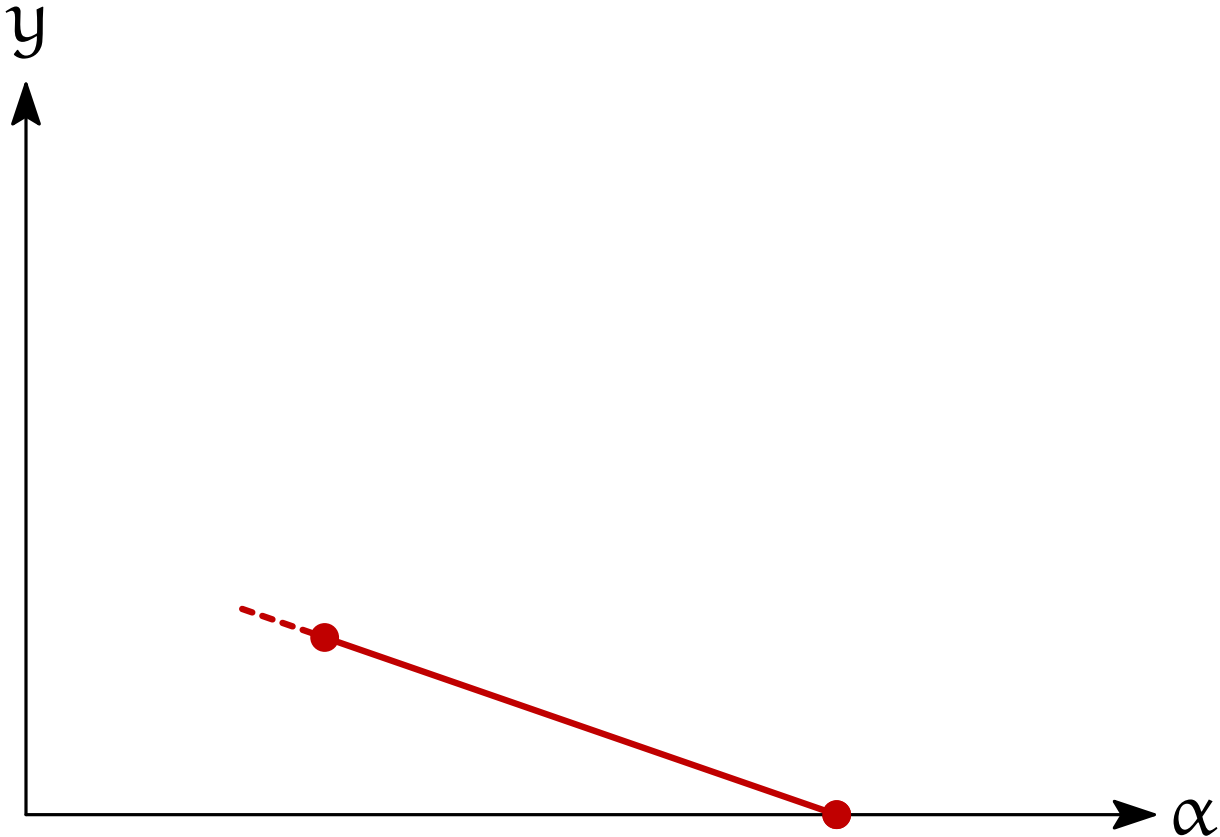
$$y = f(\alpha)$$



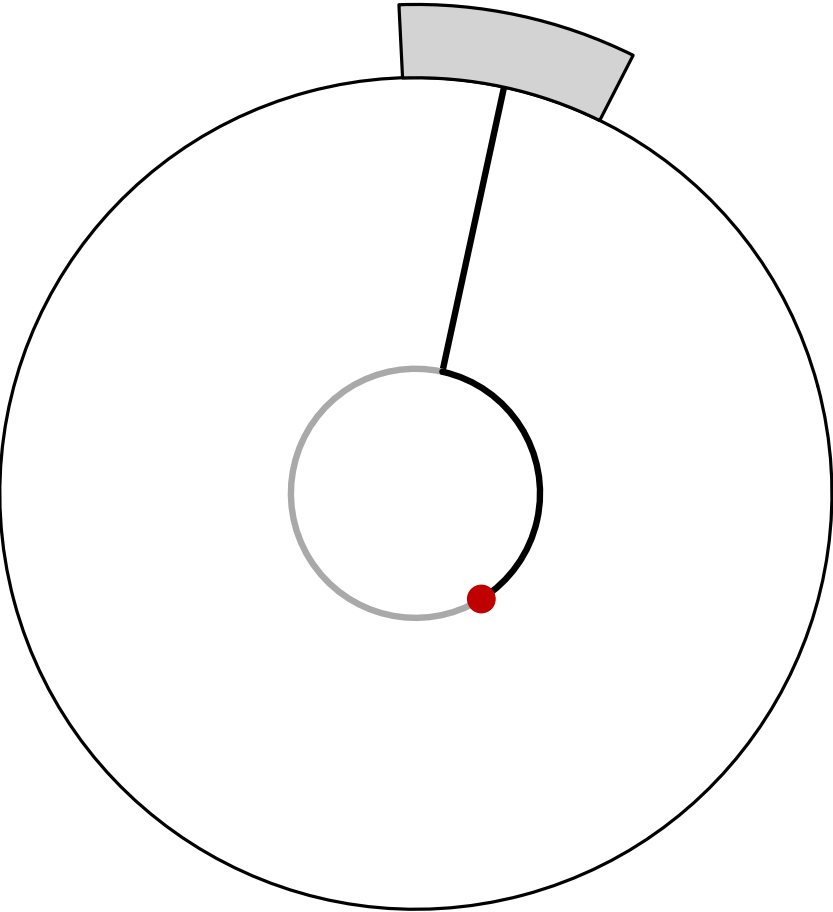
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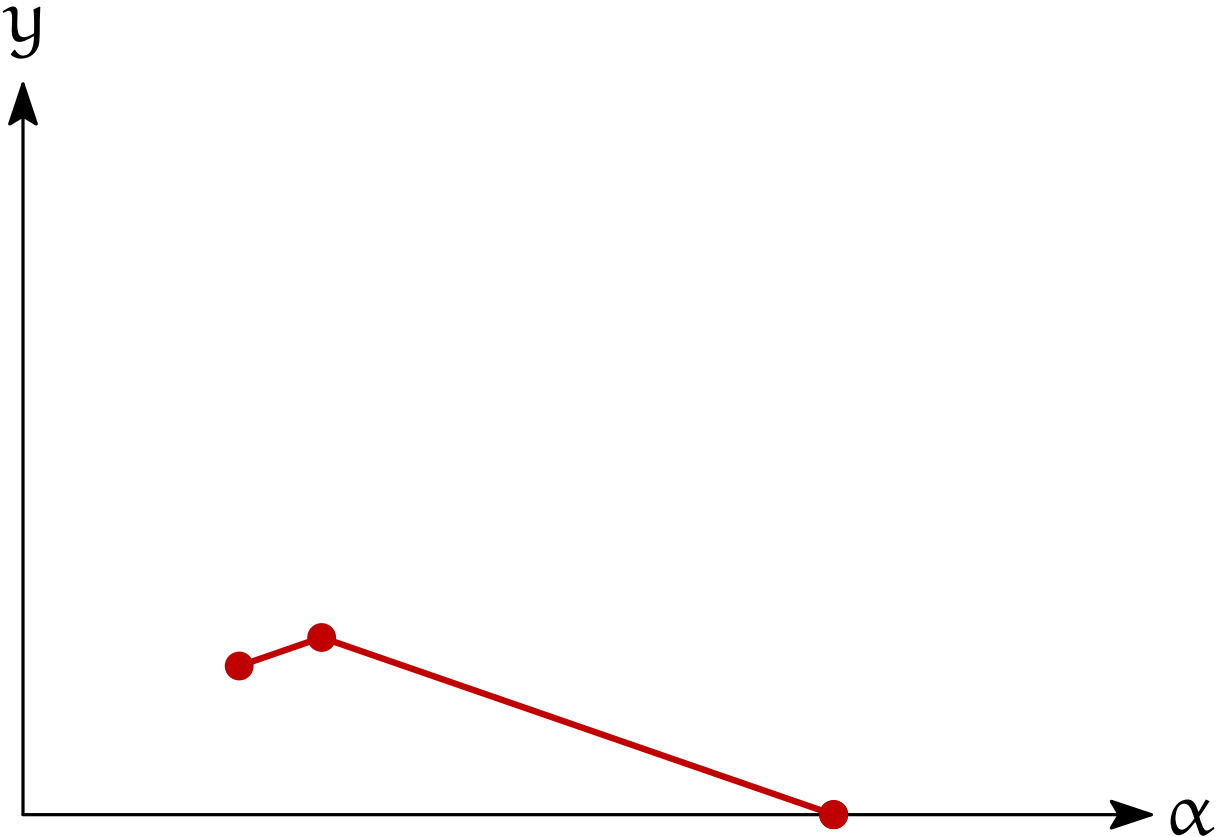
$y = f(\alpha)$



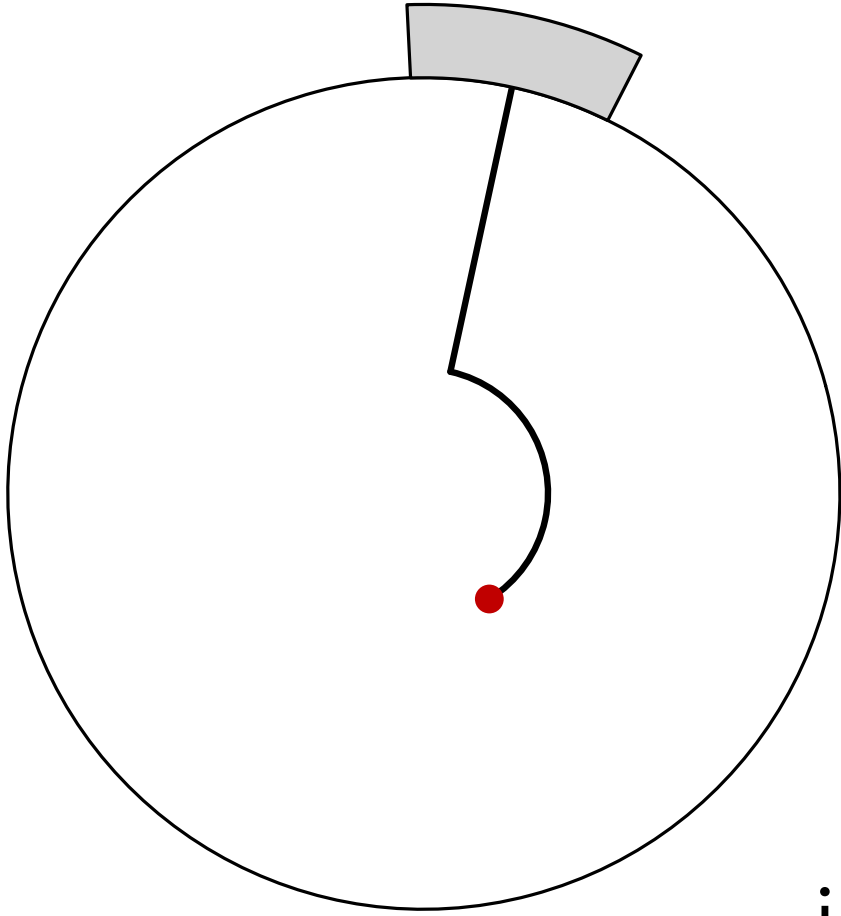
Rotating the Labels



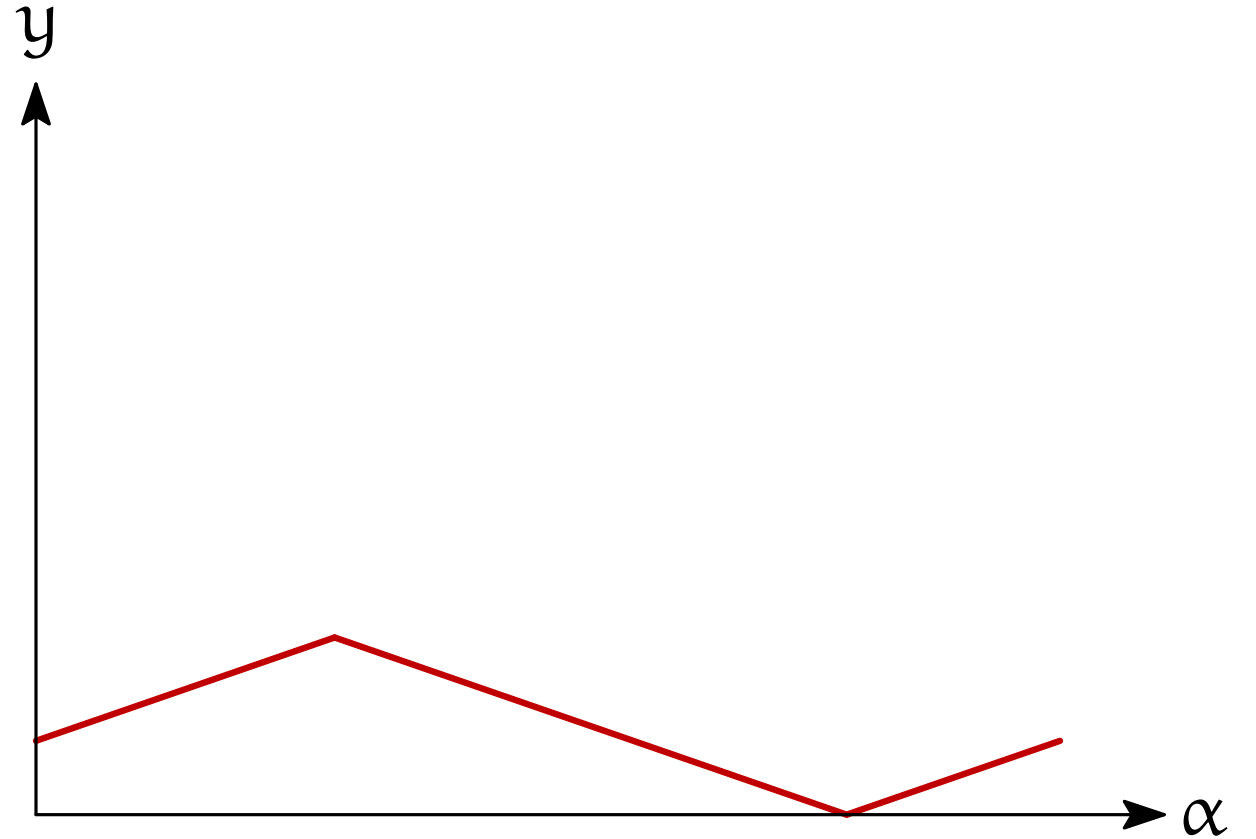
$y = f(\alpha)$



Rotating the Labels



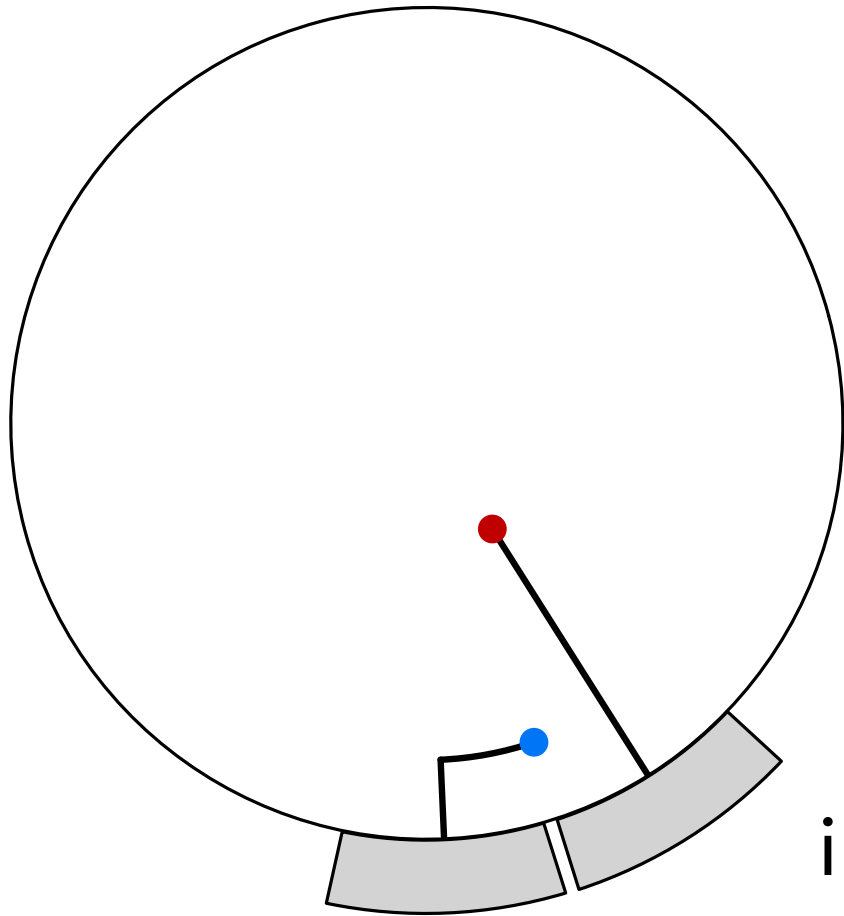
$$y = f(\alpha)$$



innermost:

- always shorter path

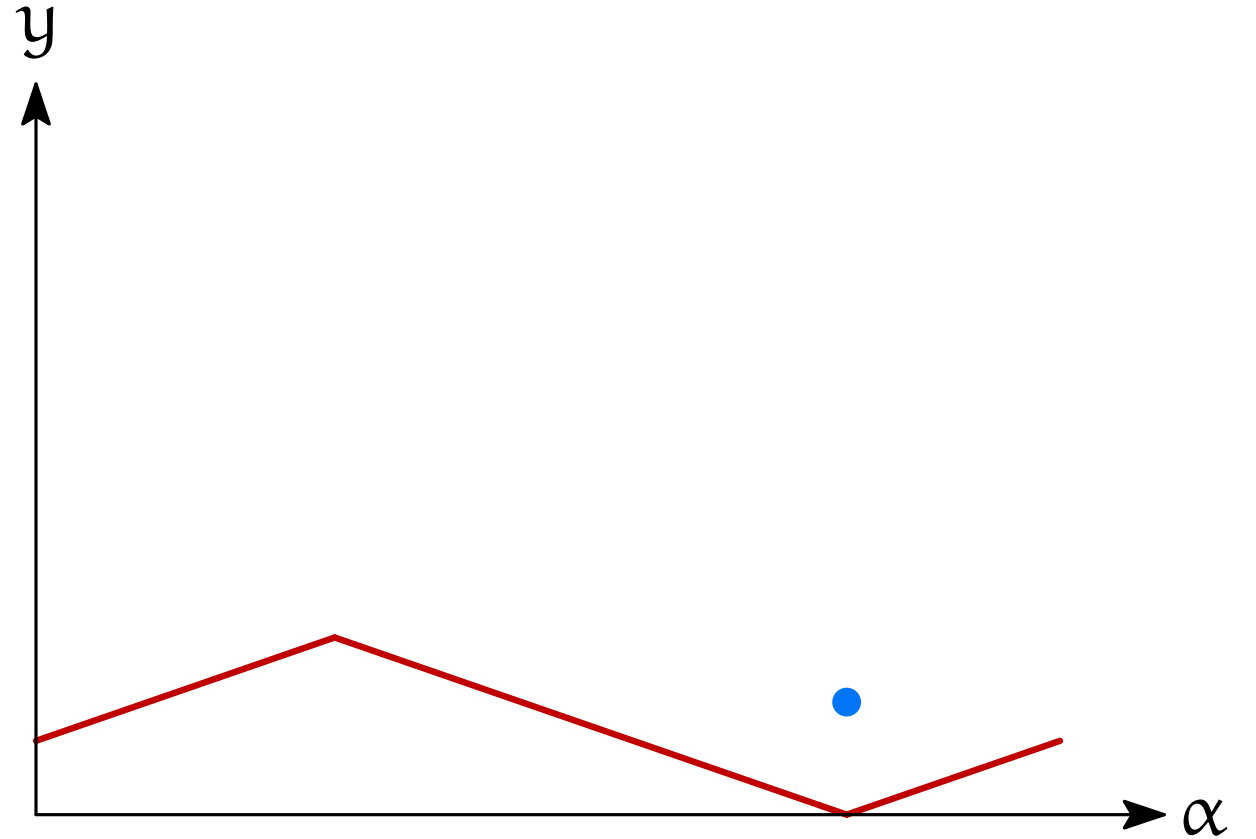
Rotating the Labels



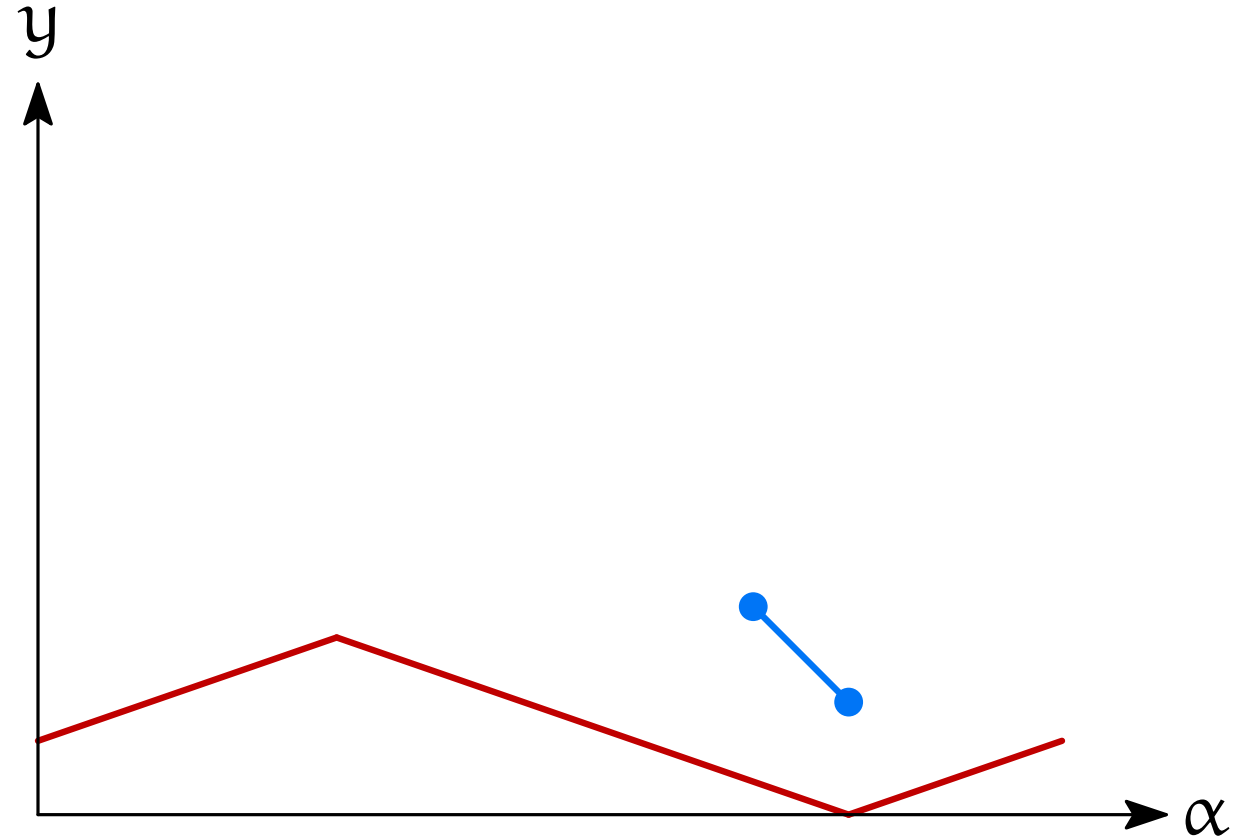
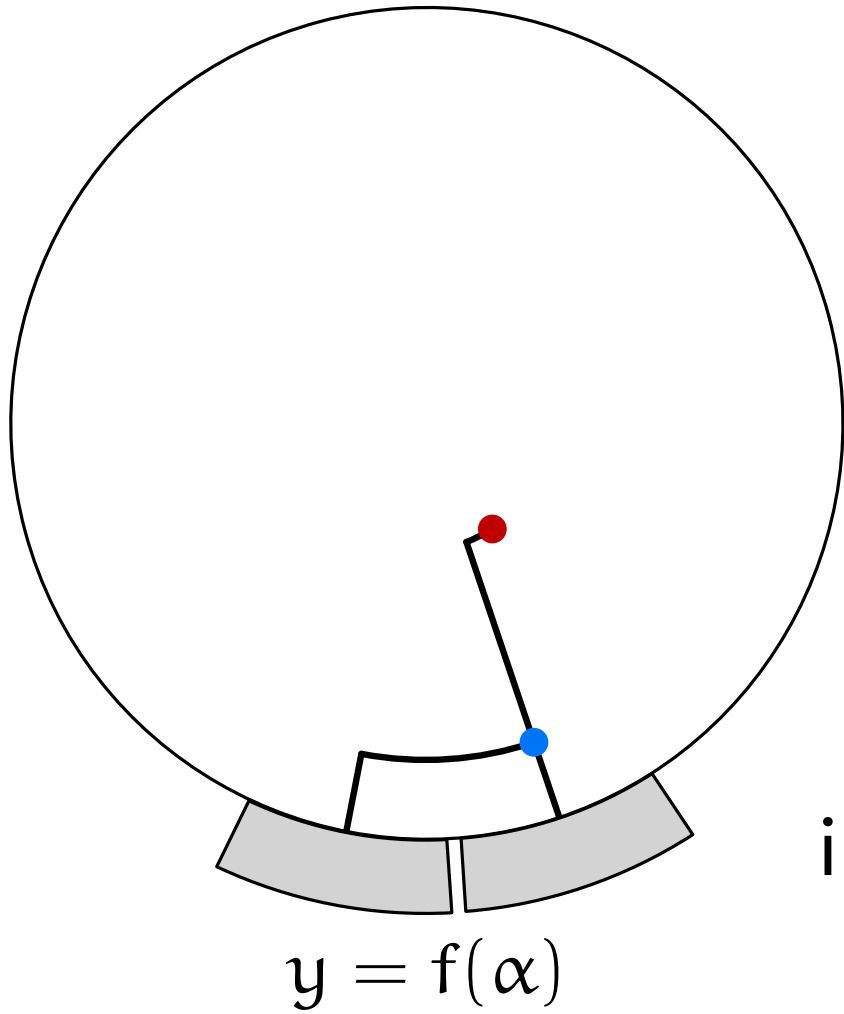
$$y = f(\alpha)$$

innermost:

- always shorter path



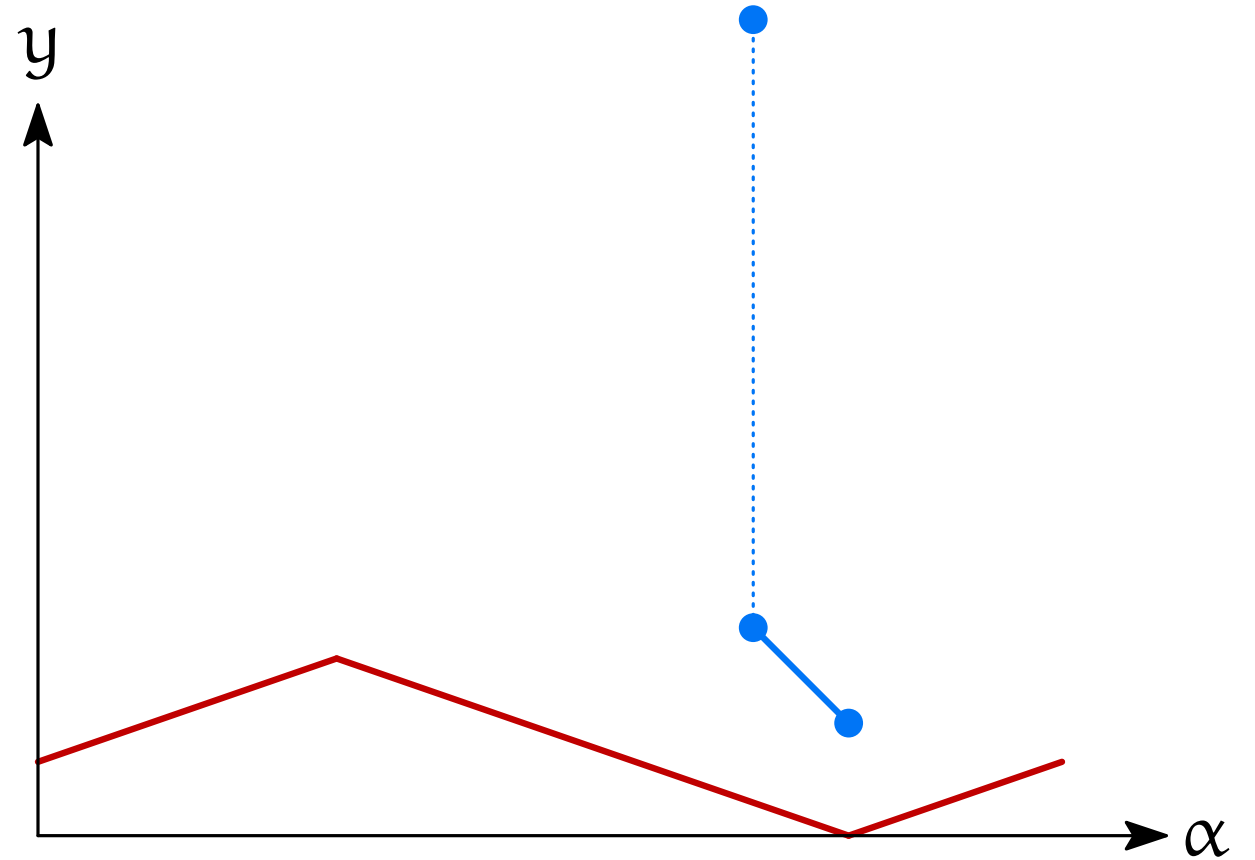
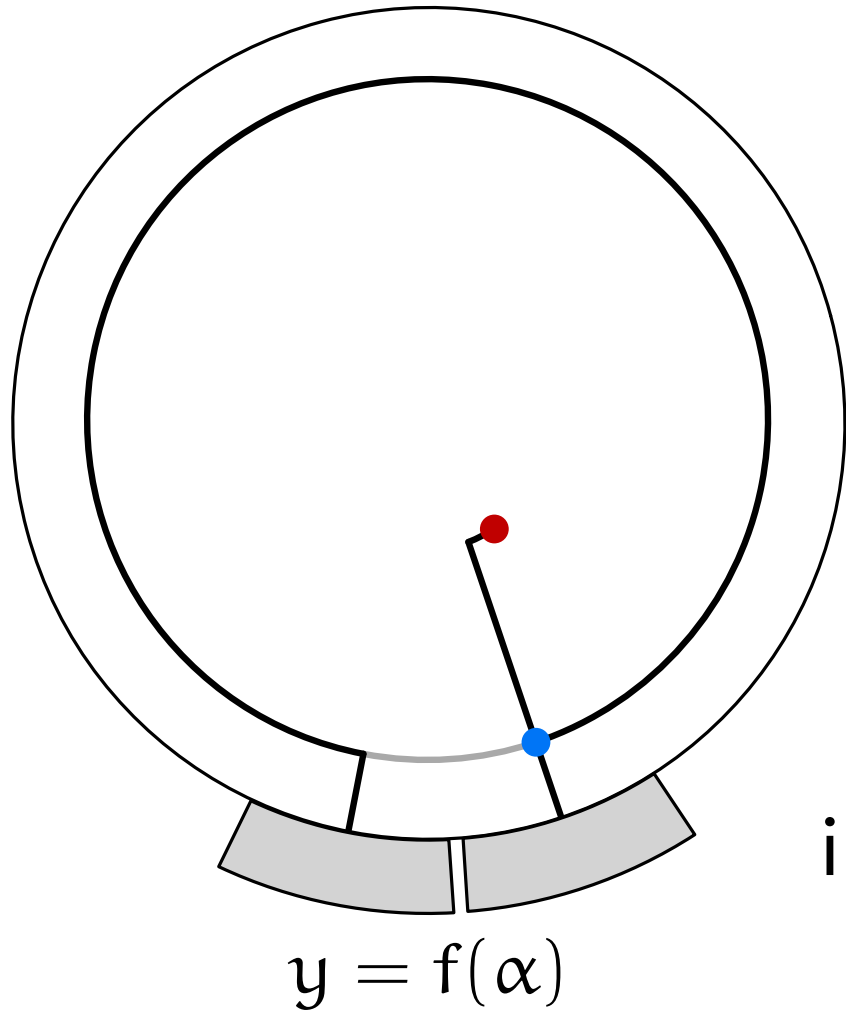
Rotating the Labels



innermost:

- always shorter path

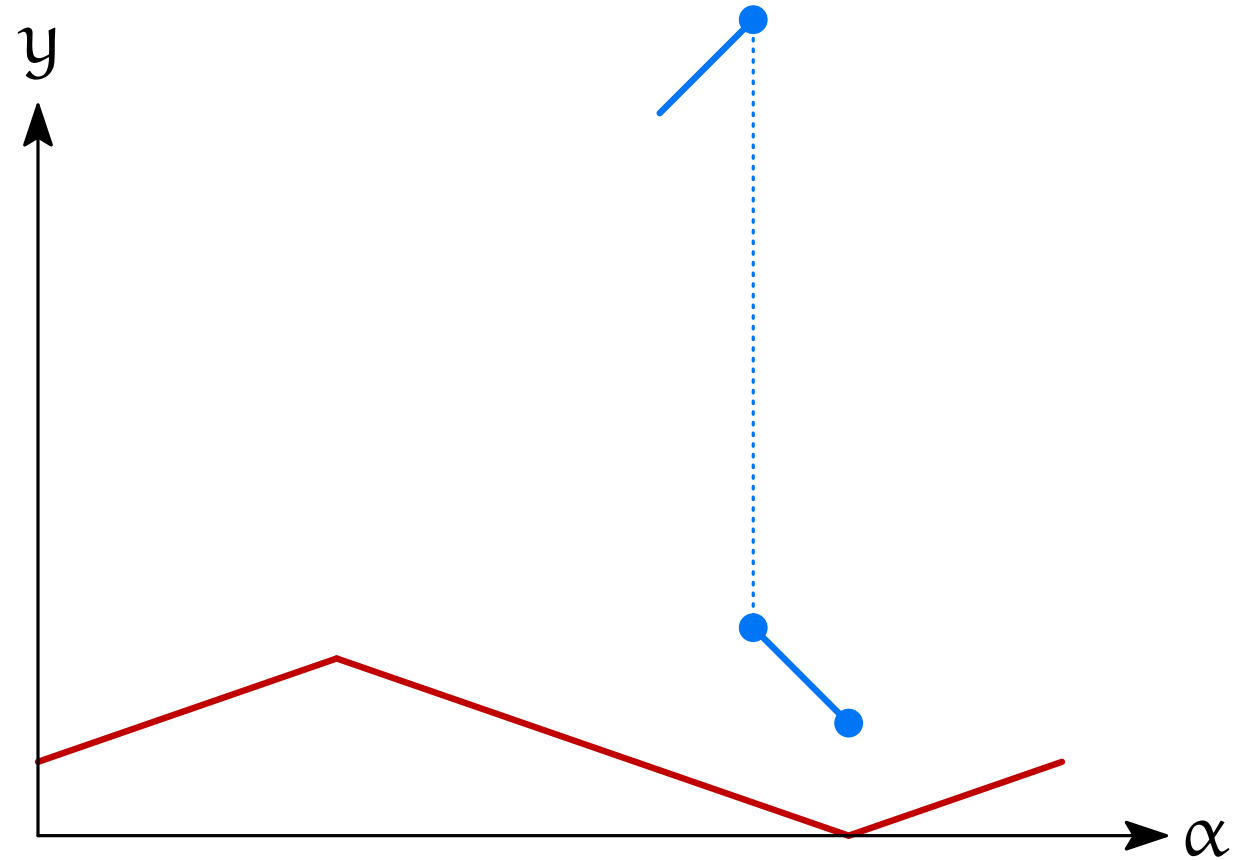
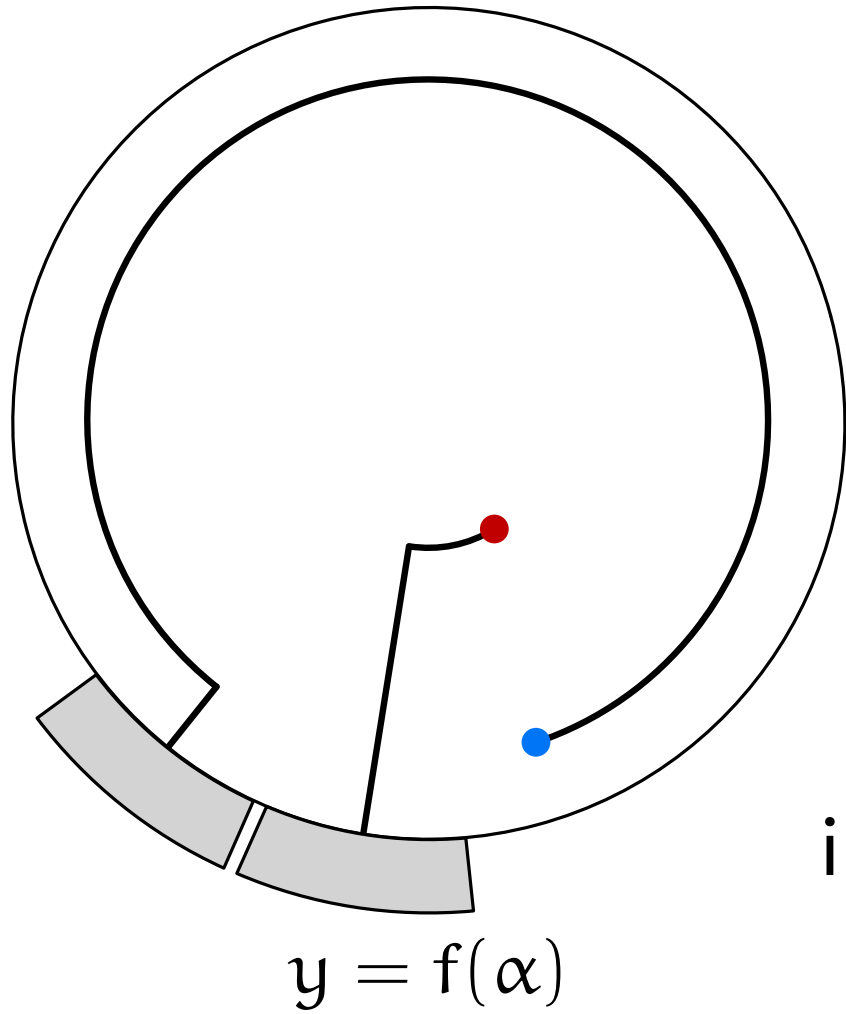
Rotating the Labels



innermost:

- always shorter path

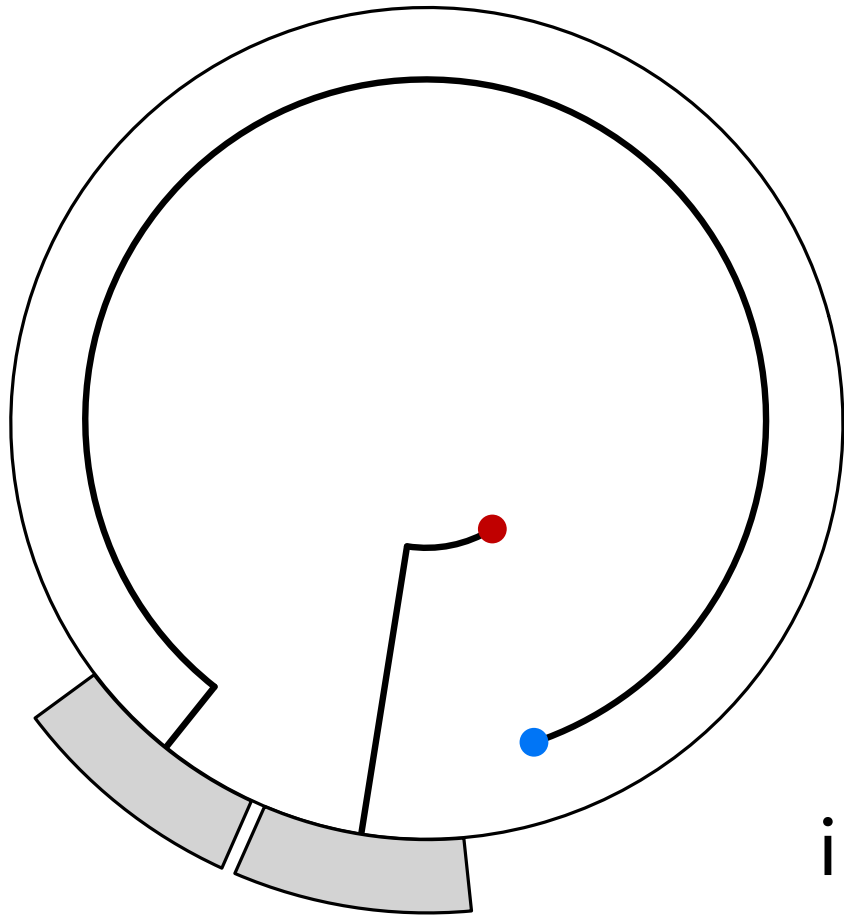
Rotating the Labels



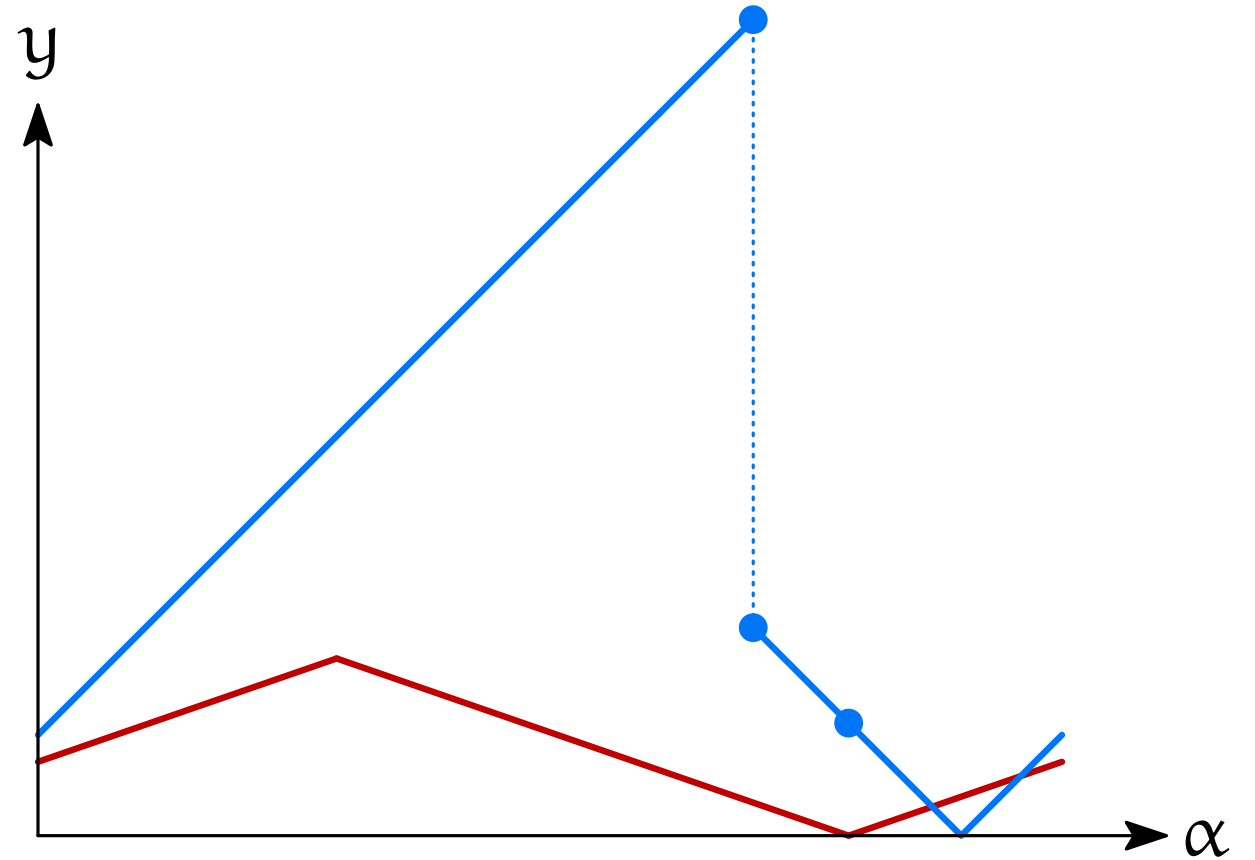
innermost:

- always shorter path

Rotating the Labels



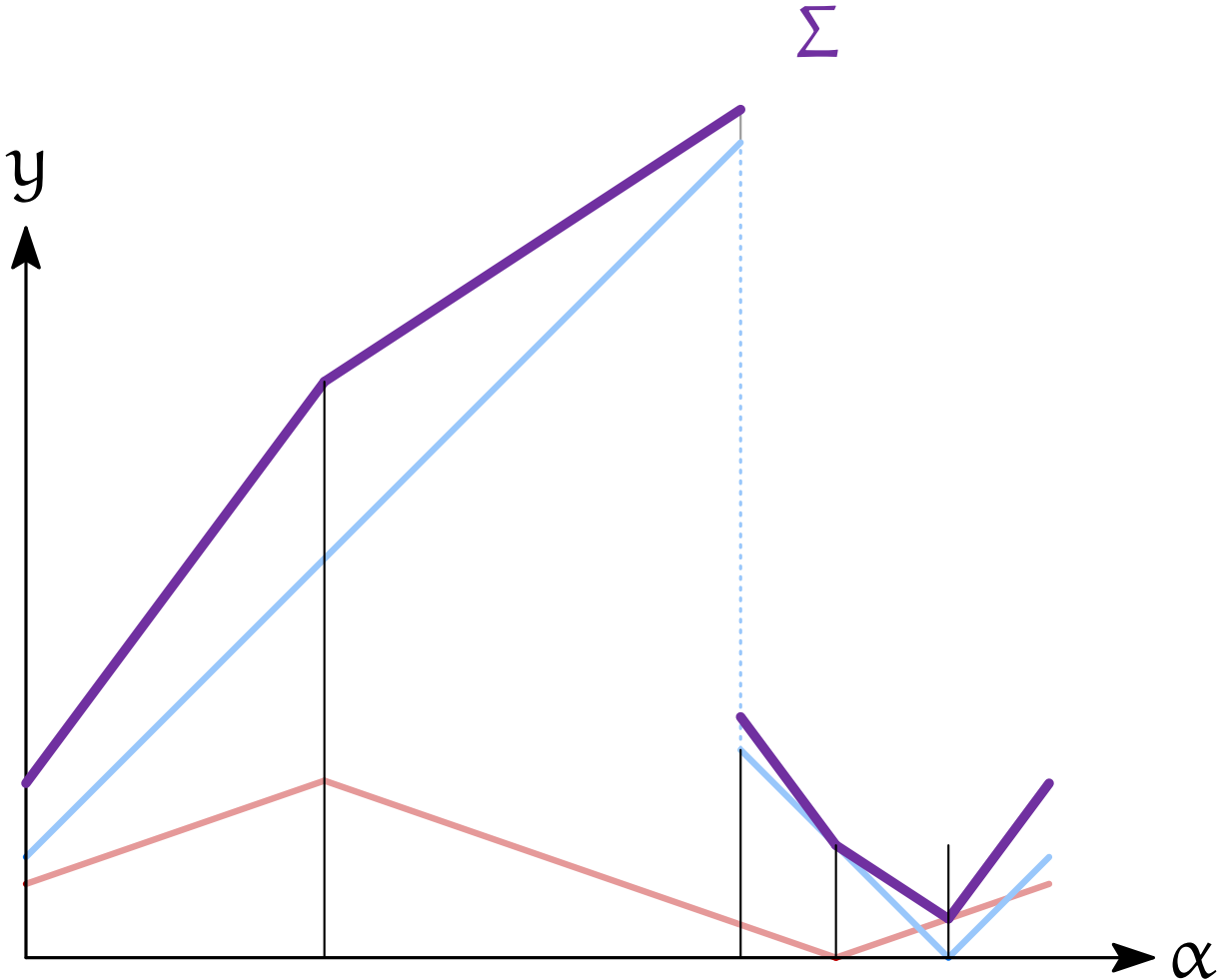
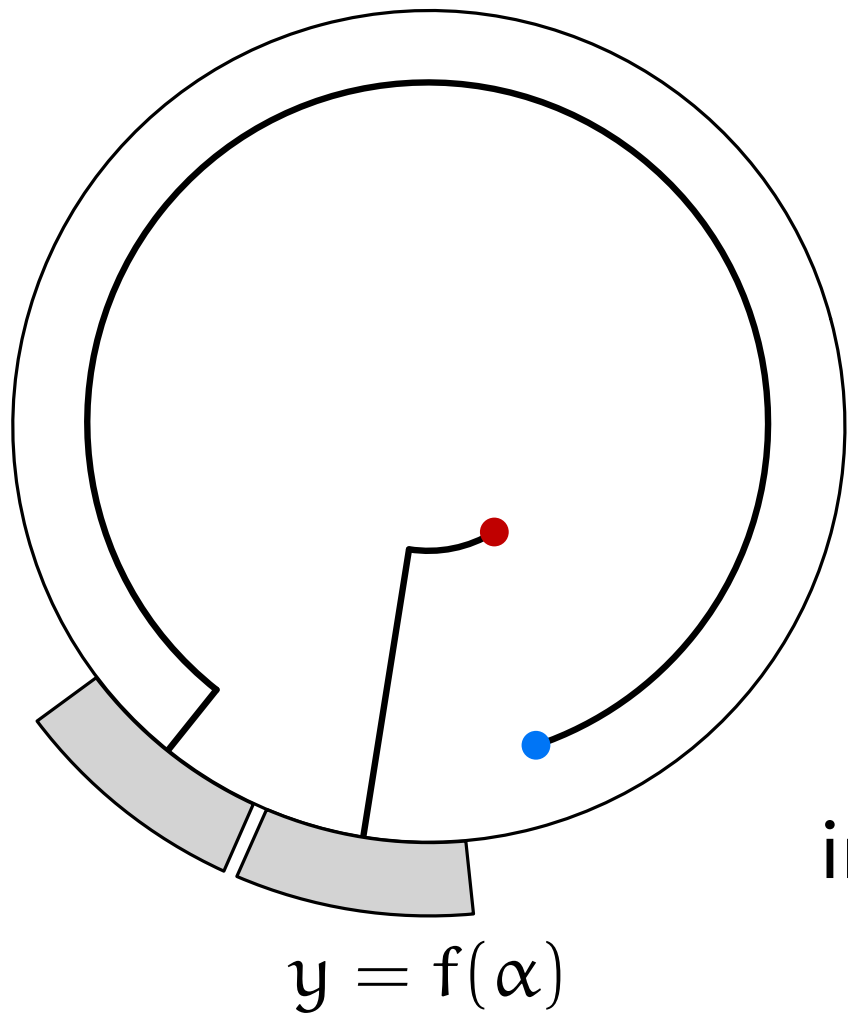
$$y = f(\alpha)$$



innermost:

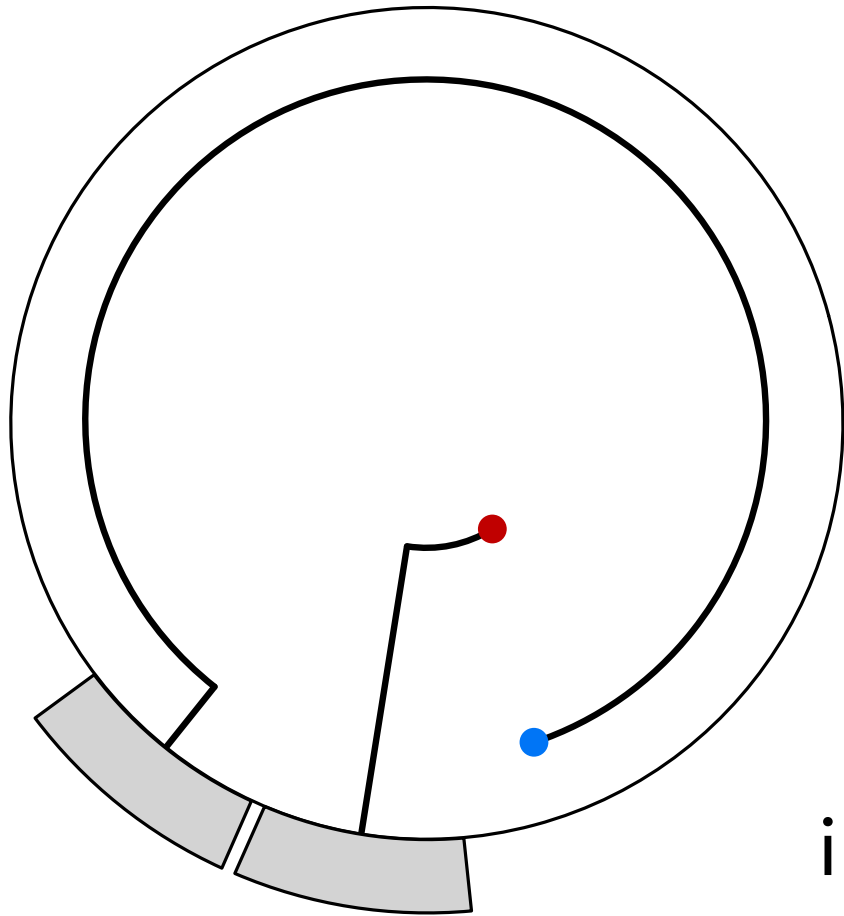
- always shorter path

Rotating the Labels

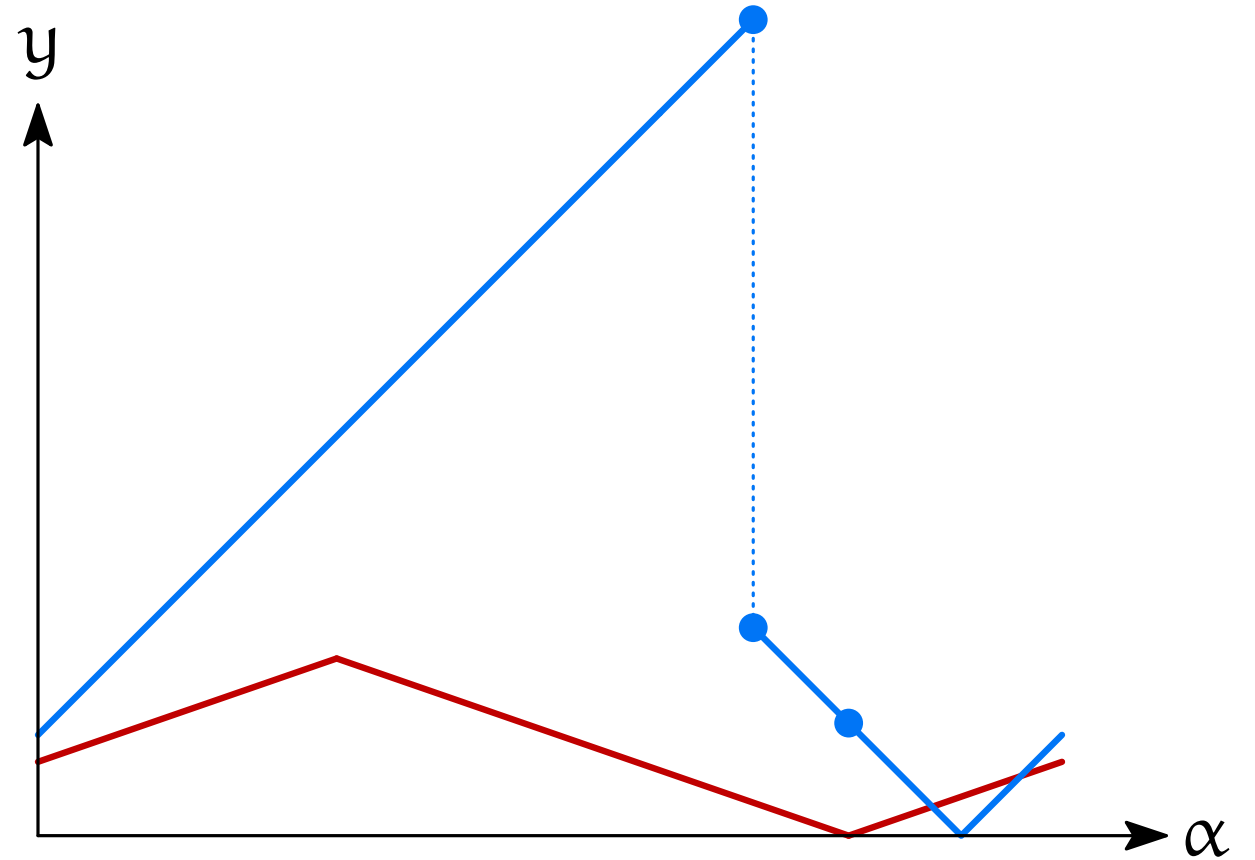


innermost:
• always shorter path

Rotating the Labels



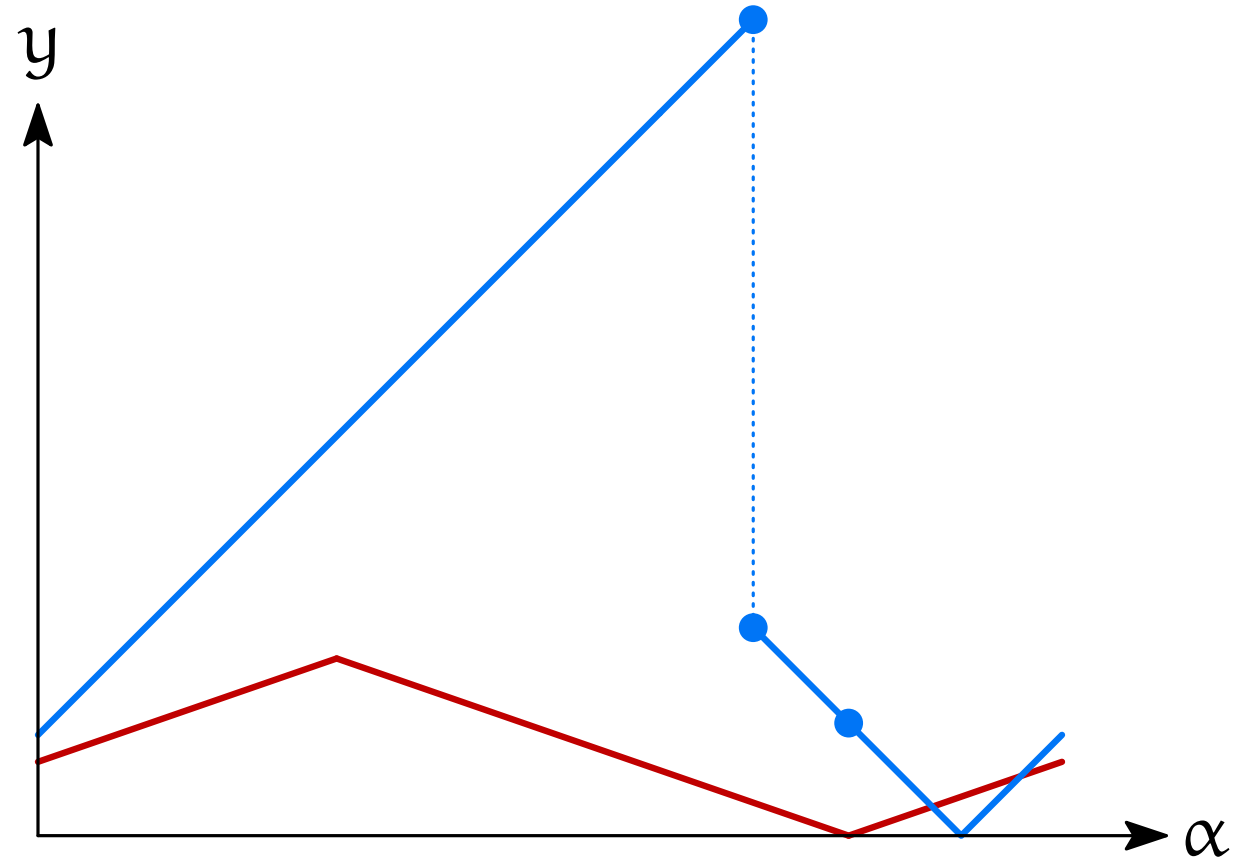
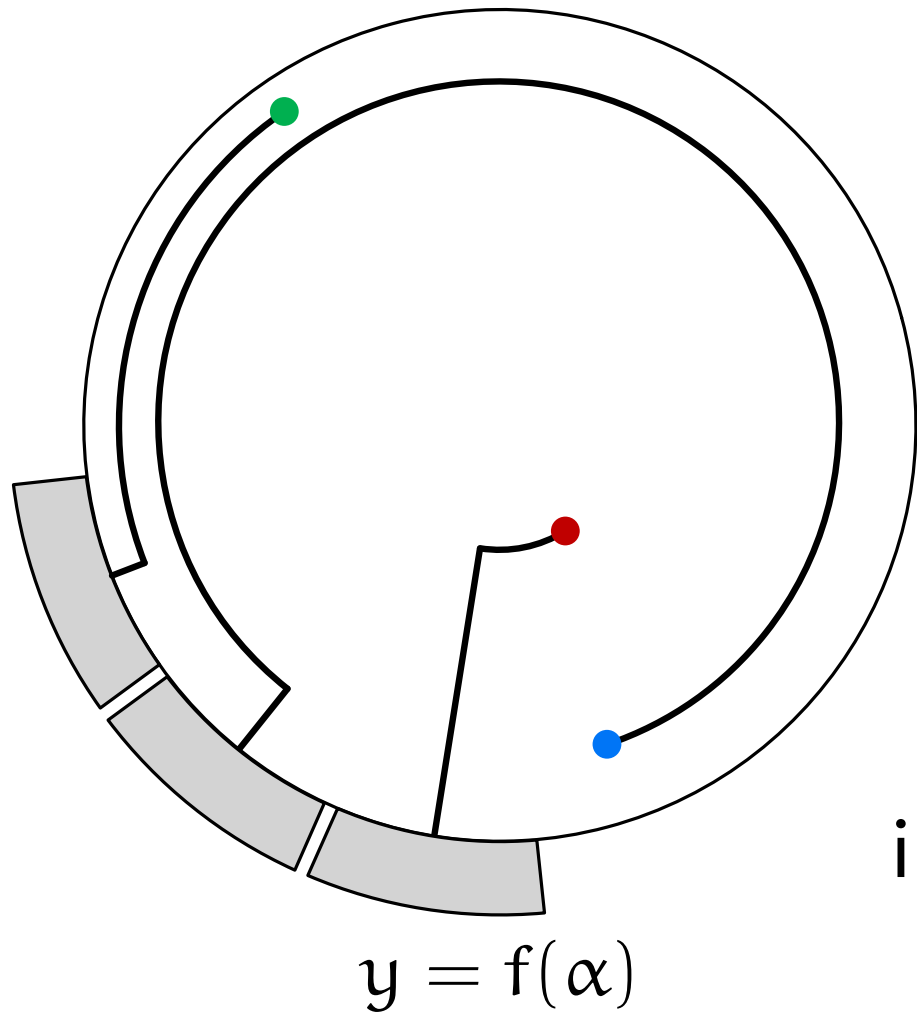
$$y = f(\alpha)$$



innermost:

- always shorter path
- decides all other leader directions

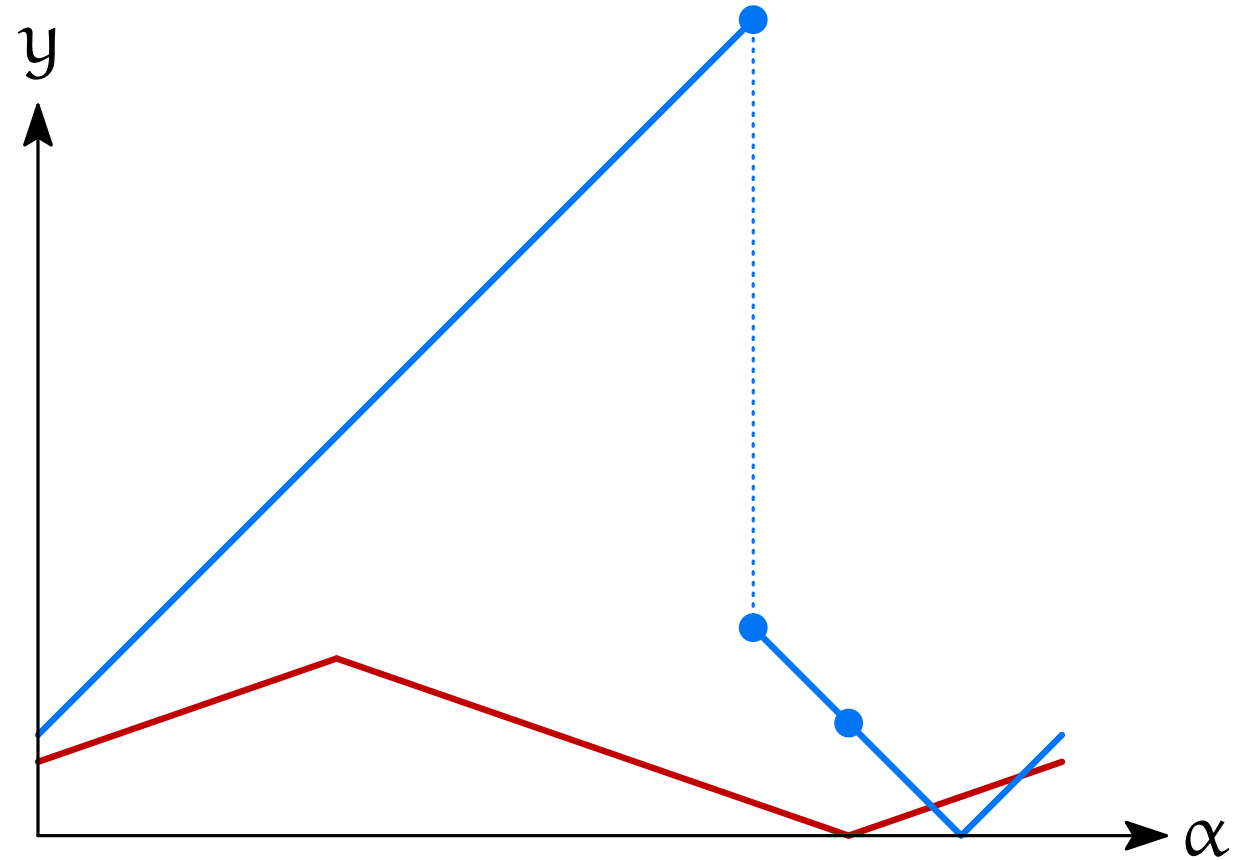
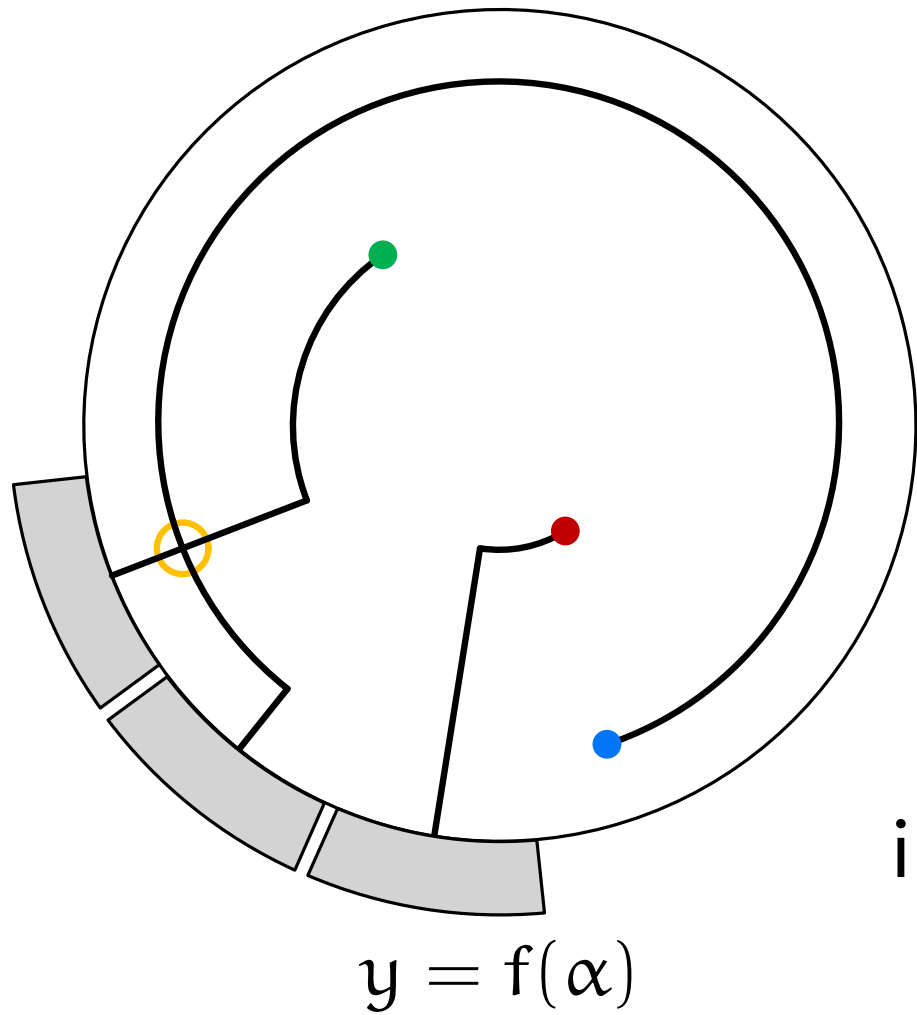
Rotating the Labels



innermost:

- always shorter path
- decides all other leader directions

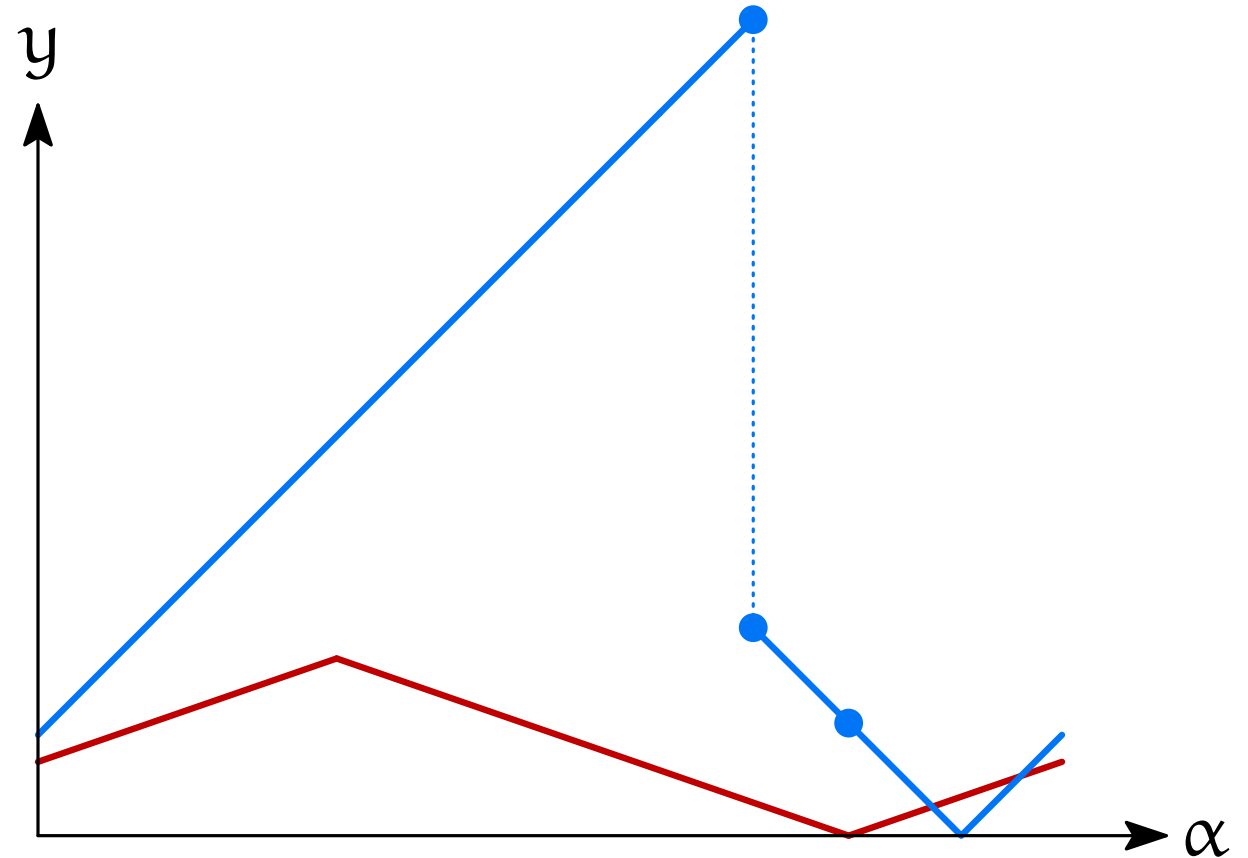
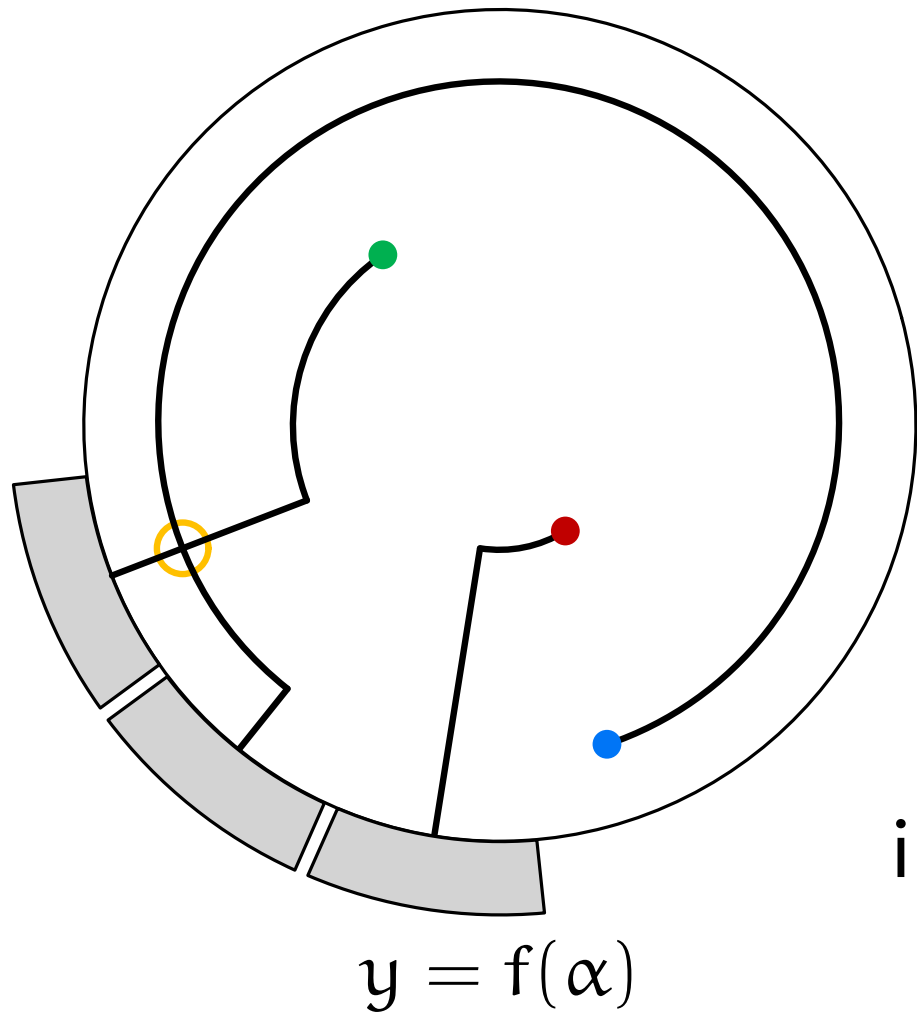
Rotating the Labels



innermost:

- always shorter path
- decides all other leader directions

Rotating the Labels

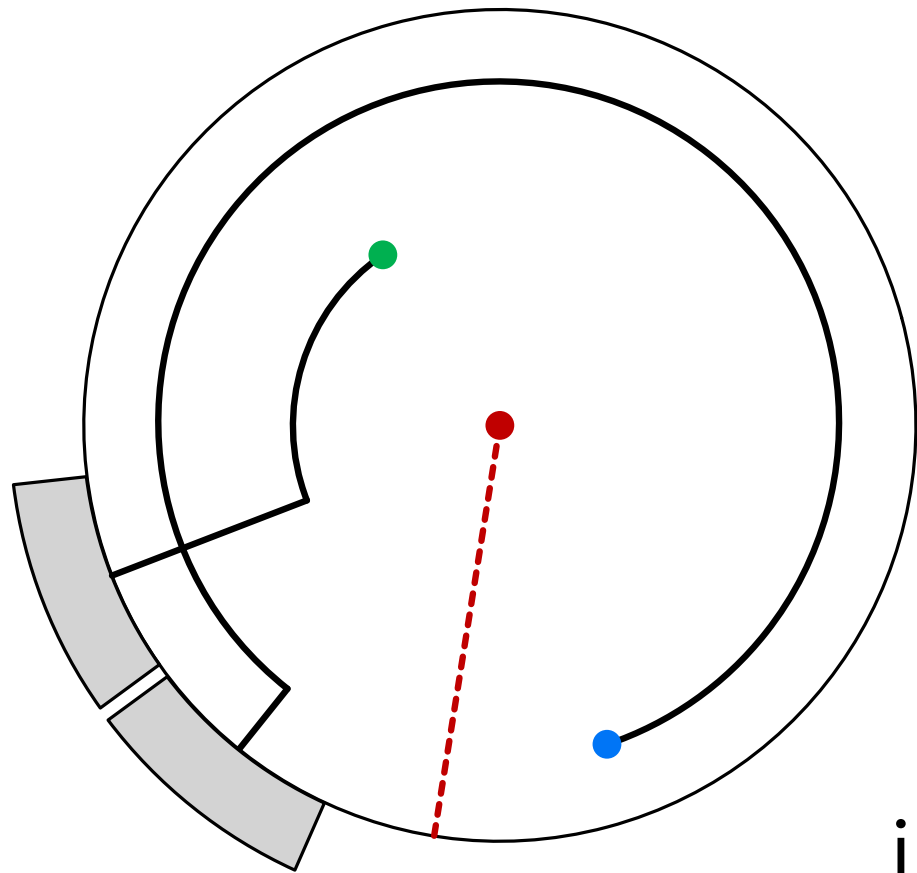


innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**

Rotating the Labels

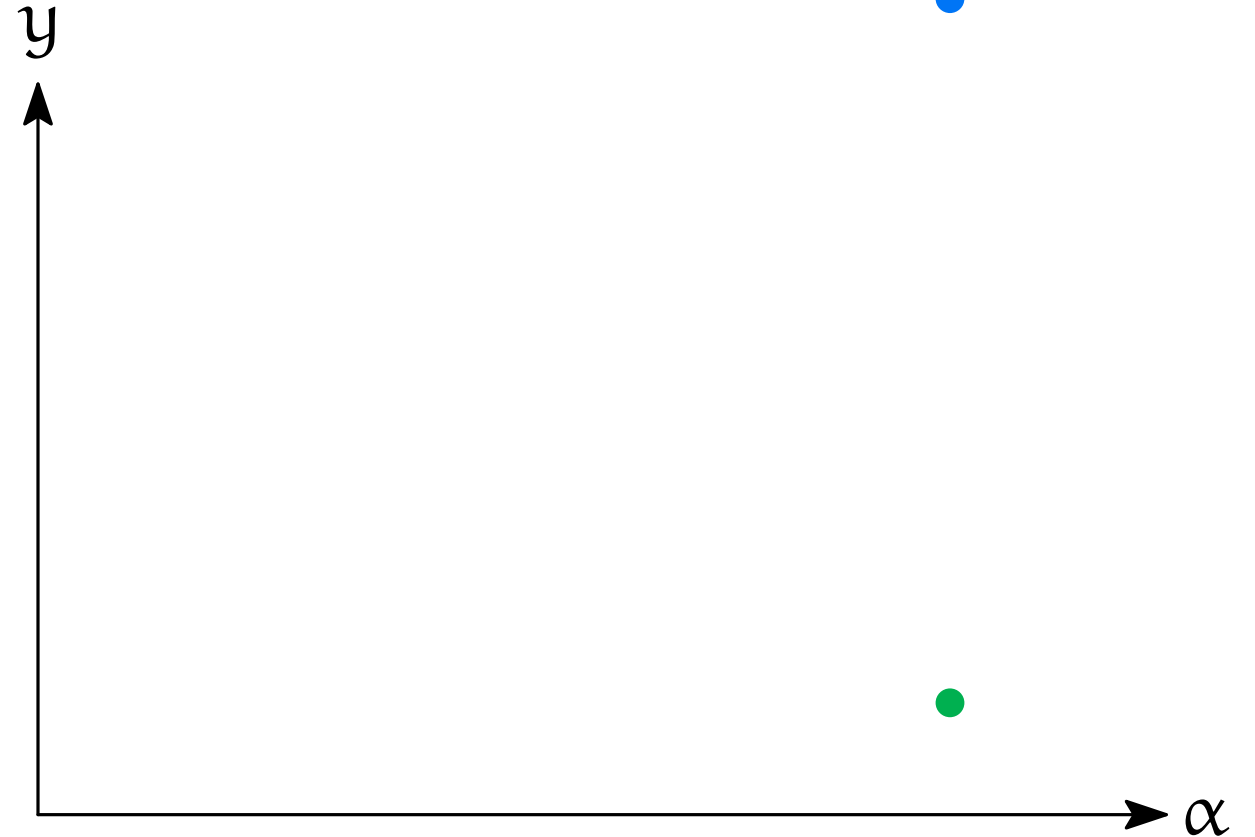


$$y = f(\alpha)$$

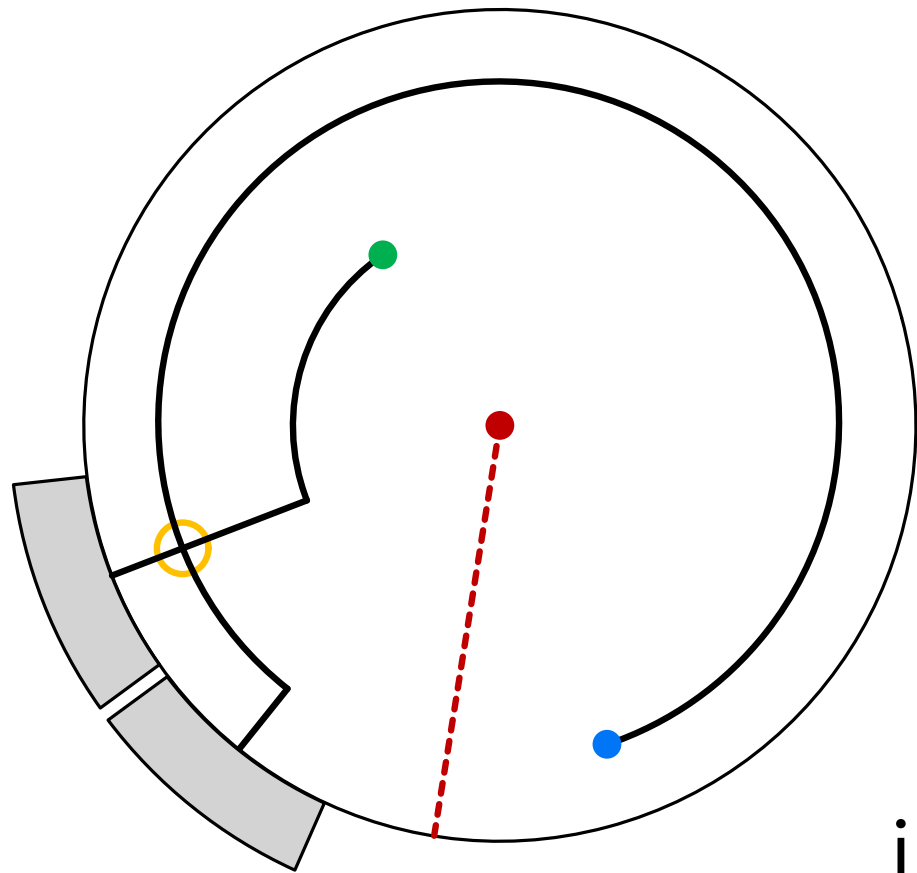
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

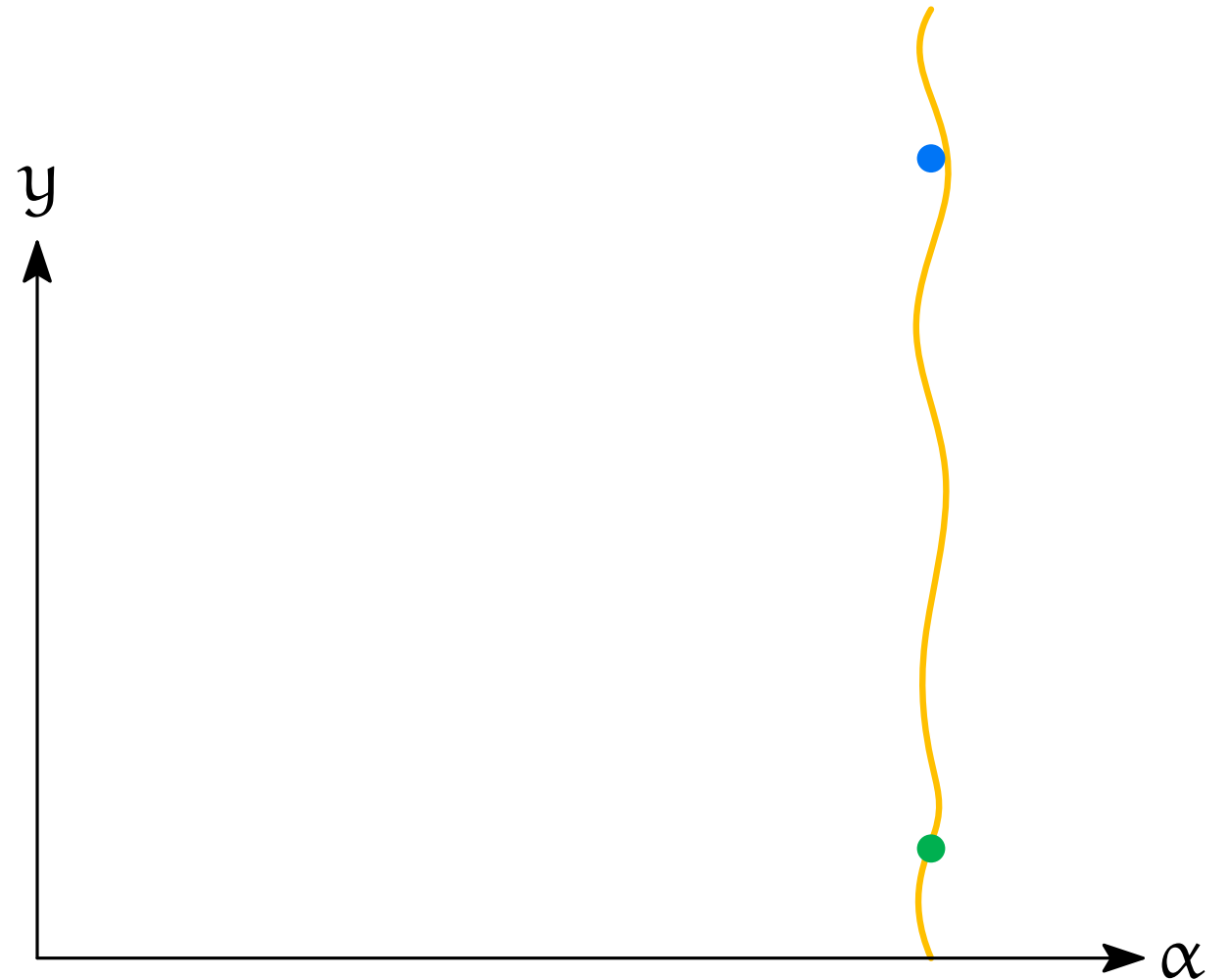


$$y = f(\alpha)$$

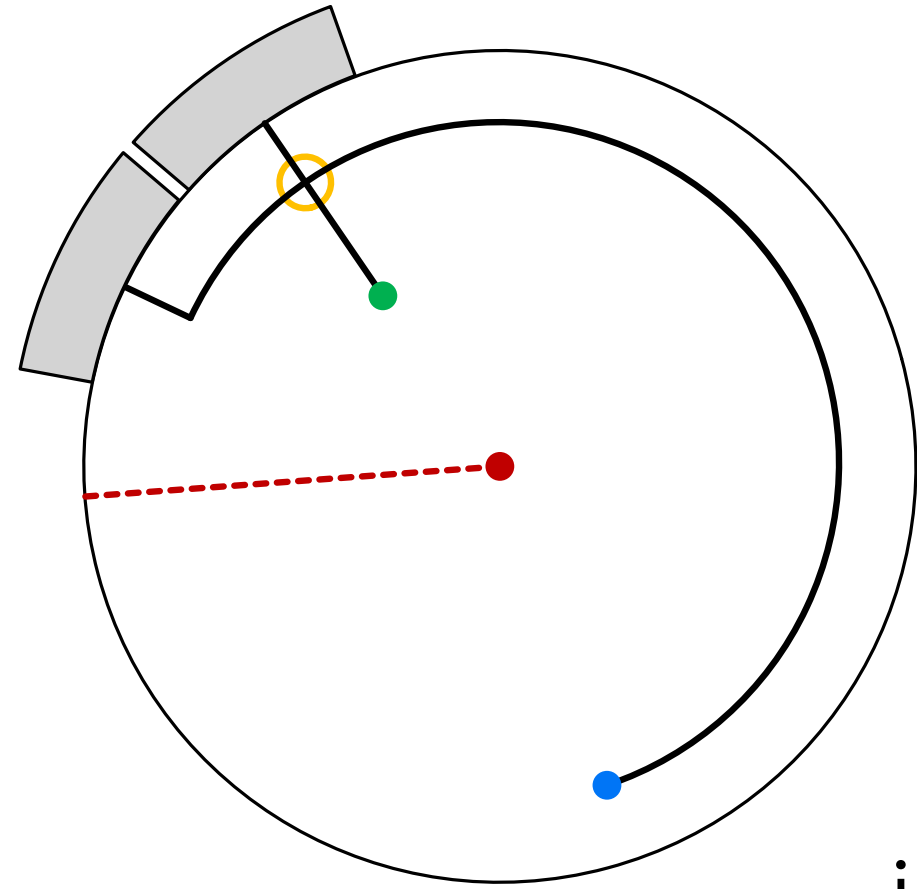
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

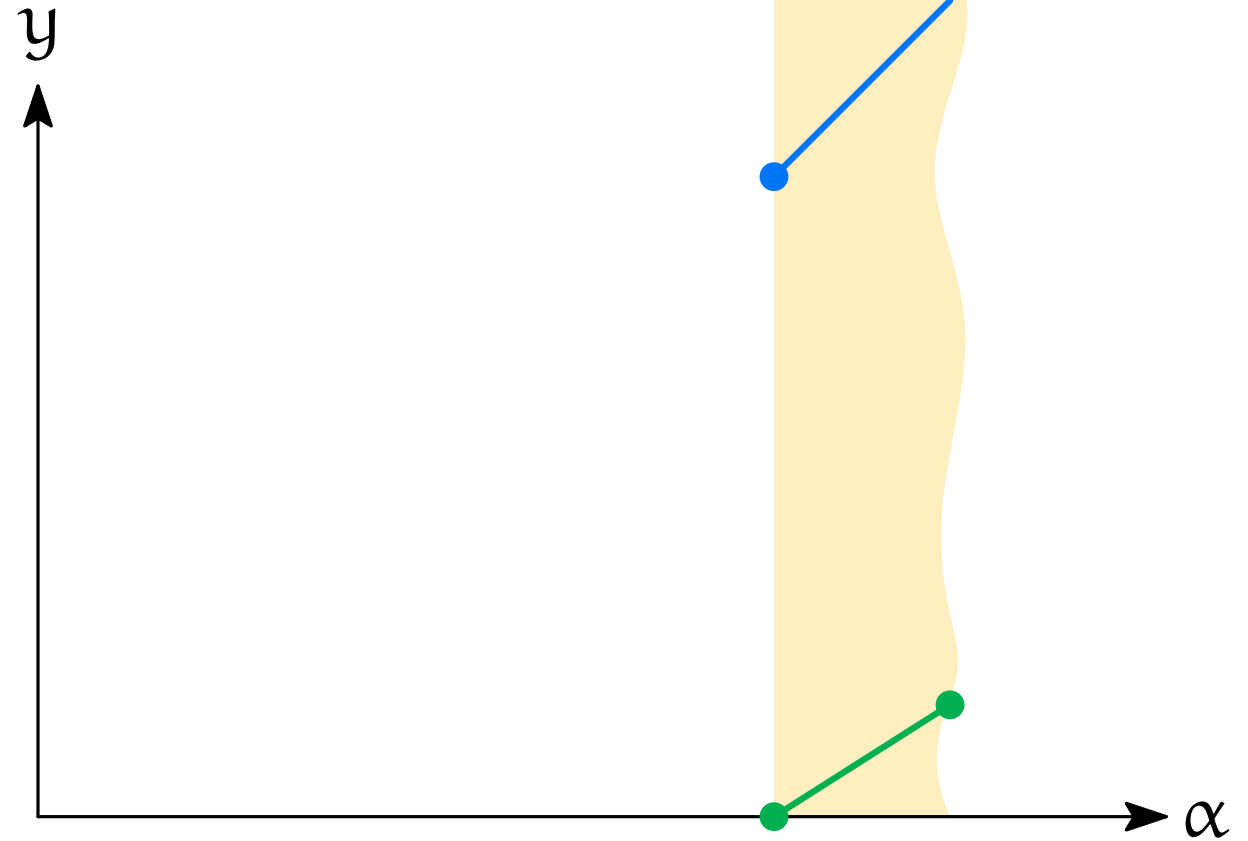


$$y = f(\alpha)$$

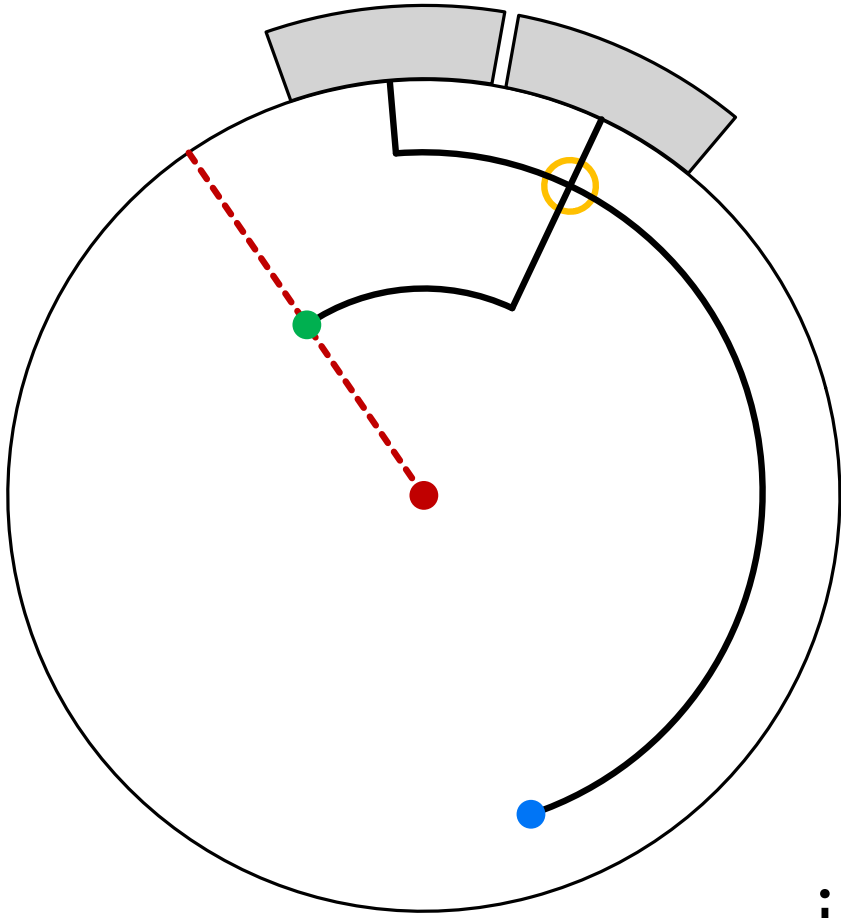
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

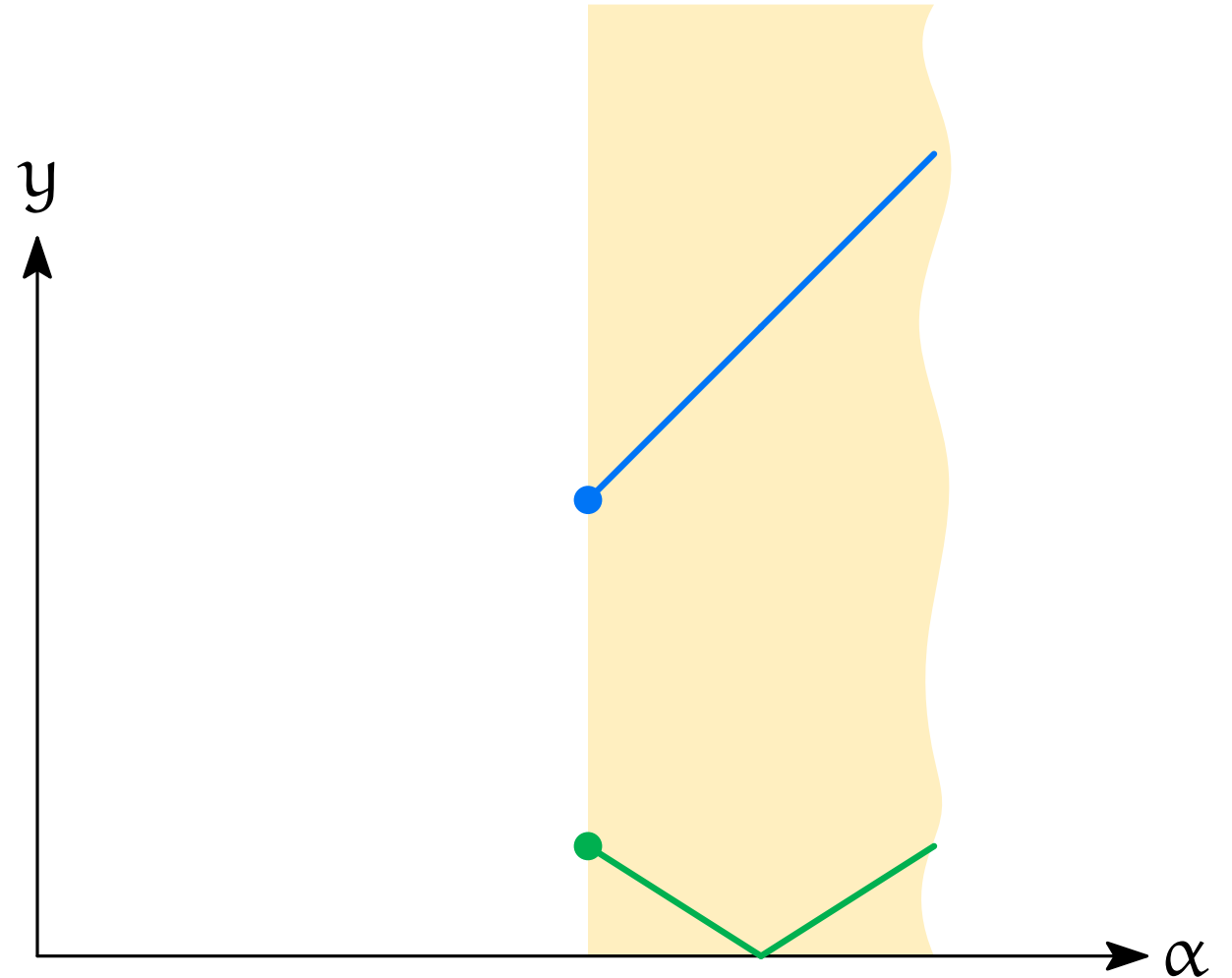


$$y = f(\alpha)$$

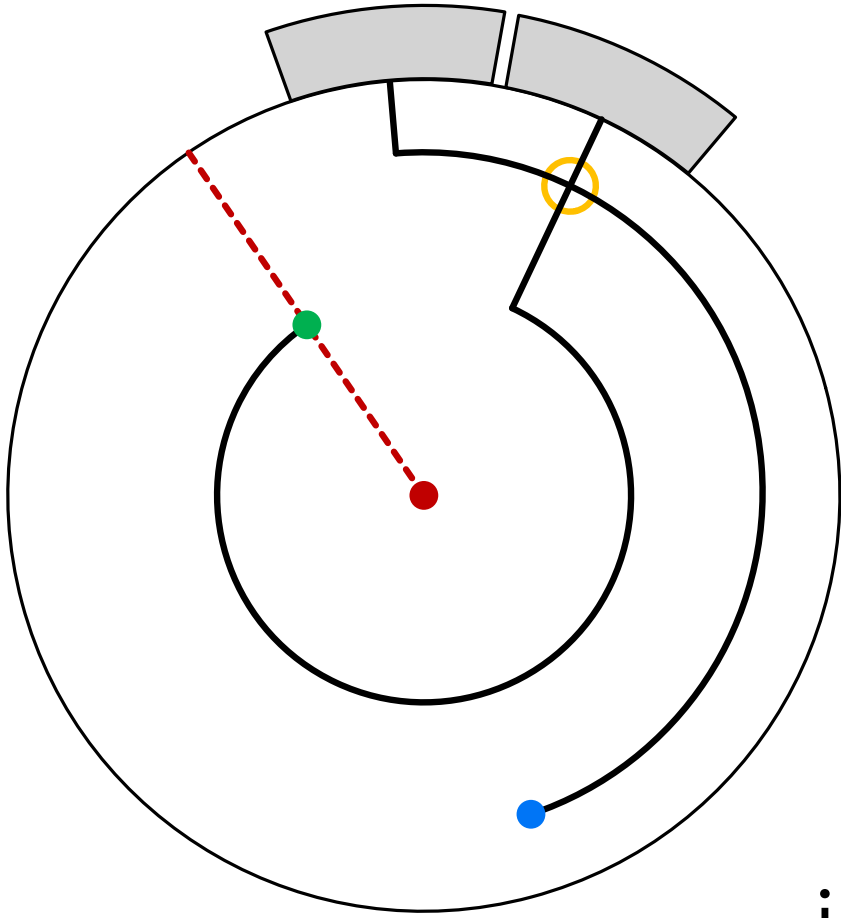
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

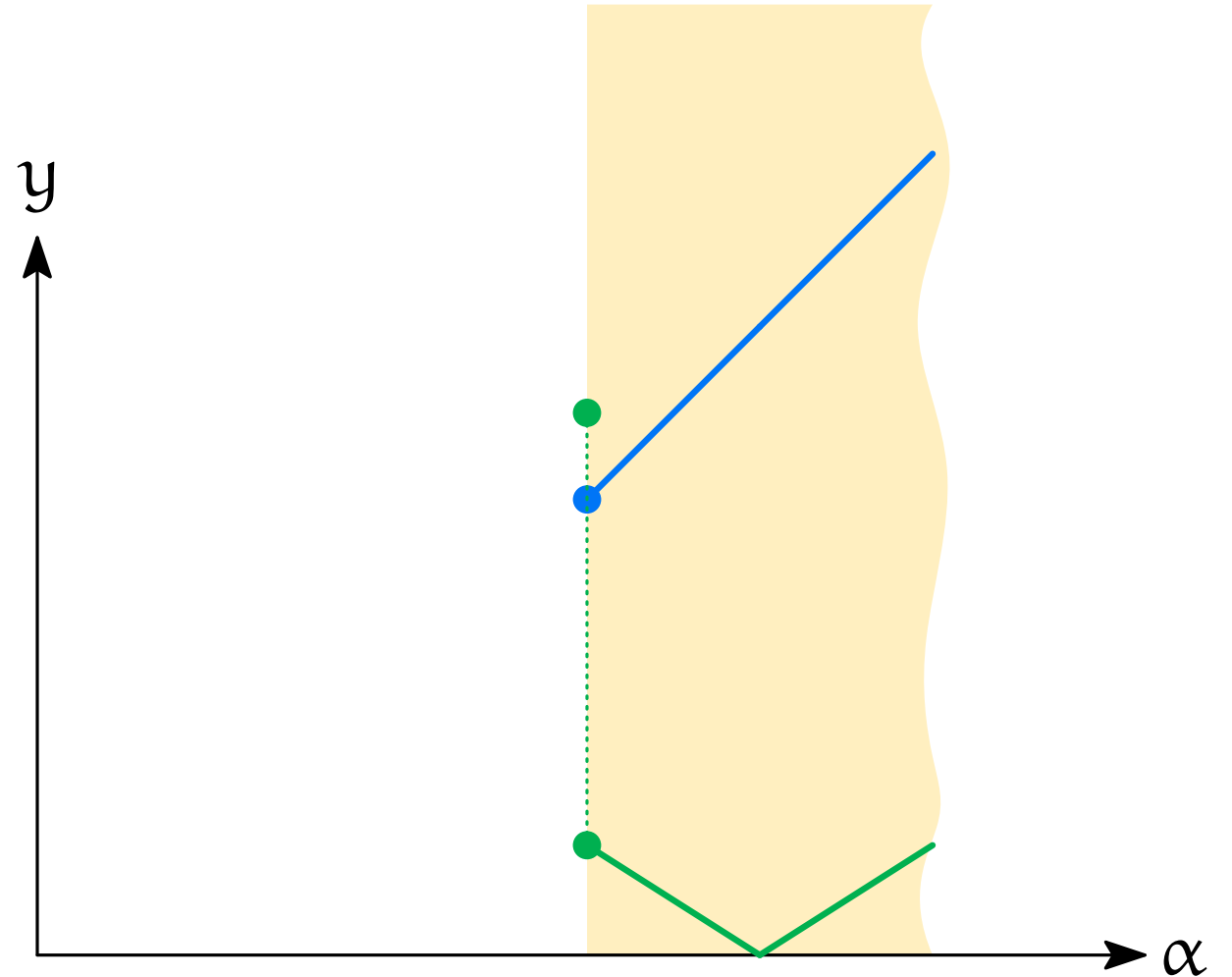


$$y = f(\alpha)$$

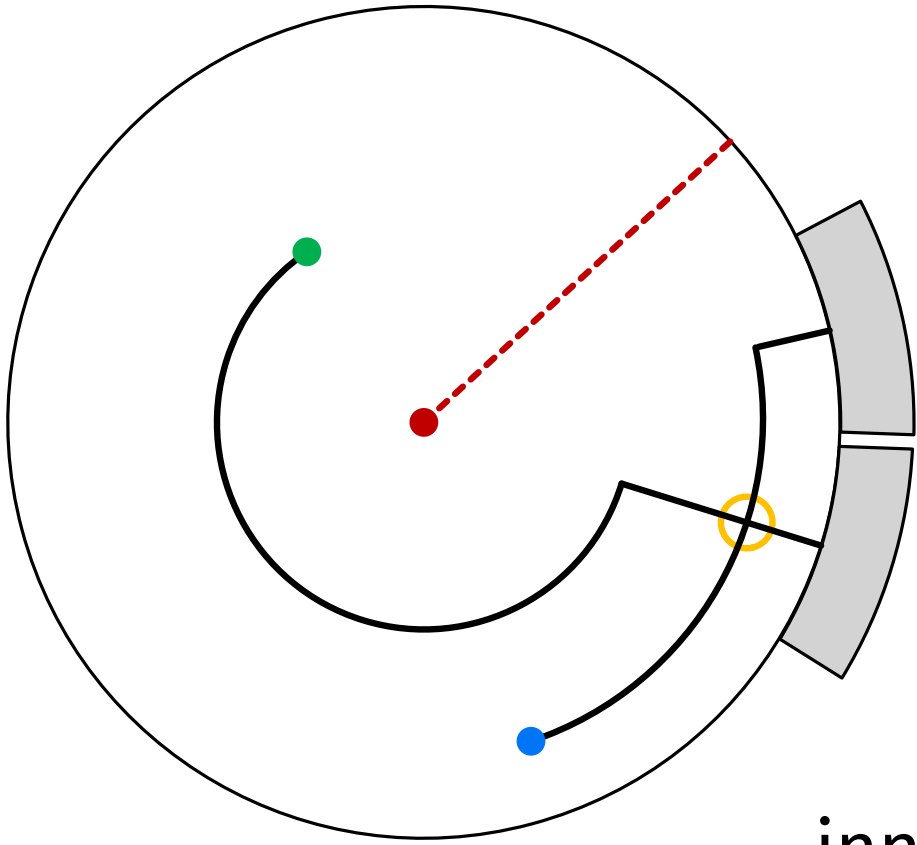
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

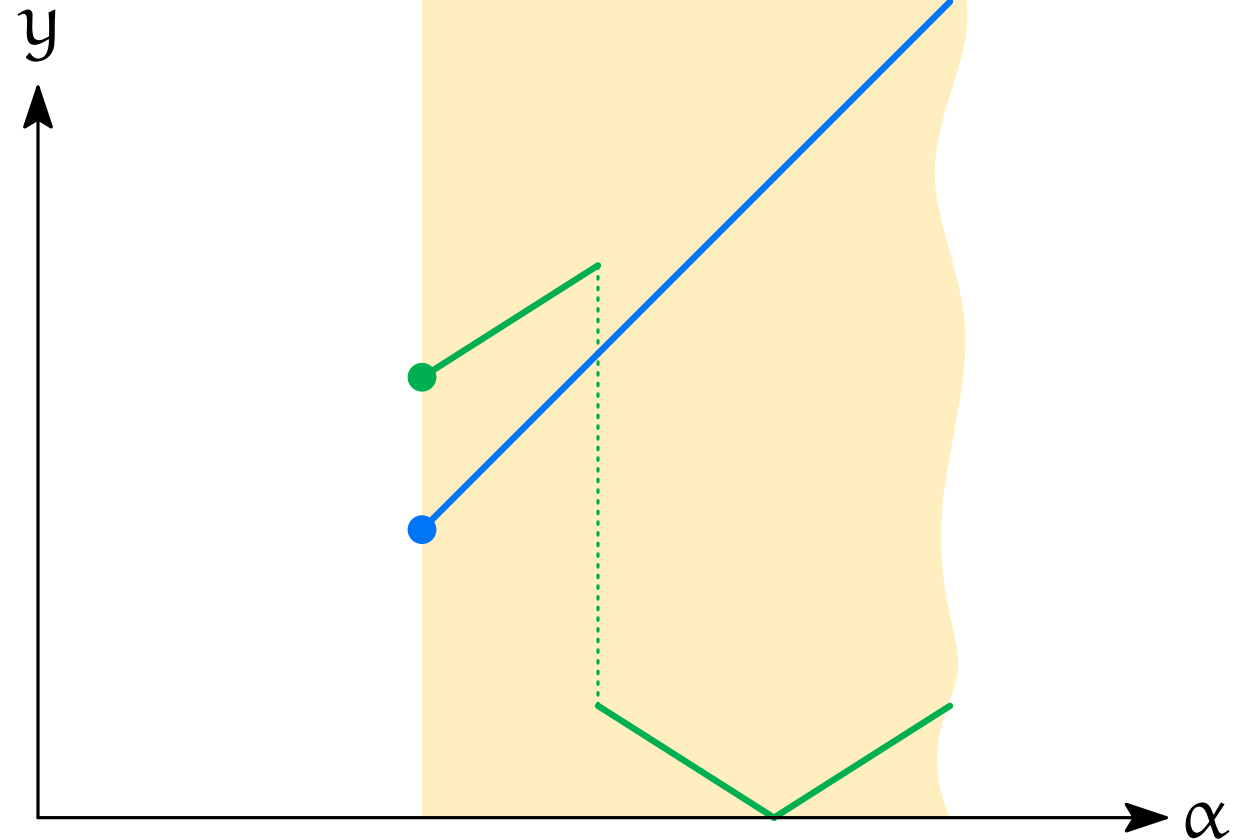


$$y = f(\alpha)$$

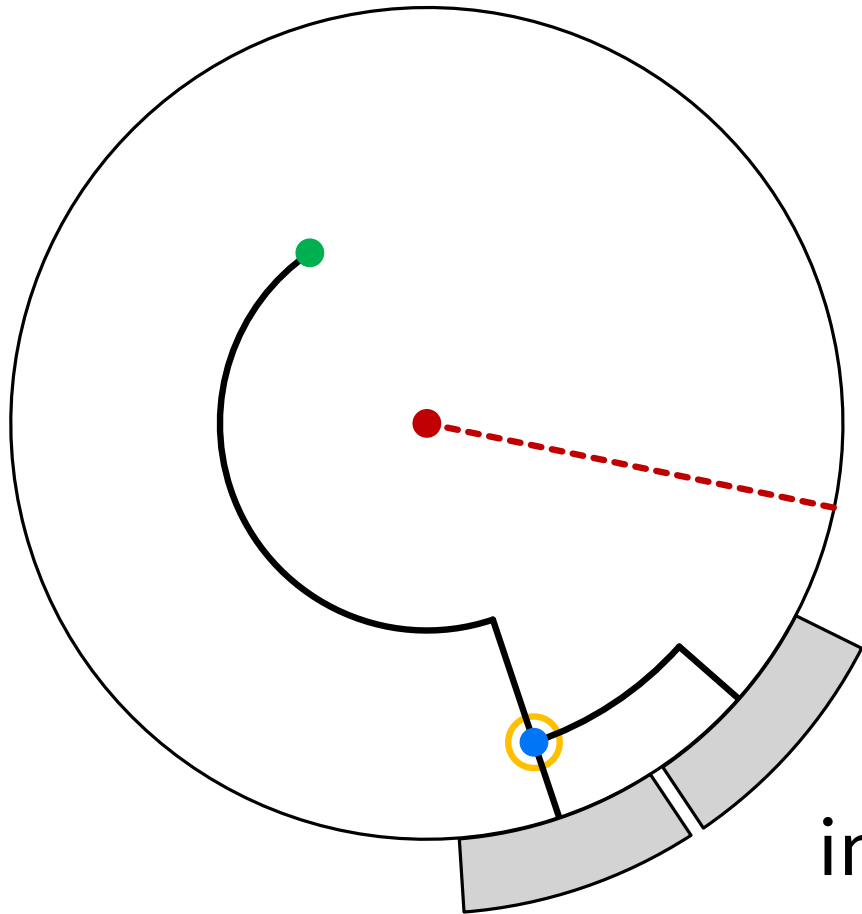
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

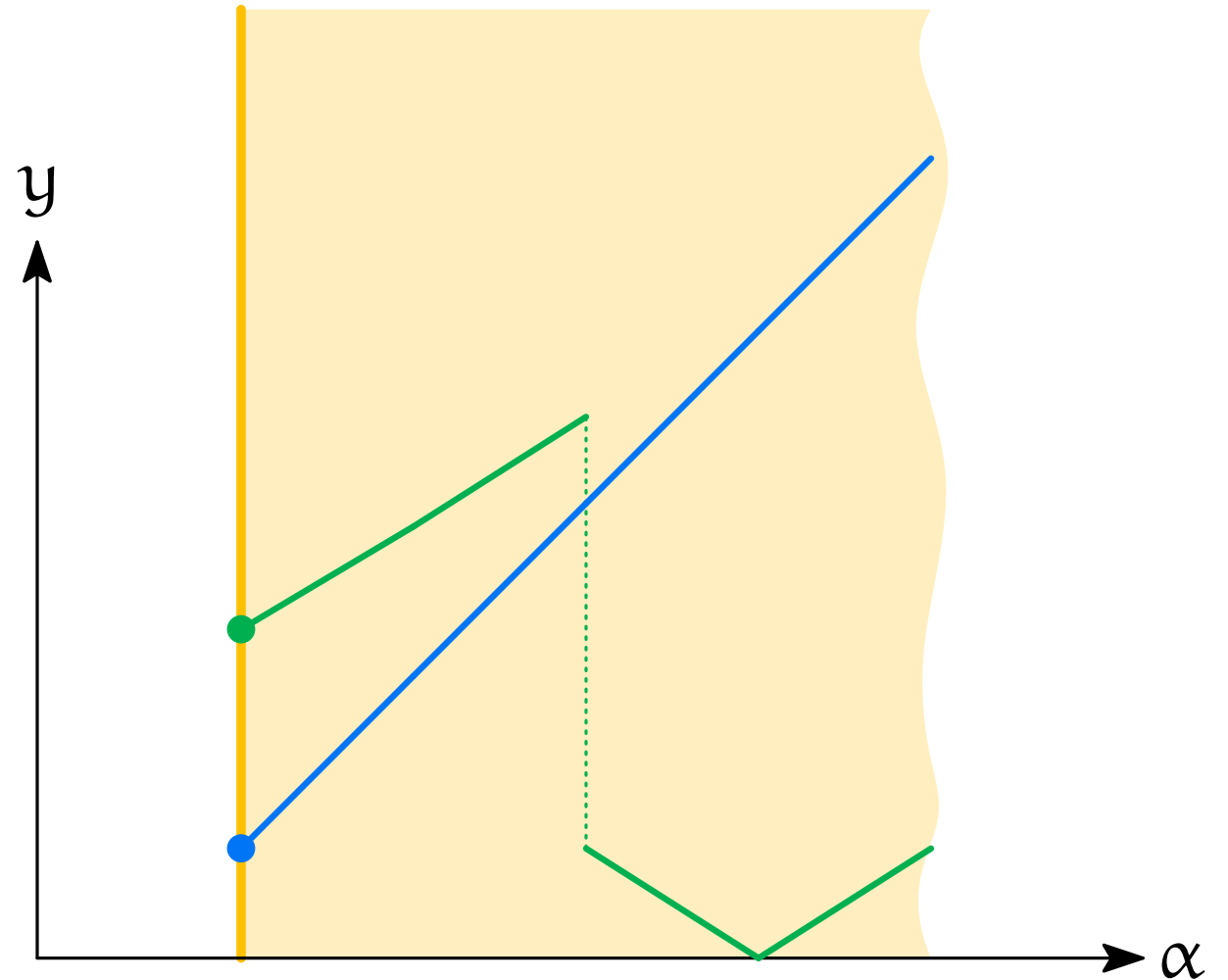


$$y = f(\alpha)$$

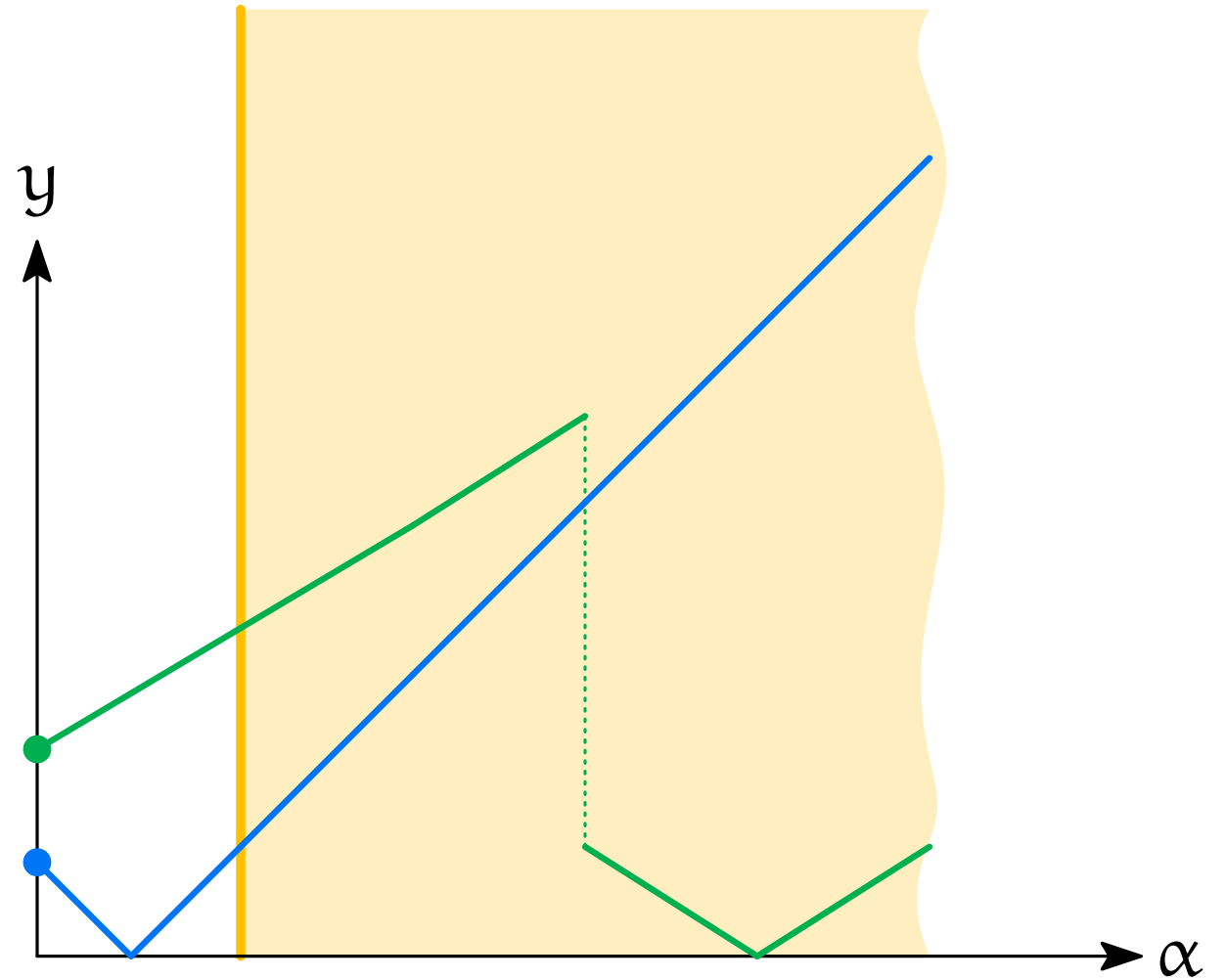
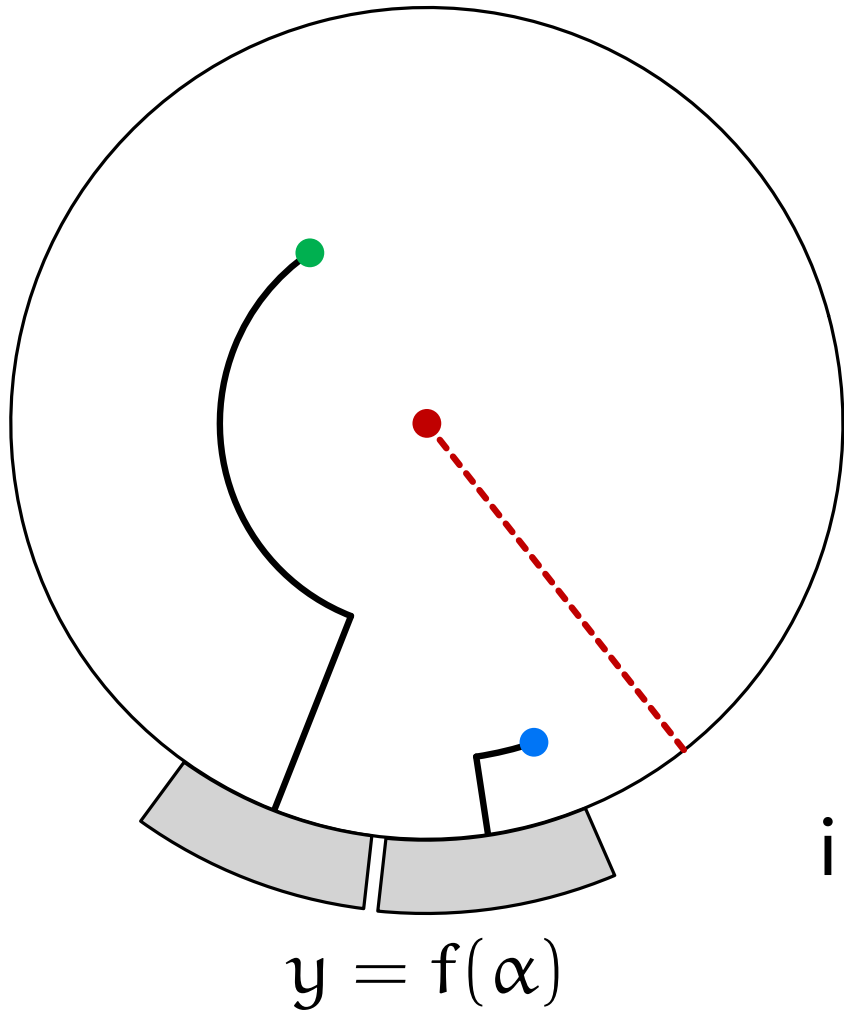
innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**



Rotating the Labels

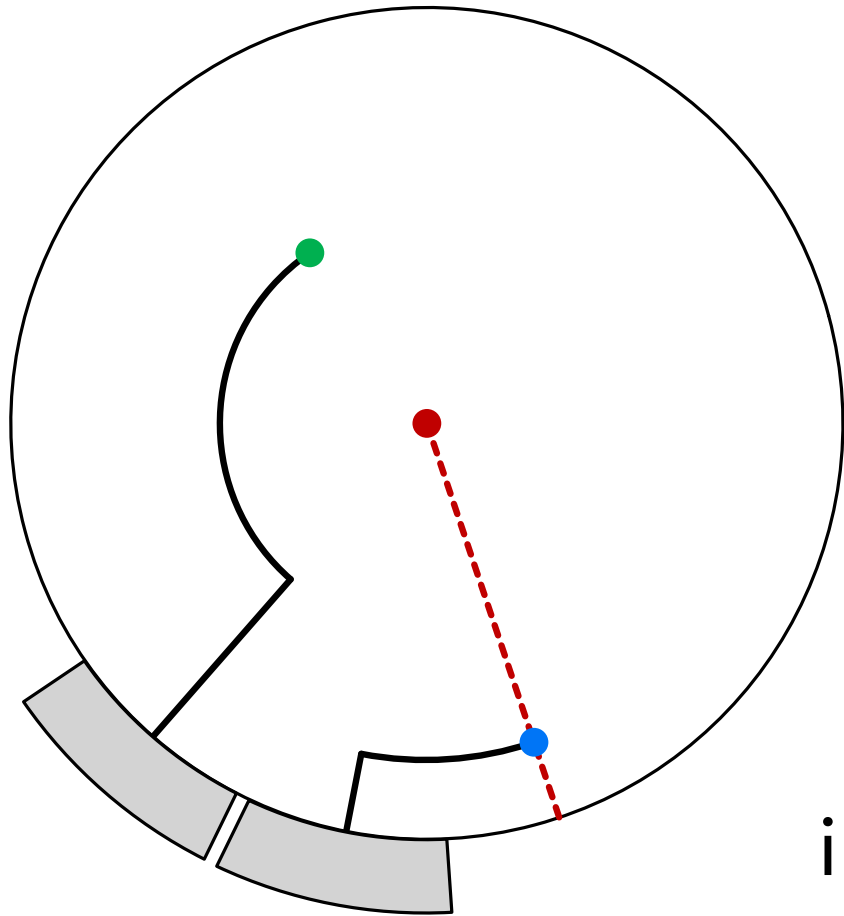


innermost:

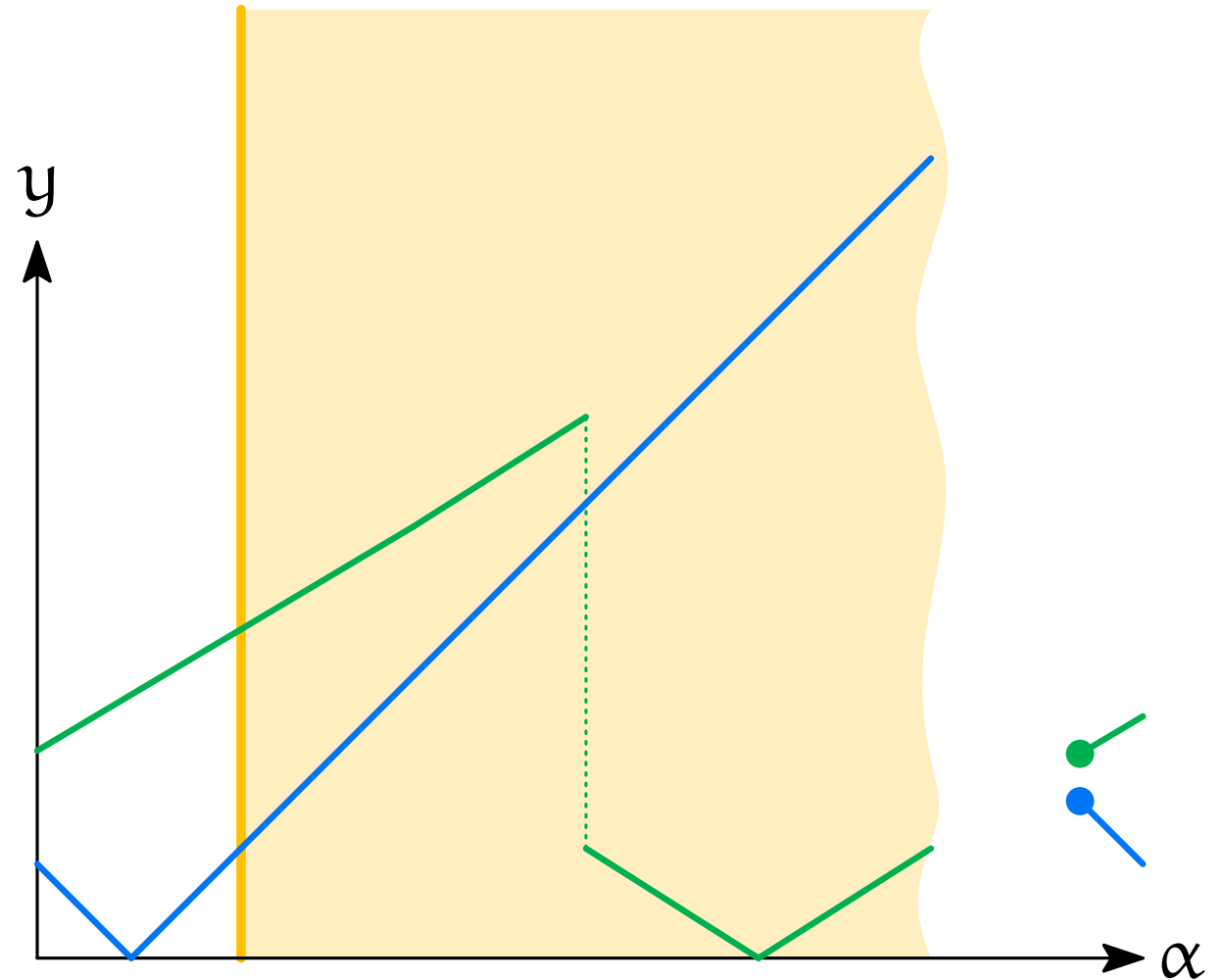
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**

Rotating the Labels



$$y = f(\alpha)$$

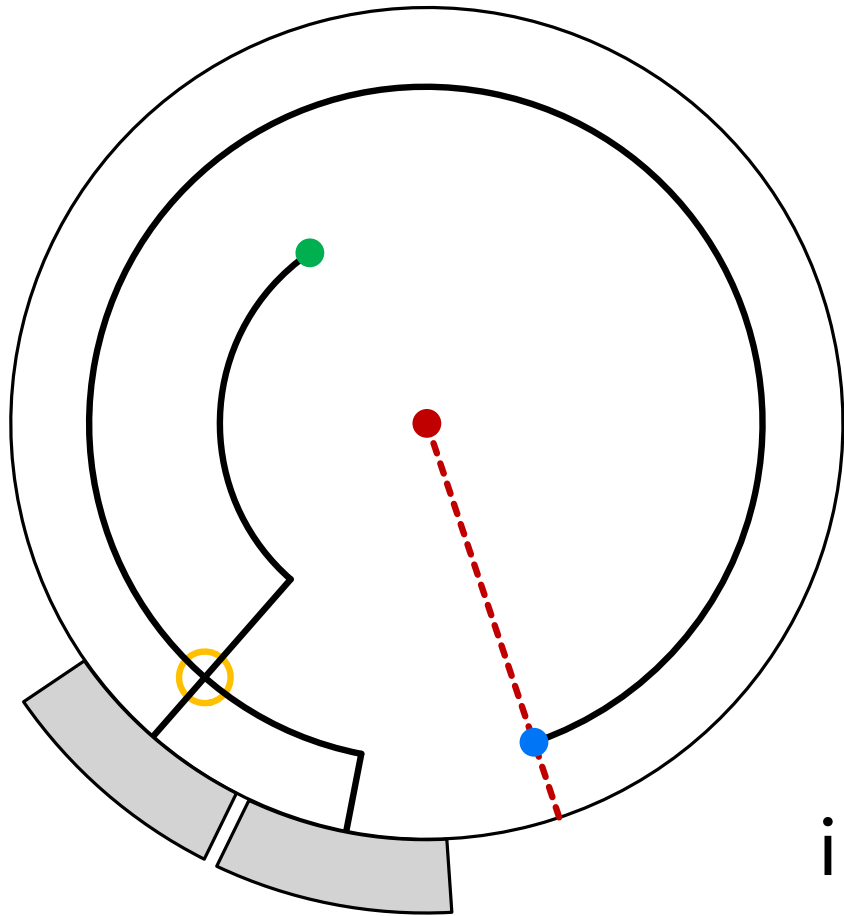


innermost:

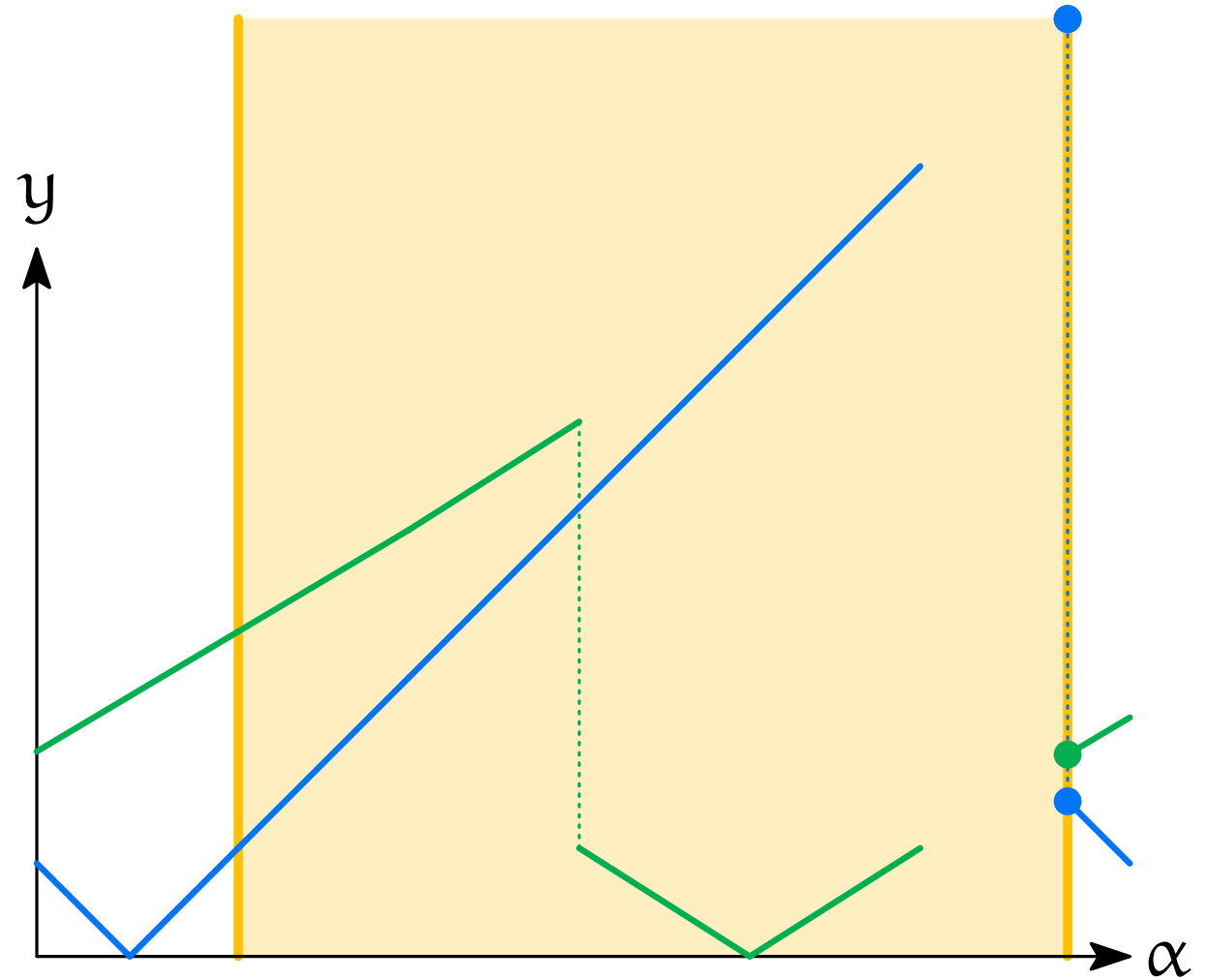
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**

Rotating the Labels



$$y = f(\alpha)$$

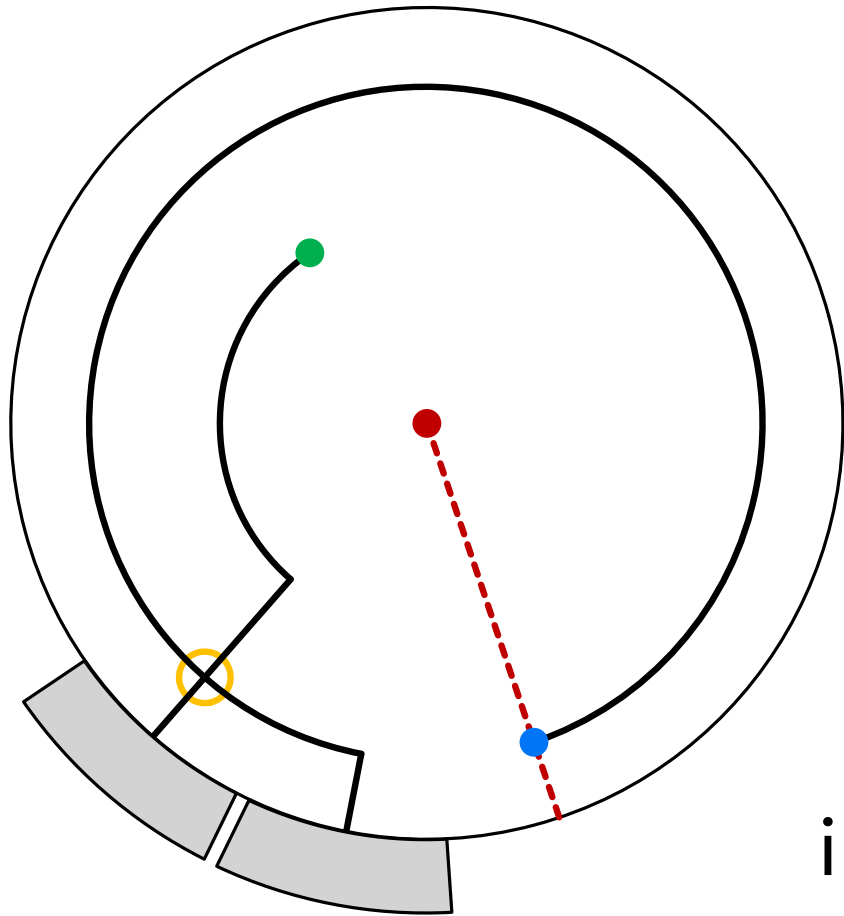


innermost:

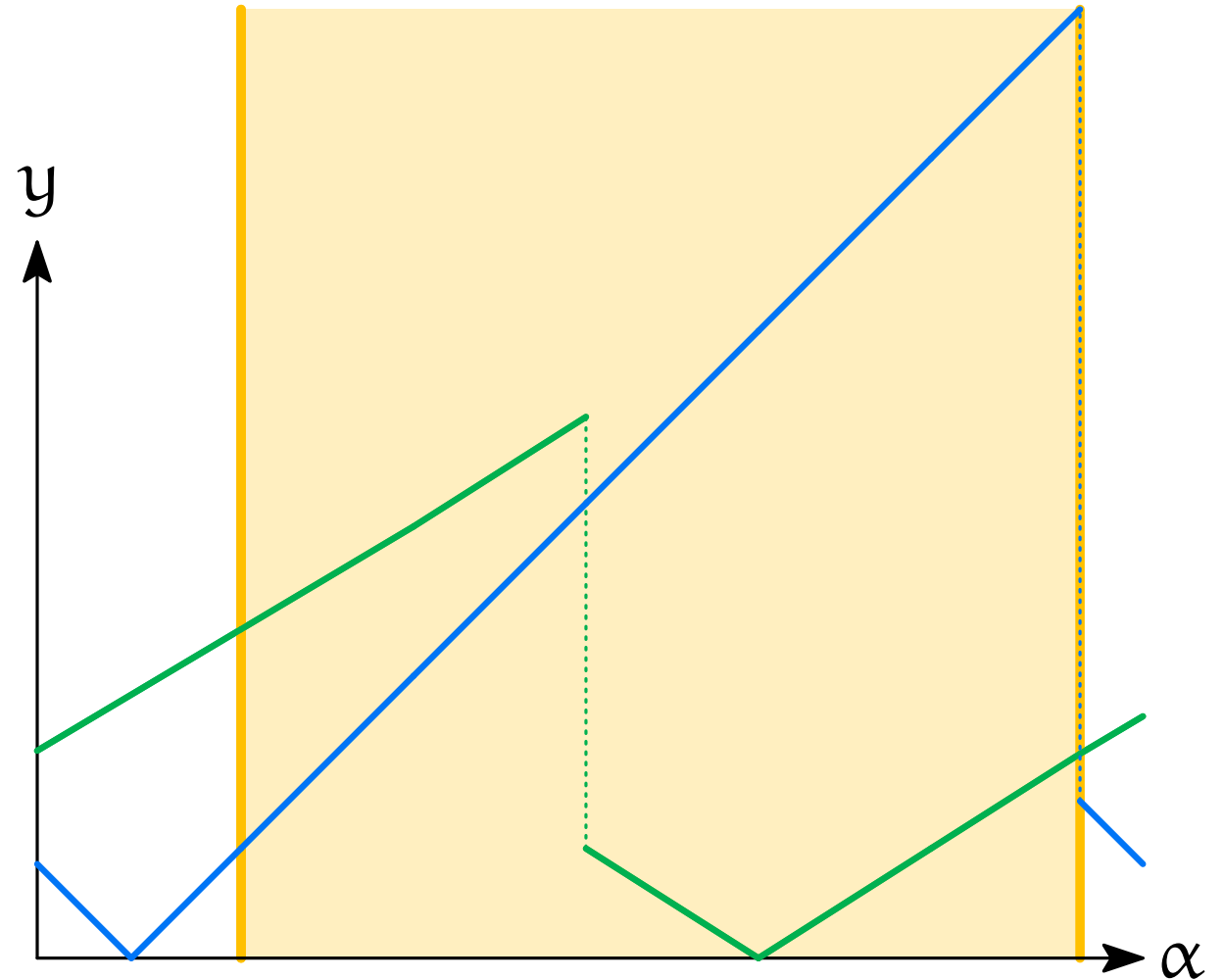
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**

Rotating the Labels



$$y = f(\alpha)$$

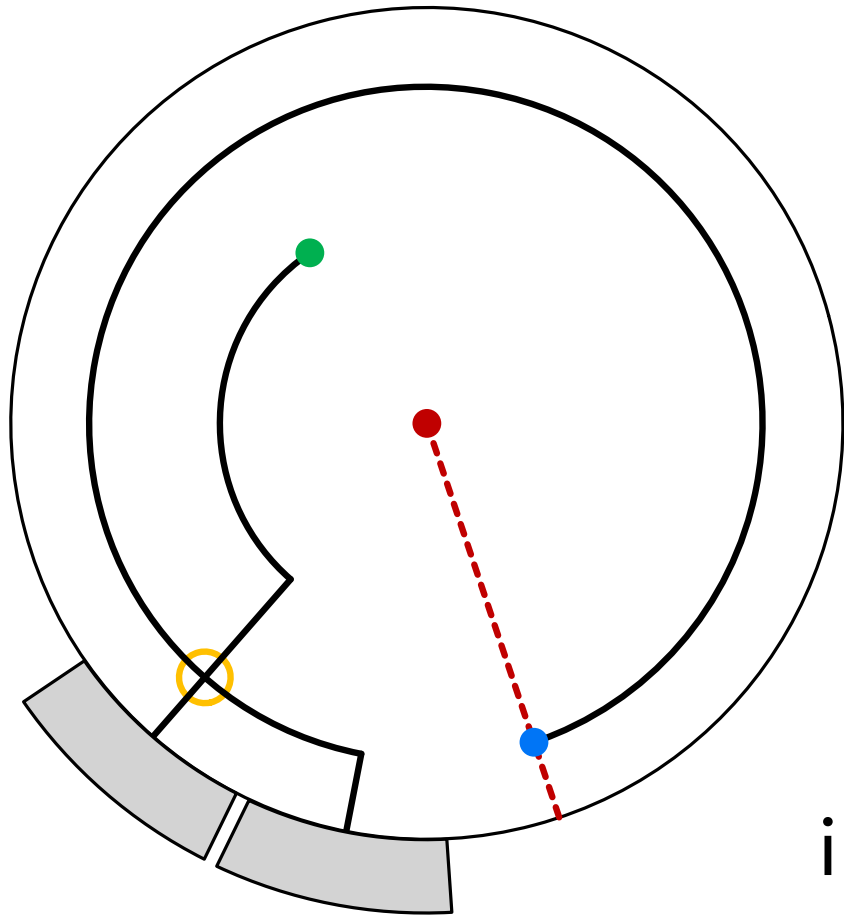


innermost:

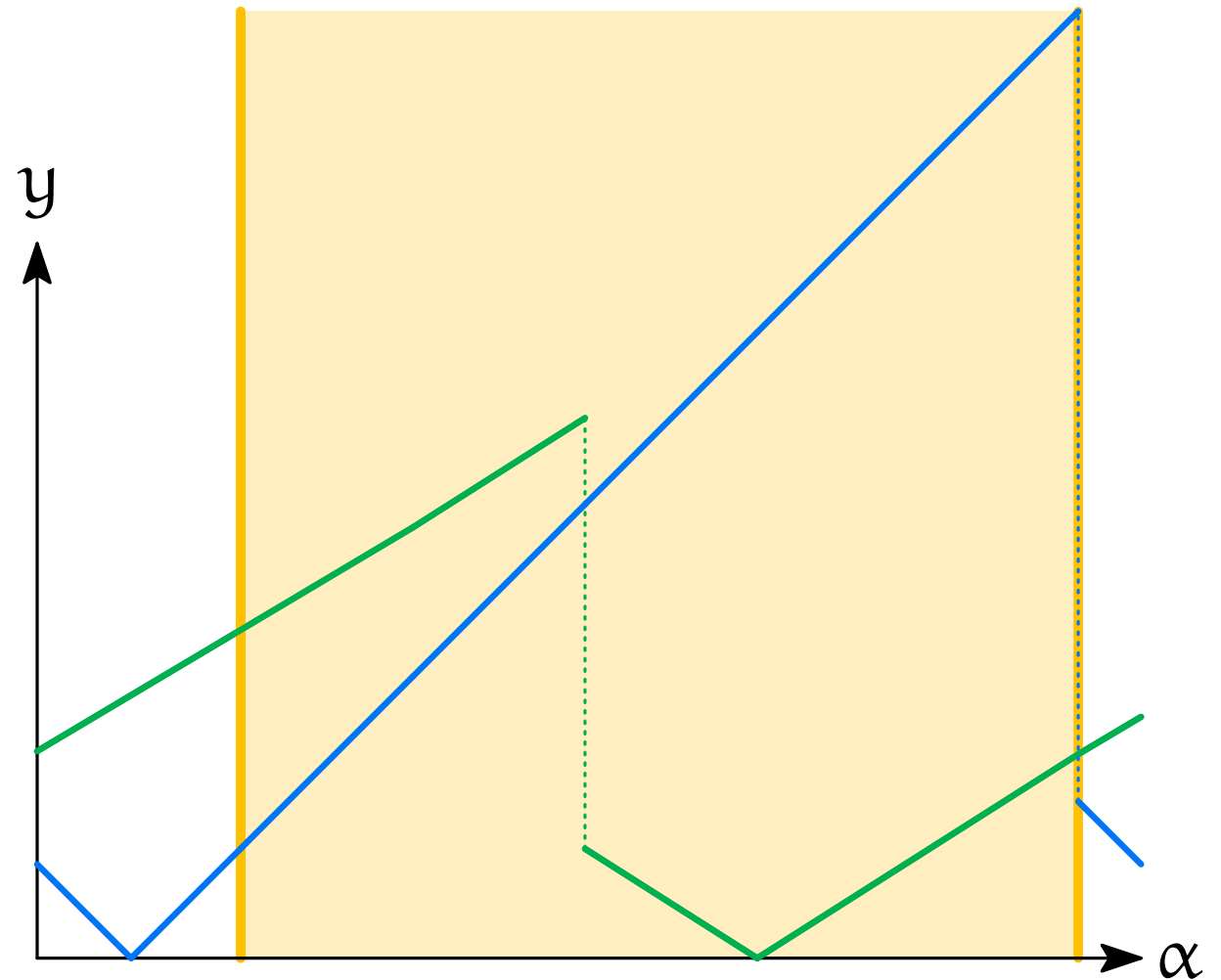
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range**

Rotating the Labels



$$y = f(\alpha)$$

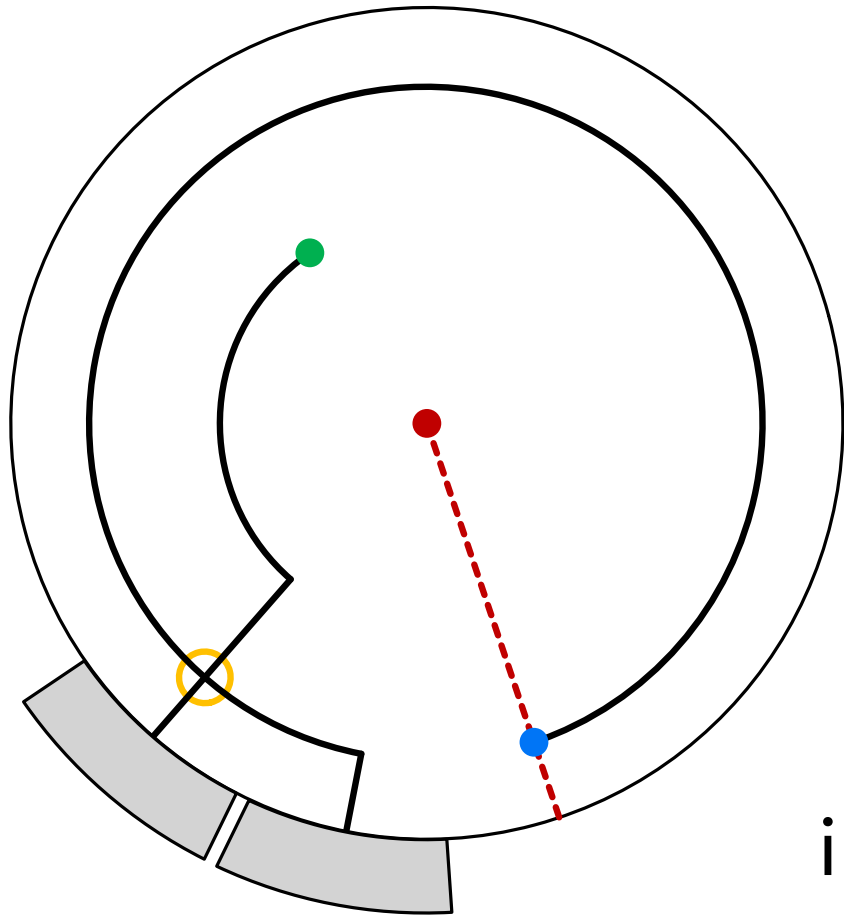


innermost:

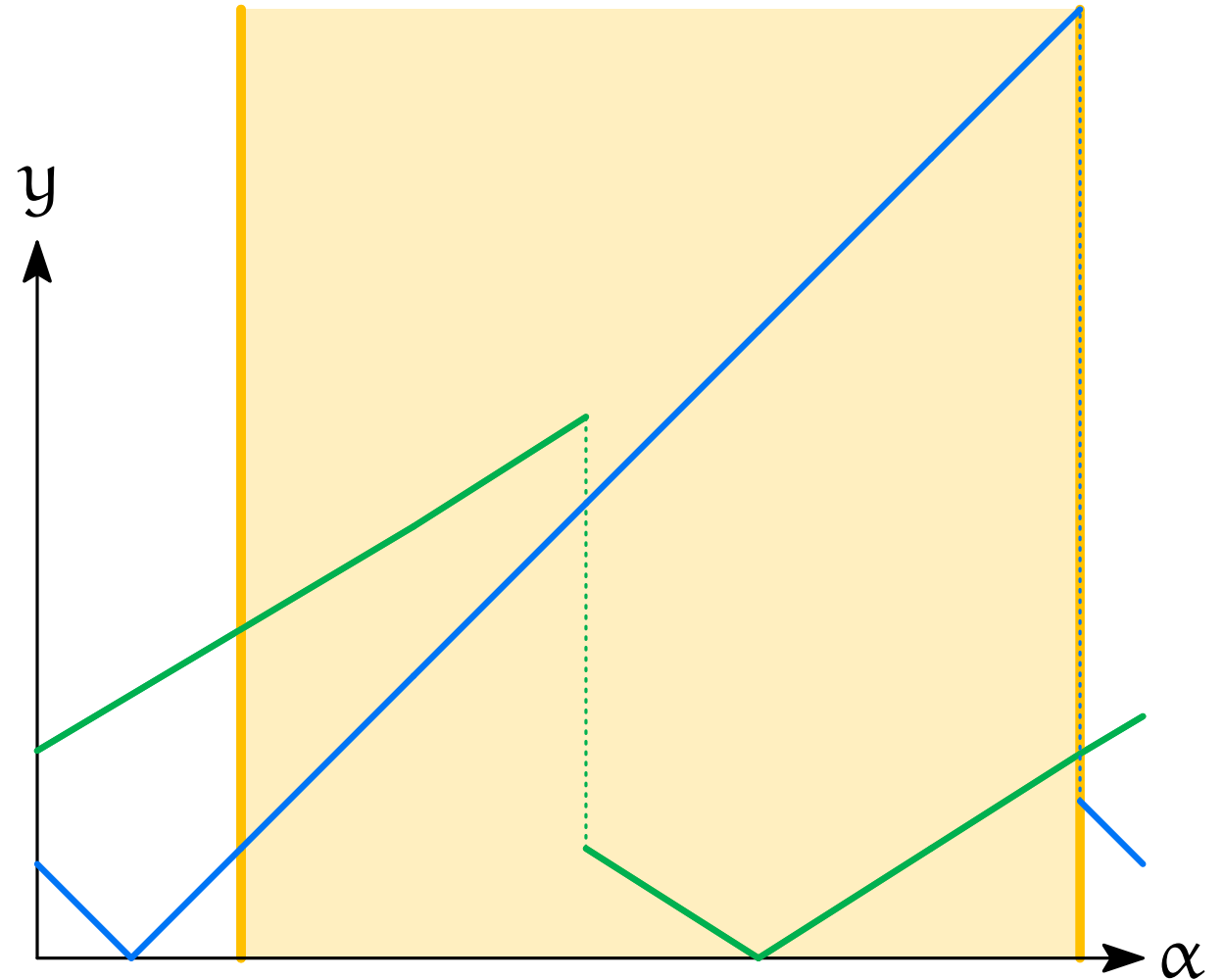
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$

Rotating the Labels



$$y = f(\alpha)$$



innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$
Find minimum of **sum of functions**

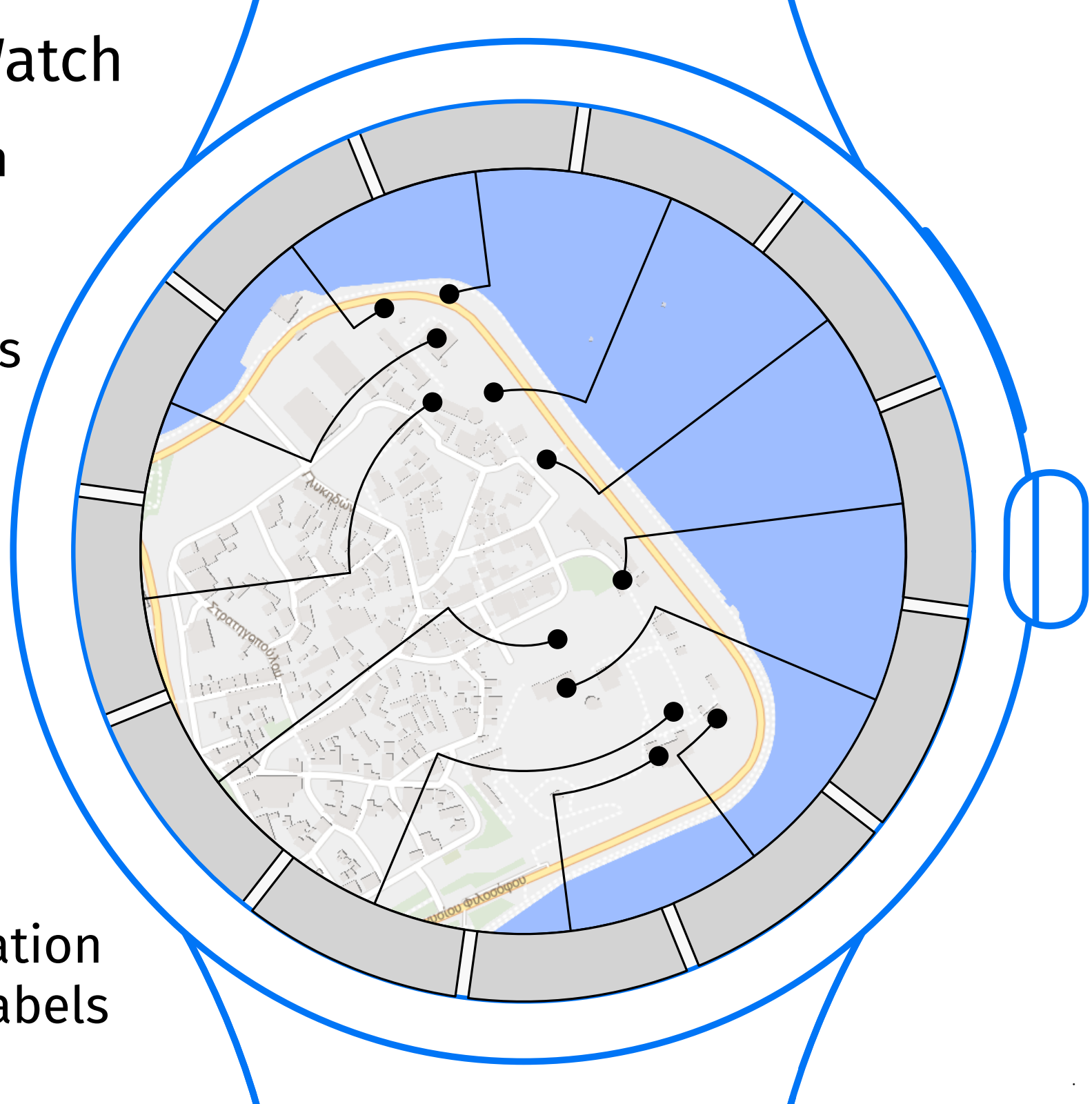
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

under:

- continuous rotation
- fixed order of labels



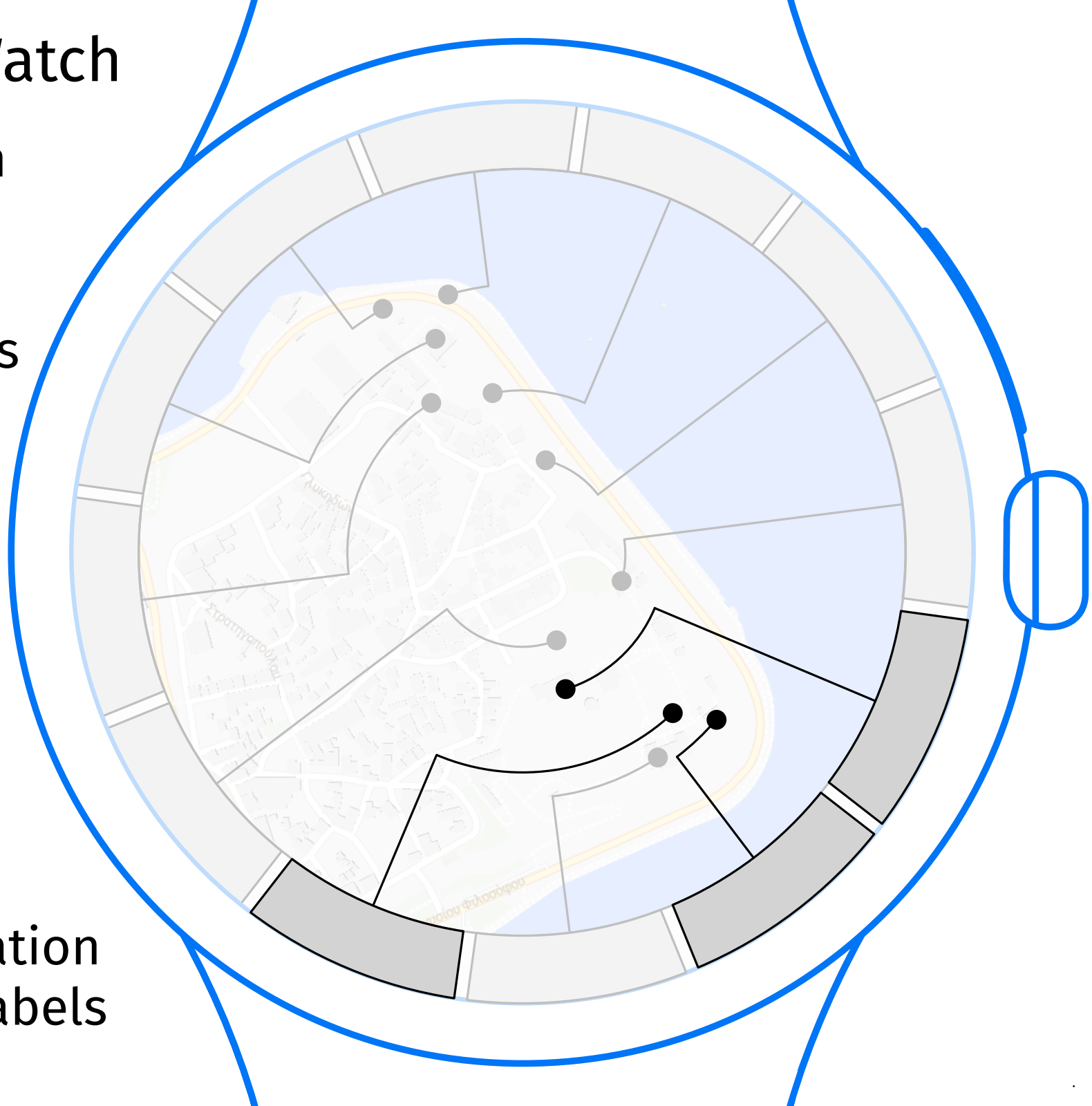
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

under:

- continuous rotation
- fixed order of labels



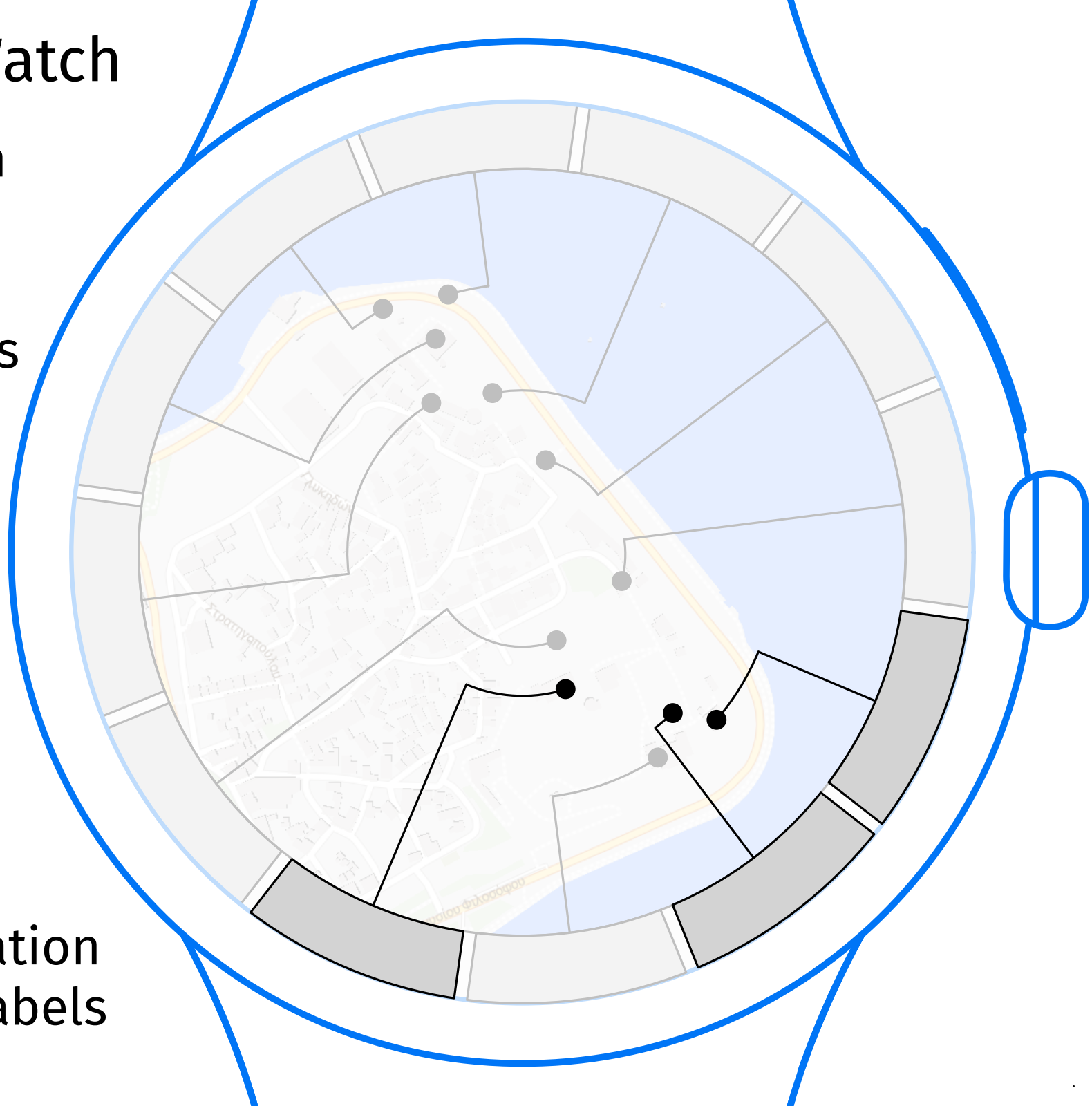
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

under:

- continuous rotation
- fixed order of labels



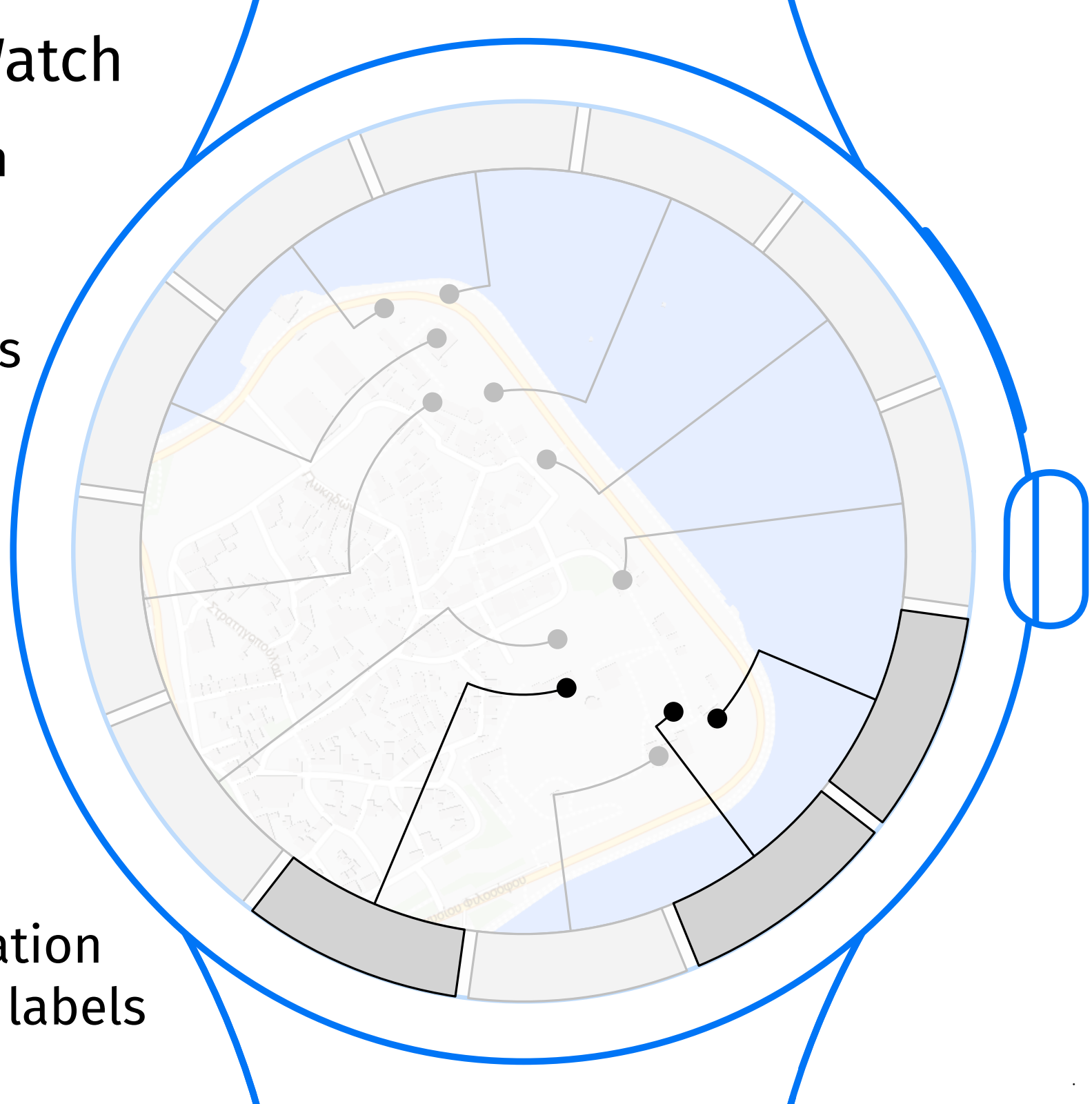
Labeling on a Smart Watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

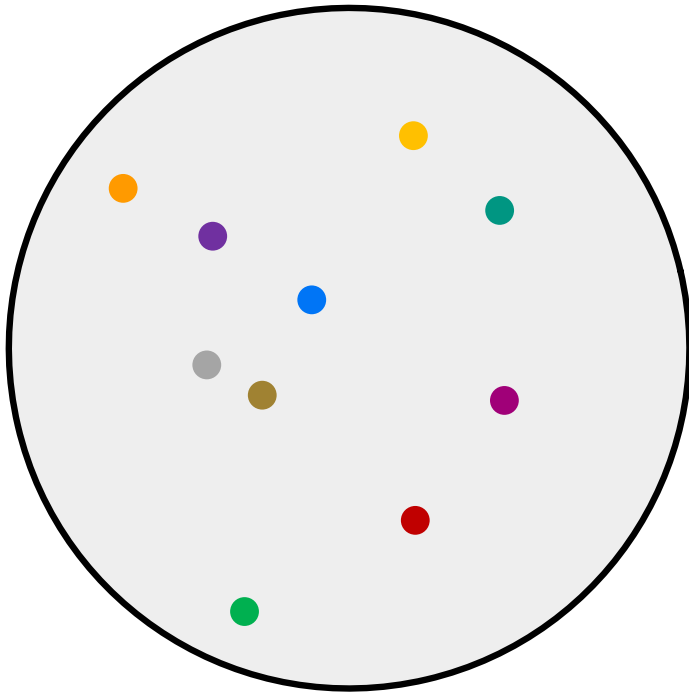
goal: minimize sum of leader lengths

under:

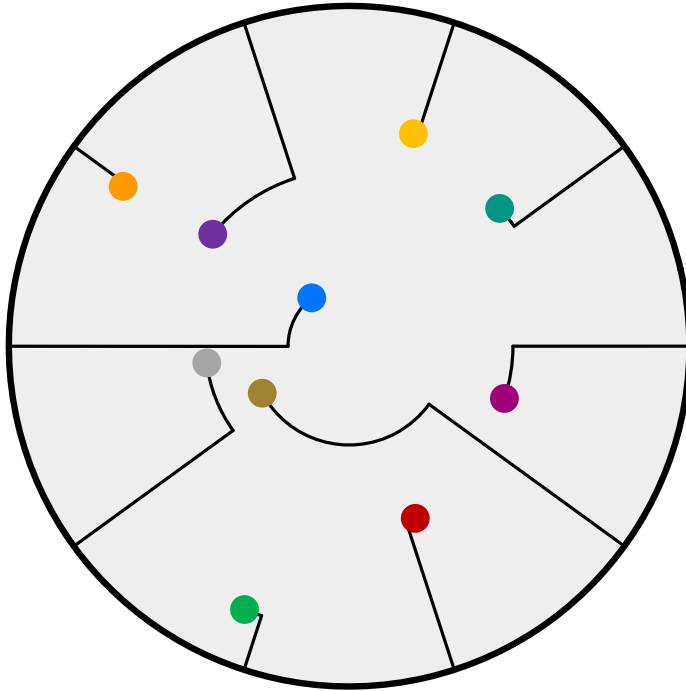
- continuous rotation
- **permutation** of labels



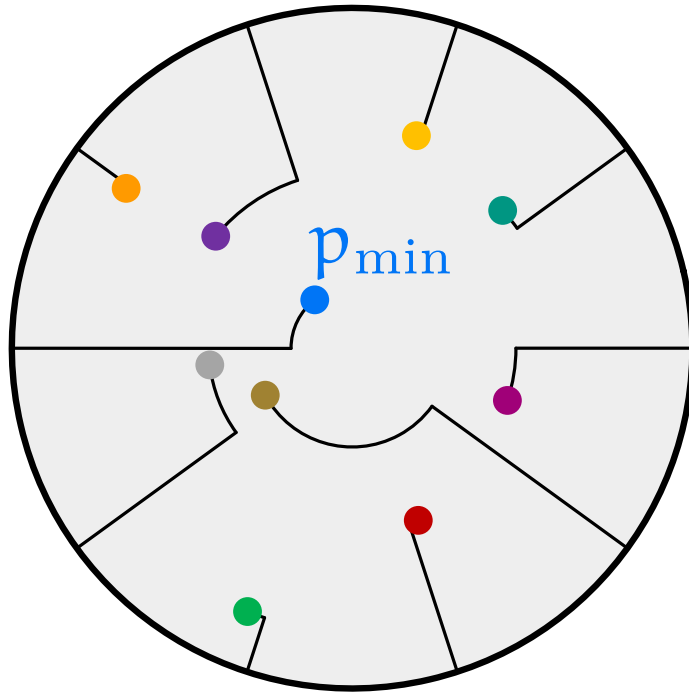
Permuting the Labels



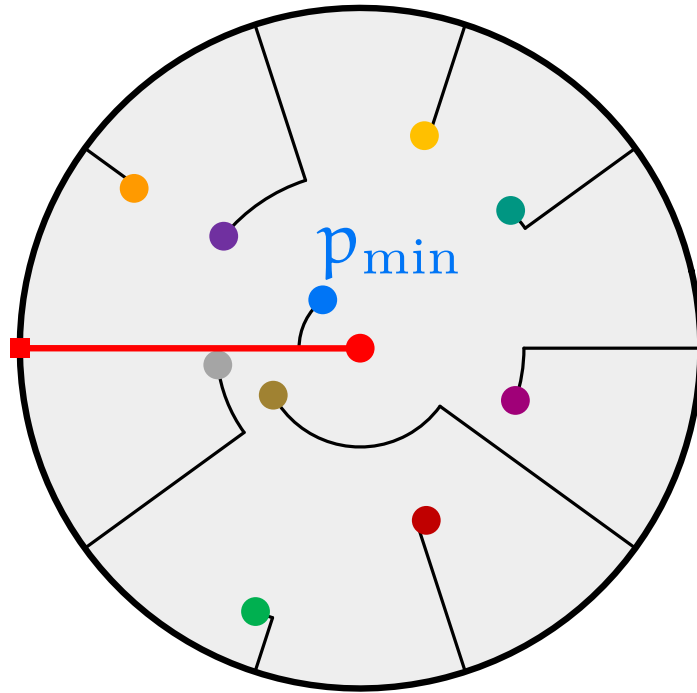
Permuting the Labels



Permuting the Labels

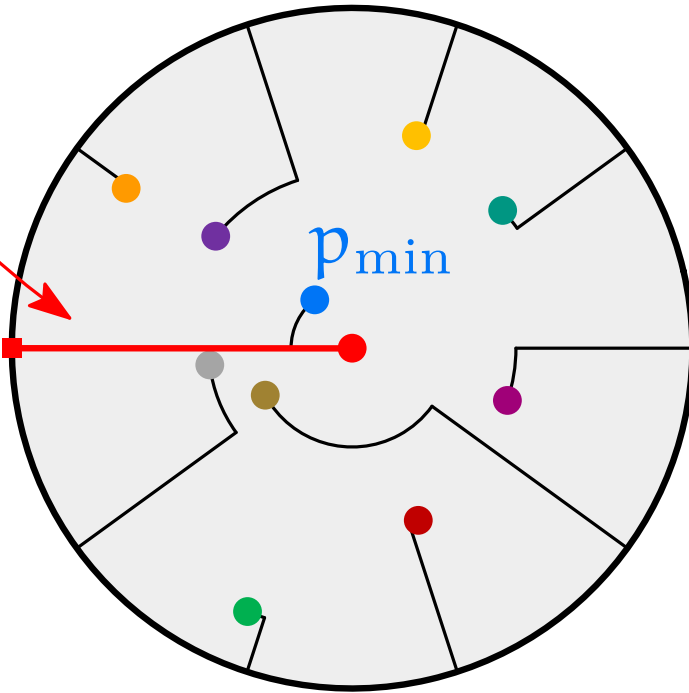


Permuting the Labels



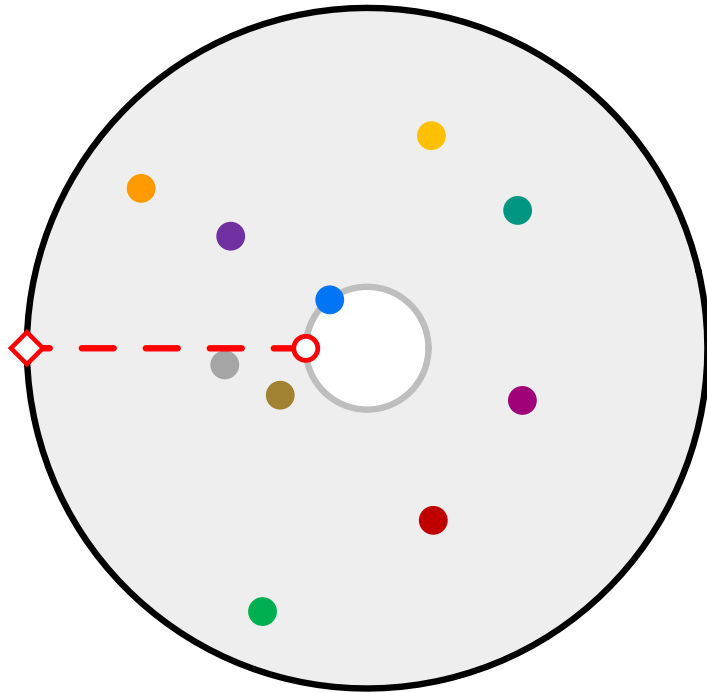
Permuting the Labels

$O(n^2)$ relevant possibilities
for port of $p_{\min}$



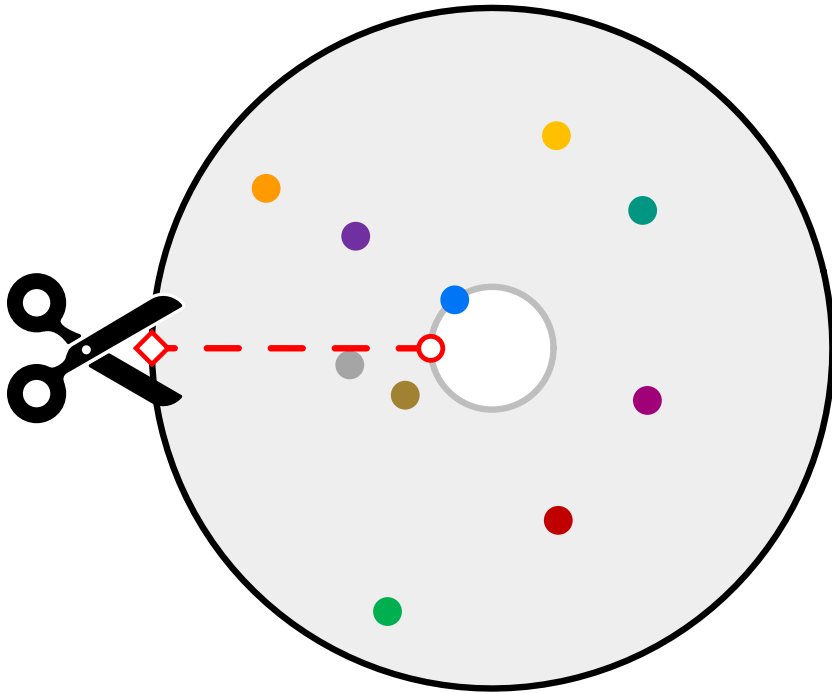
Permuting the Labels

$O(n^2)$ relevant possibilities
for port of $p_{\min}$



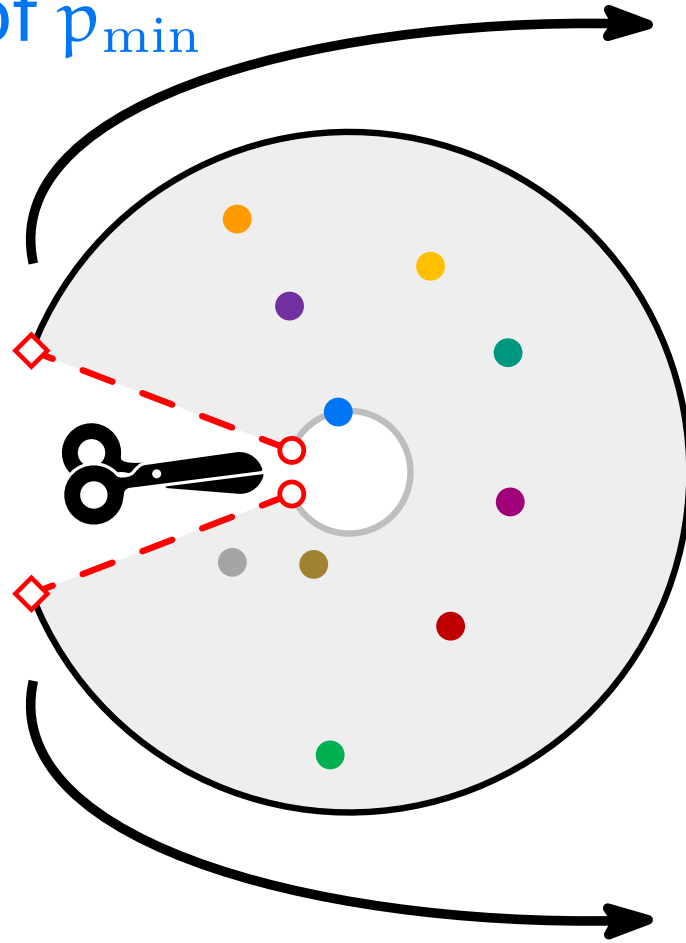
Permuting the Labels

$O(n^2)$ relevant possibilities
for port of $p_{\min}$



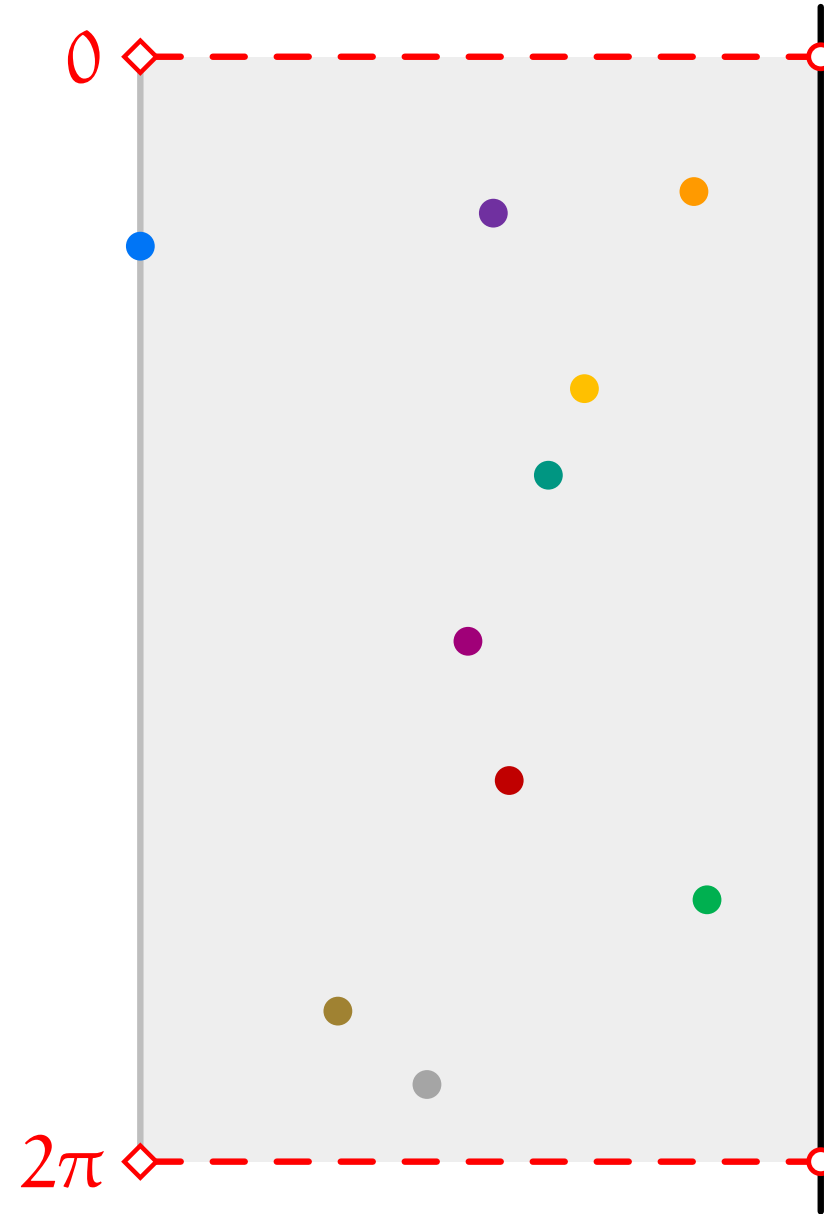
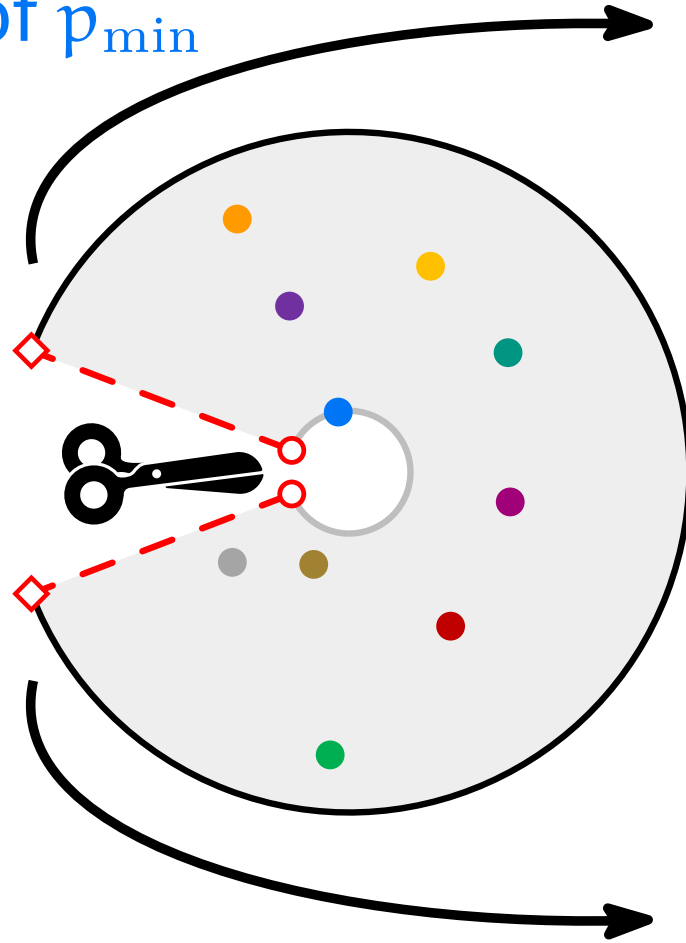
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



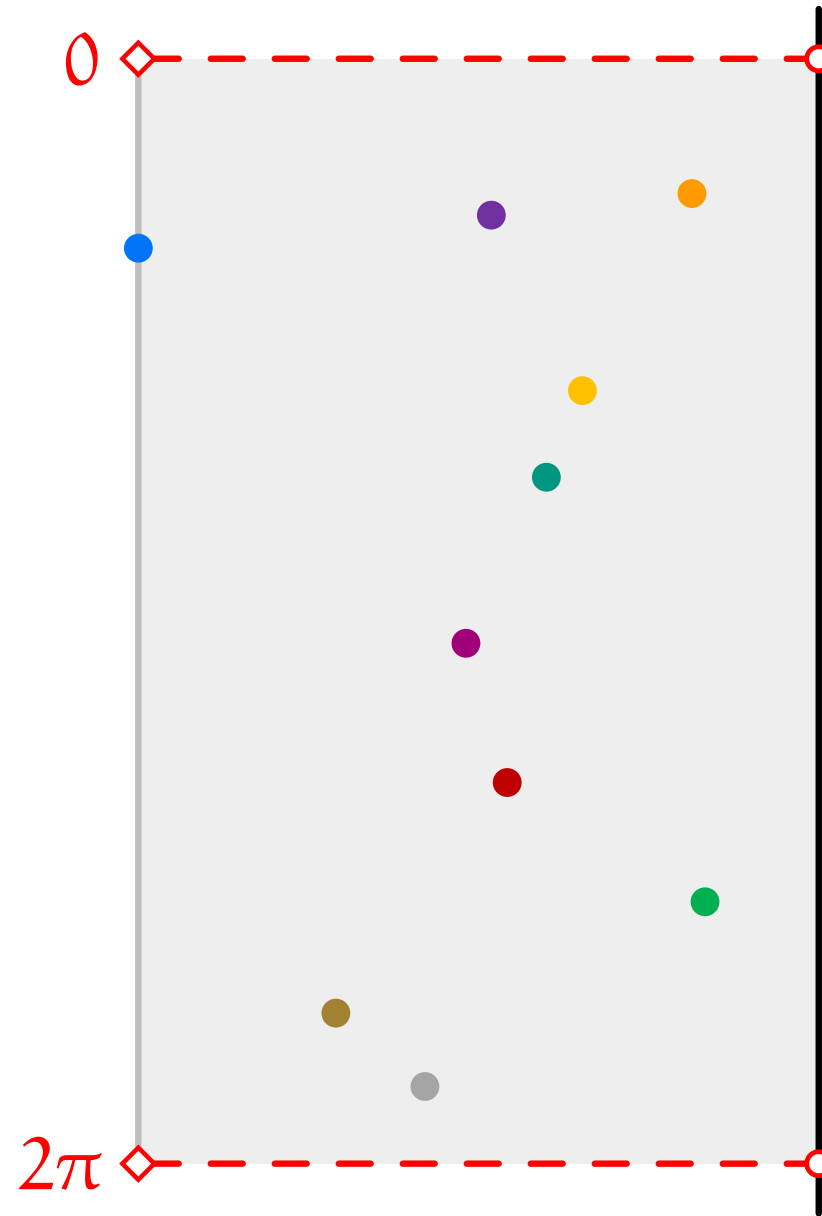
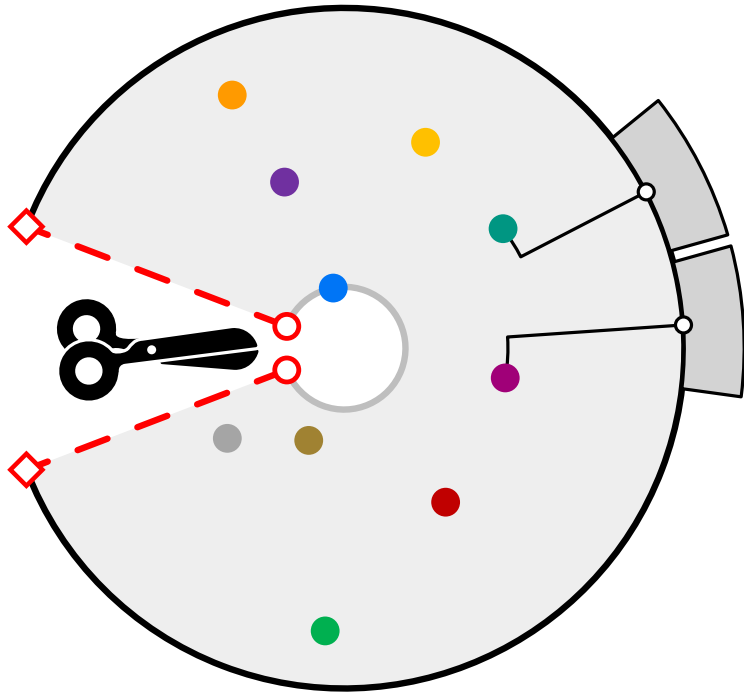
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



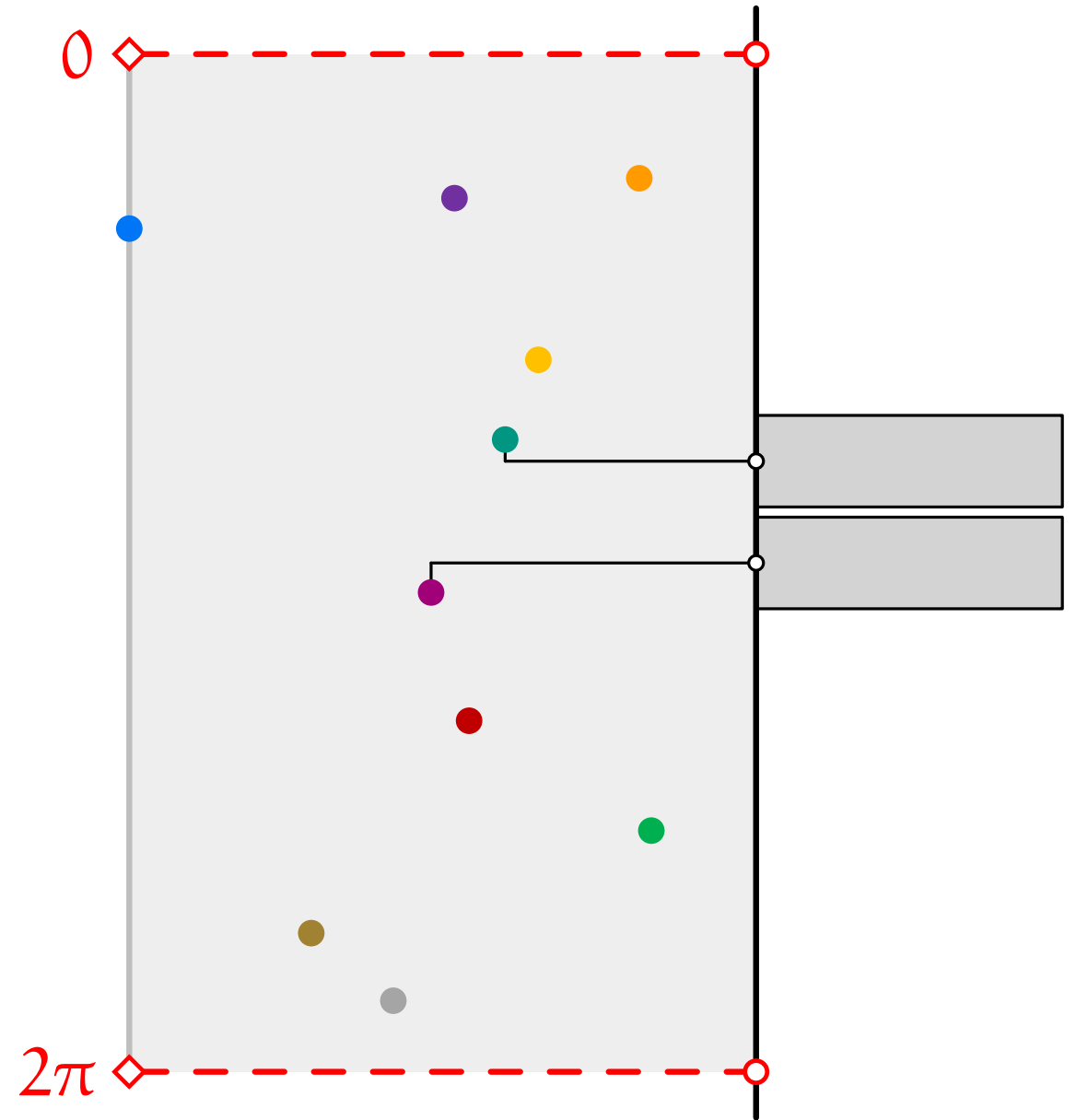
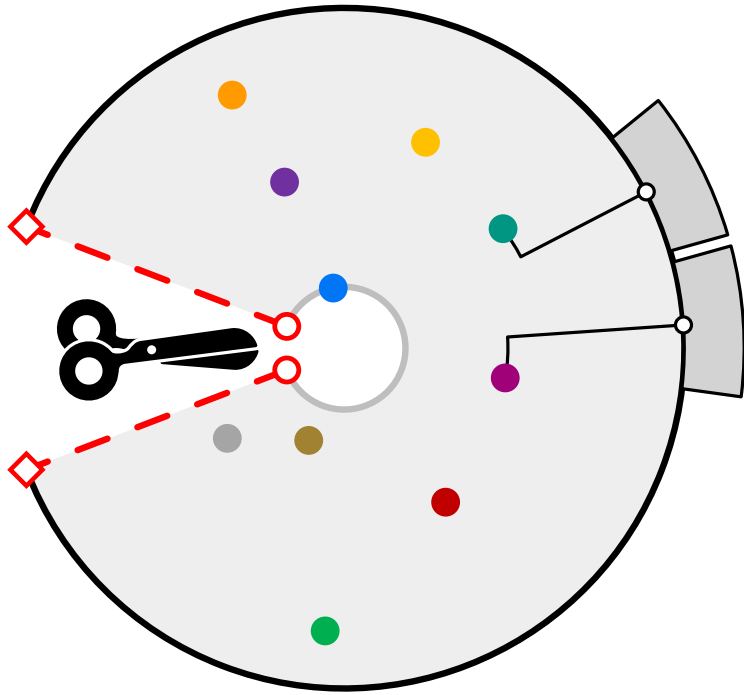
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



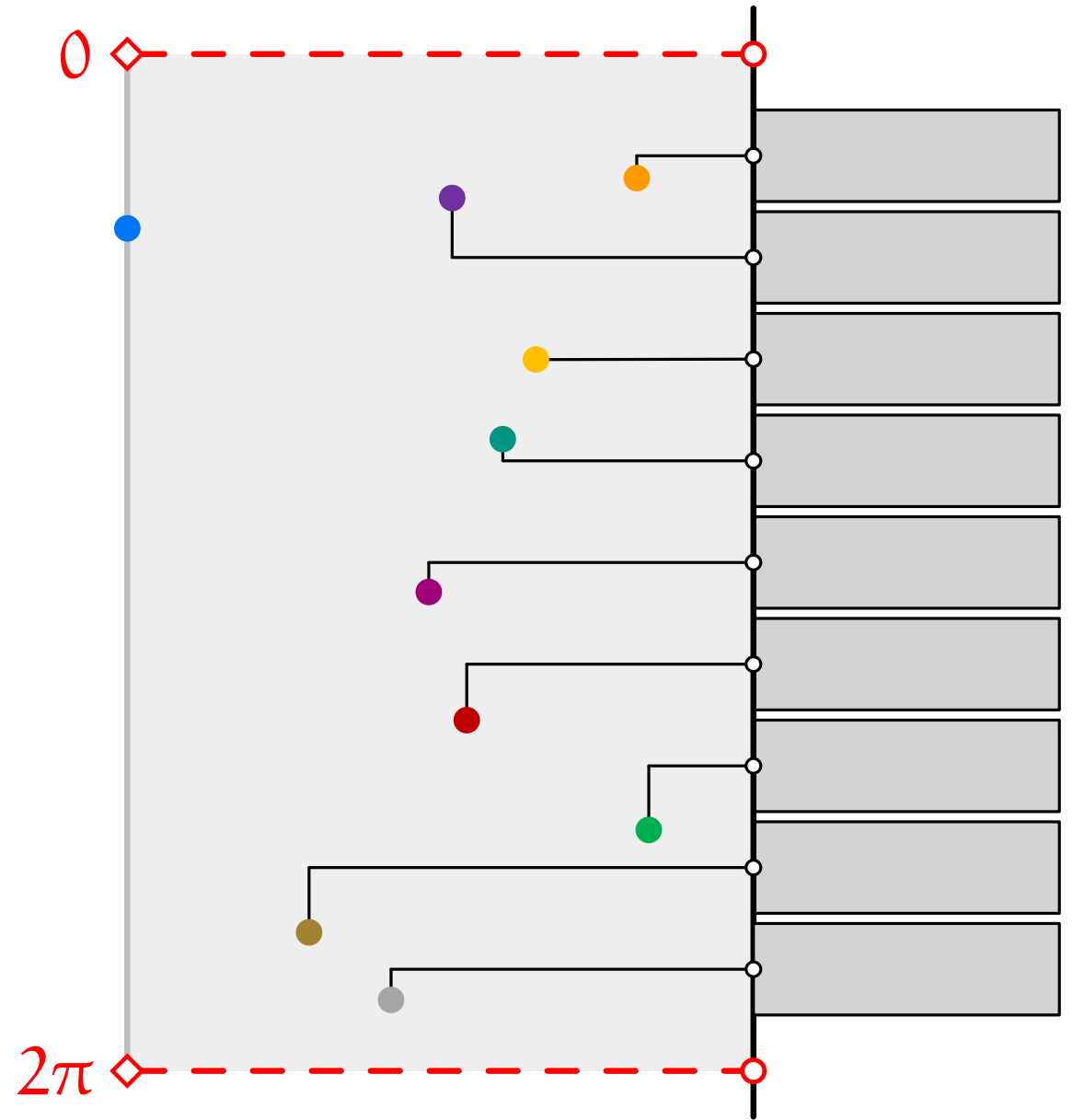
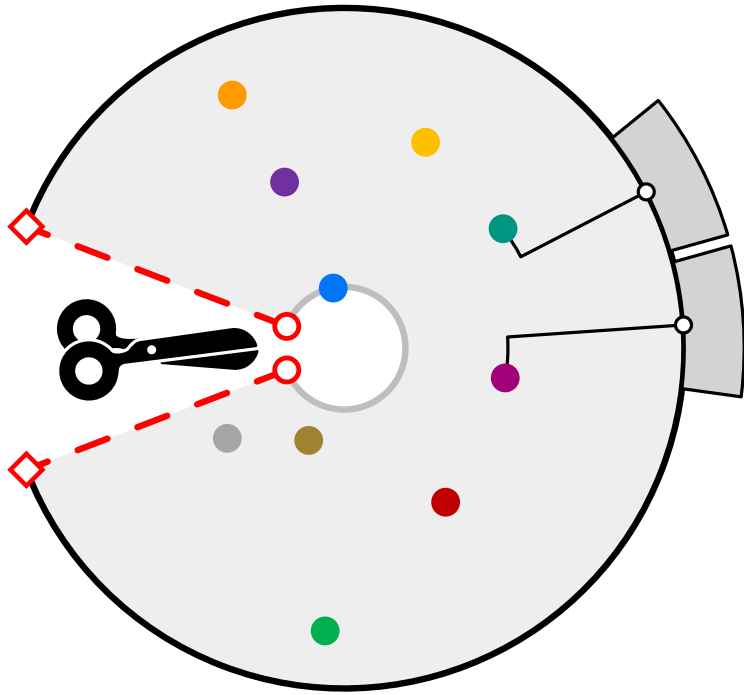
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



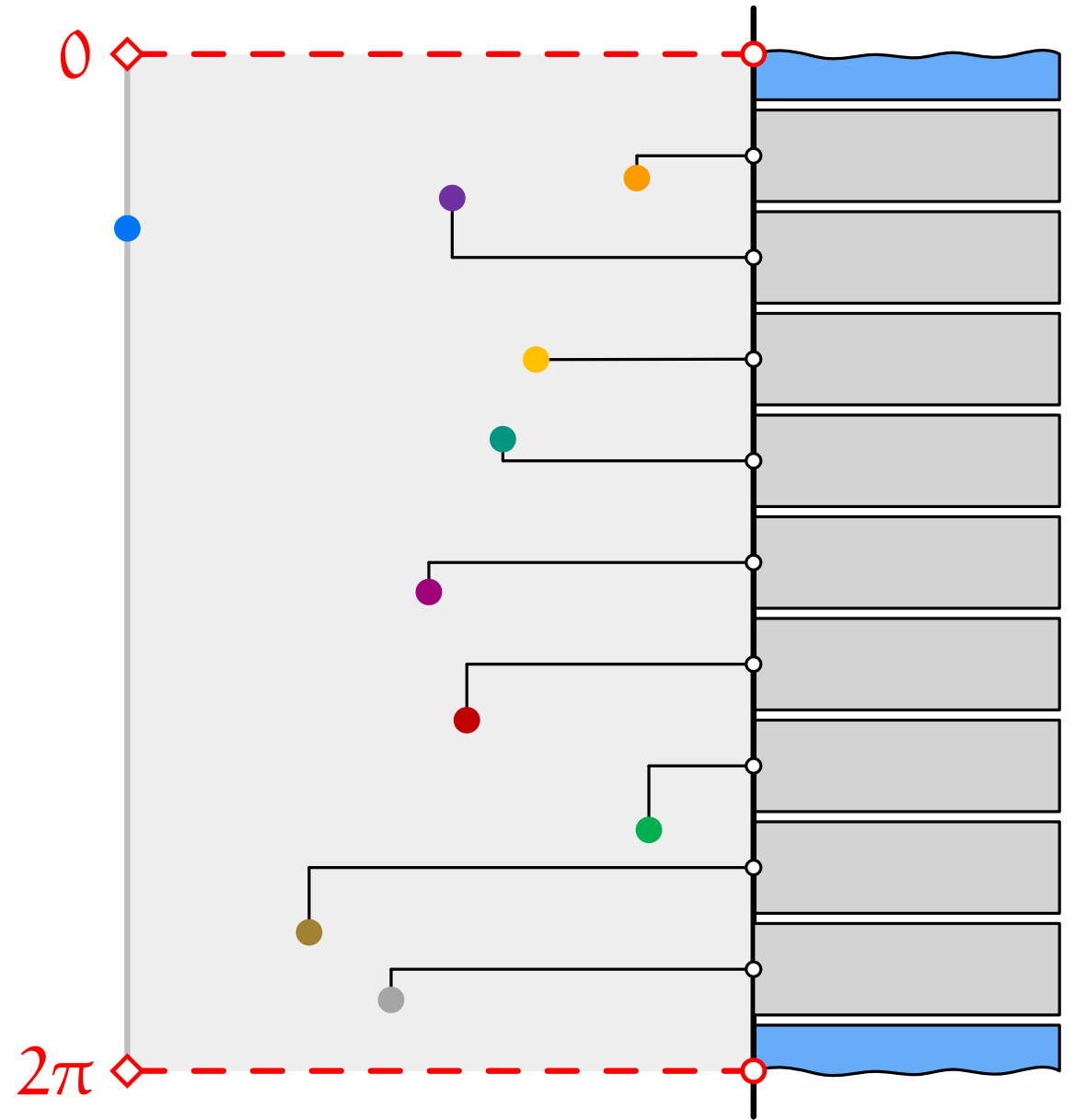
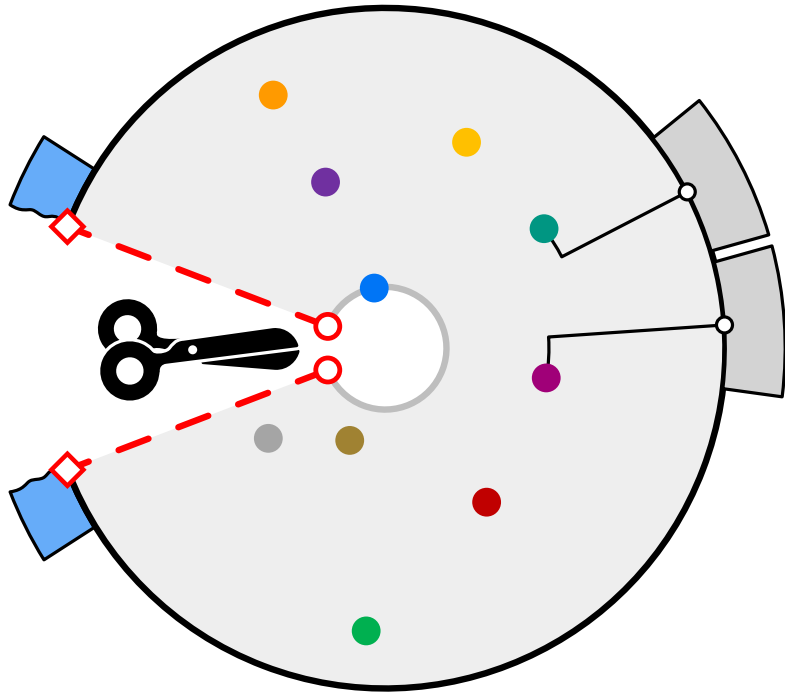
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



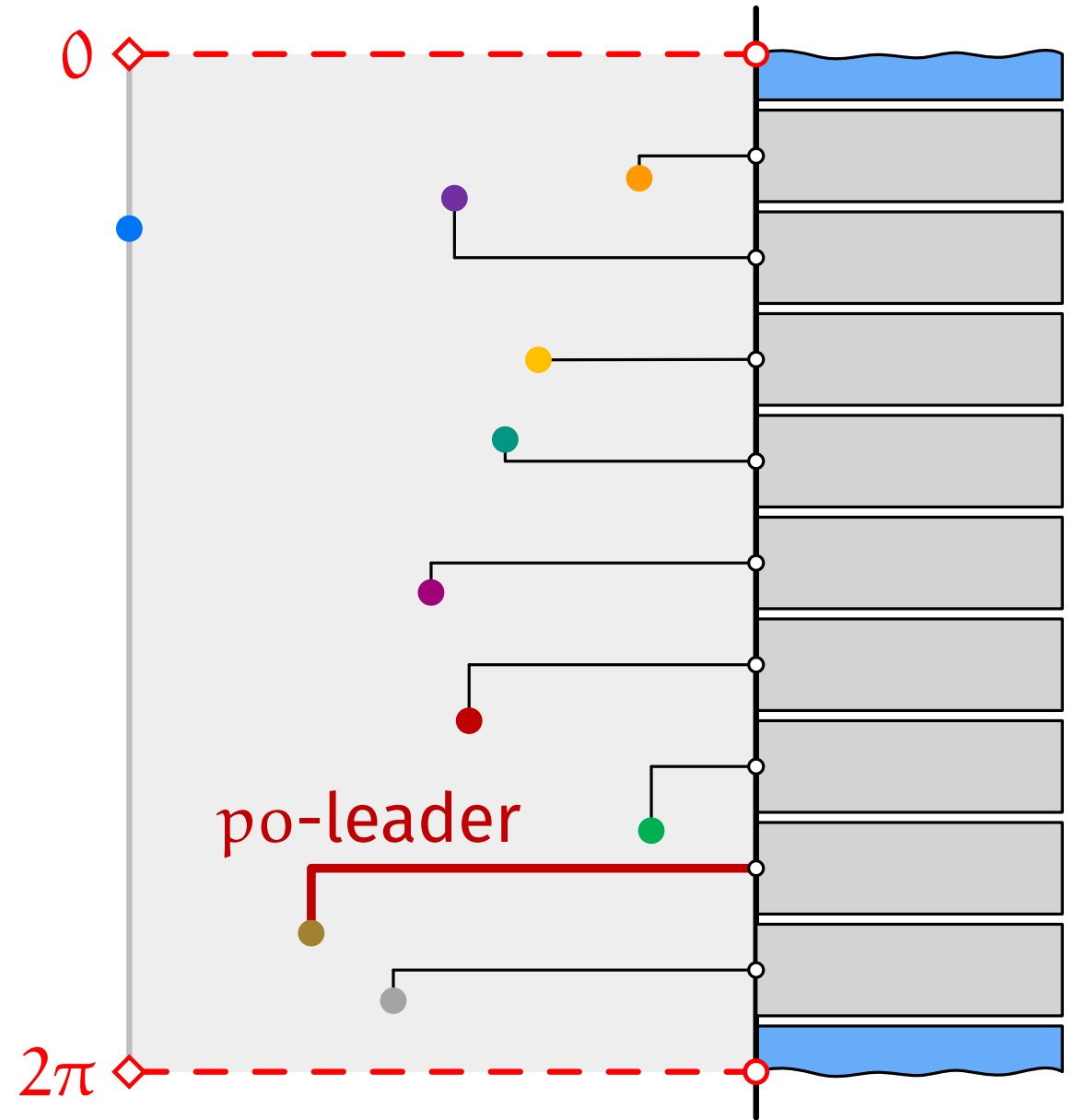
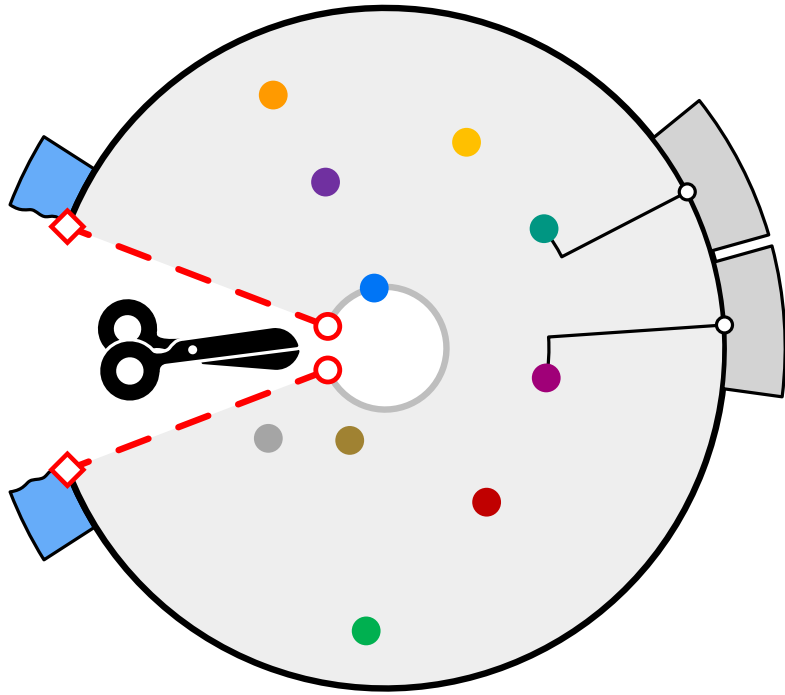
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



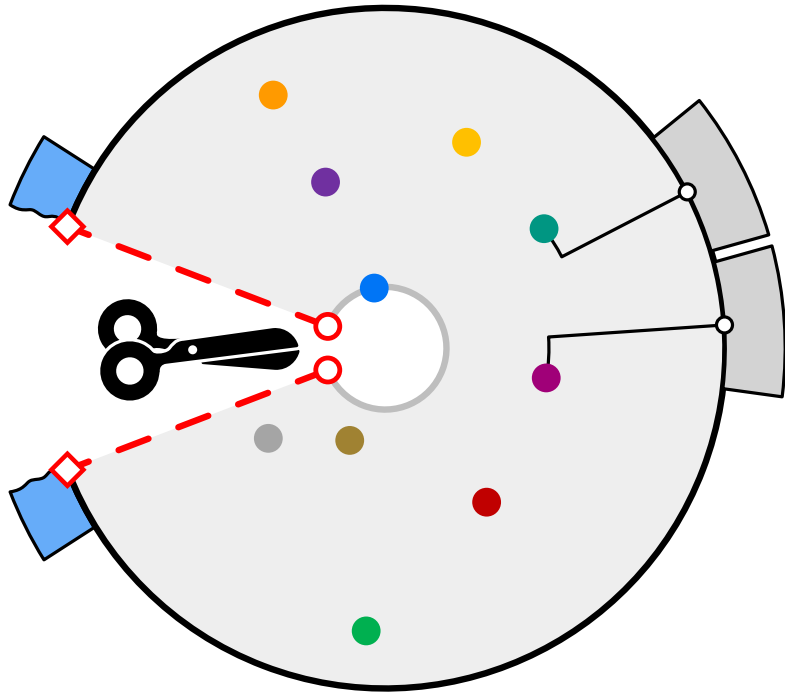
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



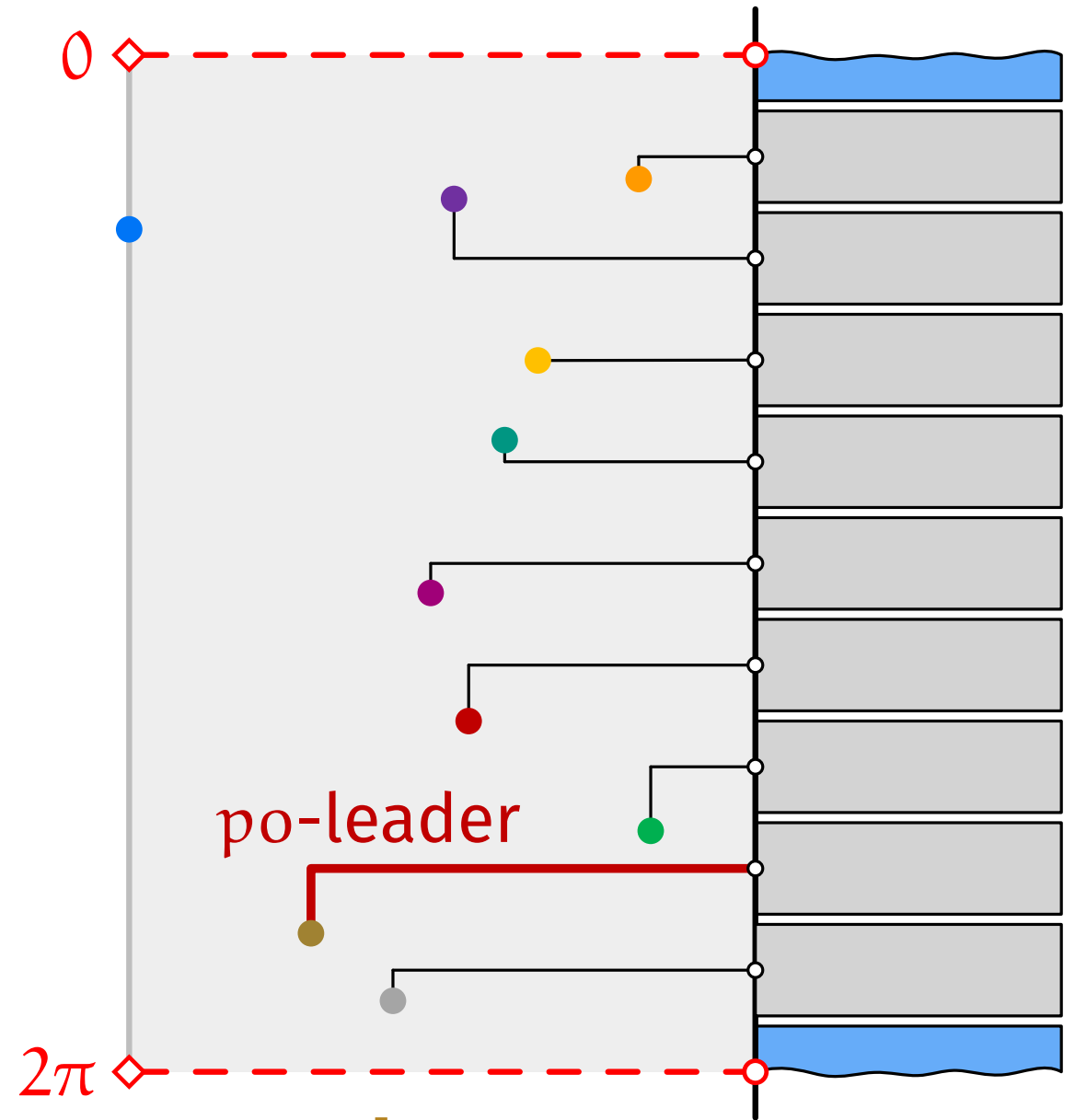
Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



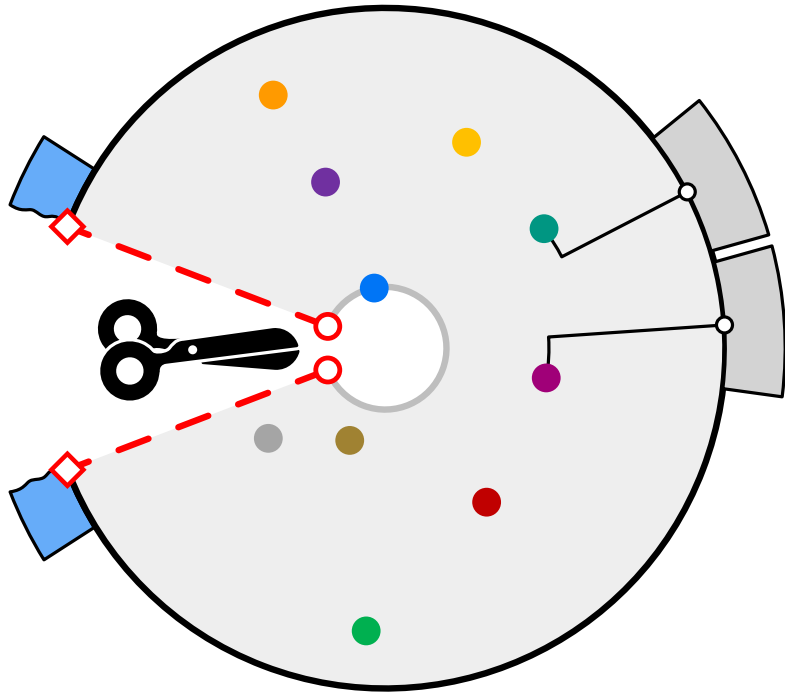
one sided po-boundary labeling
arbitrary badness function

$O(n^3)$ time [Benkert, Haverkort, Kroll, Nöllenburg, '09]



Permuting the Labels

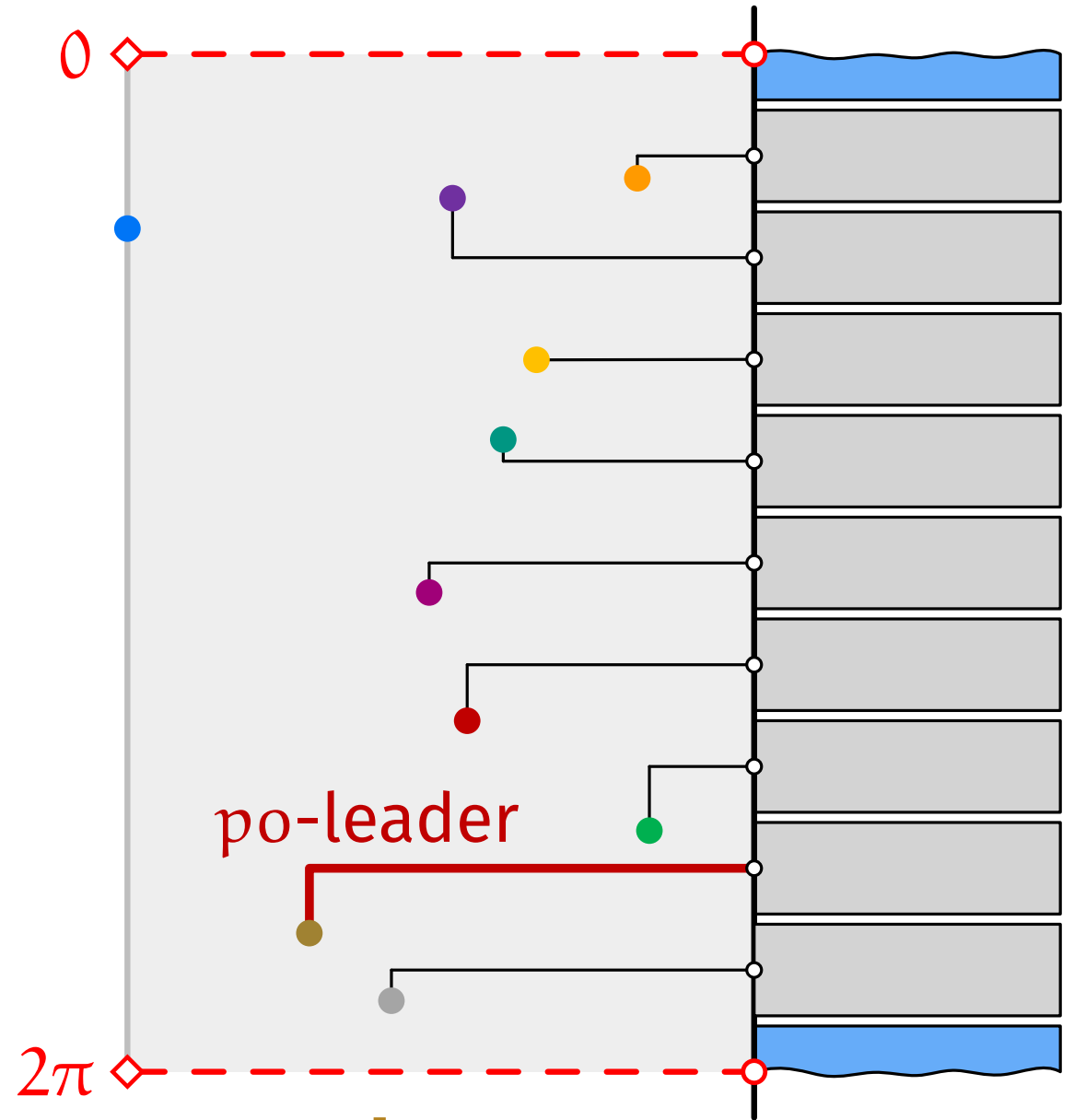
$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



one sided po-boundary labeling
arbitrary badness function

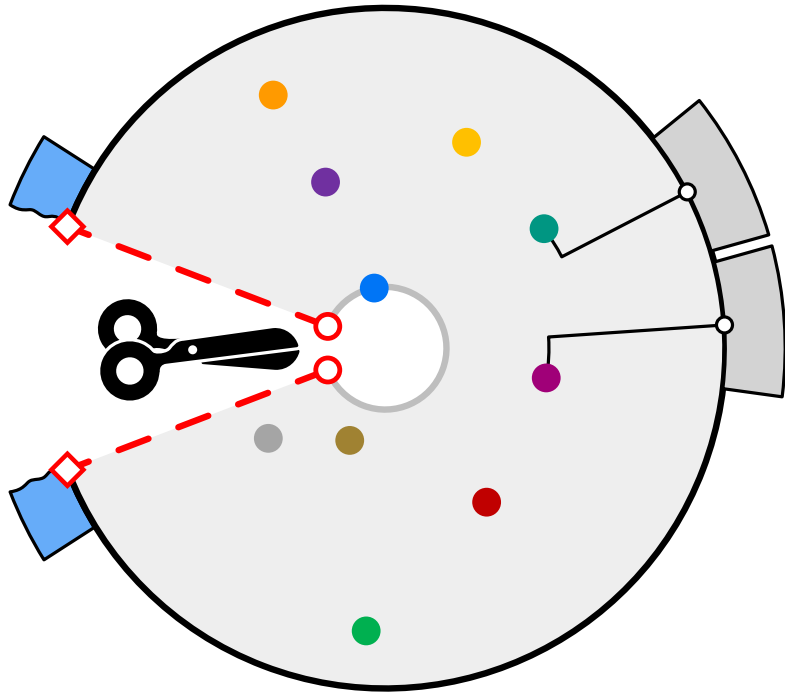
$O(n^3)$ time [Benkert, Haverkort, Kroll, Nöllenburg, '09]

→ $O(n^5)$ runtime



Permuting the Labels

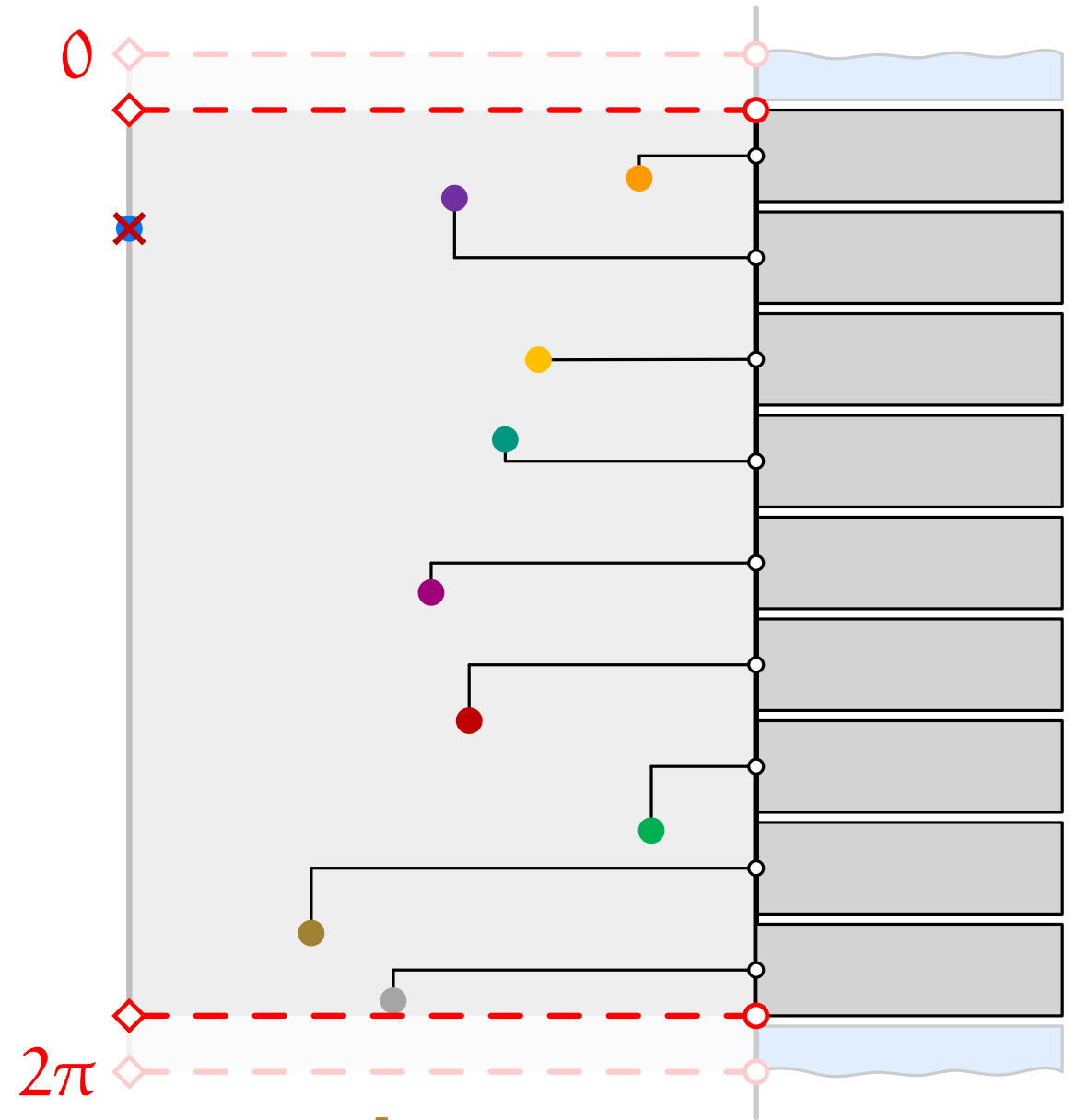
$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



one sided po-boundary labeling
arbitrary badness function

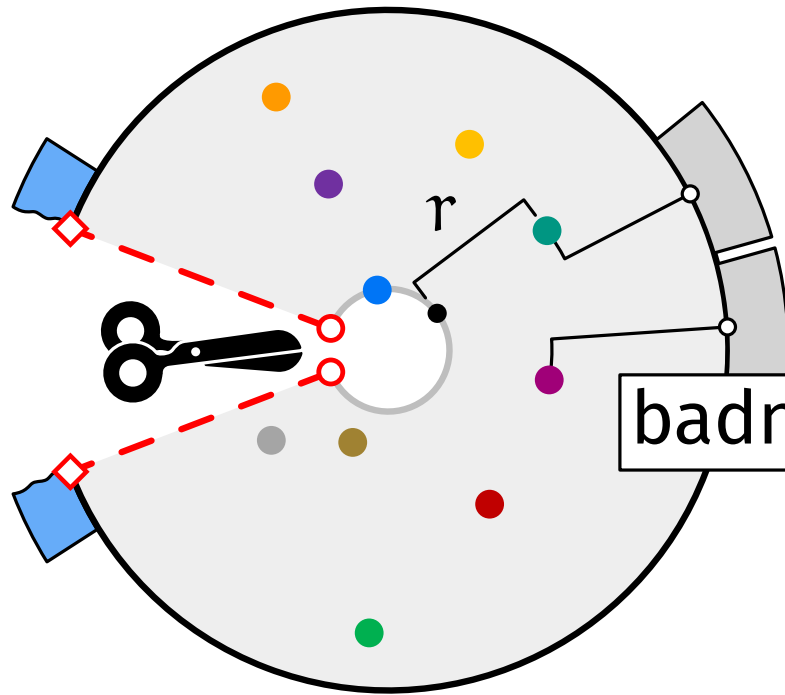
$O(n^3)$ time [Benkert, Haverkort, Kroll, Nöllenburg, '09]

$\rightarrow O(n^5)$ runtime

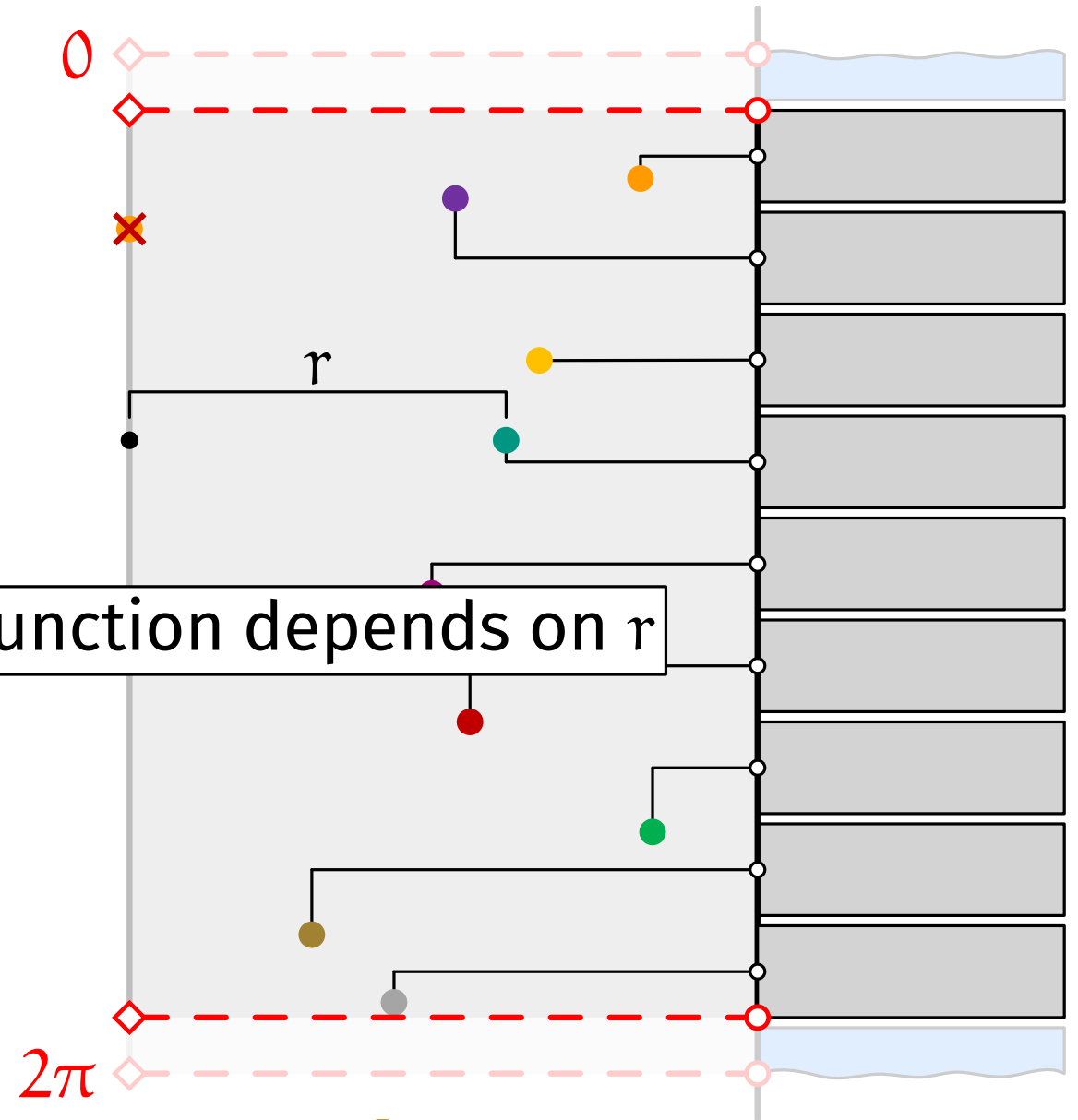


Permuting the Labels

$O(n^2)$ relevant possibilities
for **port of** $p_{\min}$



badness function depends on r



one sided po-boundary labeling
arbitrary badness function

$O(n^3)$ time [Benkert, Haverkort, Kroll, Nöllenburg, '09]

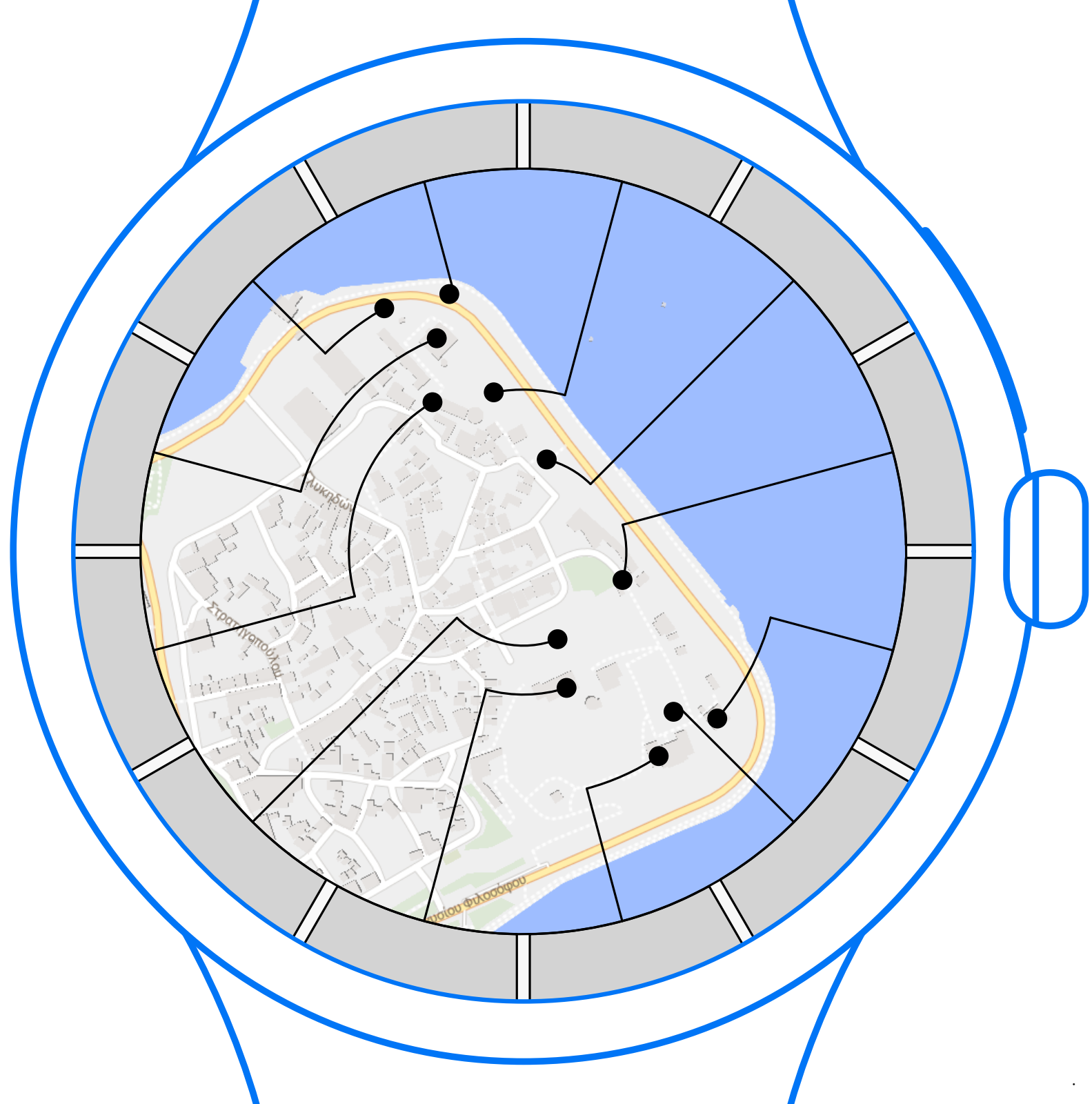
$\rightarrow O(n^5)$ runtime

Problem Variants

label order:

↻ free order

↓ fixed order

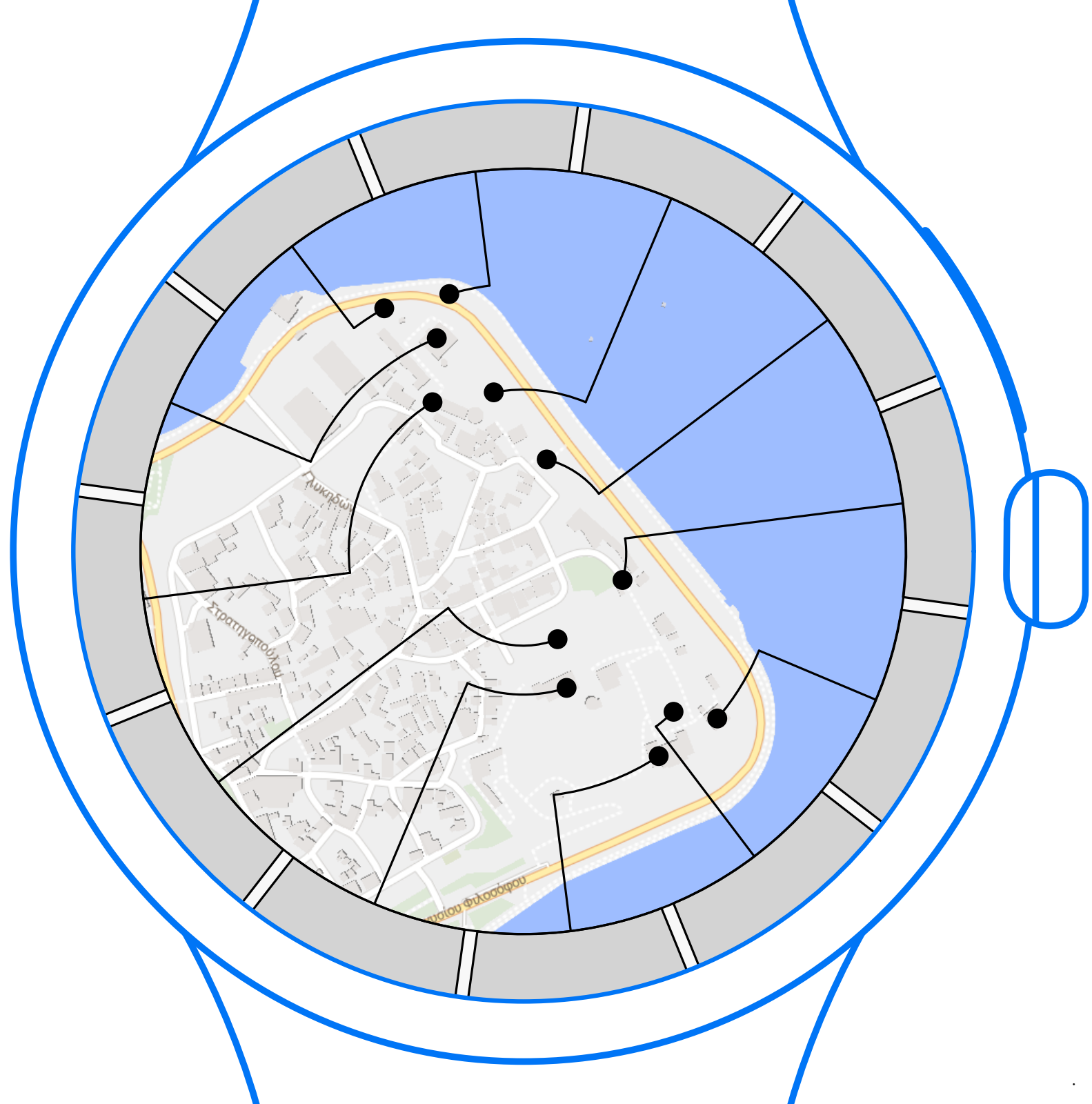


Problem Variants

label order:

↻ free order

↓ fixed order



Problem Variants

label order:

↻ free order

↓ fixed order

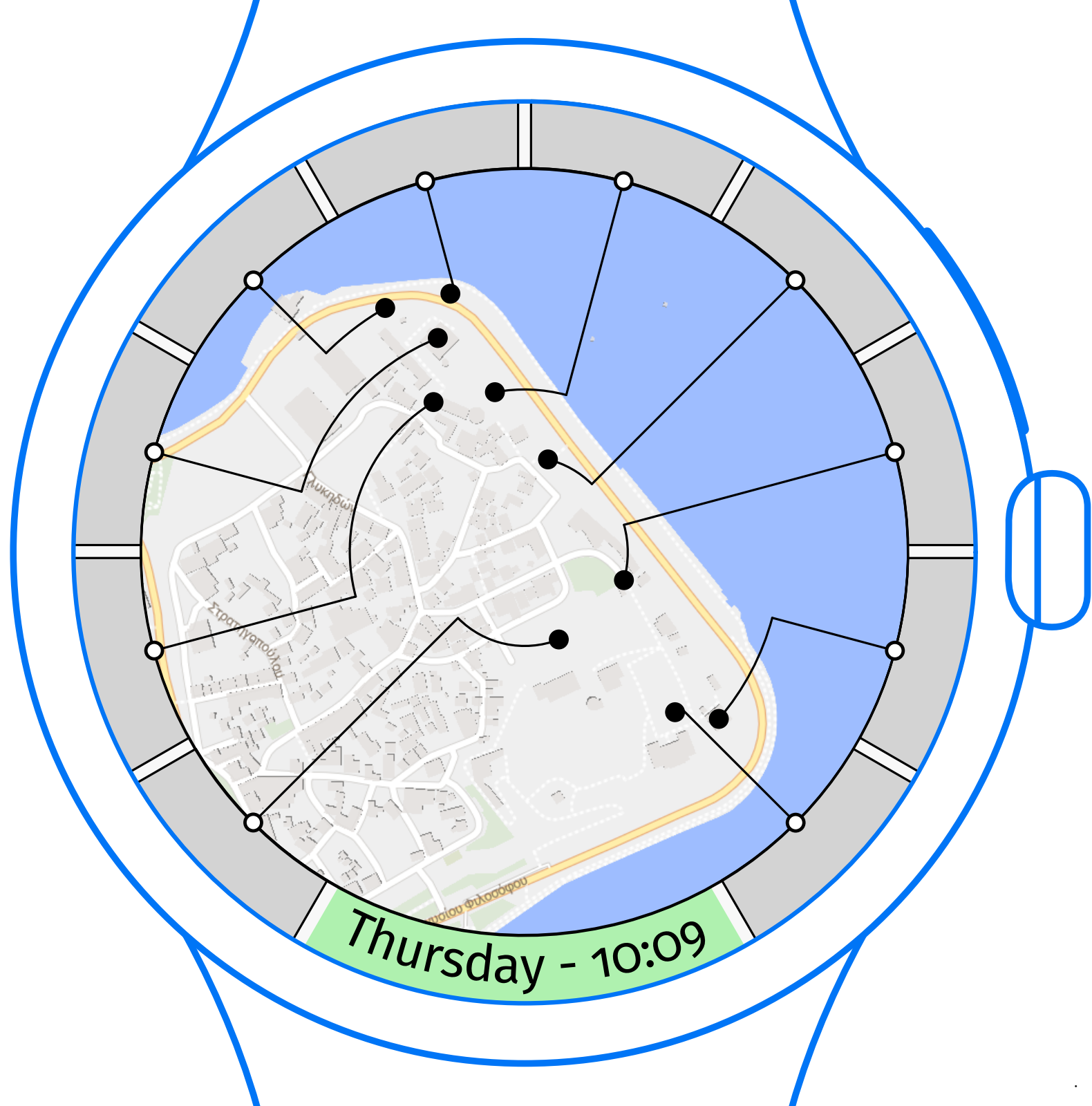


Problem Variants

label order:

↻ free order

↓ fixed order



Problem Variants

label order:

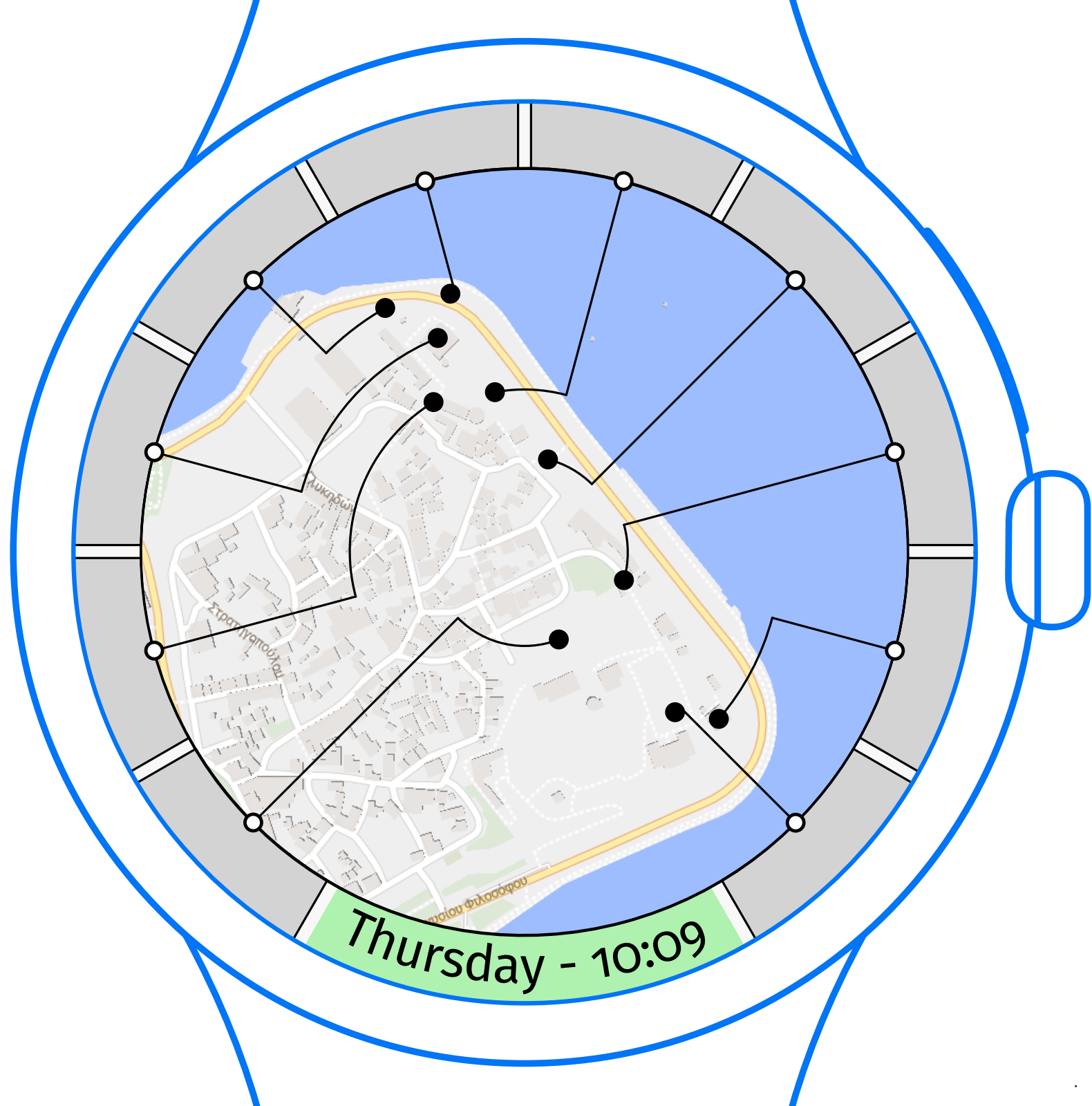
↻ free order

↓ fixed order

rotation:

🔑 continuous rotation

🔒 locked candidates C



Problem Variants

label order:

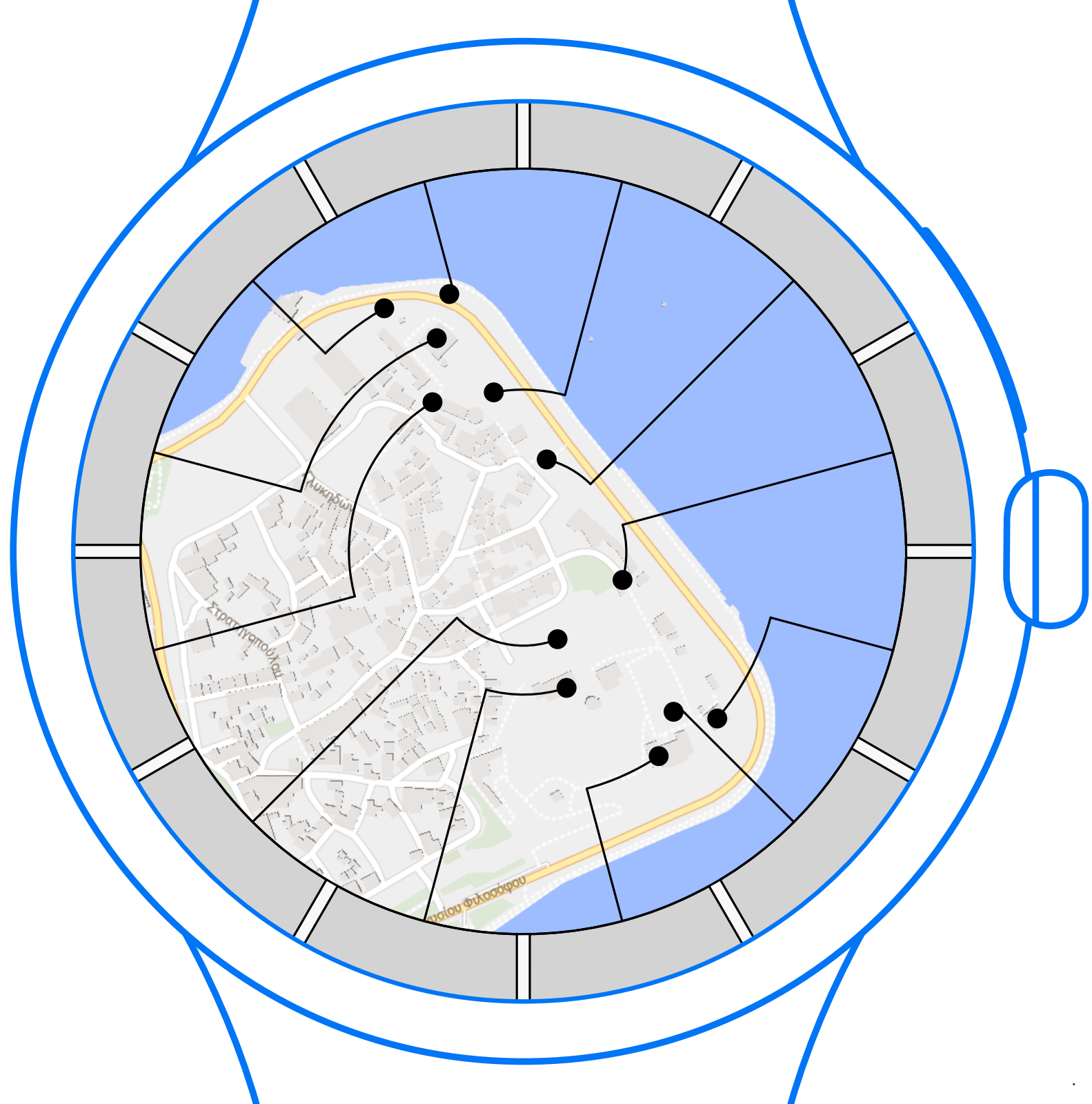
 free order

 fixed order

rotation:

 continuous rotation

 locked candidates C



Problem Variants

label order:

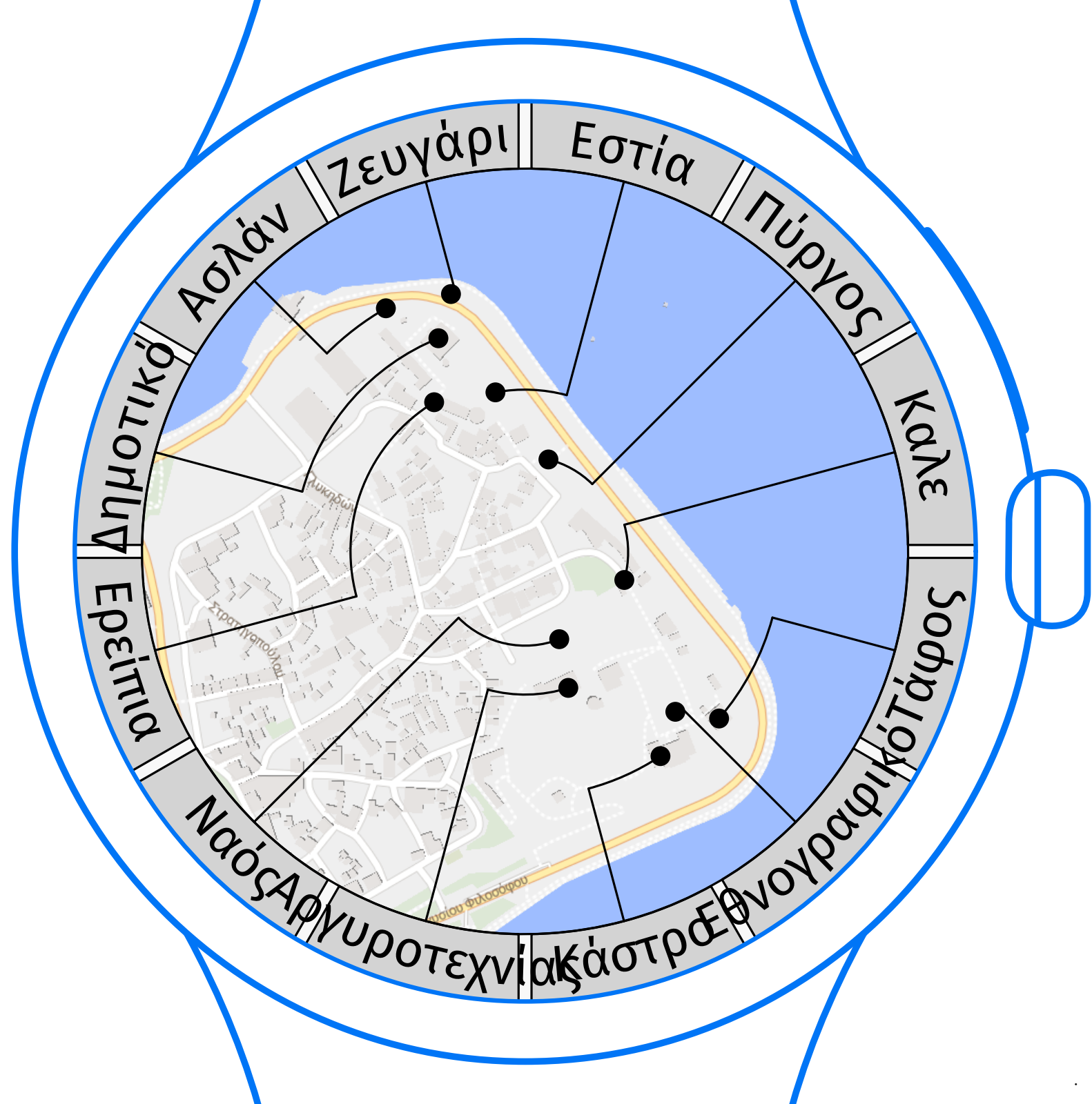
↻ free order

↓ fixed order

rotation:

🔑 continuous rotation

🔒 locked candidates C



Problem Variants

label order:

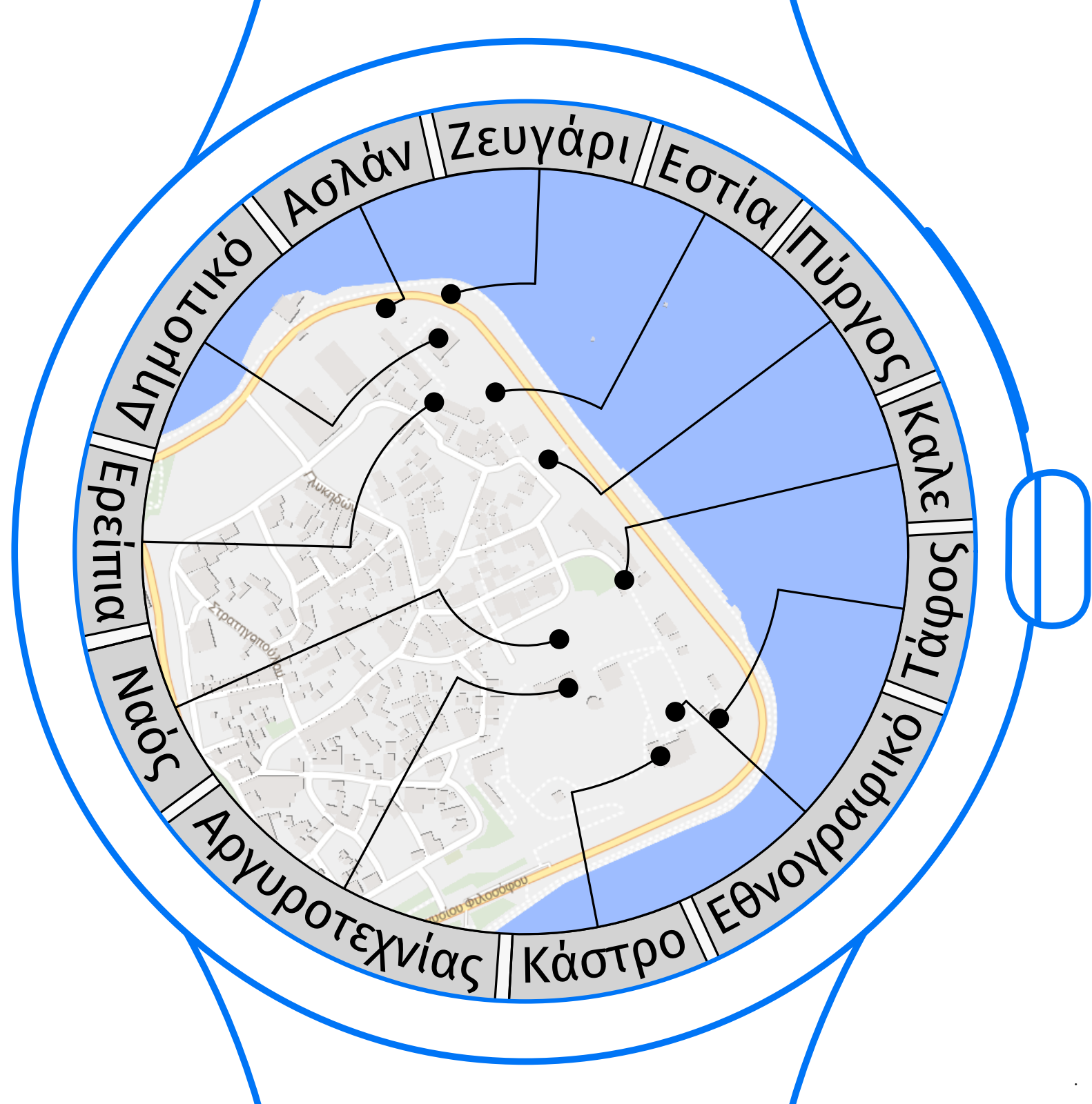
 free order

 fixed order

rotation:

 continuous rotation

 locked candidates C



Problem Variants

label order:

 free order

 fixed order

rotation:

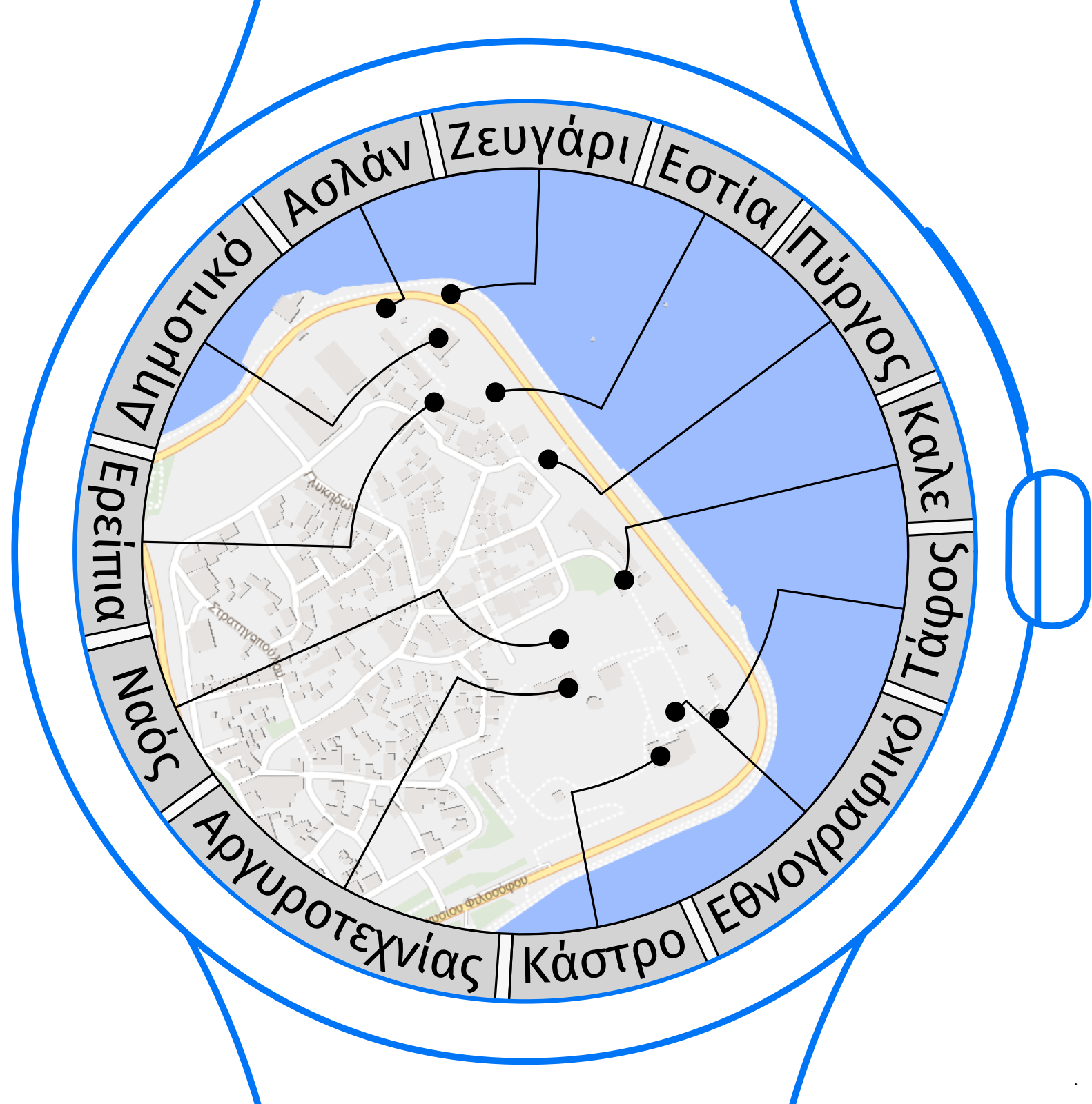
 continuous rotation

 locked candidates C

label size:

 uniform

 non-uniform



Problem Variants

label order:

 free order

 fixed order

rotation:

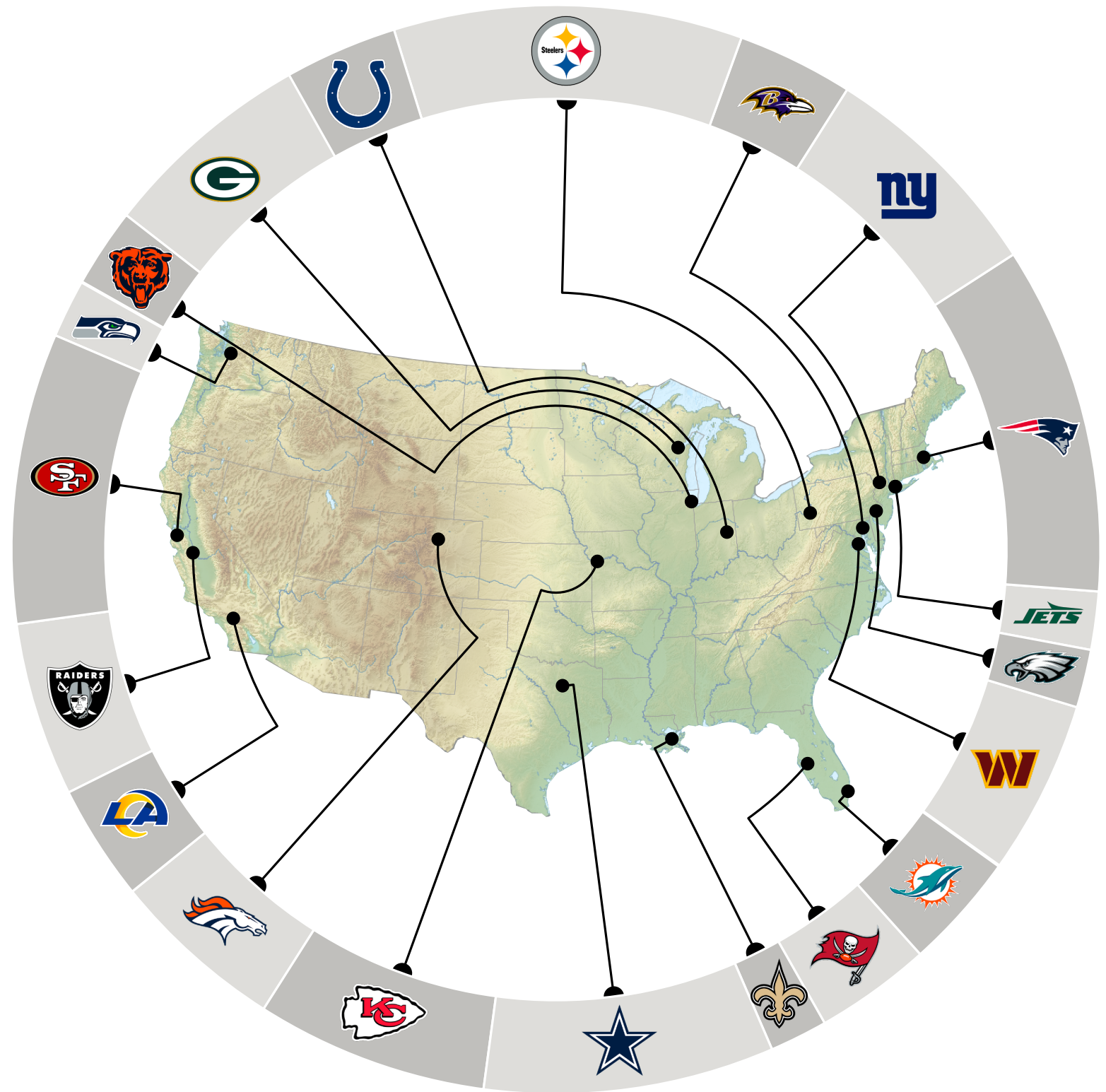
 continuous rotation

 locked candidates C

label size:

 uniform

 non-uniform



Problem Variants

label order:

 free order

 fixed order

rotation:

 continuous rotation

 locked candidates C

label size:

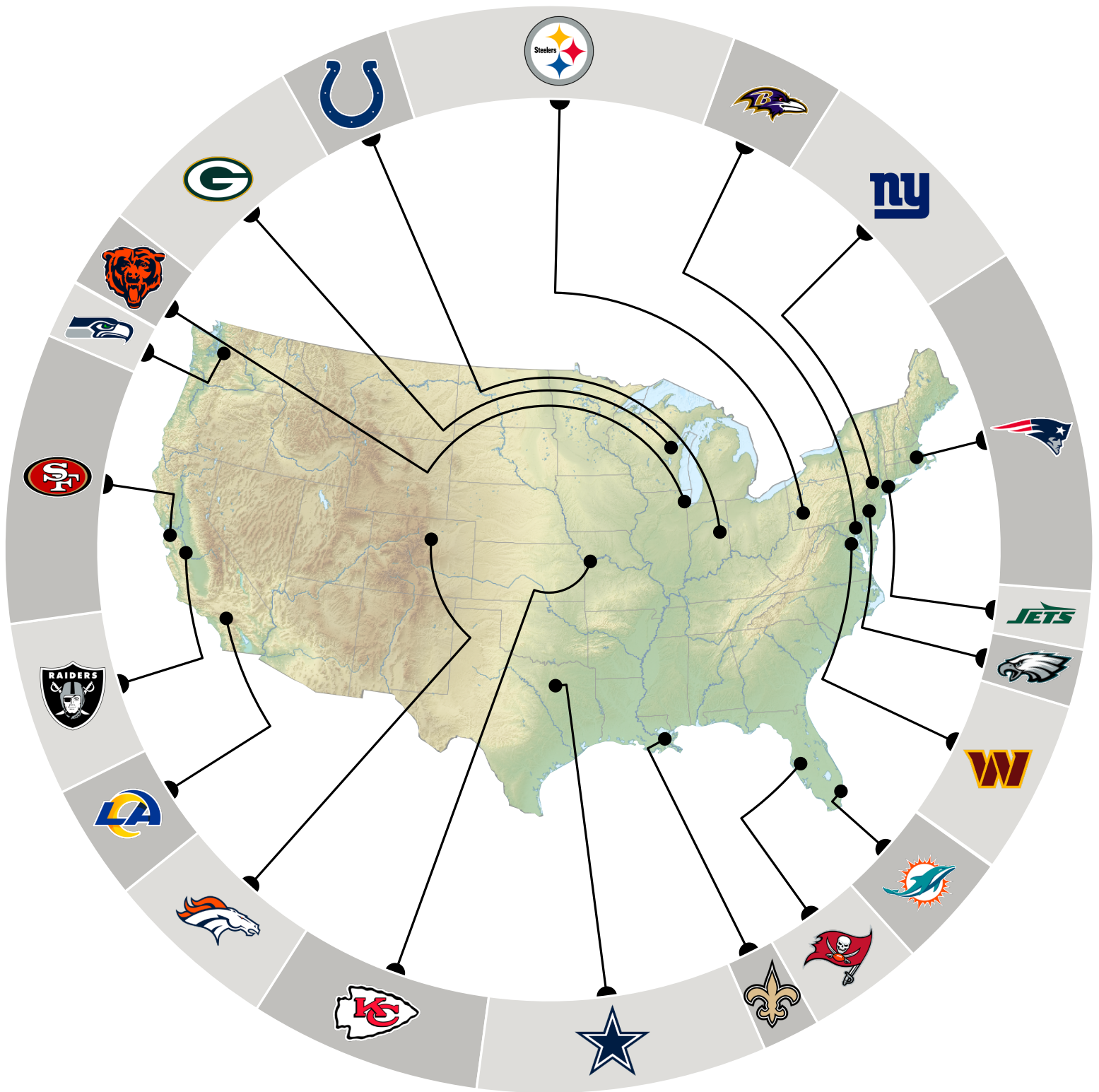
 uniform

 non-uniform

leader type:

 OR-leader

 SL-leader



Problem Variants

label order:

 free order

 fixed order

rotation:

 continuous rotation

 locked candidates C

label size:

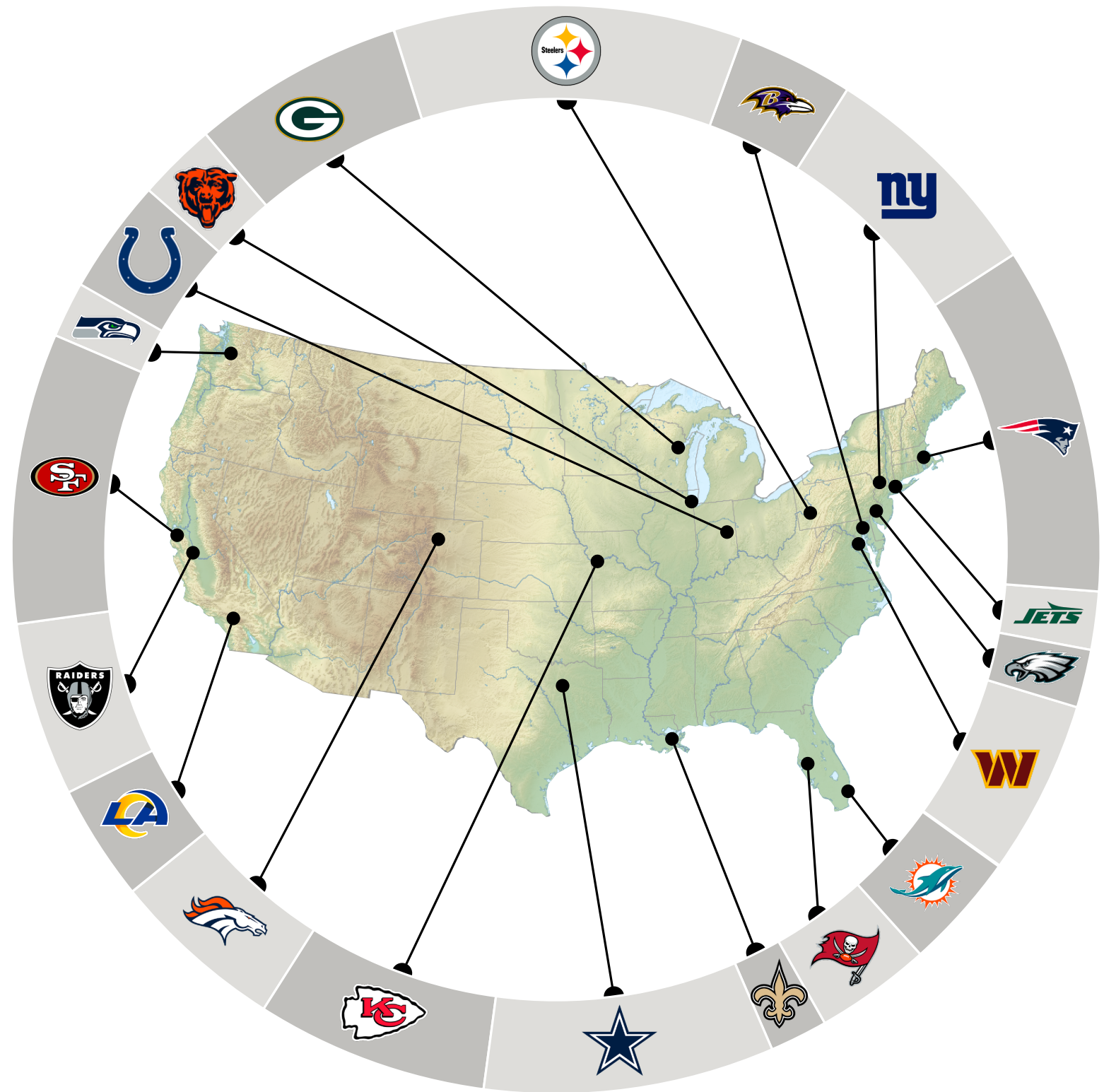
 uniform

 non-uniform

leader type:

 OR-leader

 SL-leader



Conclusion

Conclusion

rotation →	locked  (set C)		continuous 	

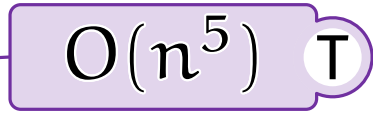
Conclusion

rotation → order →	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 

Conclusion

rotation →	locked  (set C)		continuous 	
order →	fixed 	free 	fixed 	free 
label size ↓				
uniform 				
non-uniform 				

Conclusion

rotation →	locked  (set C)		continuous 	
order →	fixed 	free 	fixed 	free 
label size ↓				
uniform 				$O(n^5)$ 
non-uniform 				

reduction to one-sided po-labeling

Conclusion

rotation →	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
order →				
label size ↓				
uniform 				$O(n^5)$ 
non-uniform 		$O(C^4)$		

reduction to one-sided po-labeling

Conclusion

rotation →	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
order →				
label size ↓				
uniform 				$O(n^5)$ 
non-uniform 		$O(C^4)$		

reduction to one-sided po-labeling

Conclusion

rotation →	locked  (set C)		continuous 	
order →	fixed 	free 	fixed 	free 
label size ↓				
uniform 				$O(n^5)$ 
non-uniform 		$O(C^4)$		NP-hard

reduction to one-sided po-labeling

reduction from Partition shows weak NP-Completeness

Conclusion

rotation →	locked  (set C)		continuous 	
order →	fixed 	free 	fixed 	free 
label size ↓				
uniform 			$O(n^2)$ 	$O(n^5)$ 
non-uniform 		$O(C^4)$	$O(n^2)$ 	NP-hard

minimum of a sum
of linear functions

reduction to one-
sided po-labeling

reduction from Partition shows
weak NP-Completeness

Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$		$O(n^2)$ 	$O(n^5)$ 
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard

brute force

minimum of a sum
of linear functions

reduction to one-
sided po-labeling

reduction from Partition shows
weak NP-Completeness

Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ T	$O(n^5)$ T
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ T	NP-hard

brute force

min-weight matching between features and ports

minimum of a sum of linear functions

reduction to one-sided po-labeling

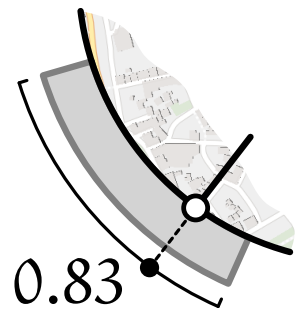
reduction from Partition shows weak NP-Completeness

Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard

Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard

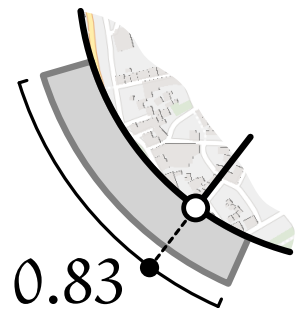


- port ratios → in particular free, non-uniform

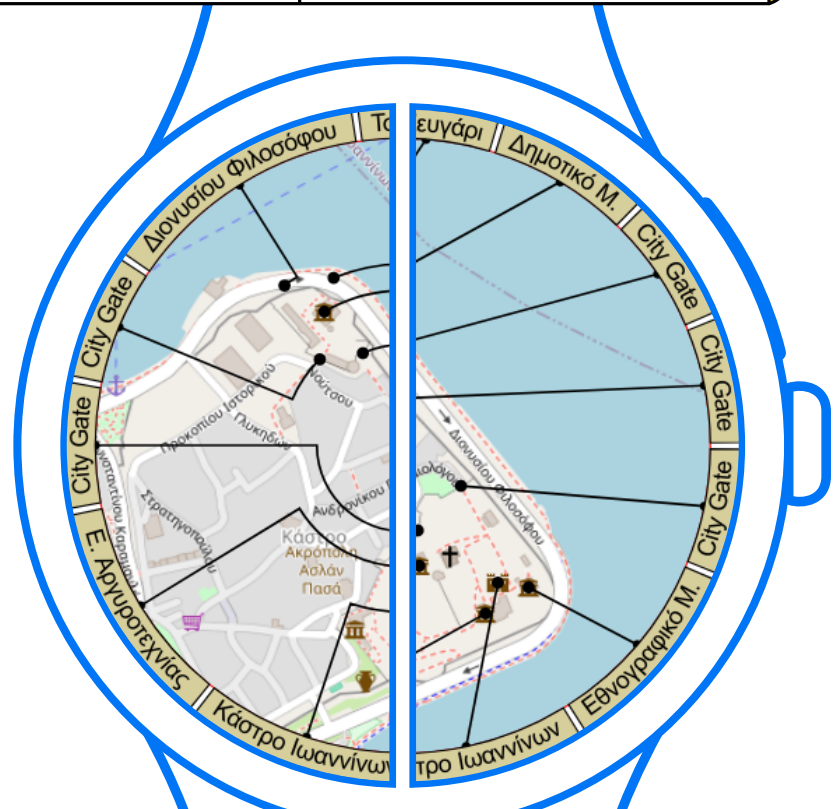
Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard

OR



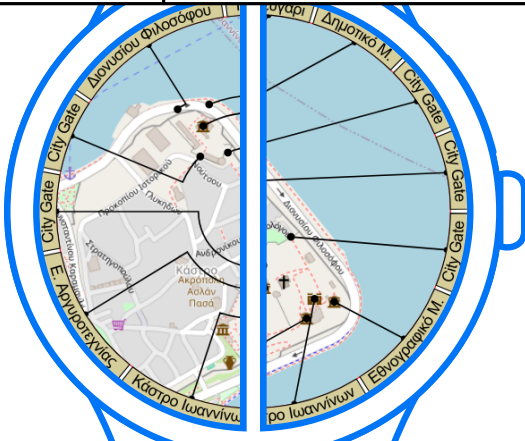
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend



Conclusion

rotation →	locked  (set C)		continuous 		
	fixed 	free 	fixed 	free 	
order →					
label size ↓					
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 	OR
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard	
uniform 					SL
non-uniform 					

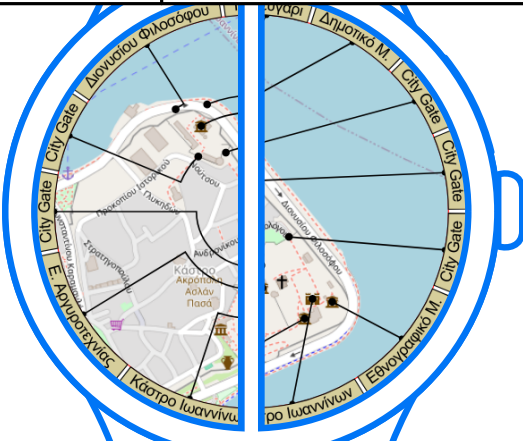
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend



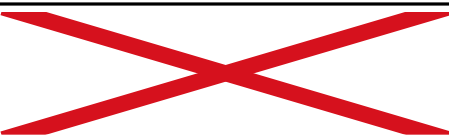
Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 		
	fixed 	free 	fixed 	free 	
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 	OR
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard	
uniform 	$O(n^2 C)$	$O(n C ^2)$			SL
non-uniform 	$O(n^2 C)$			NP-hard	

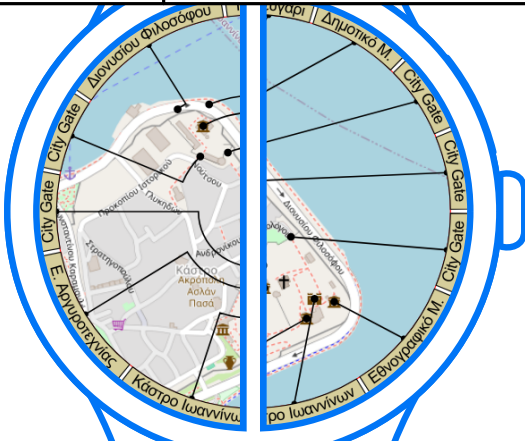
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend



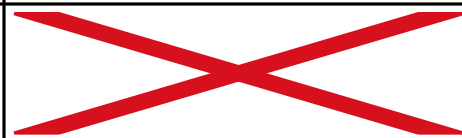
Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 		
	fixed 	free 	fixed 	free 	
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 	OR
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard	
uniform 	$O(n^2 C)$	$O(n C ^2)$			SL
non-uniform 	$O(n^2 C)$			NP-hard	

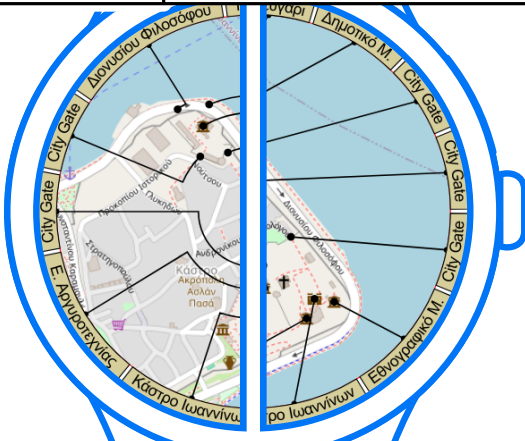
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend



Conclusion

rotation → order → label size ↓	locked  (set C)		continuous 	
	fixed 	free 	fixed 	free 
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard
uniform 	$O(n^2 C)$	$O(n C ^2)$	 $O(n^2)?$	
non-uniform 	$O(n^2 C)$		 $O(n^2)?$	NP-hard

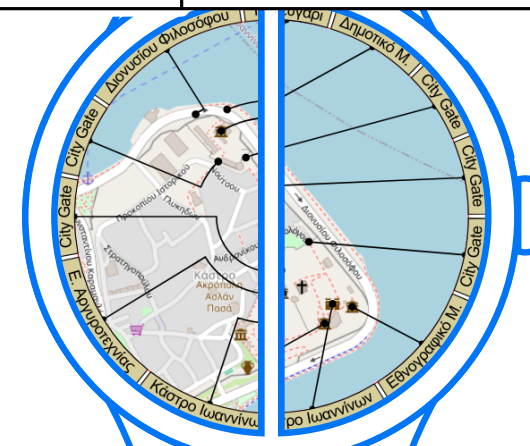
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend



Conclusion

rotation →	locked  (set C)		continuous 		
	order →	fixed 	free 	fixed 	
label size ↓					
uniform 	$O(n^2 C)$	$O(n C ^2)$	$O(n^2)$ 	$O(n^5)$ 	OR
non-uniform 	$O(n^2 C)$	$O(C^4)$	$O(n^2)$ 	NP-hard	
uniform 	$O(n^2 C)$	$O(n C ^2)$	 $O(n^2)?$		SL
non-uniform 	$O(n^2 C)$		 $O(n^2)?$	NP-hard	

- port ratios $\rightarrow$ in particular free, non-uniform
- straight line leaders $\rightarrow$ some results extend
- user study evaluation comparing OR and SL



				C^{lock}		C^{unlock}	
				A^{lock}	A^{unlock}	A^{lock}	A^{unlock}
OR	O^{lock}	S^{lock}	$O(C n^2)$ [S. 4.1]	$O(C n^2)$ [S. 4.1]	$O(n^2)$ [S. 4.3]	$O(n^2)$ [S. 4.3]	
		S^{unlock}	$O(C n^2)$ [S. 4.1]	$O(C n^2)$ [S. 4.1]	$O(n^2)$ [S. 4.3]	$O(n^2)$ [S. 4.3]	
	O^{unlock}	S^{lock}	$O(C ^2n)$ [S. 4.2]	$O(C n^3)$ [S. 3]	$O(n^5)$ [S. 3]	$O(n^5)$ [S. 3]	
		S^{unlock}	$O(C ^4)$ [S. 3]	$O(C ^4)$ [S. 3]	w. NP-h [S. 5]	w. NP-h [S. 5]	
SL	O^{lock}	S^{lock}	$O(C n^2)$ [8, B.1]	$O(C n^2)$ [8, B.1]	$O(n^2) ?$ [8, B.3]	$O(n^2) ?$ [8, B.3]	
		S^{unlock}	$O(C n^2)$ [8, B.1]	$O(C n^2)$ [8, B.1]	$O(n^2) ?$ [8, B.3]	$O(n^2) ?$ [8, B.3]	
	O^{unlock}	S^{lock}	$O(C ^2n)$ [8, B.2]				
		S^{unlock}			w. NP-h [8, B.4]	w. NP-h [8, B.4]	
				C^{lock}		C^{unlock}	
				A^{unlock}	A^{lock}	A^{unlock}	A^{lock}
OR	O^{lock}	S^{lock}	$O(C n^2)$ [8, C.1]	$O(C ^2n^3)$ [8, C.2]	$O(n^2)$ [8, C.1]	$O(n^6)$ [8, C.2]	
		S^{unlock}		$O(C ^5n)$ [8, C.2]		$O(n^7)$ [8, C.2]	
	O^{unlock}	S^{lock}	$O(C ^2n)$ [8, C.1]	$O(C ^2n^3)$ [8, C.2]	$O(n^5)$ [8, C.1]	$O(n^6)$ [8, C.2]	
		S^{unlock}			w. NP-h [8, C.3]		
SL	O^{lock}	S^{lock}	$O(C n^2)$ [8, C.1]		$O(n^2) ?$ [8, C.1]		
		S^{unlock}					
	O^{unlock}	S^{lock}	$O(C ^2n)$ [8, C.1]				
		S^{unlock}			w. NP-h [8, C.3]		

Hardness

continuous rotation 

free order 

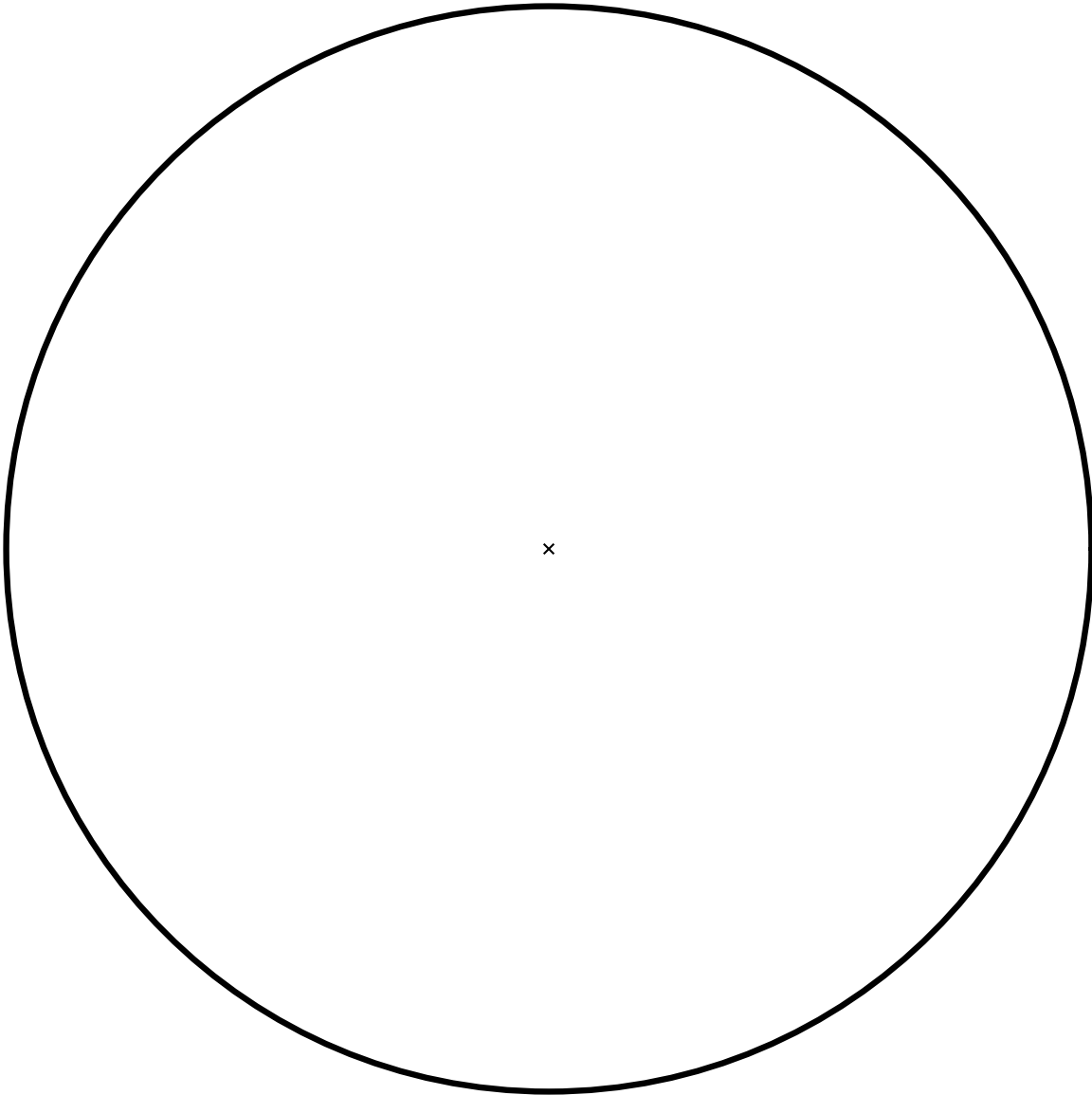
non-uniform labels 

Hardness

continuous rotation 

free order 

non-uniform labels 

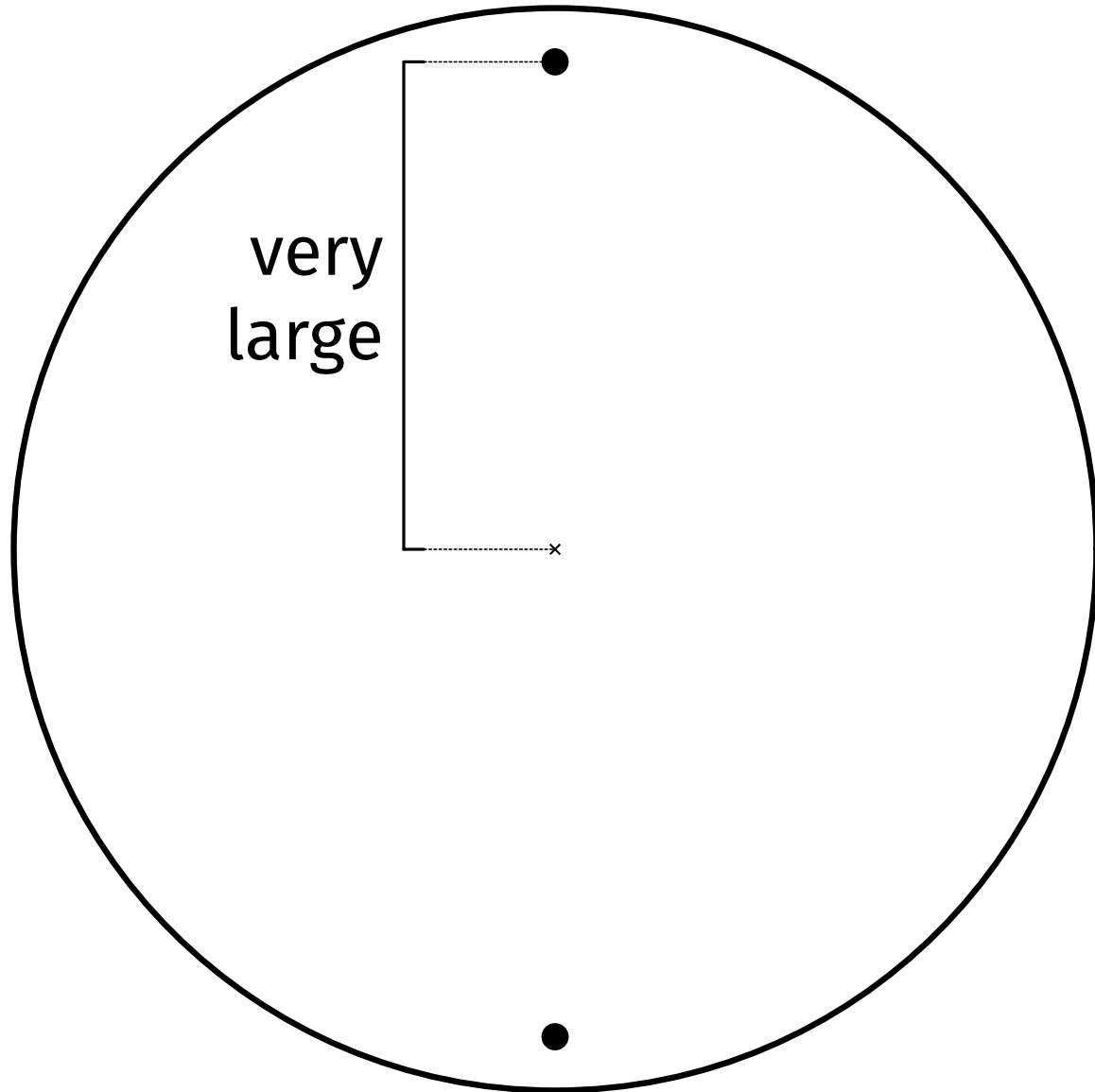


Hardness

continuous rotation 

free order 

non-uniform labels 

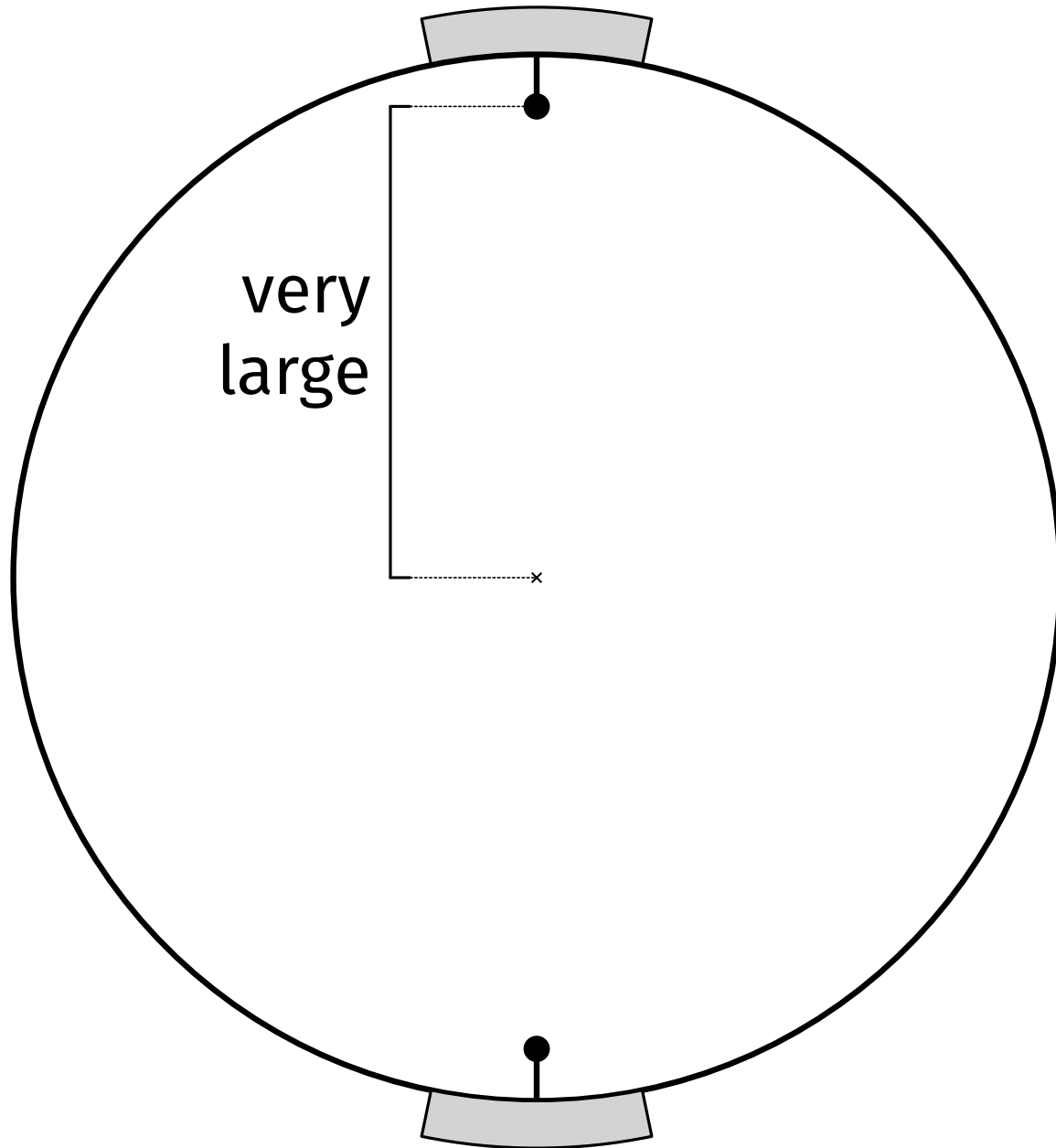


Hardness

continuous rotation 

free order 

non-uniform labels 

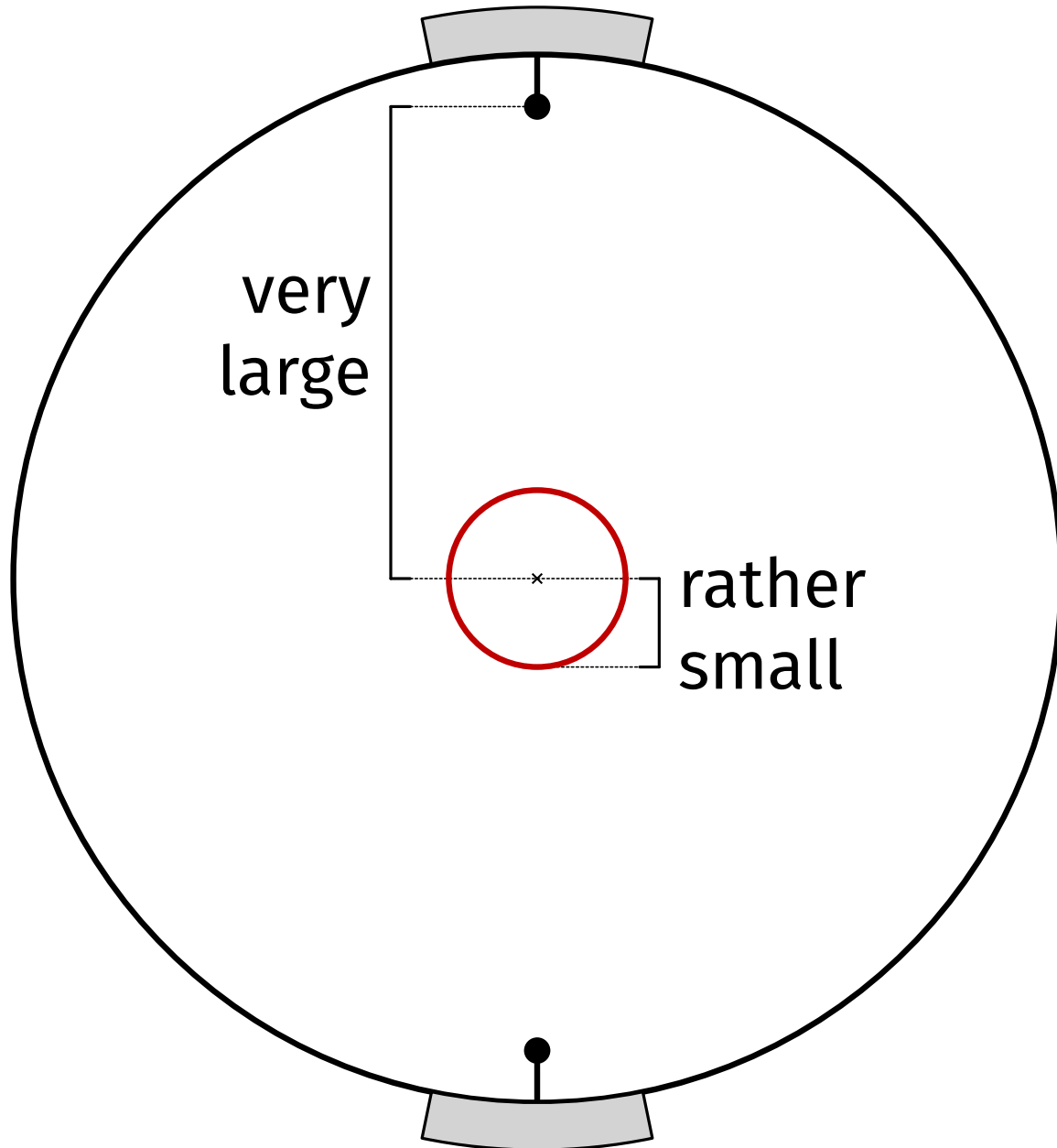


Hardness

continuous rotation 

free order 

non-uniform labels 

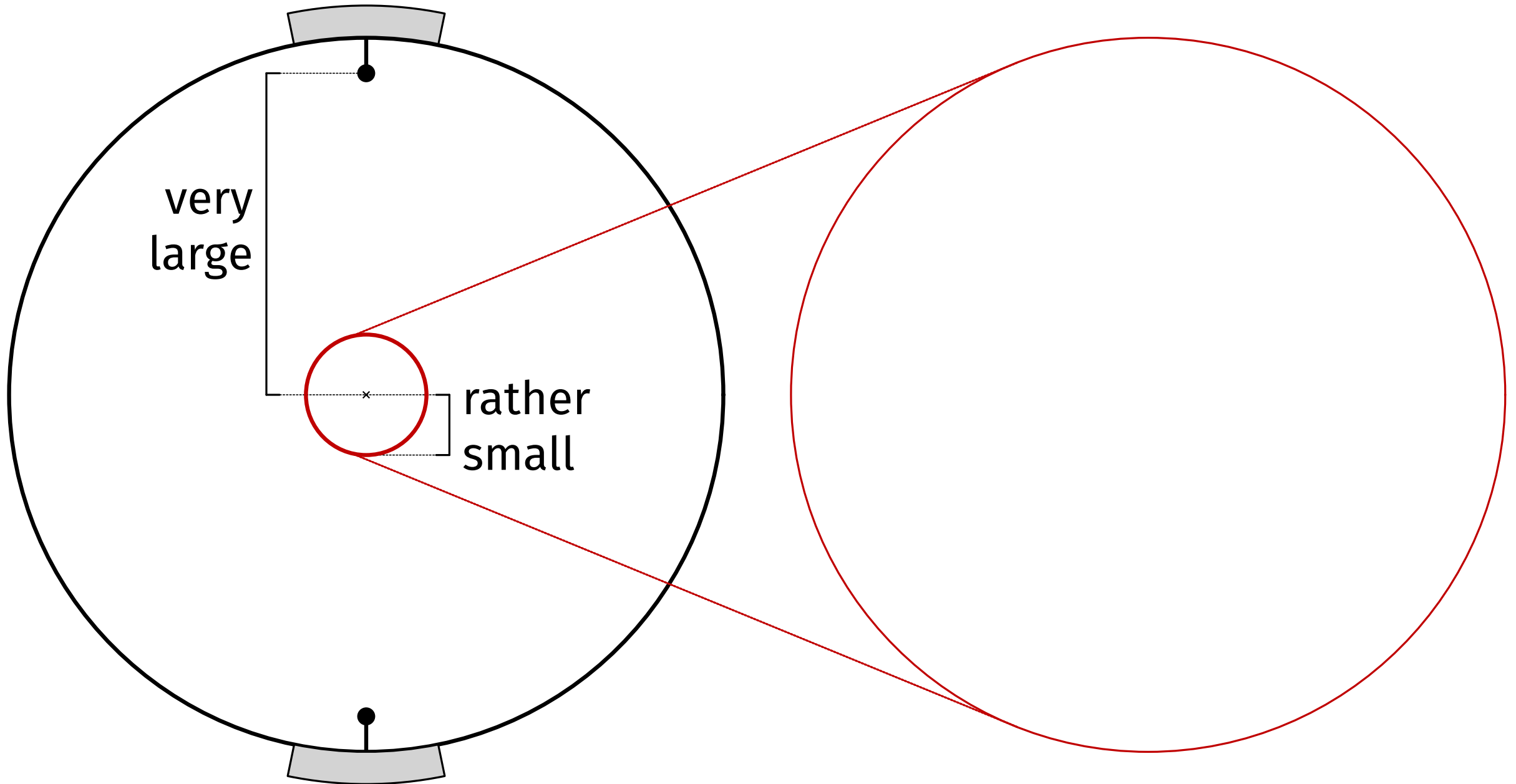


Hardness

continuous rotation 

free order 

non-uniform labels 

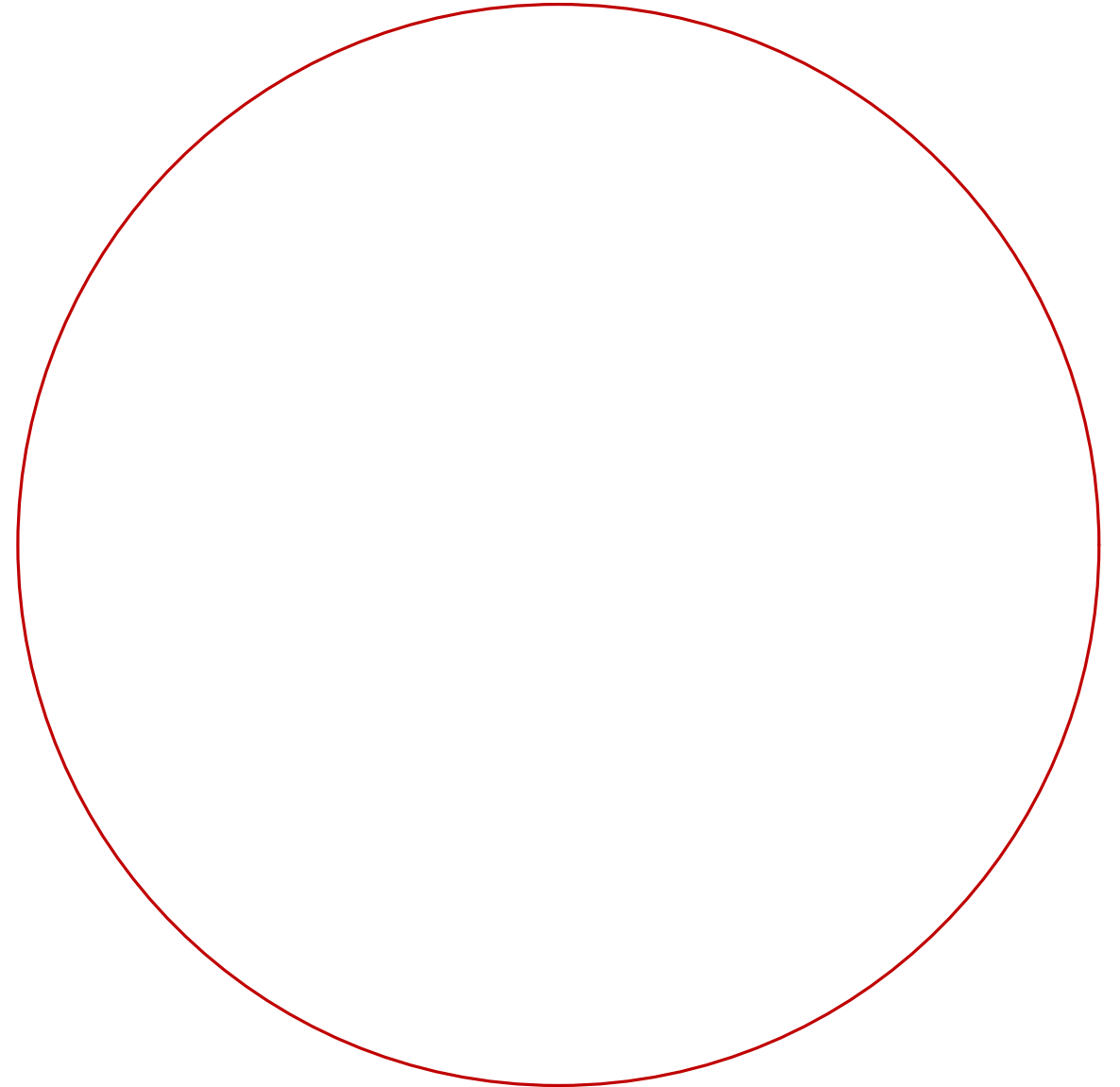
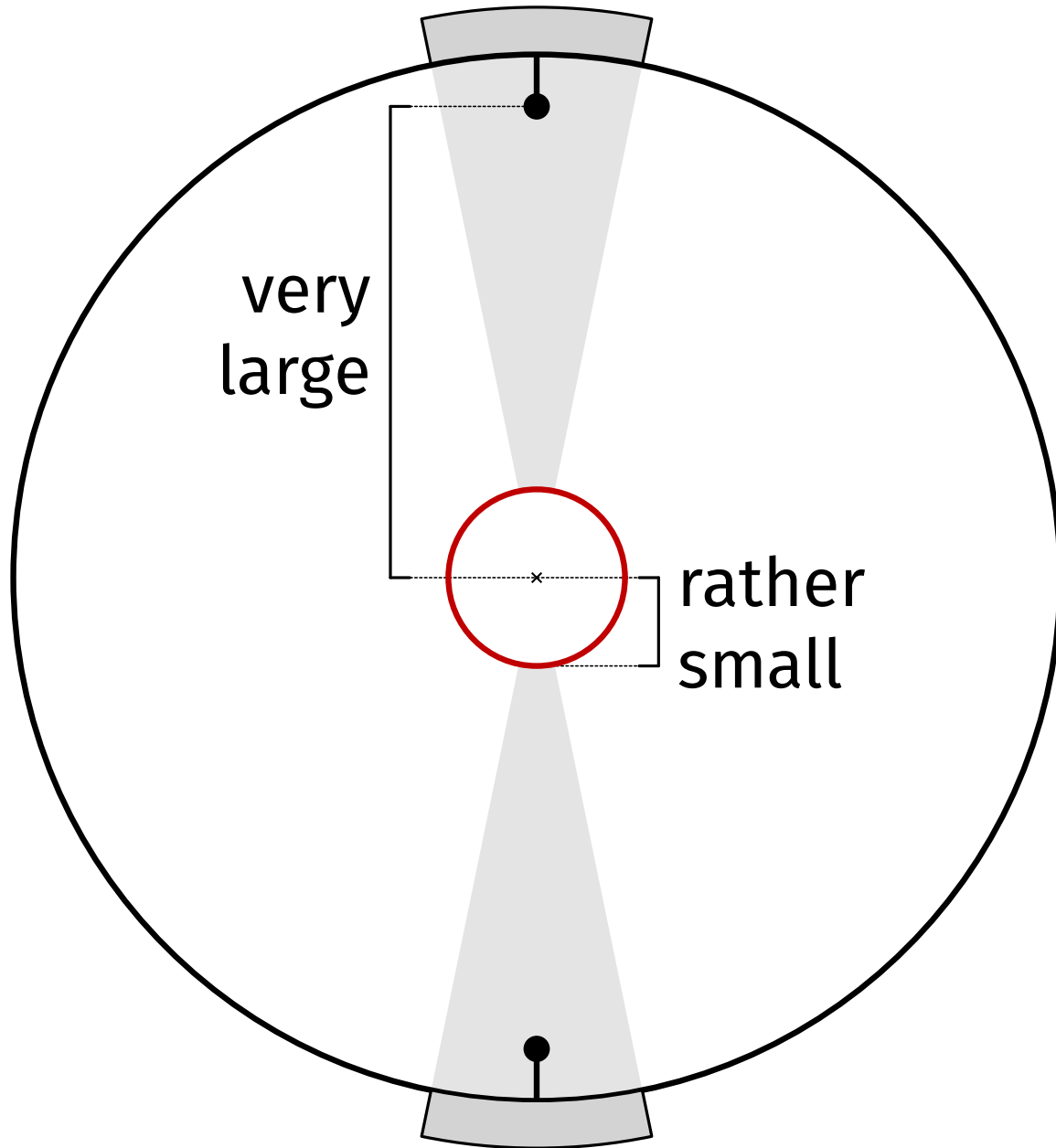


Hardness

continuous rotation 

free order 

non-uniform labels 

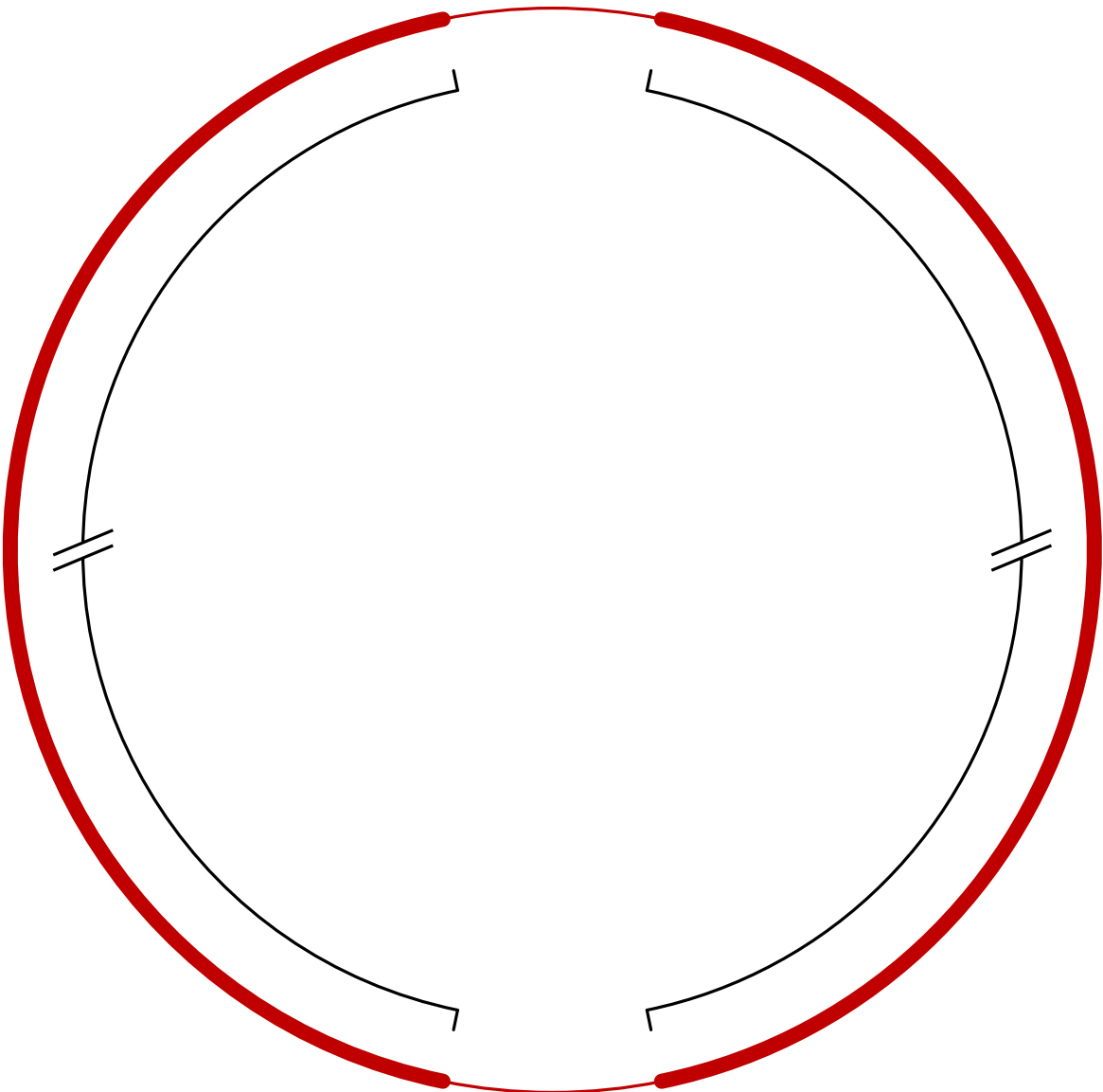
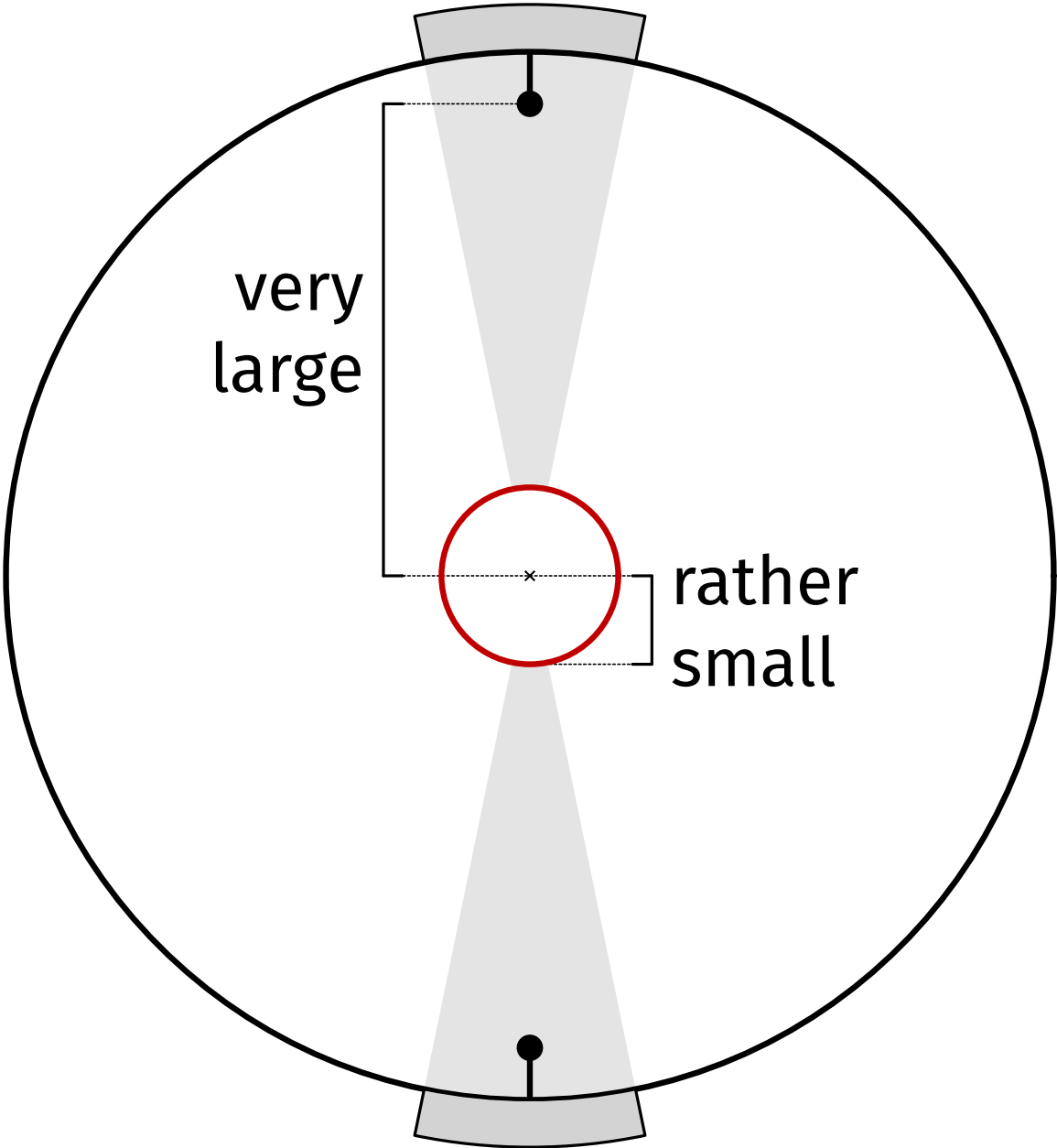


Hardness

continuous rotation 

free order 

non-uniform labels 

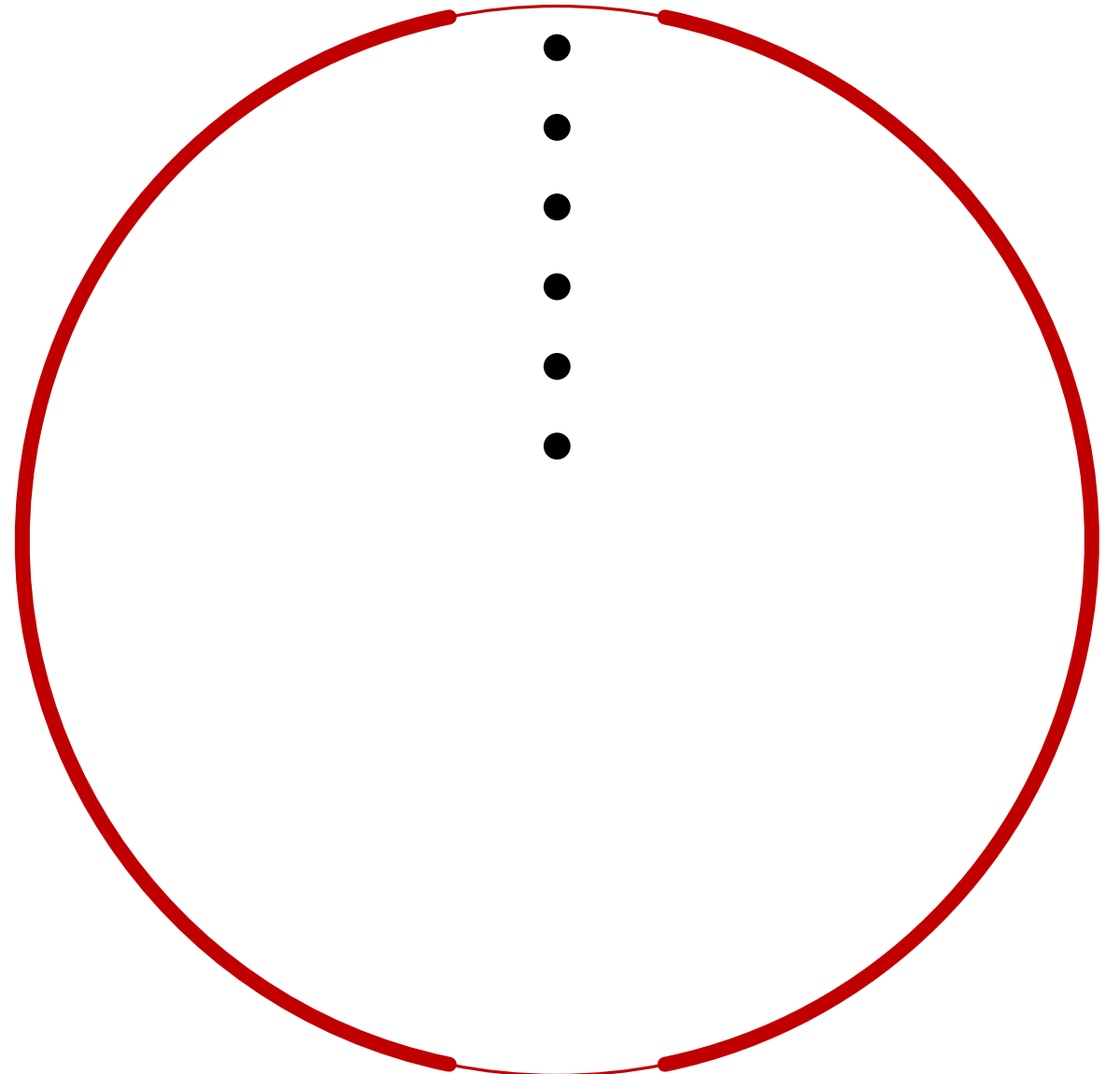
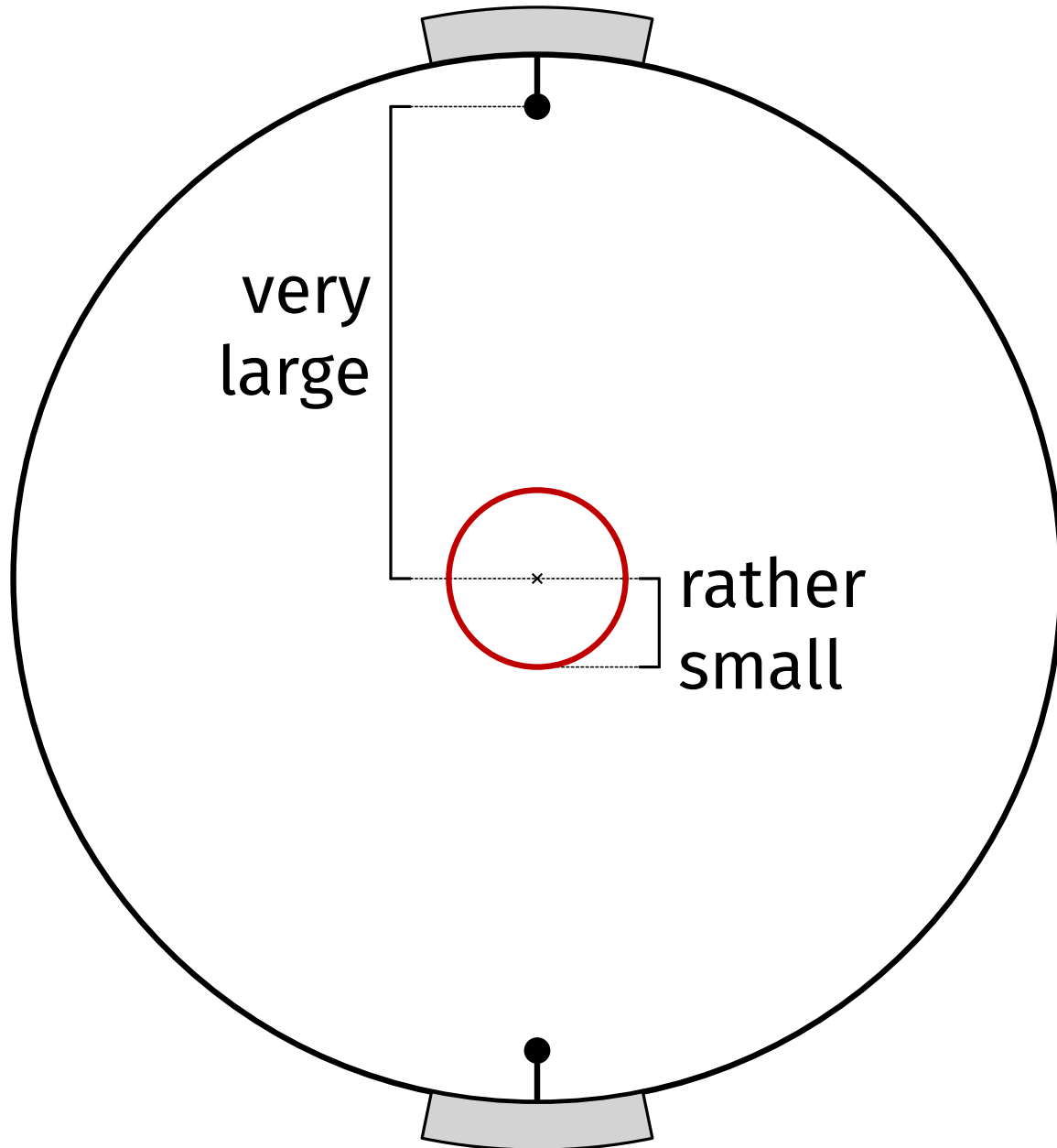


Hardness

continuous rotation 

free order 

non-uniform labels 

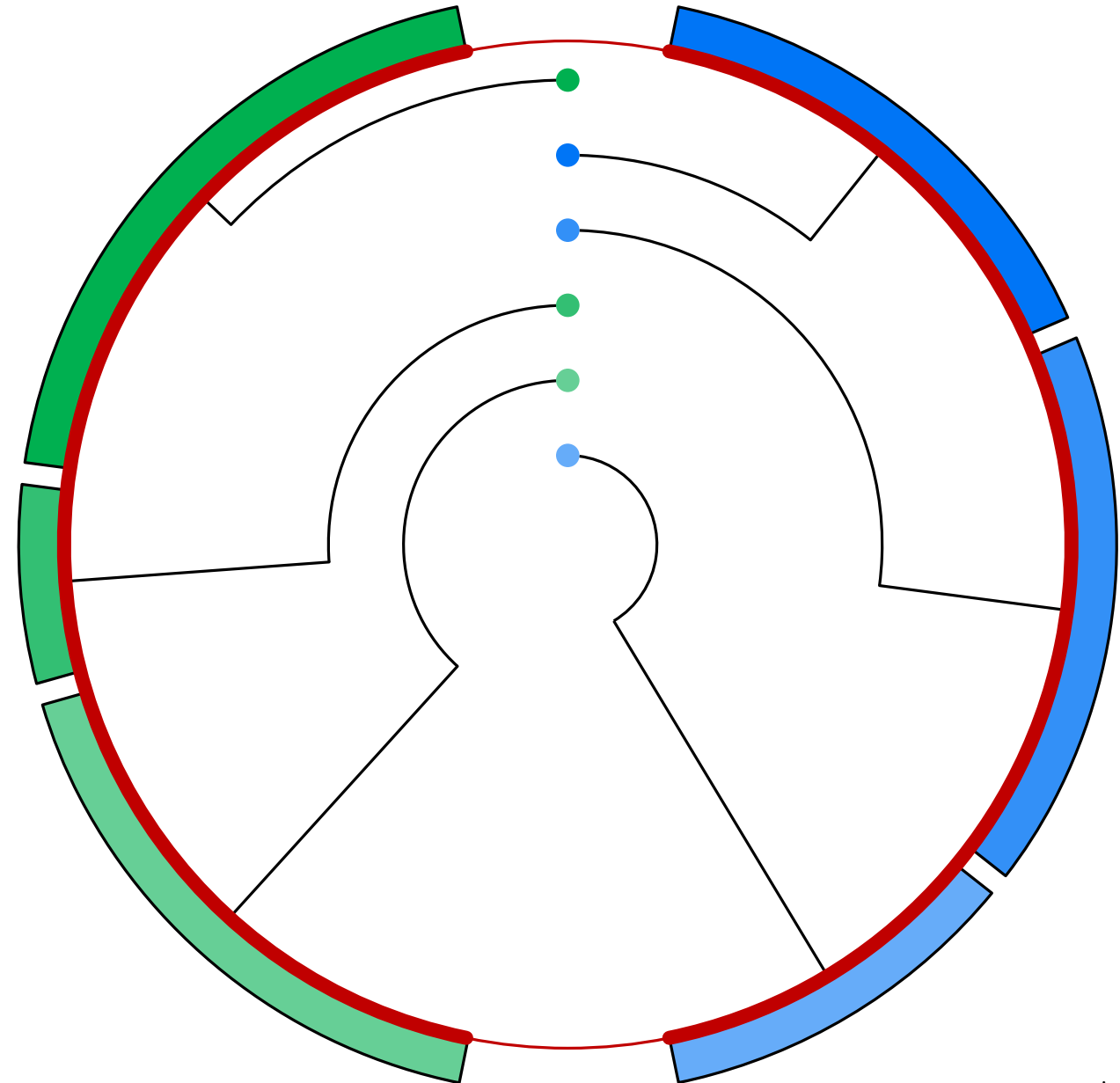
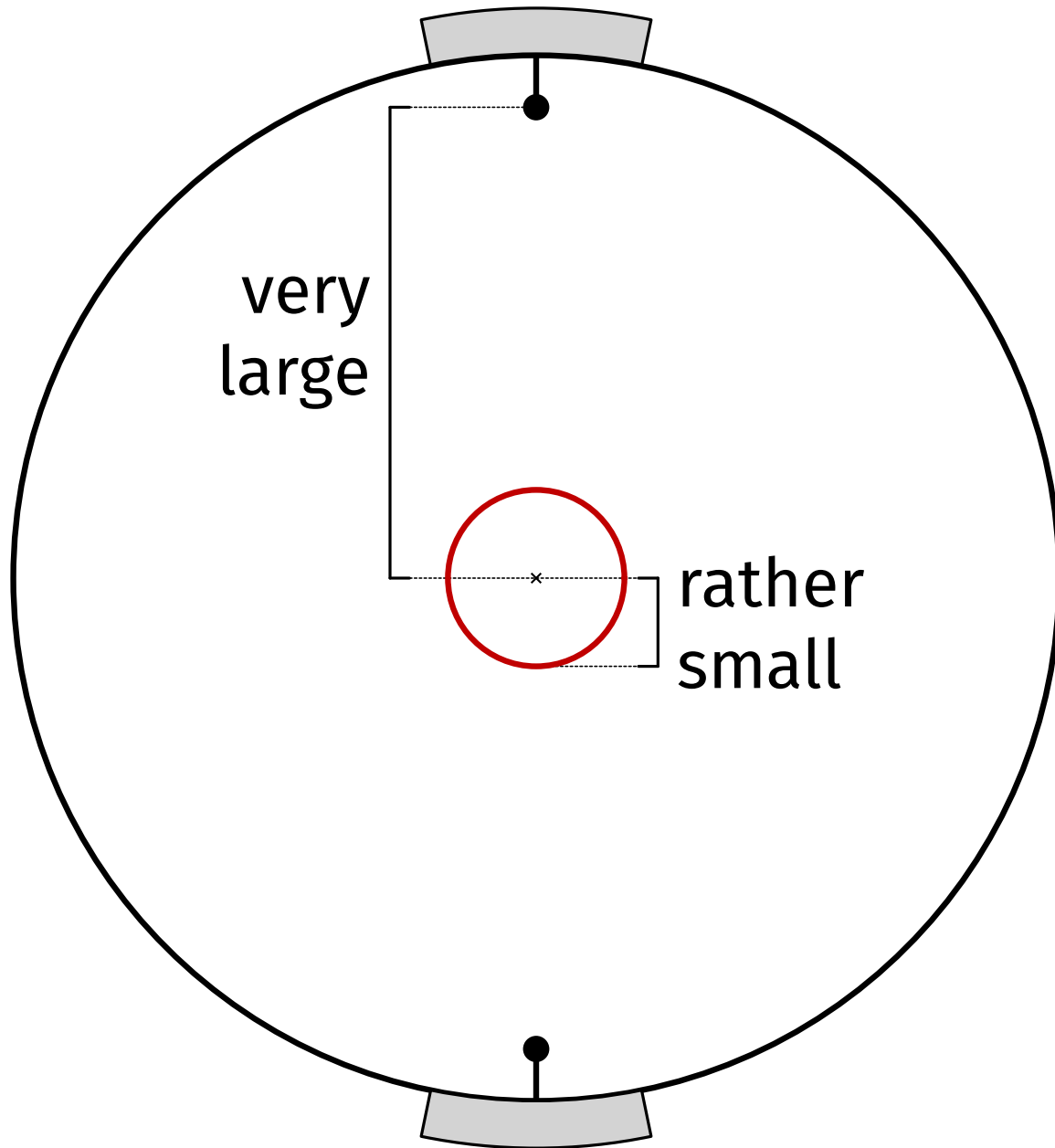


Hardness

continuous rotation 

free order 

non-uniform labels 

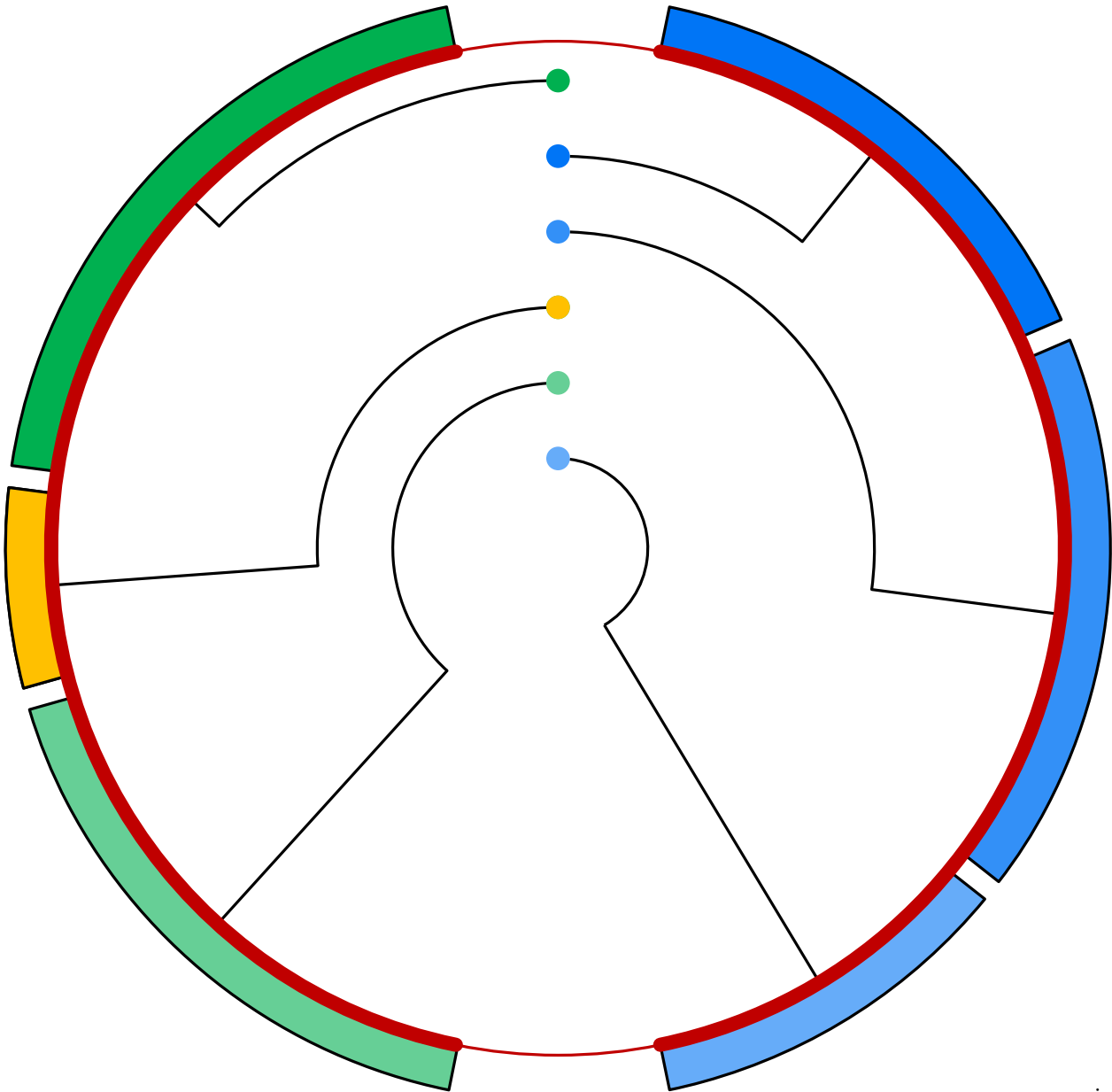
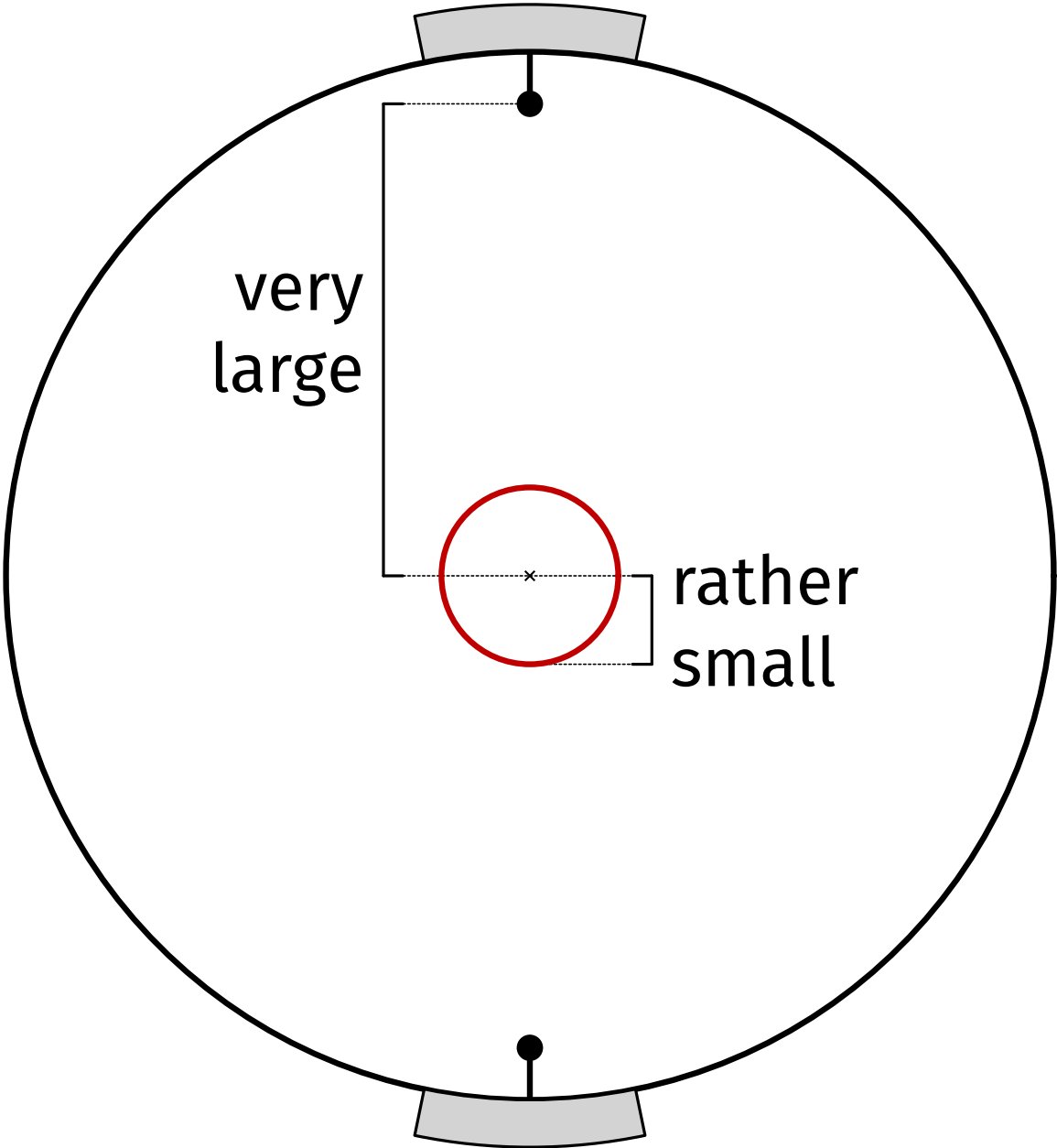


Hardness

continuous rotation 

free order 

non-uniform labels 

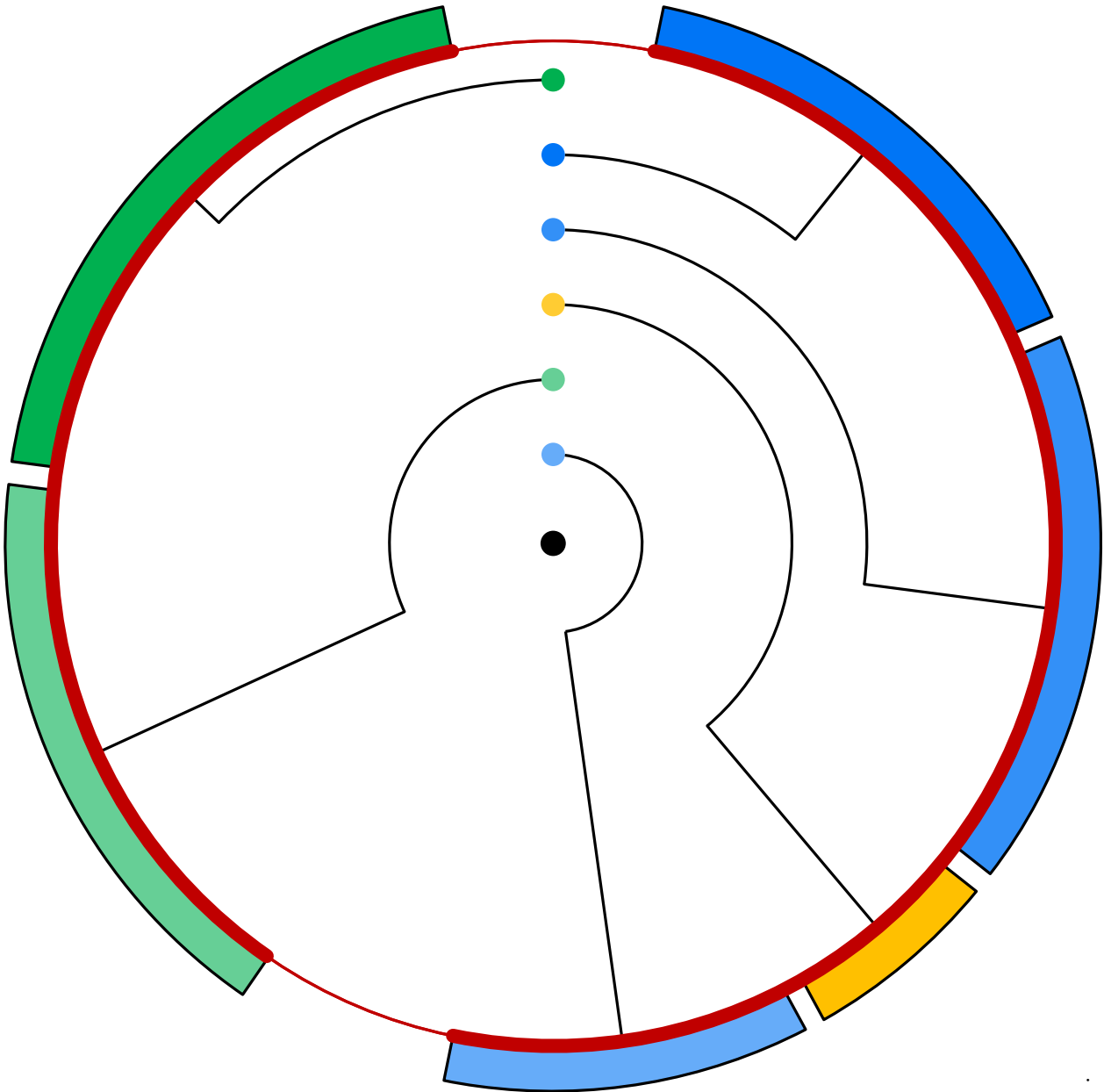
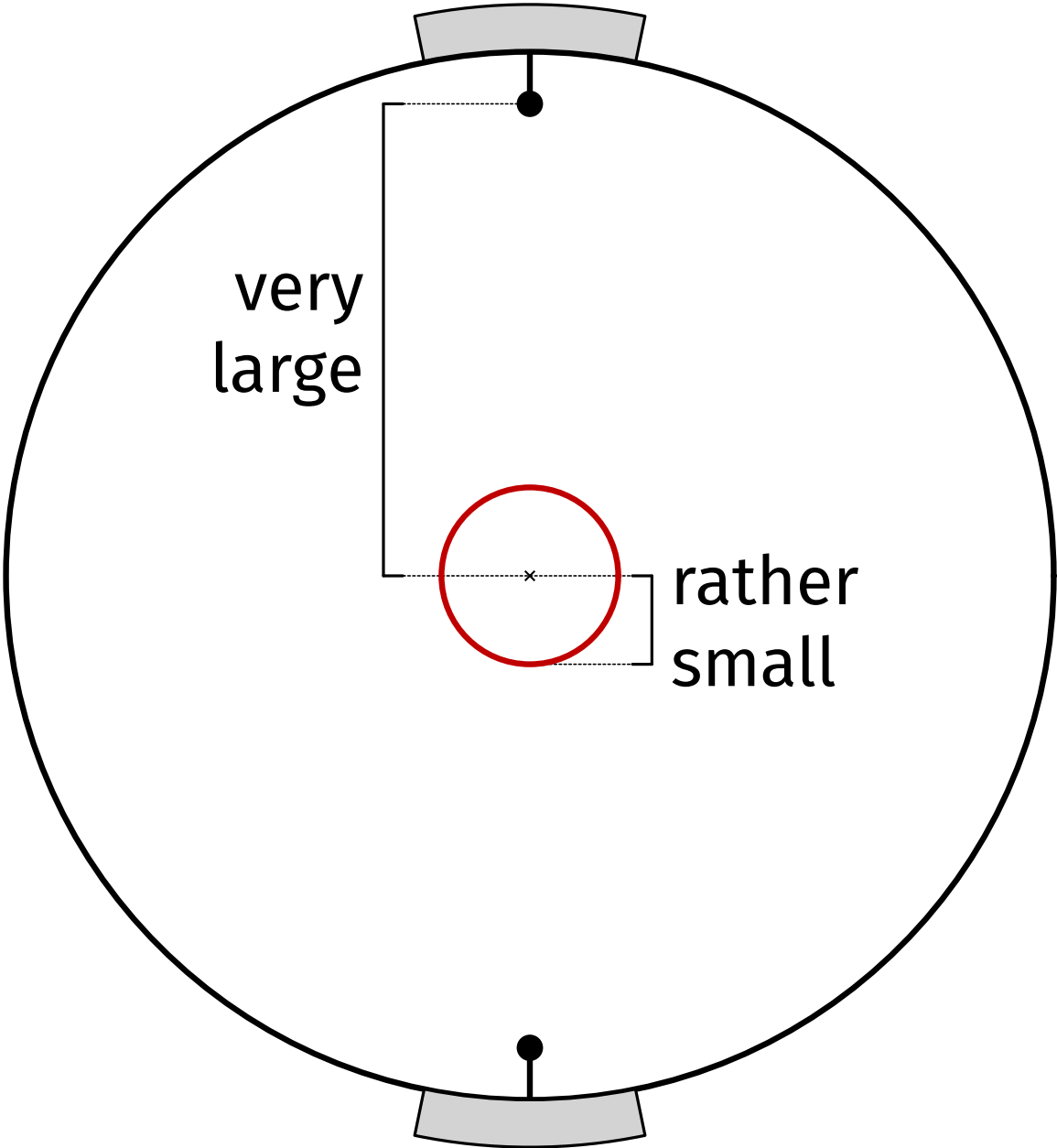


Hardness

continuous rotation 

free order 

non-uniform labels 

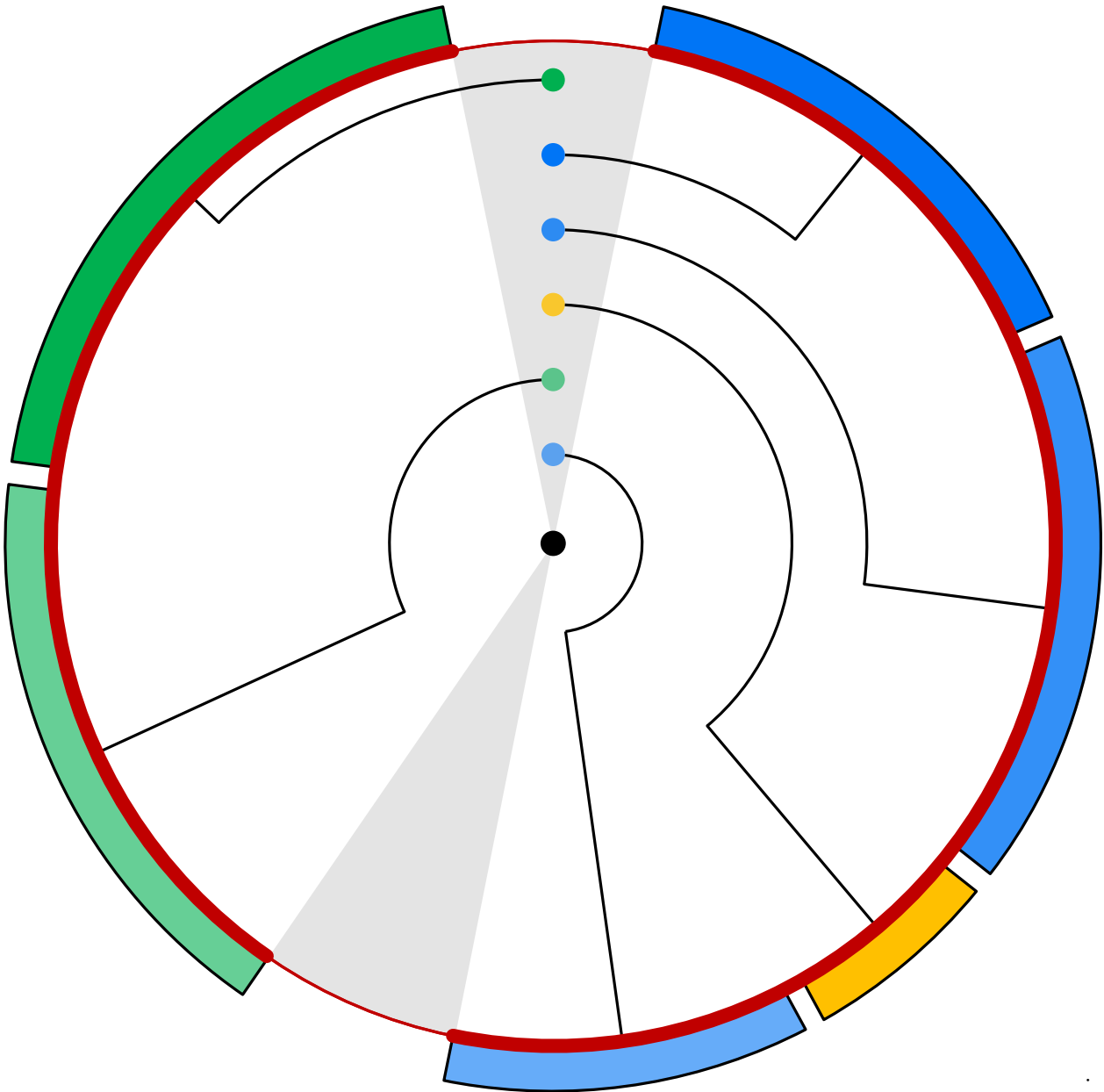
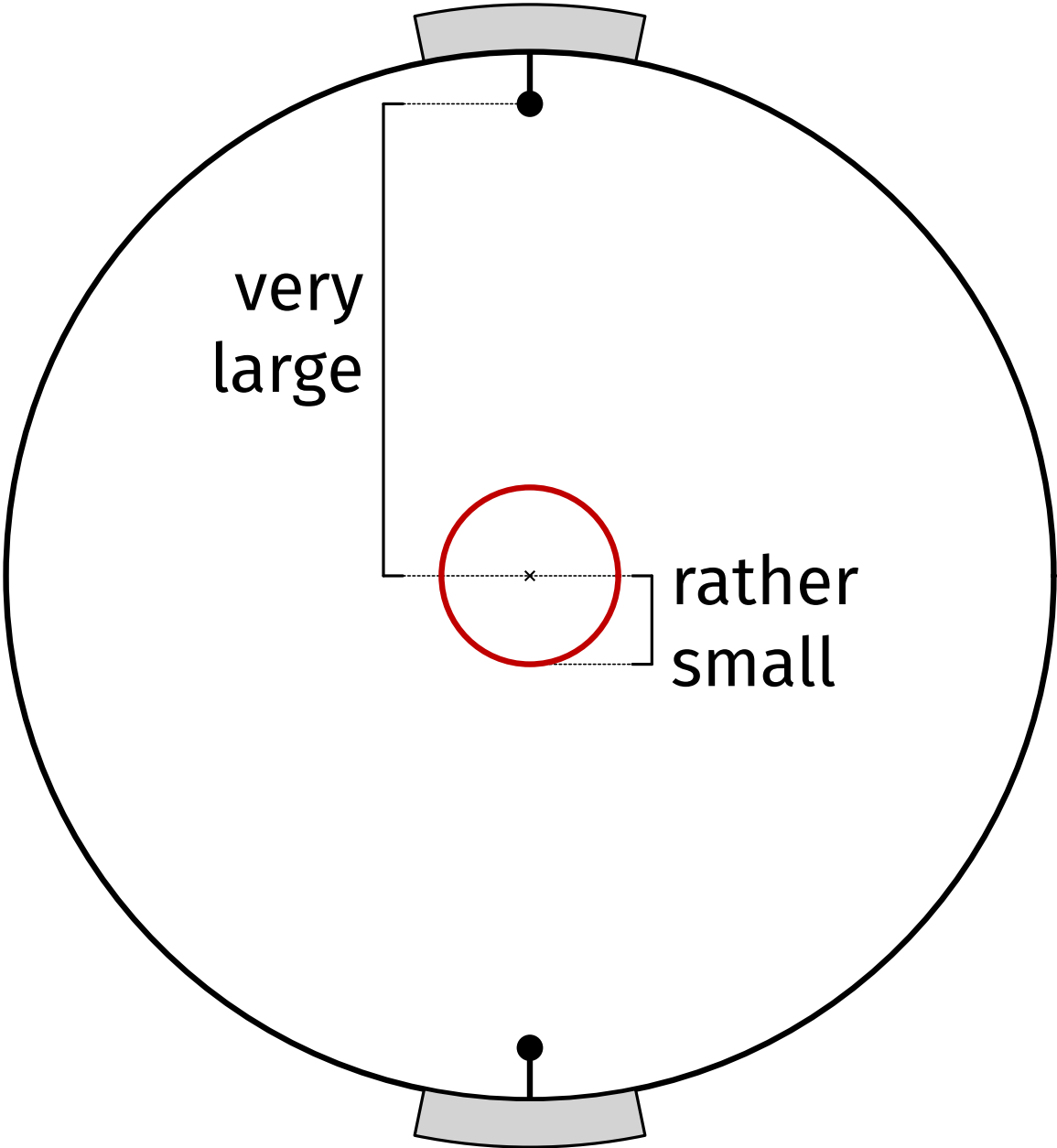


Hardness

continuous rotation 

free order 

non-uniform labels 

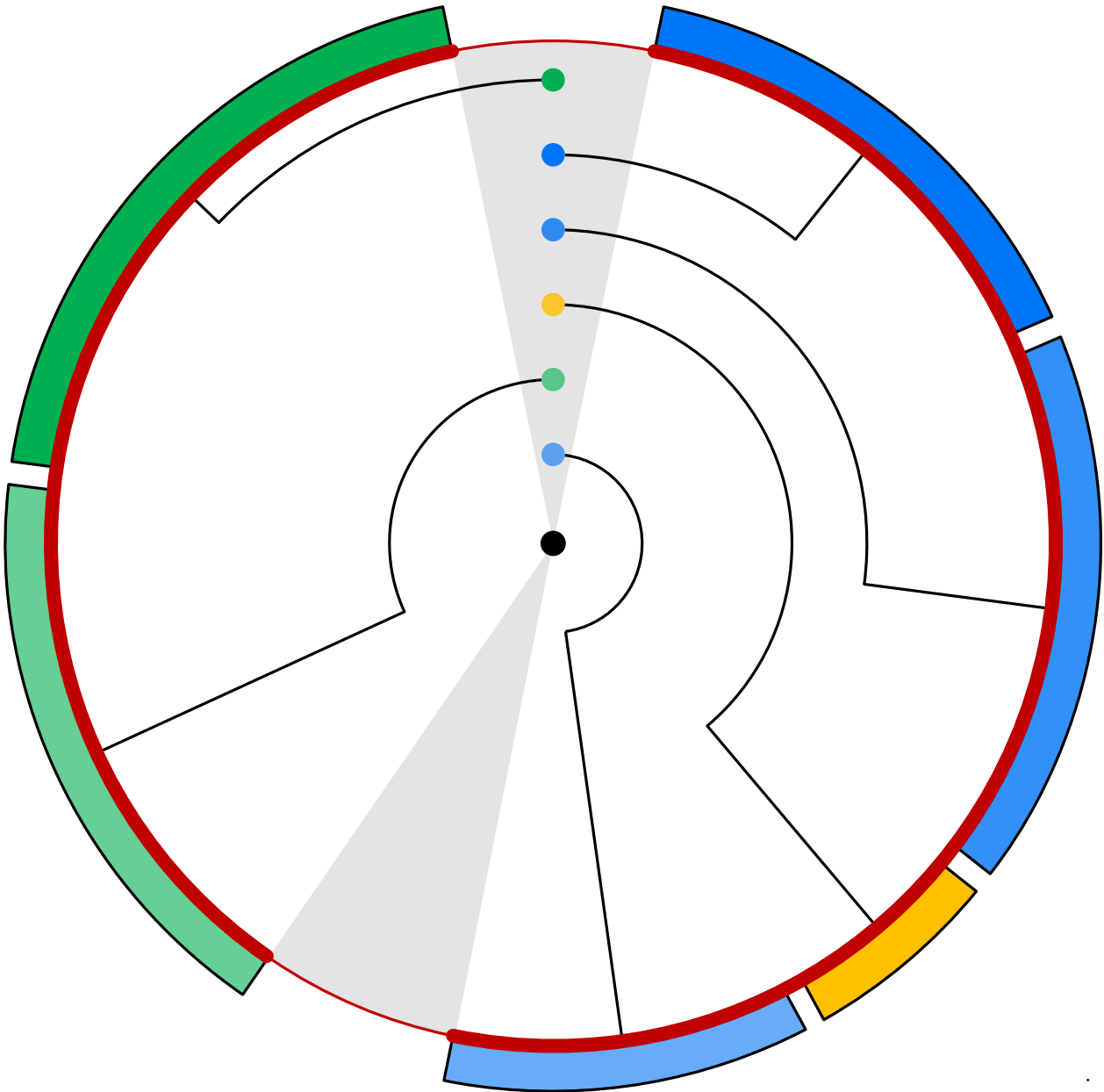
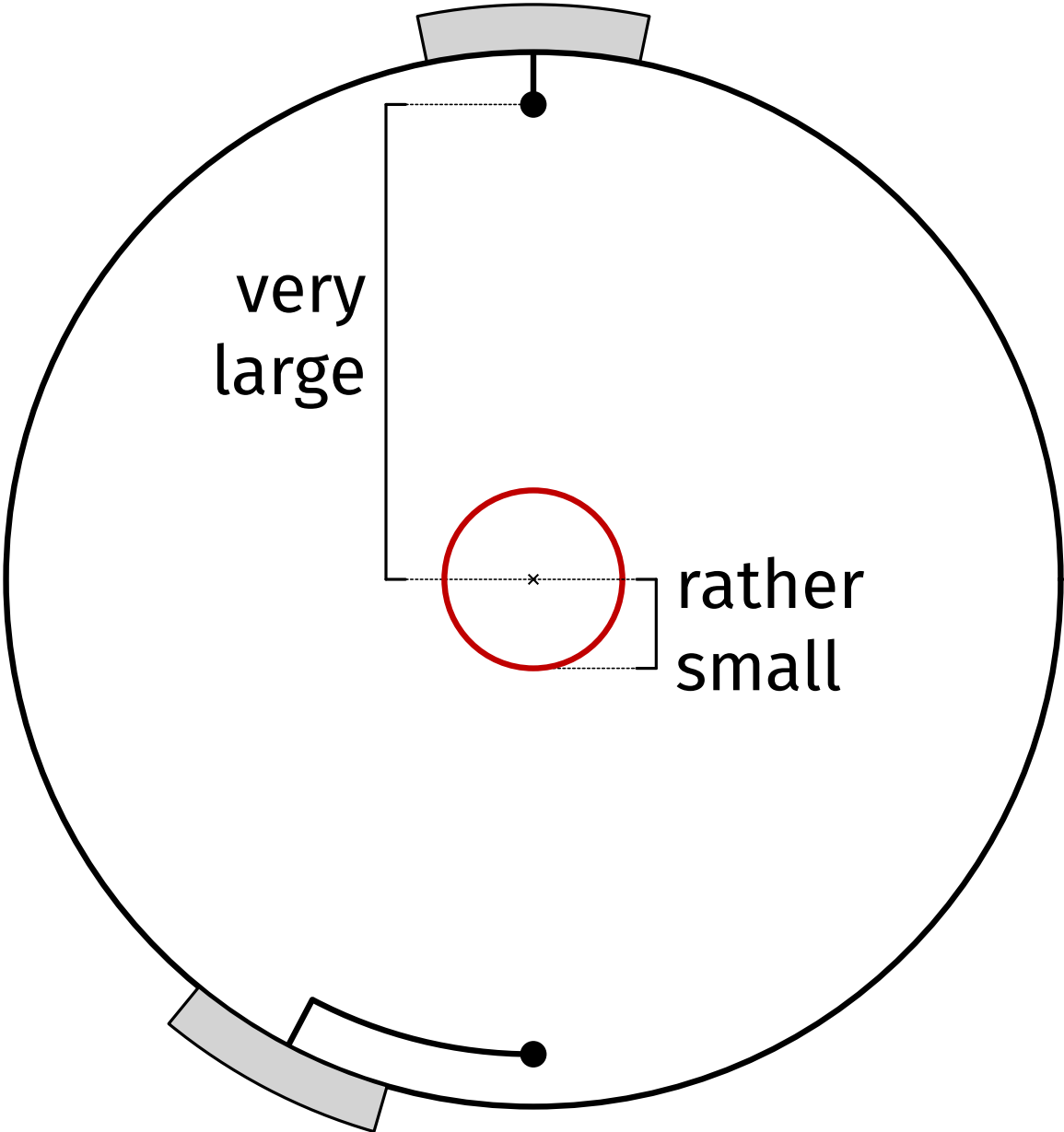


Hardness

continuous rotation 

free order 

non-uniform labels 

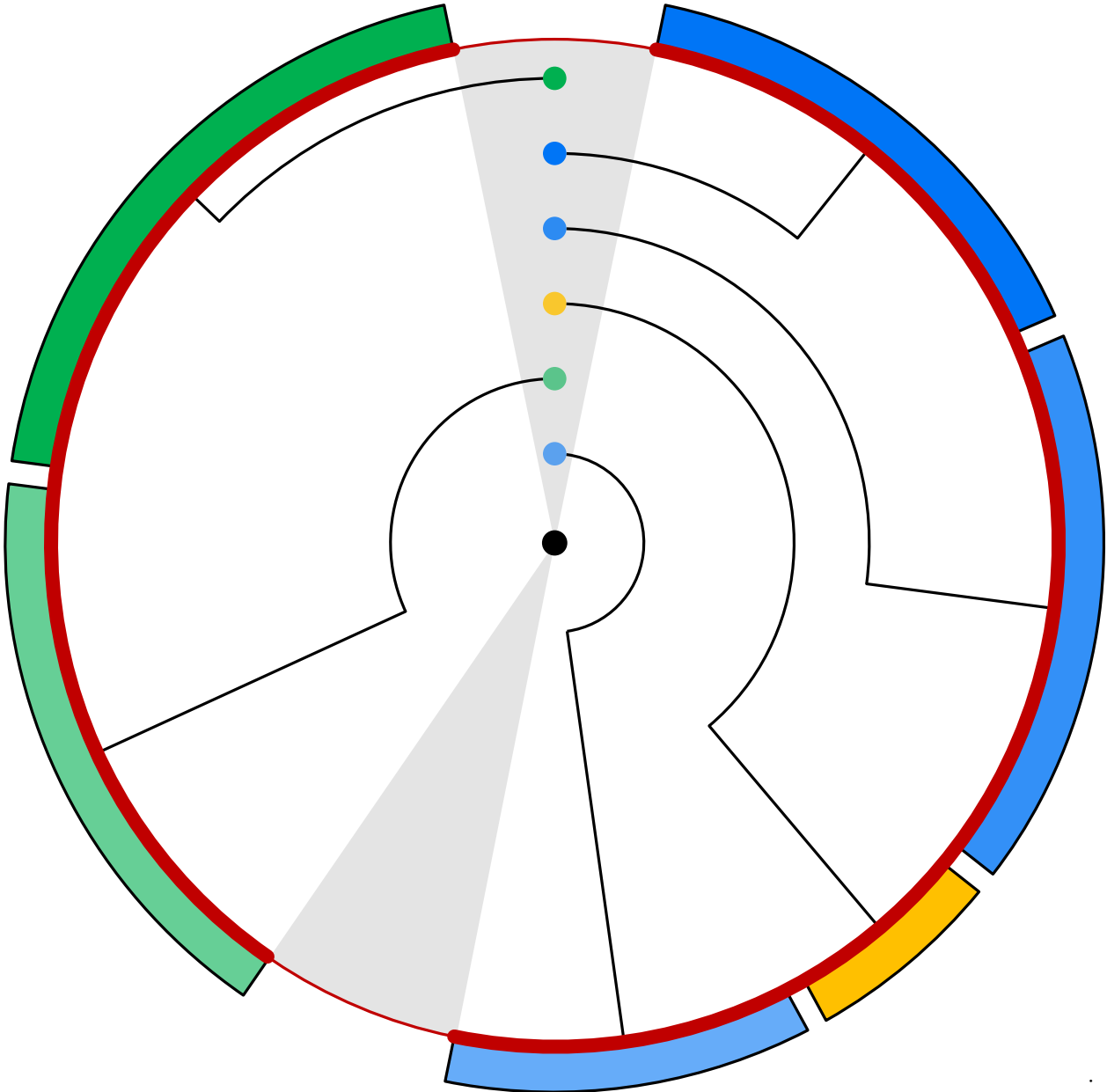
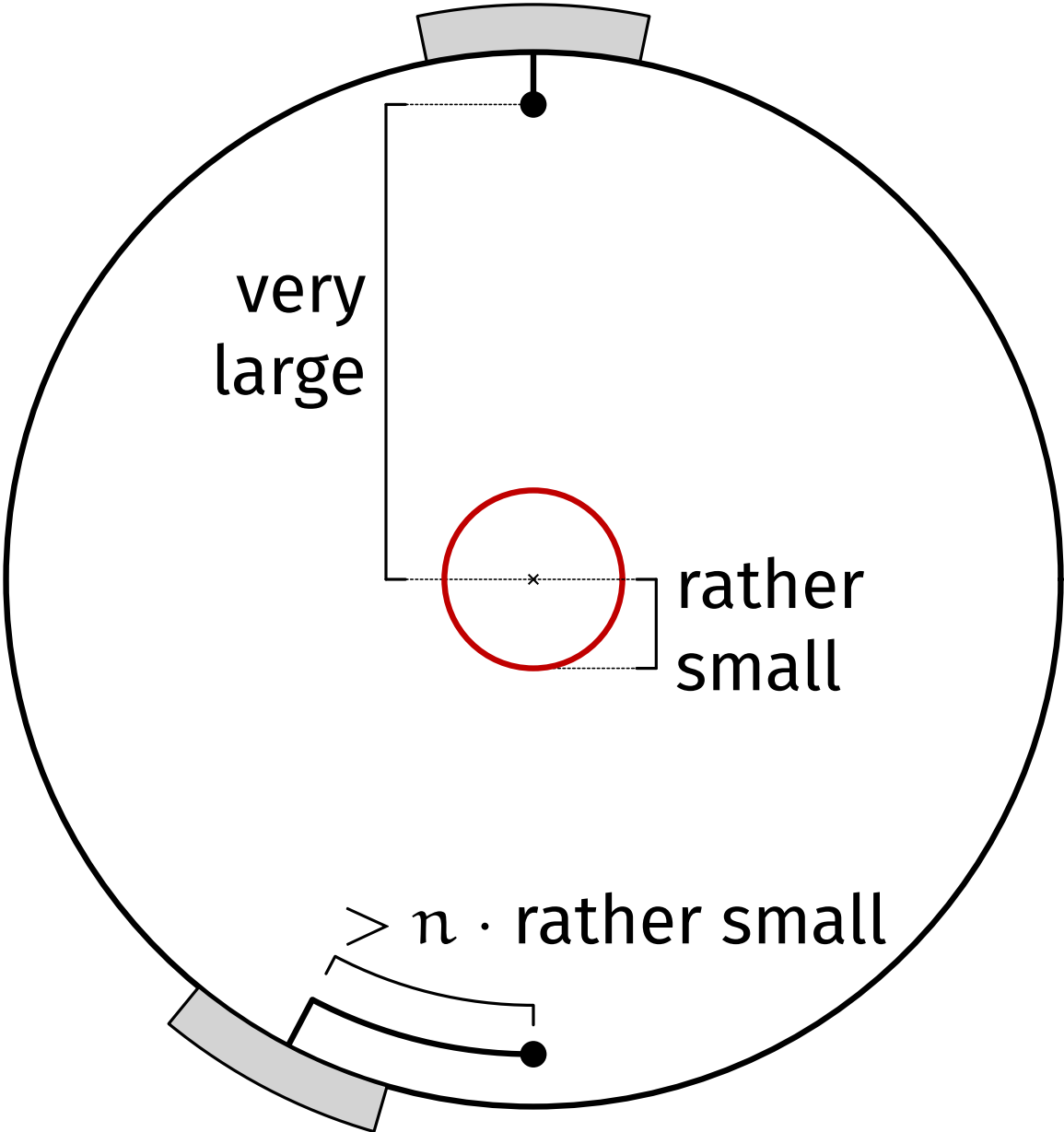


Hardness

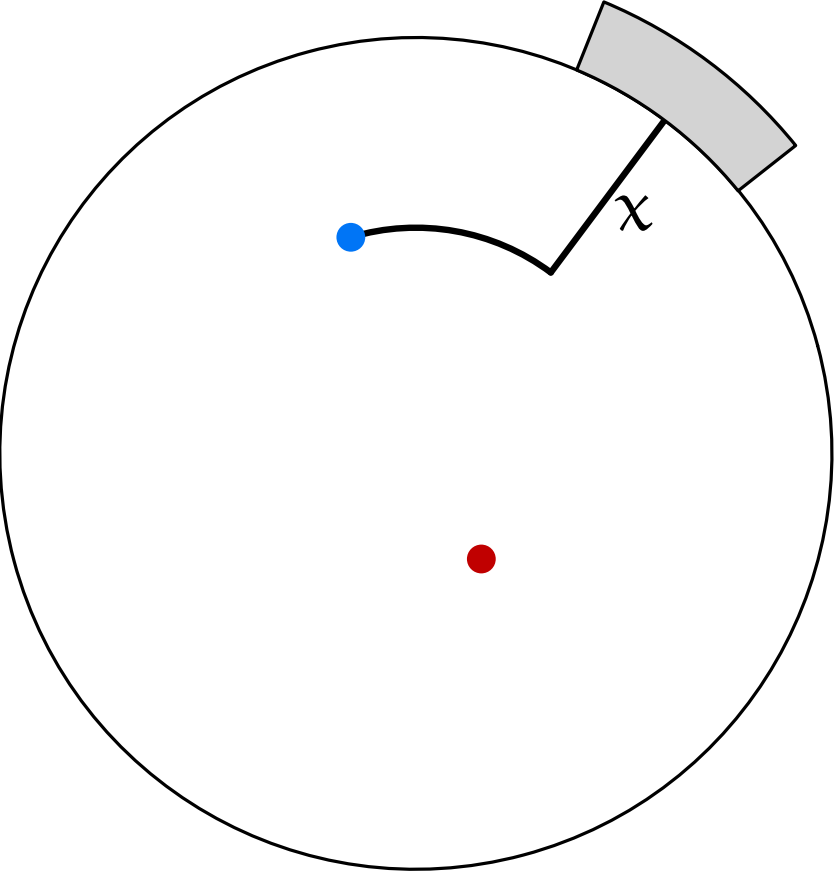
continuous rotation 

free order 

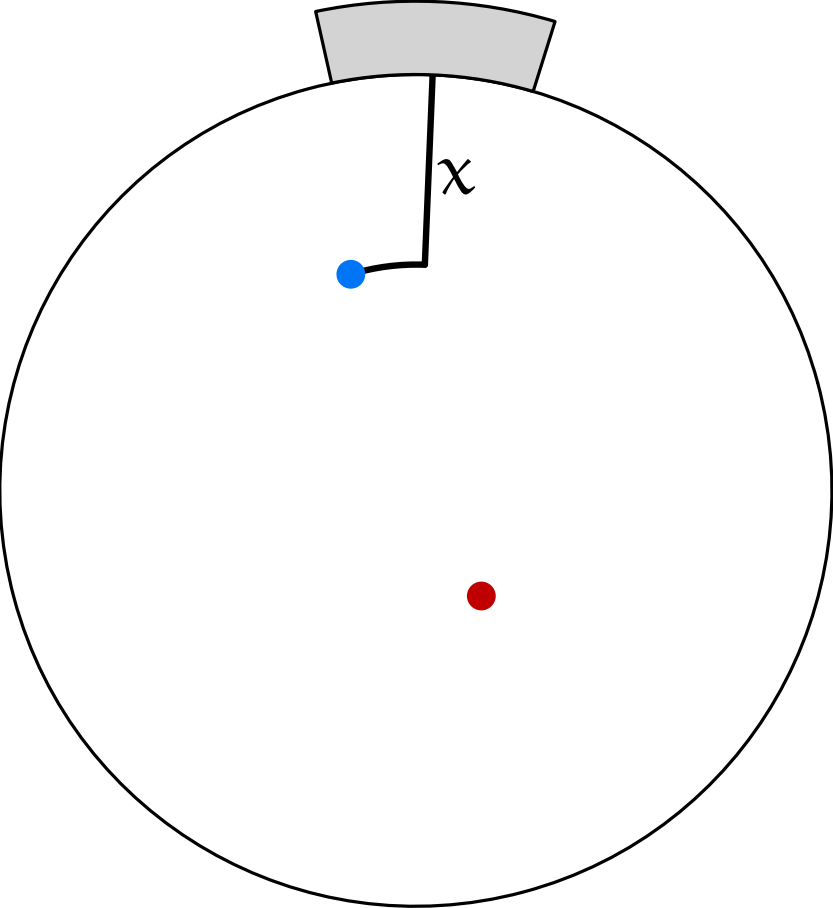
non-uniform labels 



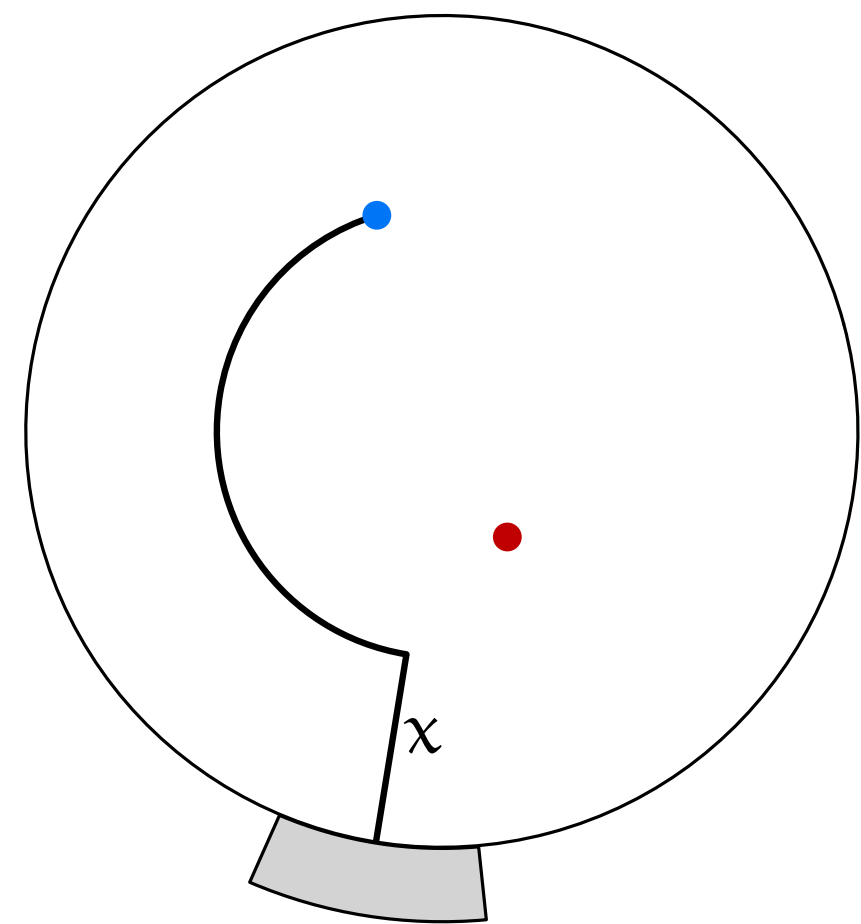
Rotating the labels



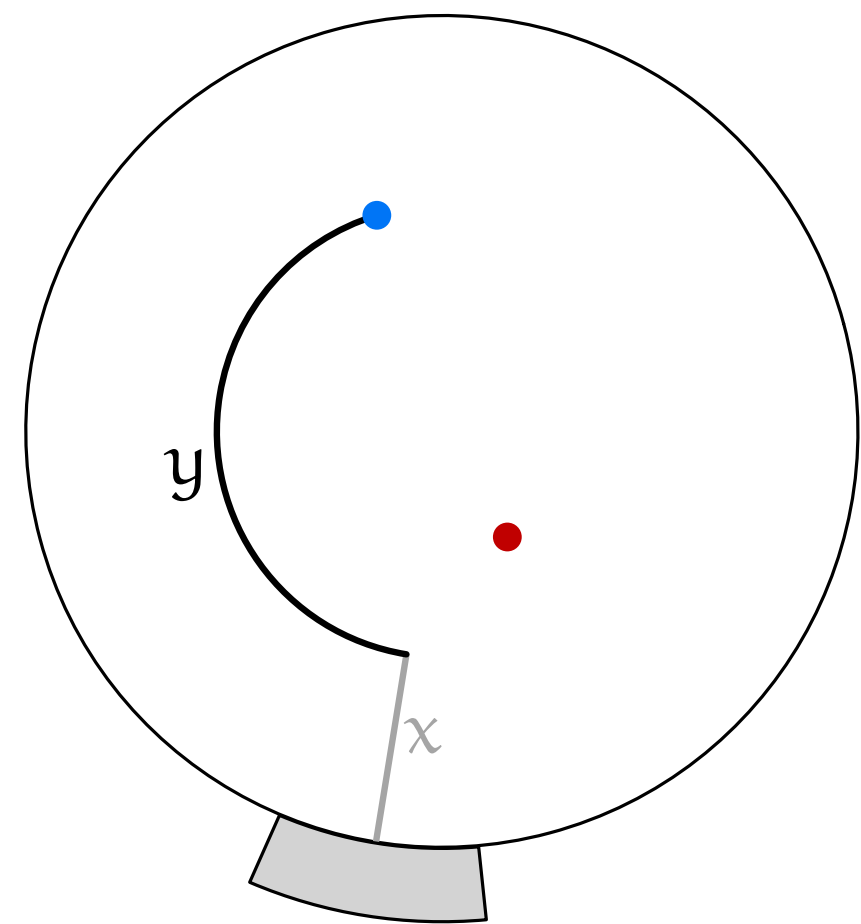
Rotating the labels



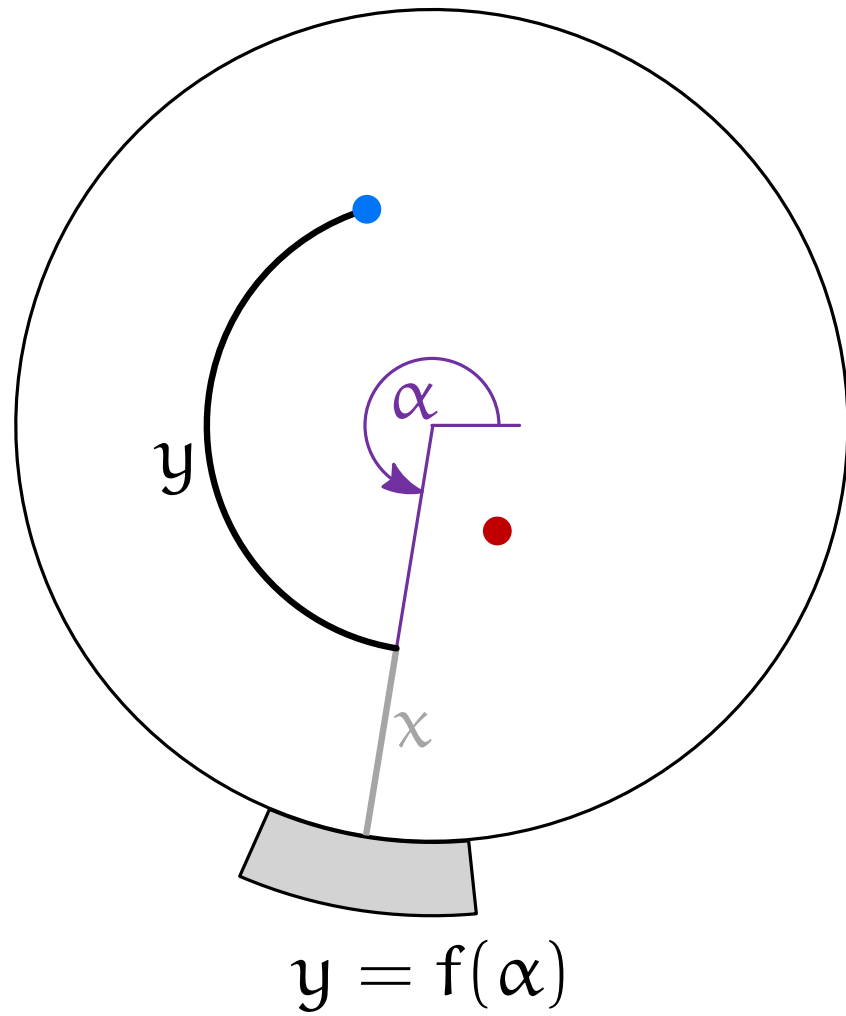
Rotating the labels



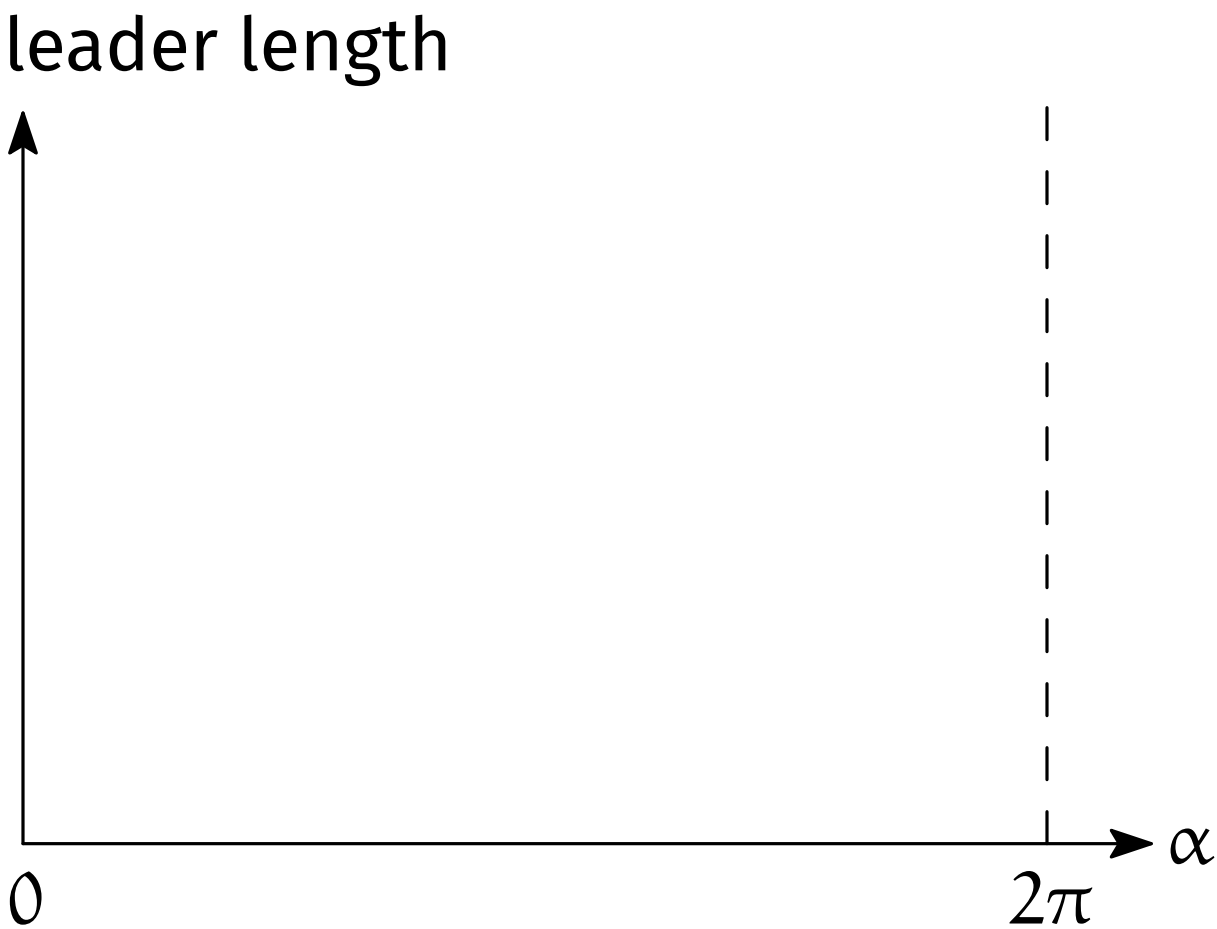
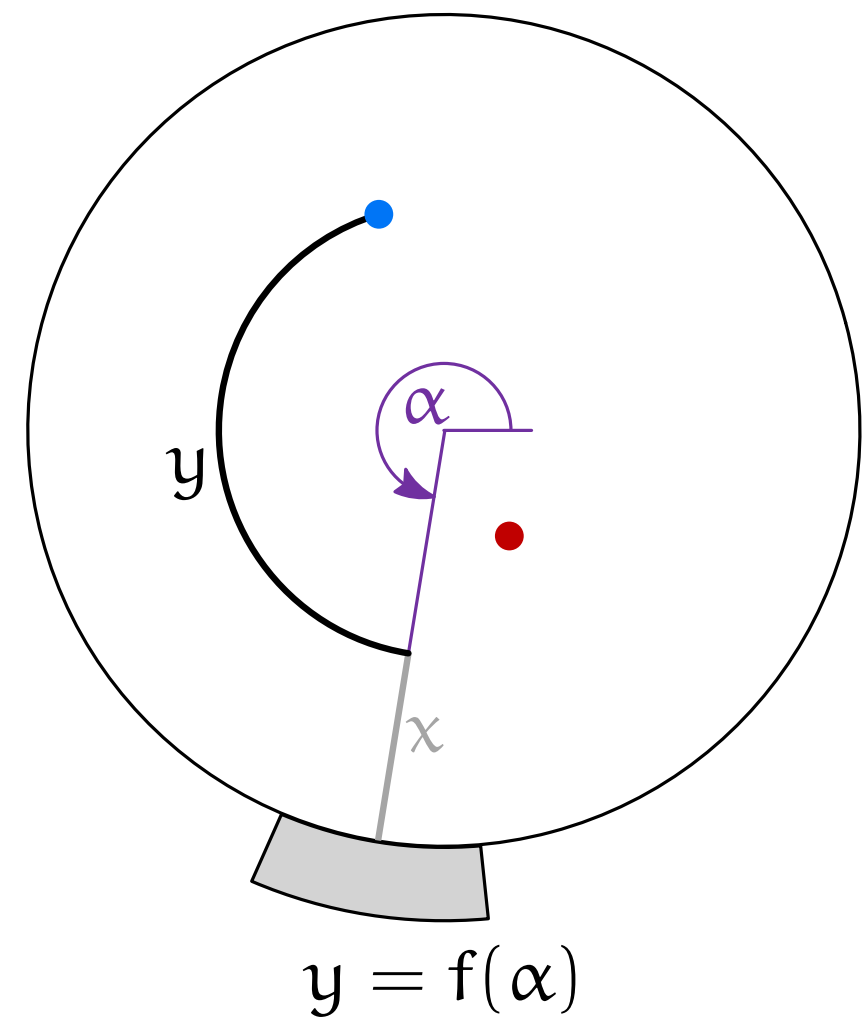
Rotating the labels



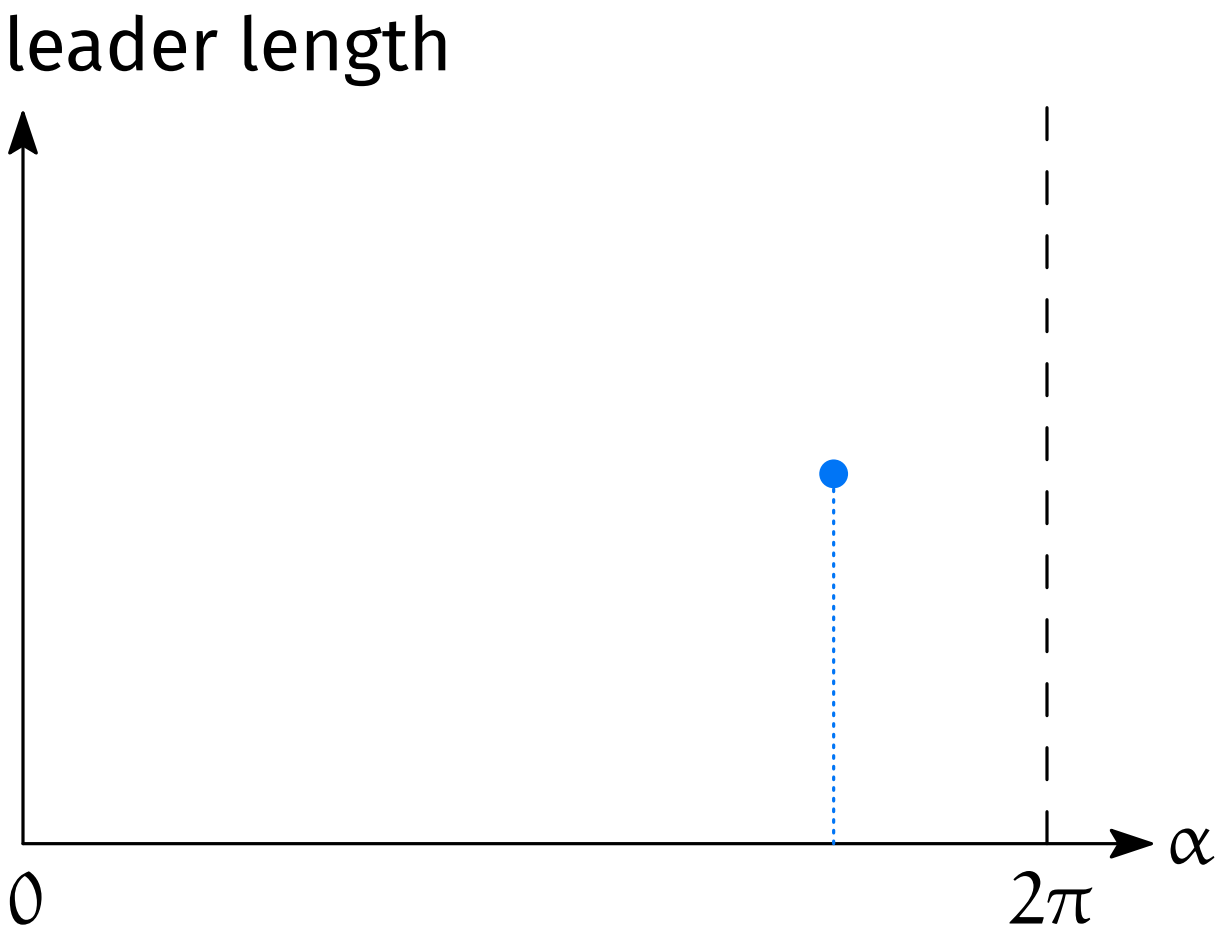
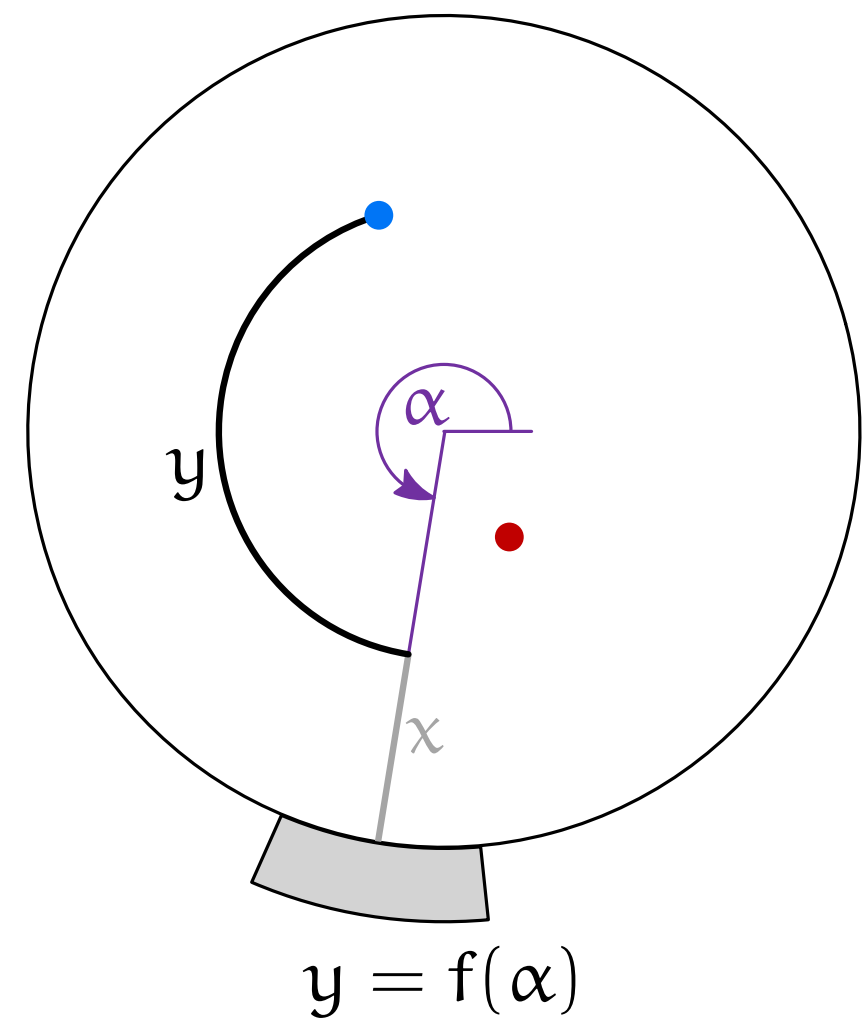
Rotating the labels



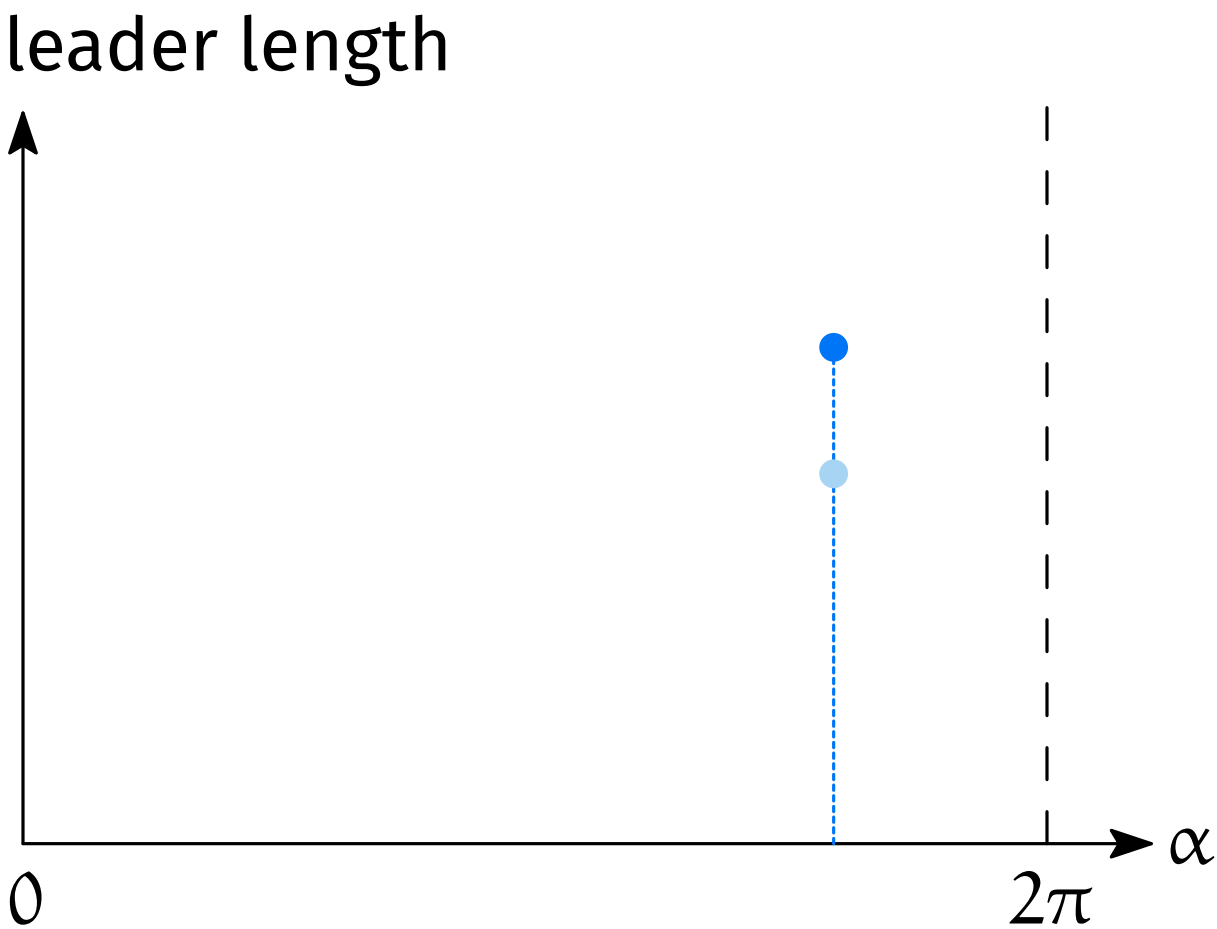
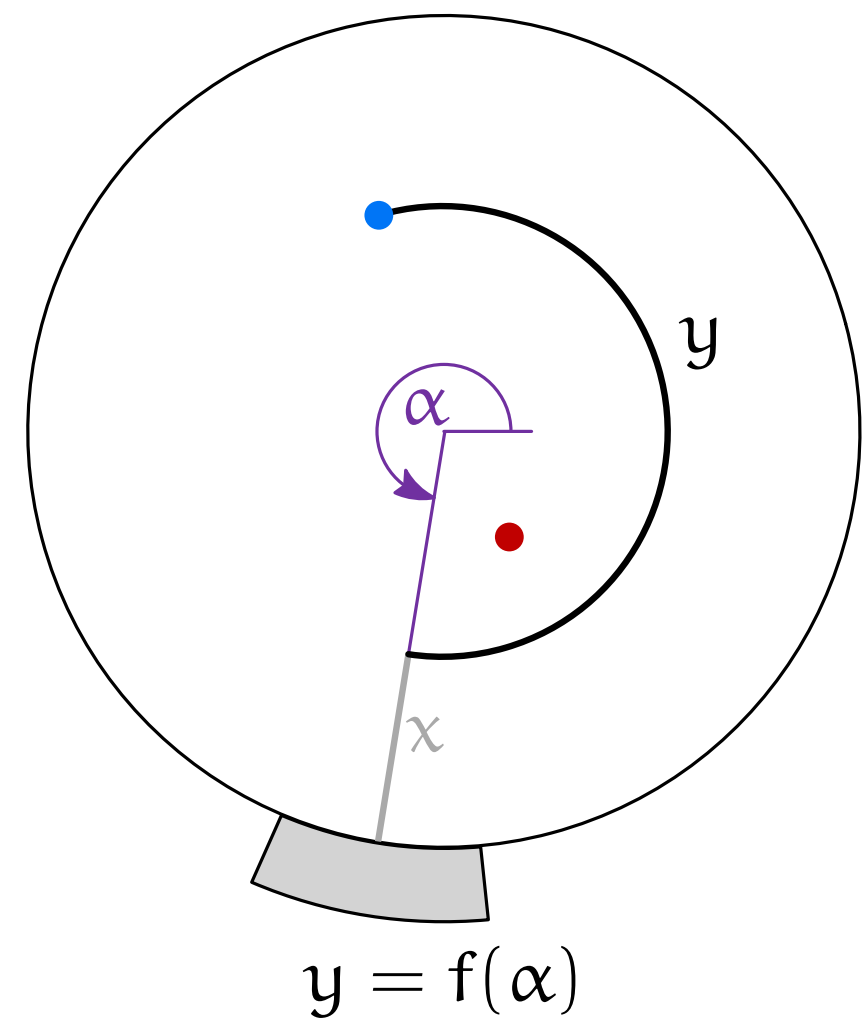
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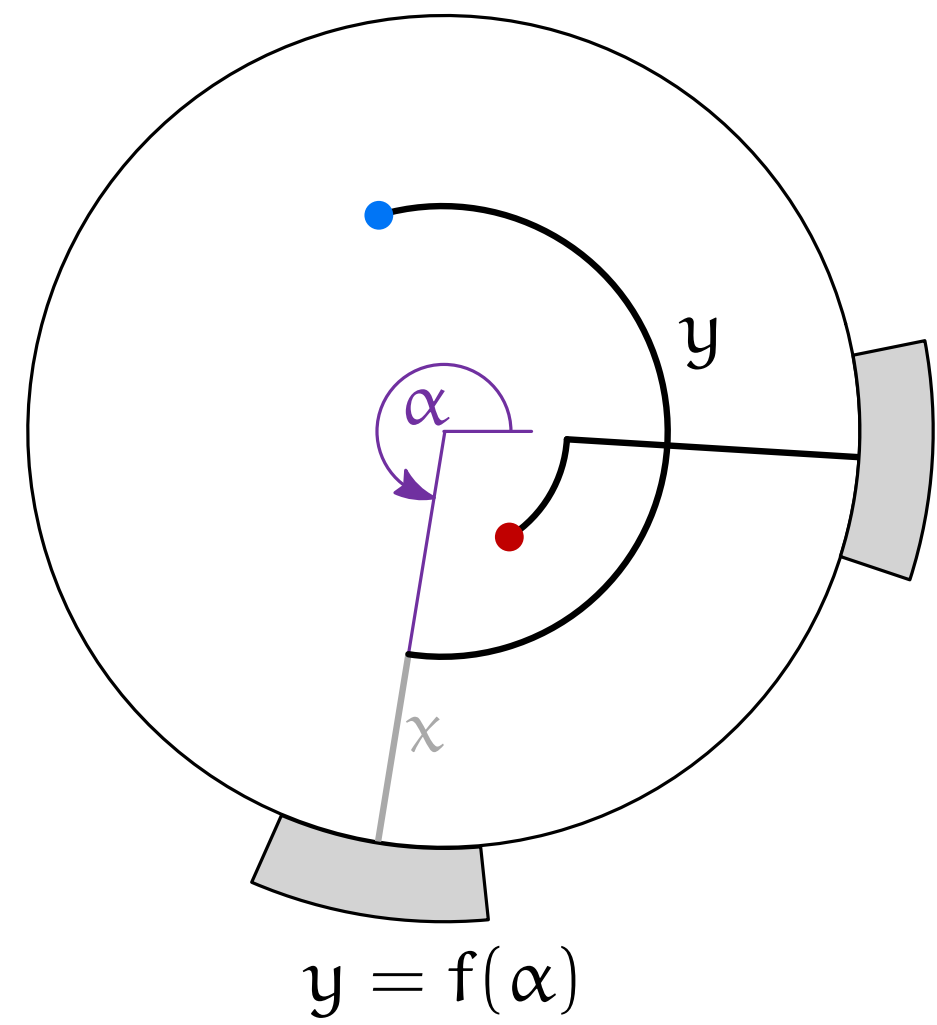
Rotating the labels



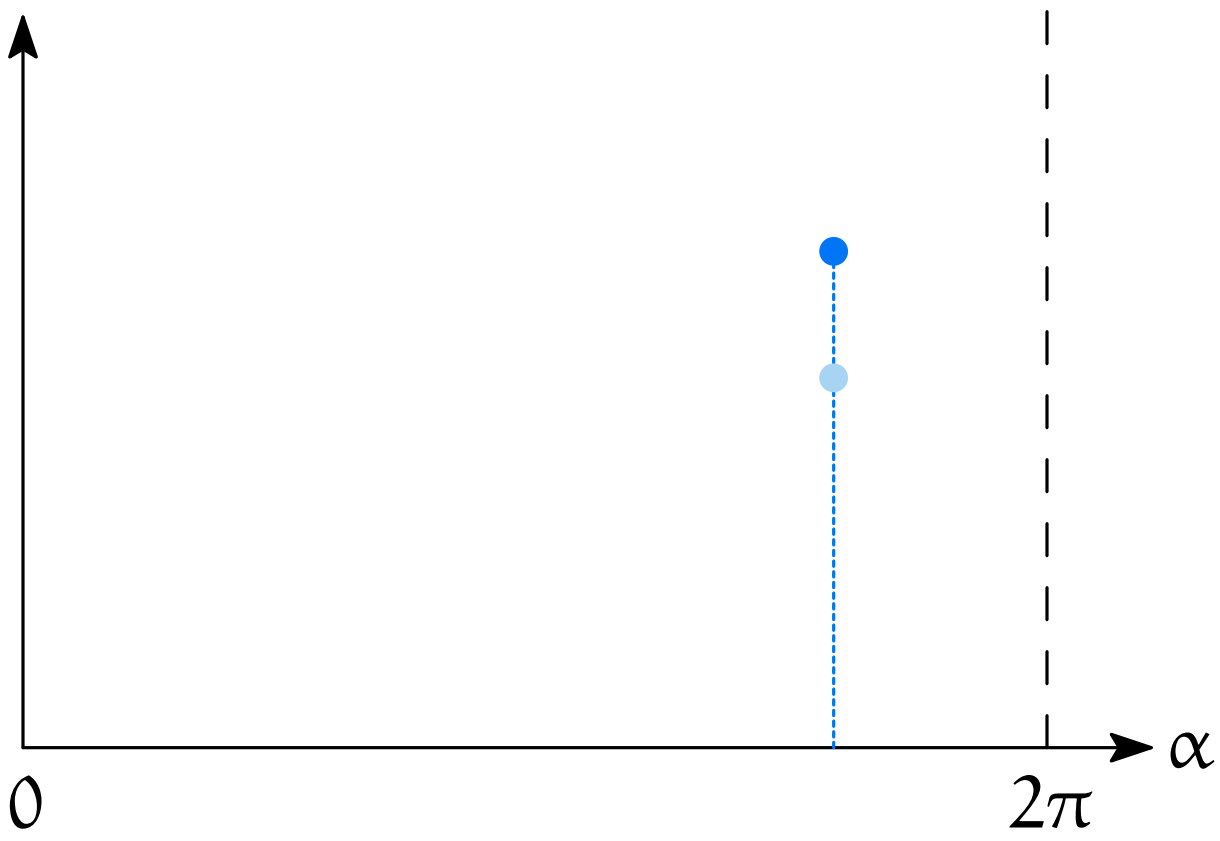
Rotating the labels



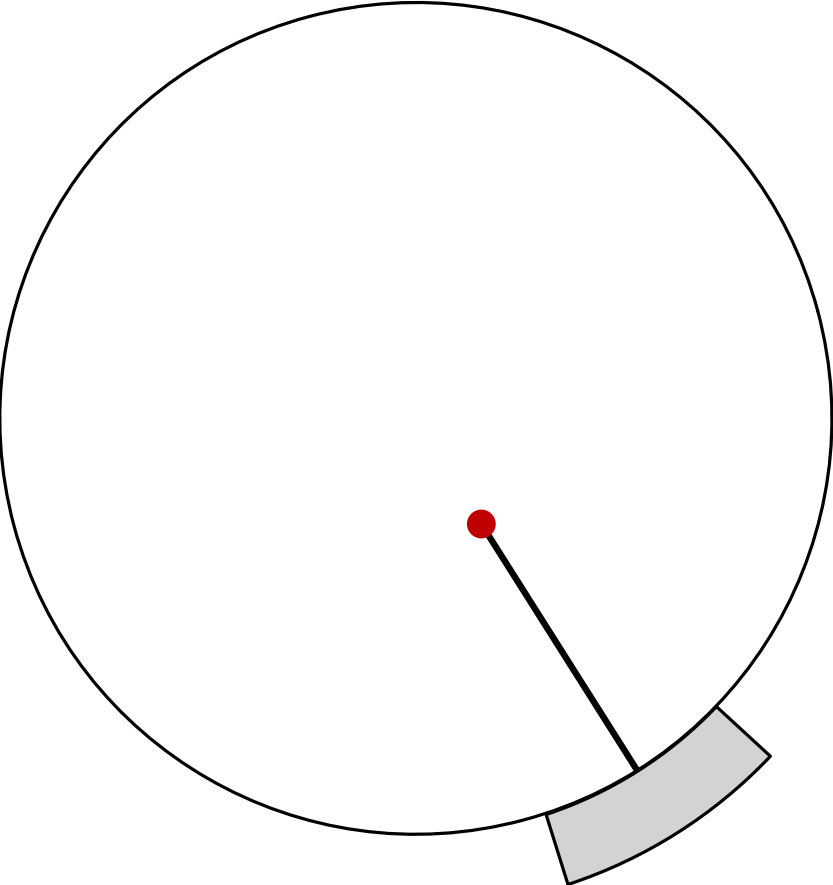
Rotating the labels



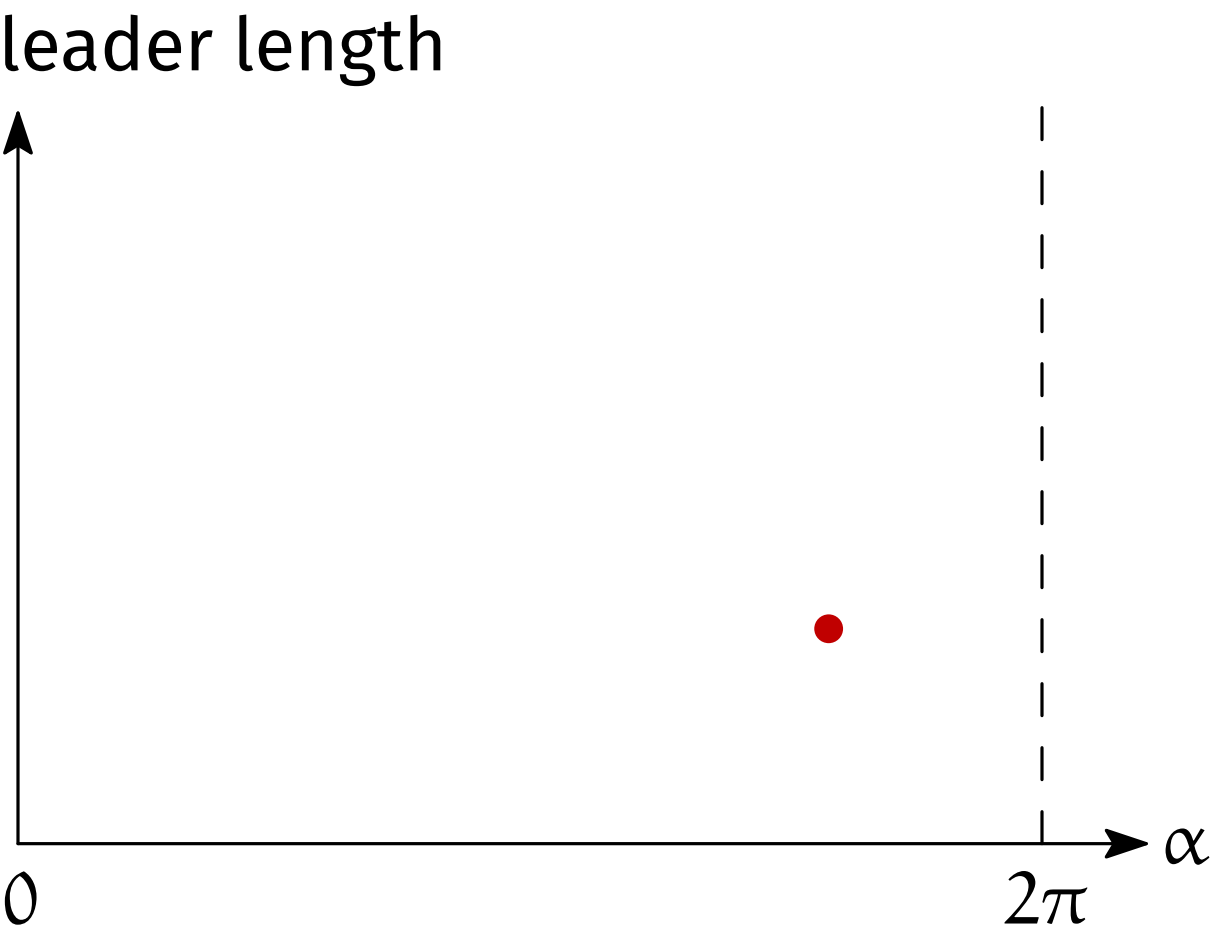
leader length



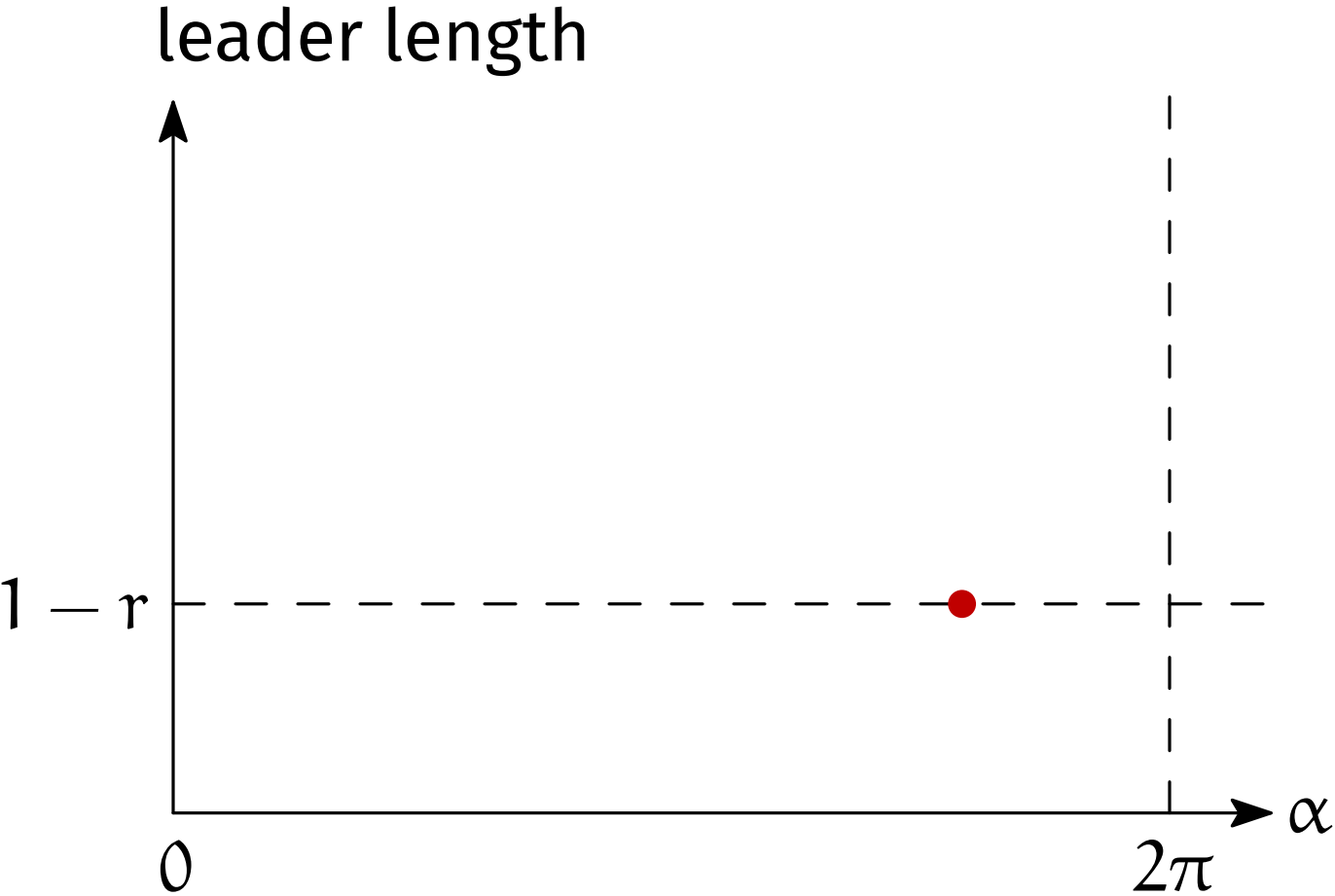
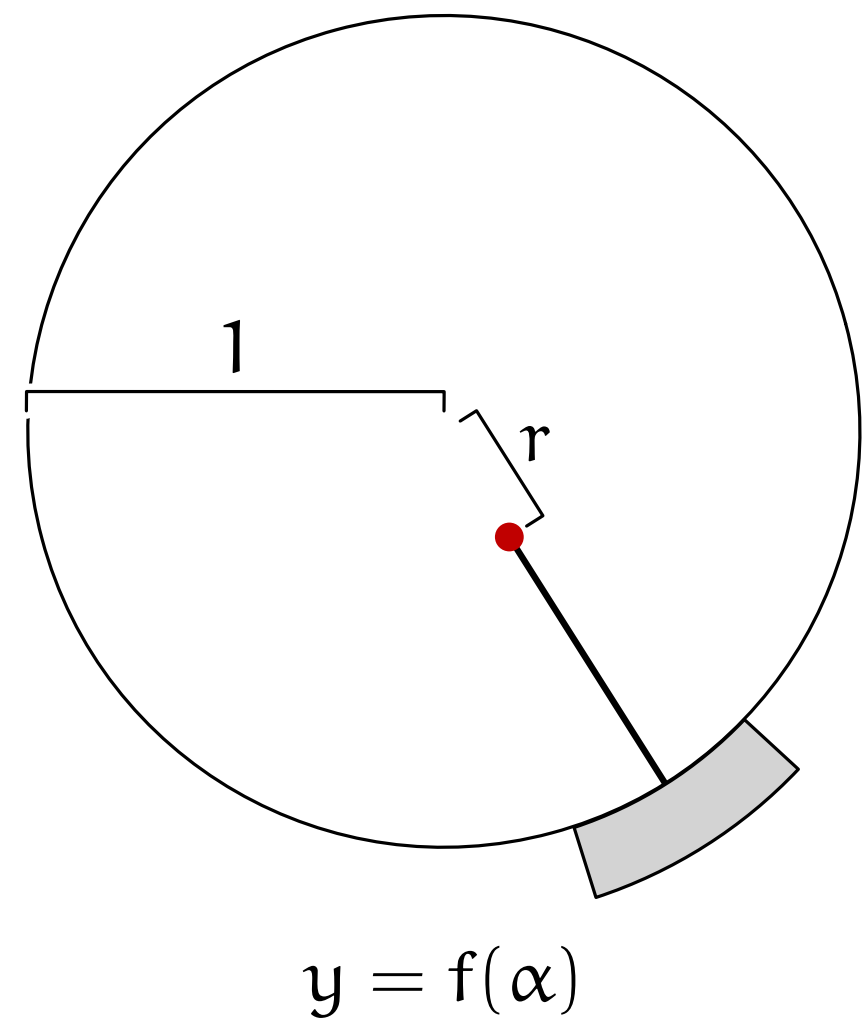
Rotating the labels



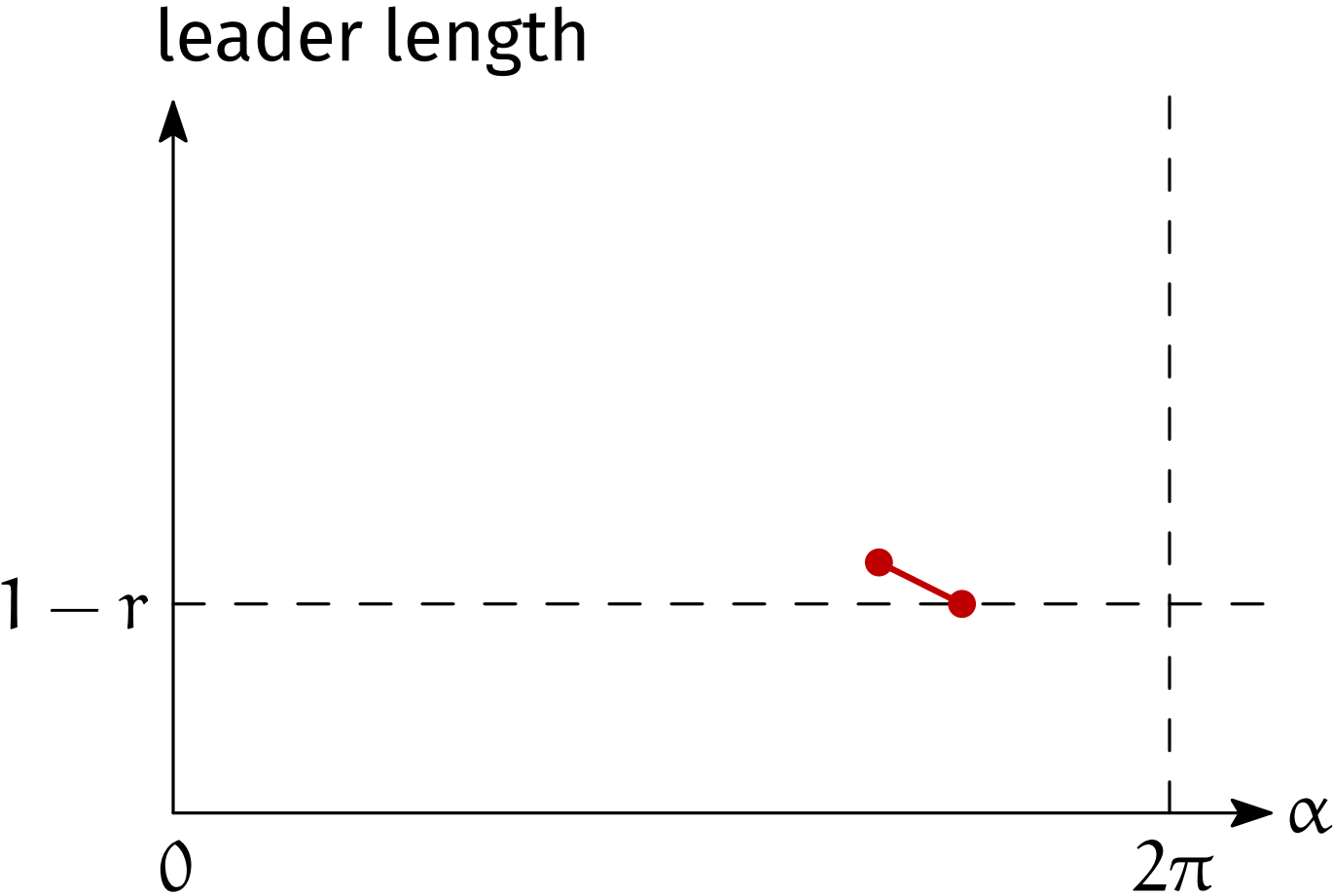
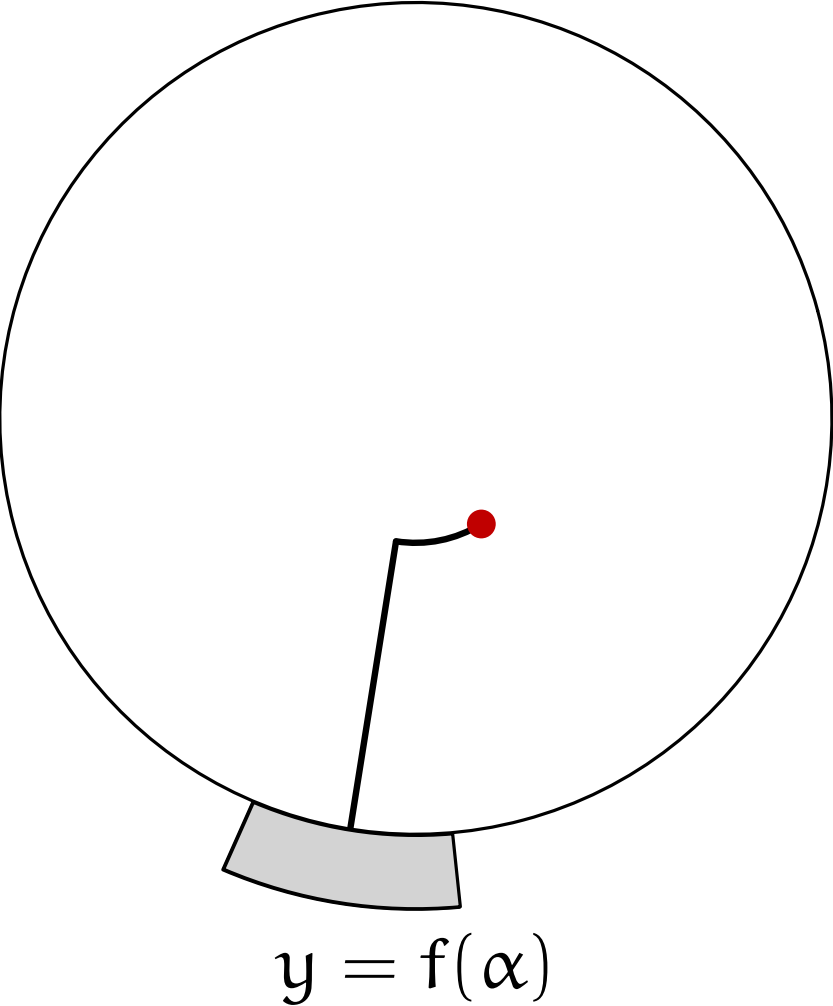
$y = f(\alpha)$



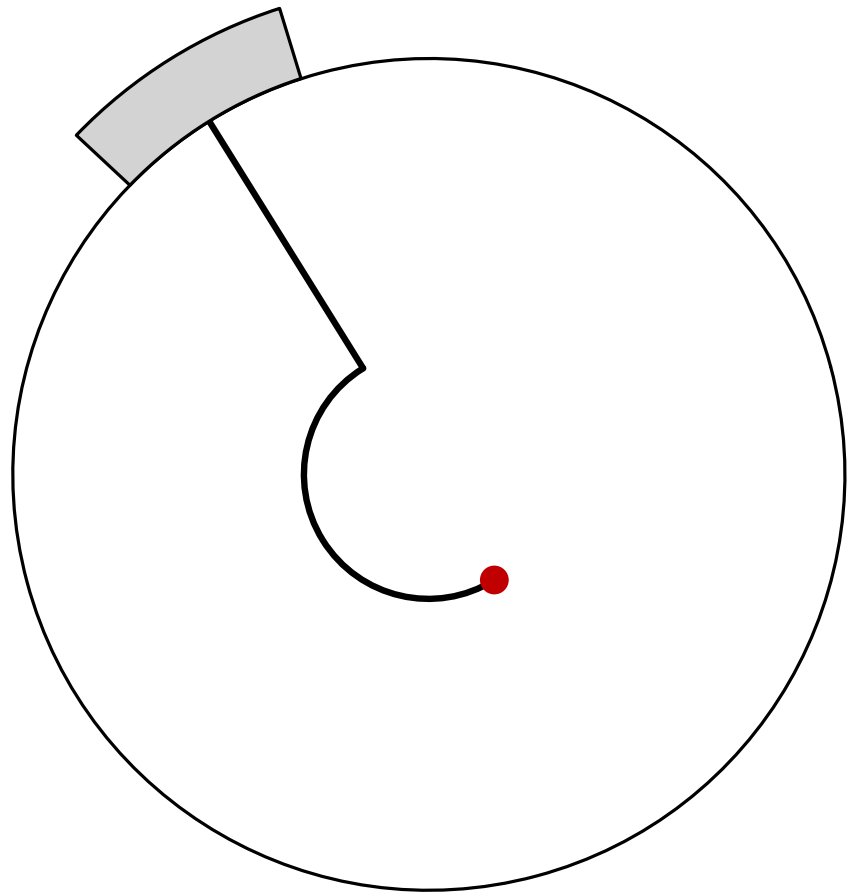
Rotating the labels



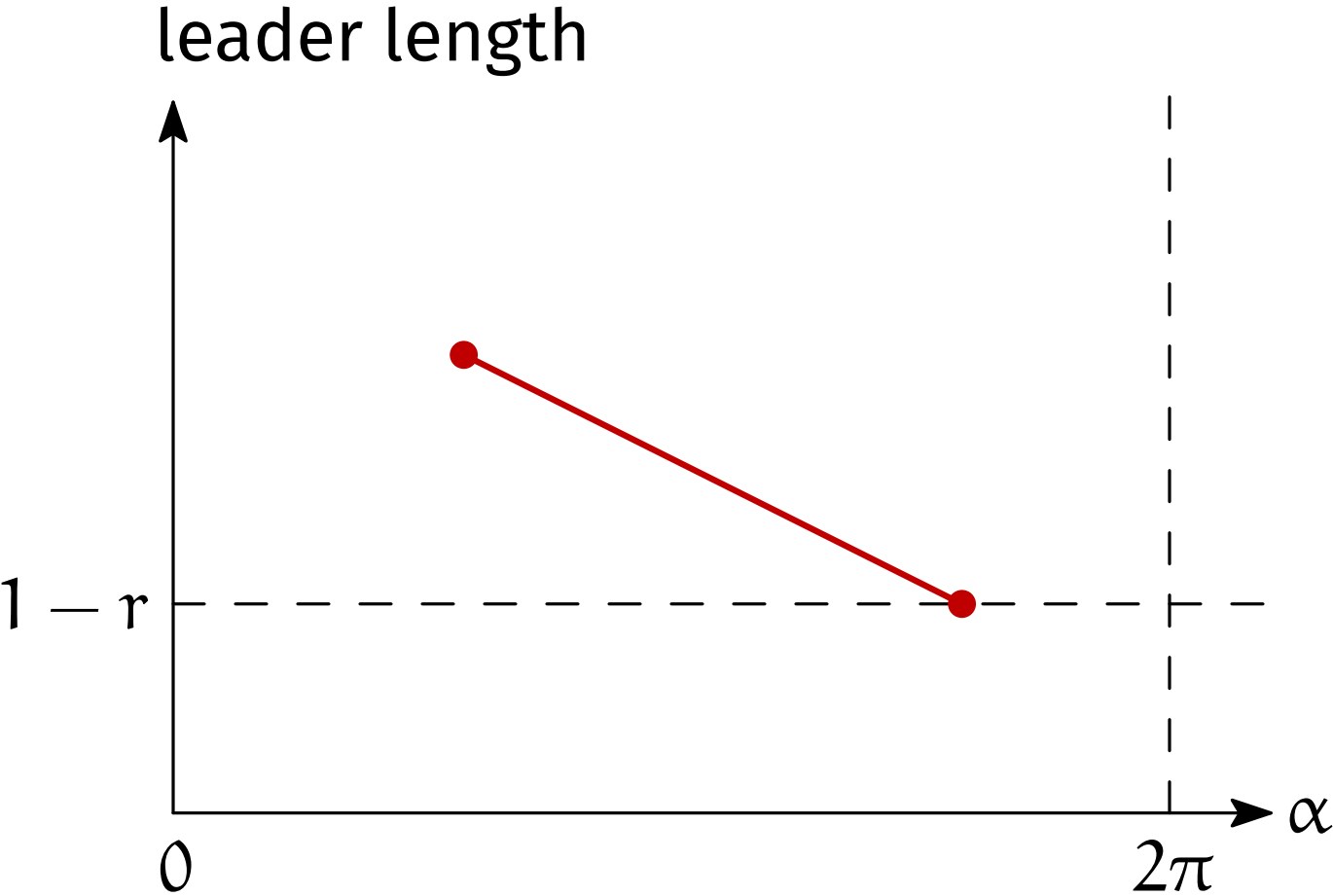
Rotating the labels



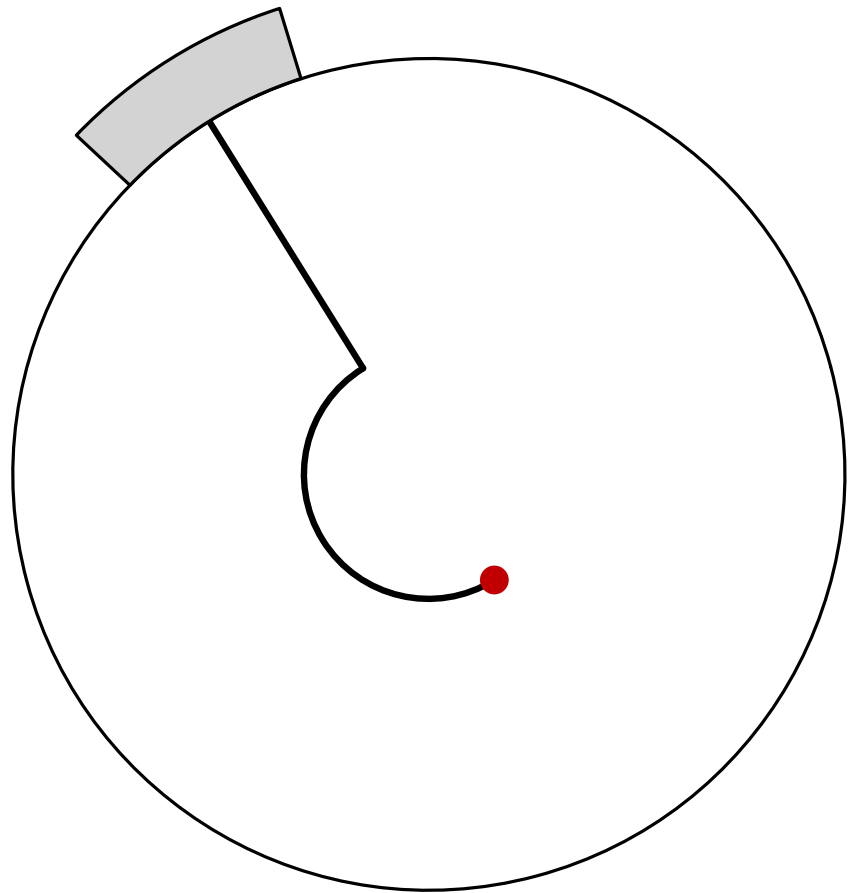
Rotating the labels



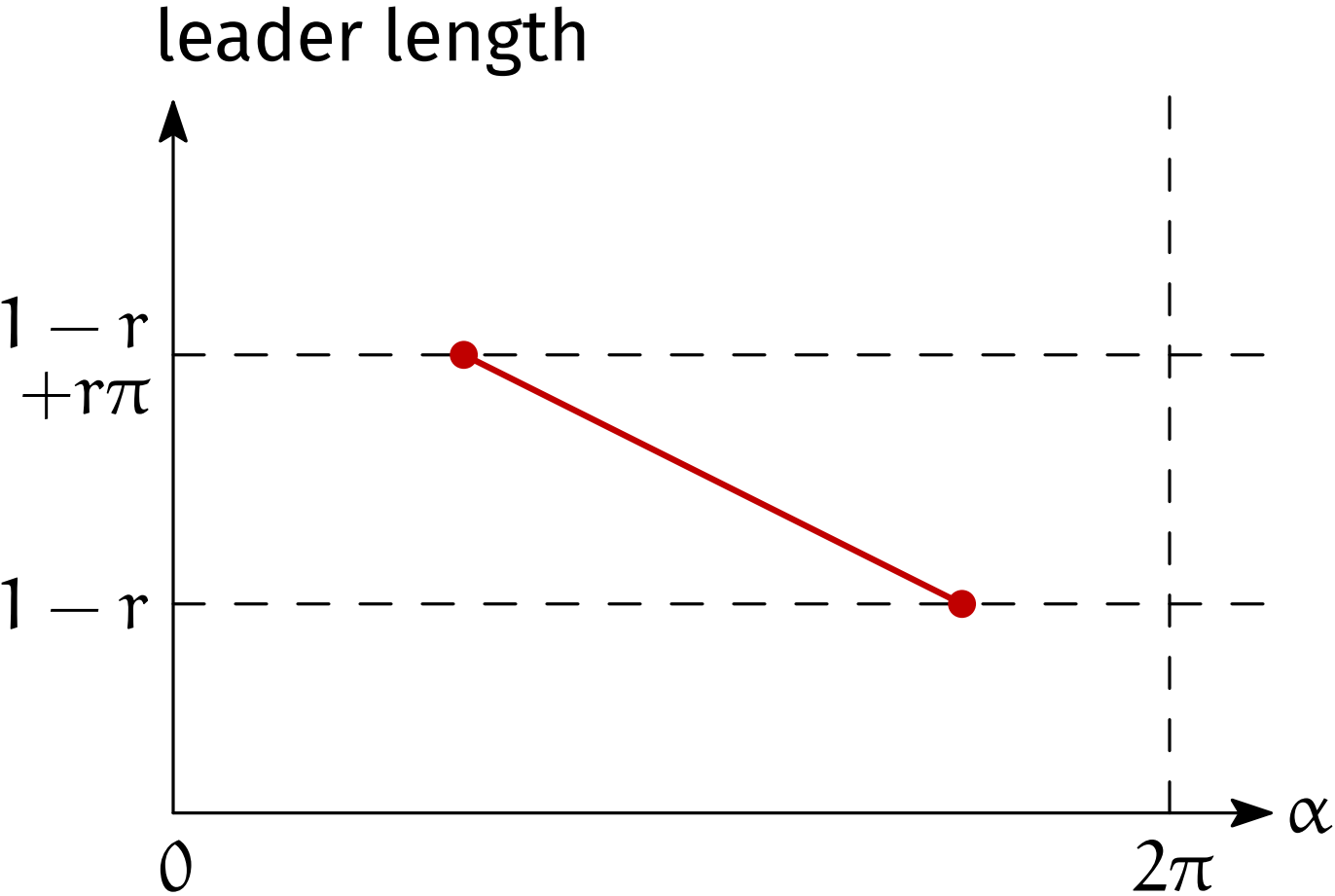
$$y = f(\alpha)$$



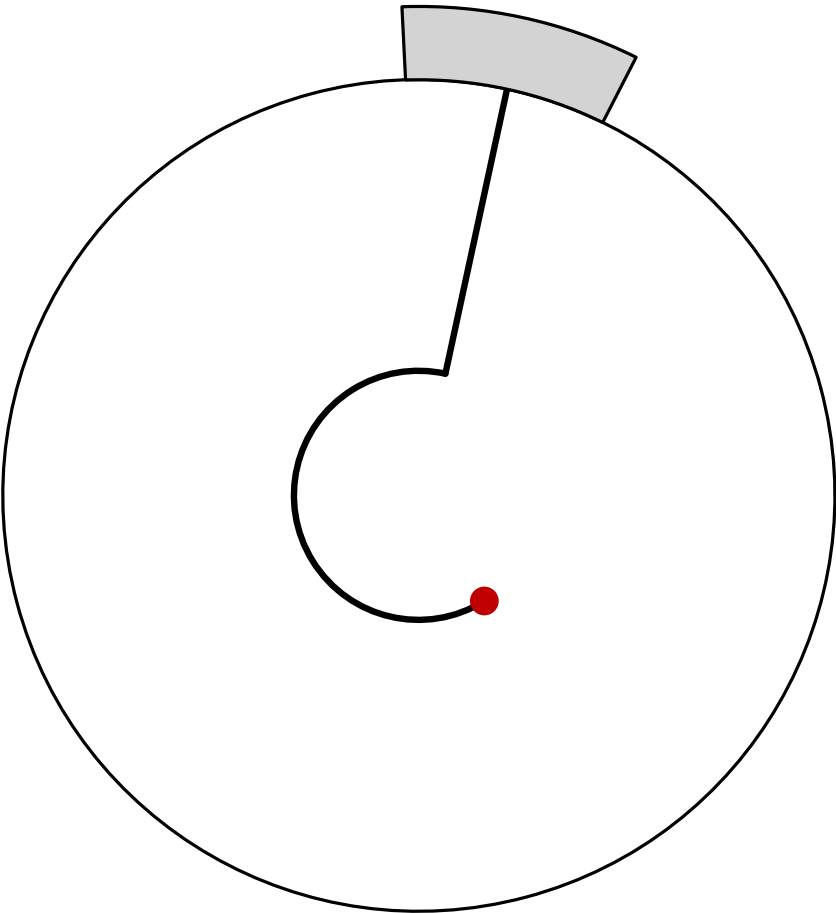
Rotating the labels



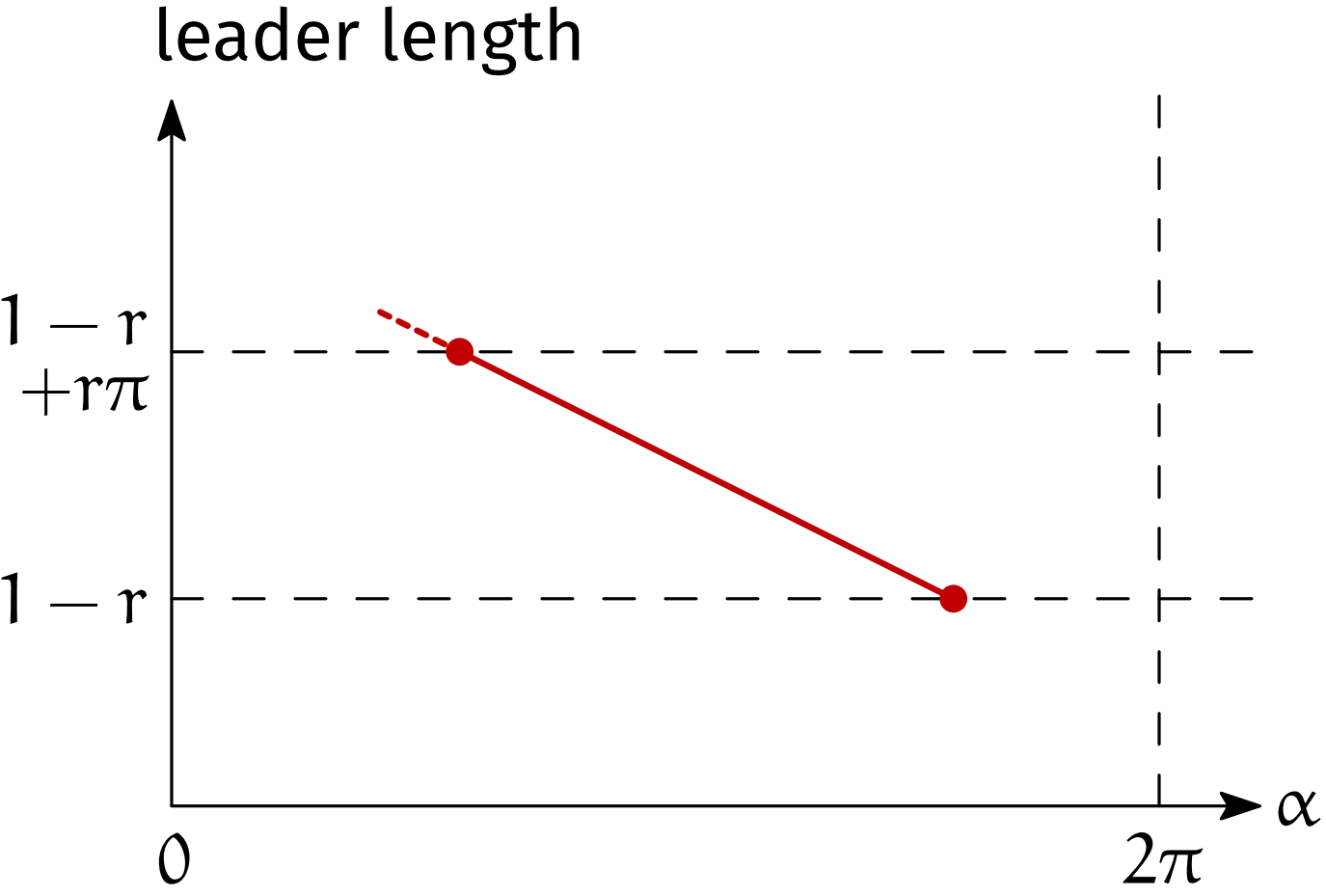
$$y = f(\alpha)$$



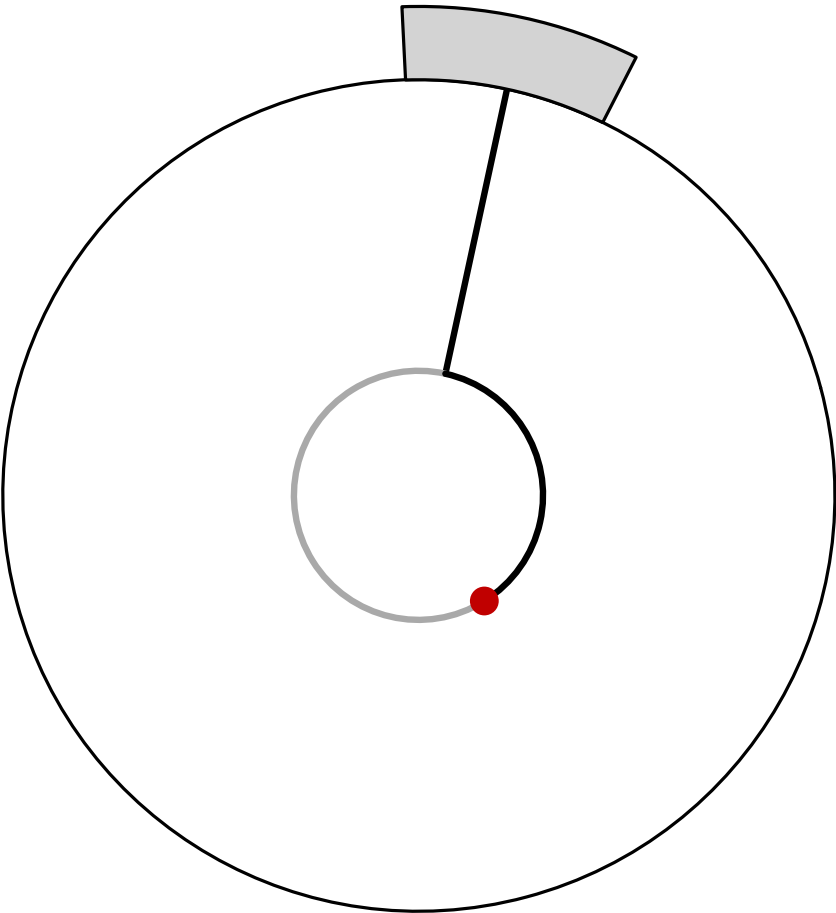
Rotating the labels



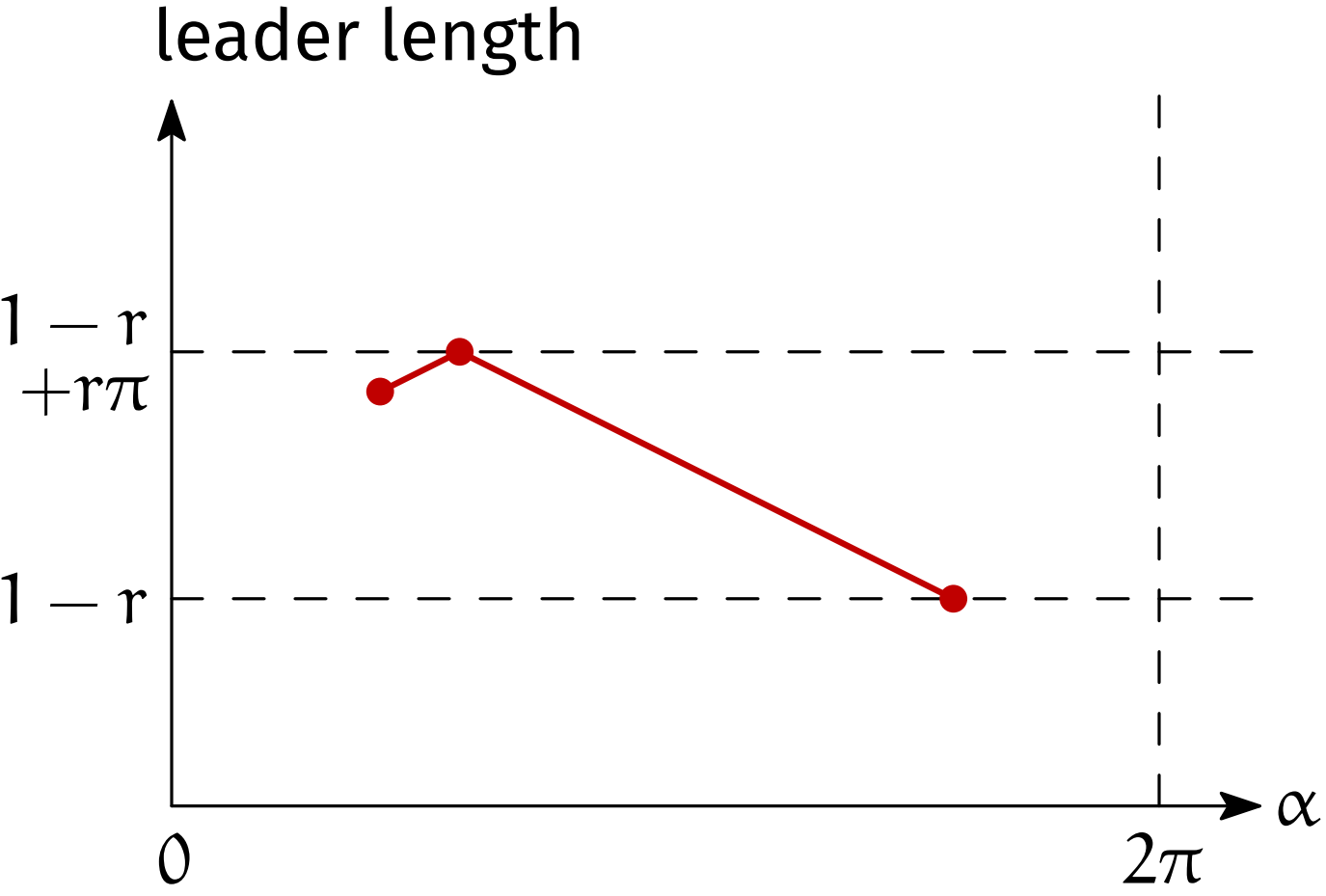
$y = f(\alpha)$



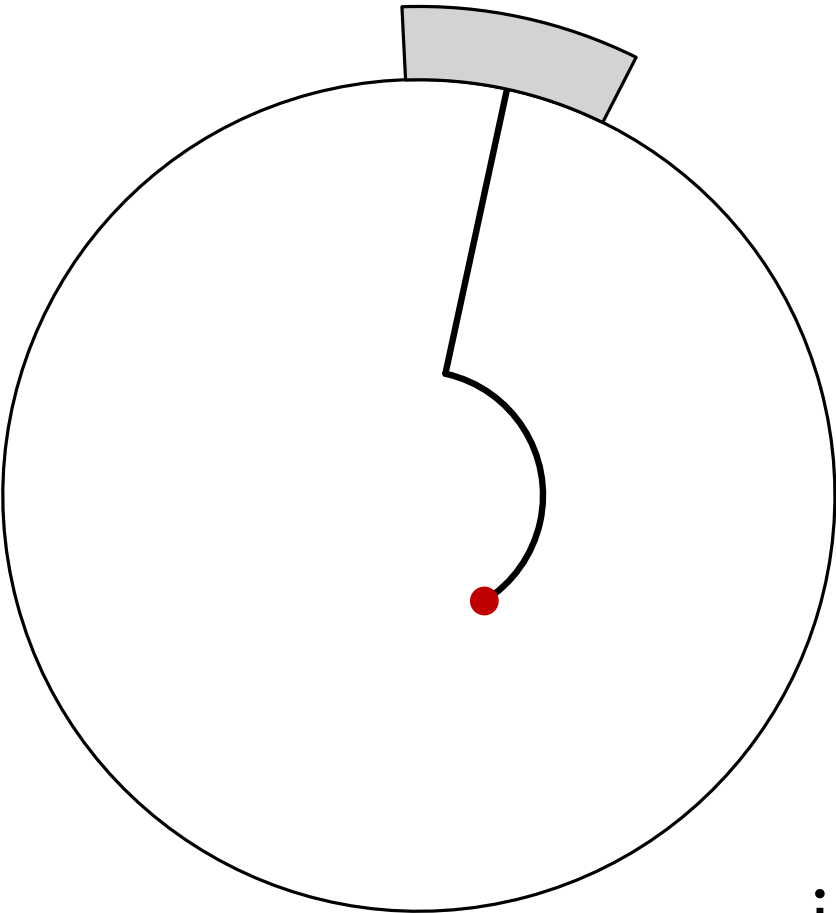
Rotating the labels



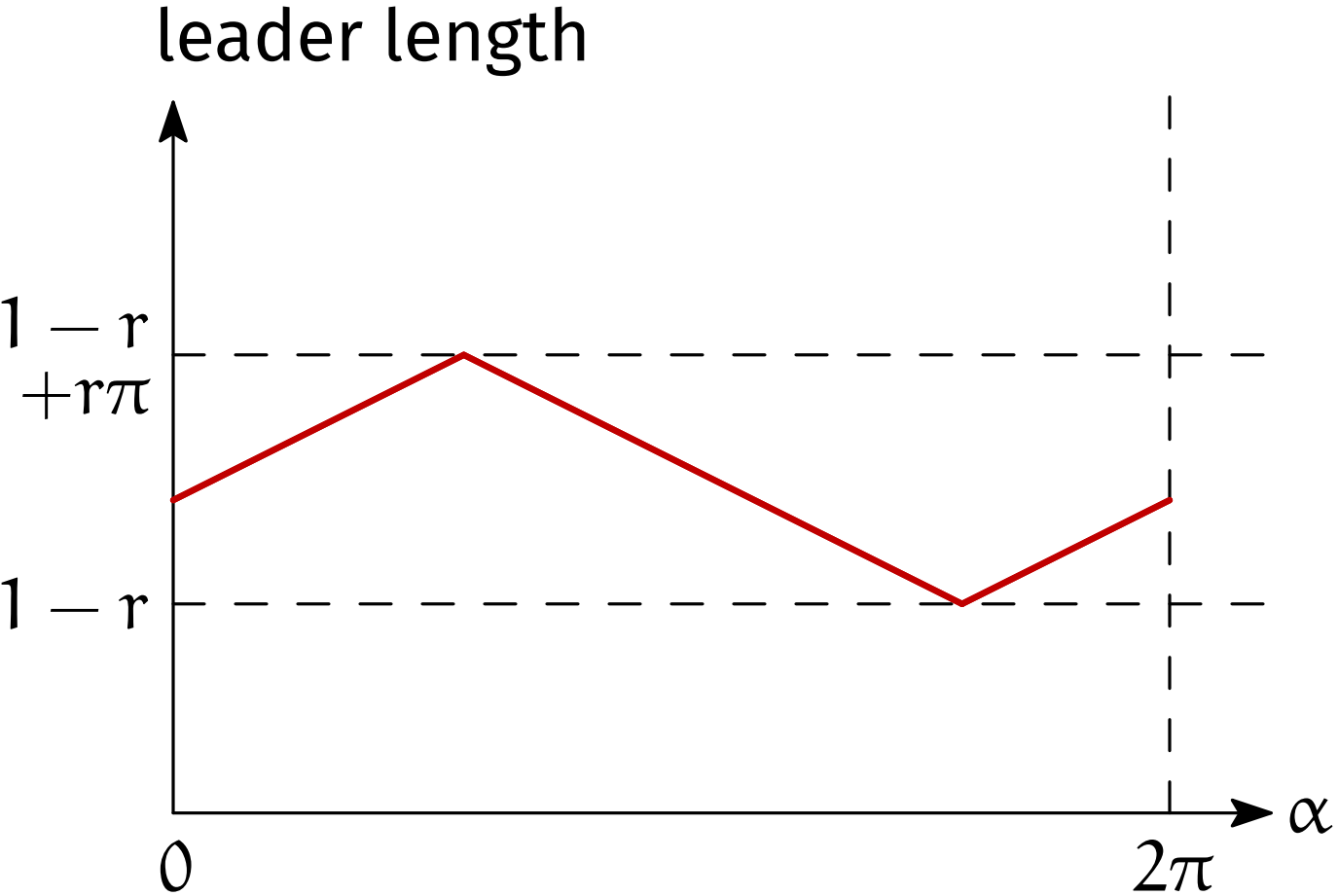
$y = f(\alpha)$



Rotating the labels

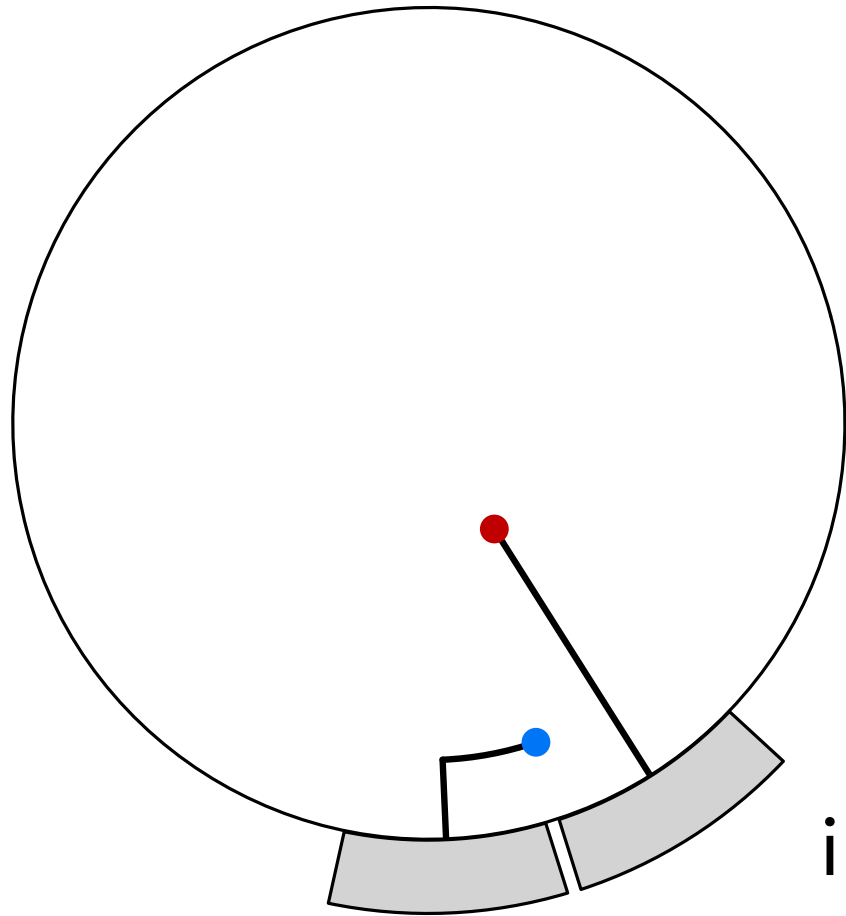


$y = f(\alpha)$



- innermost:
- always shorter path

Rotating the labels

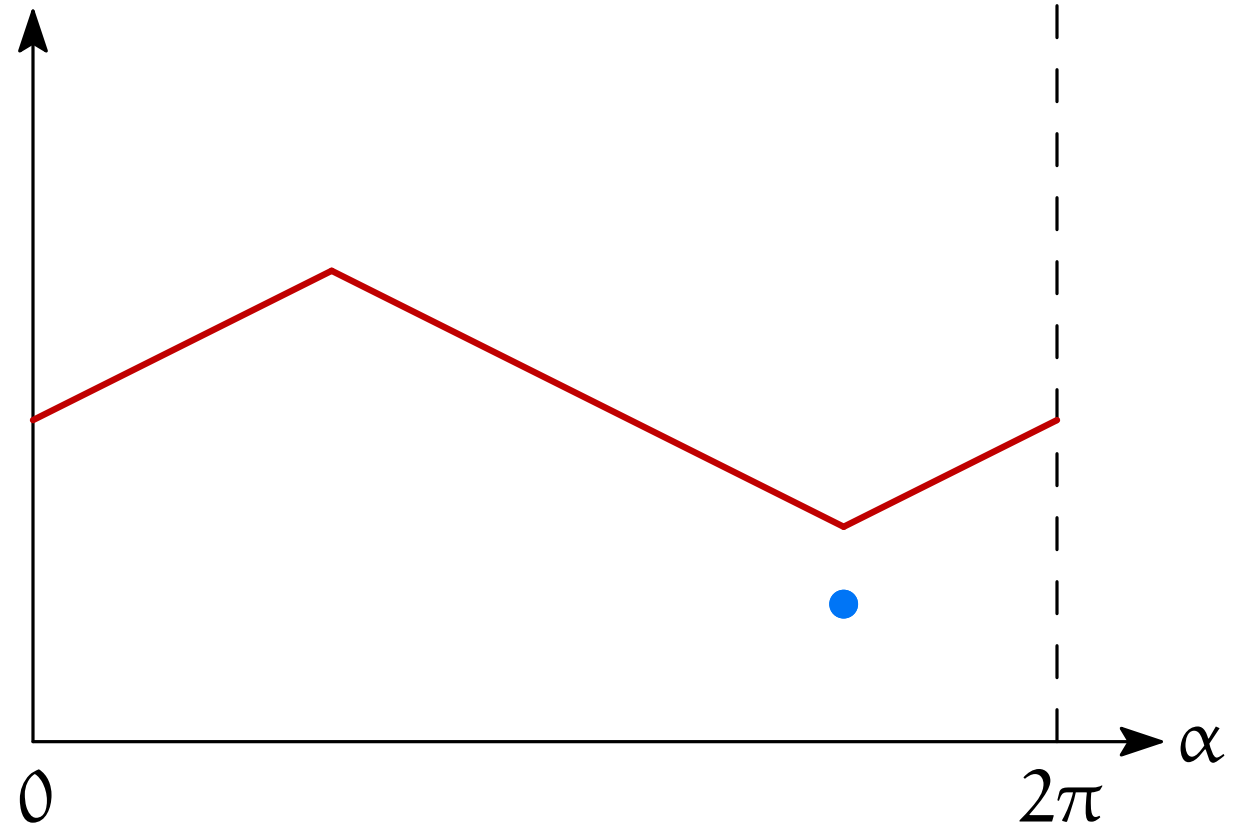


$$y = f(\alpha)$$

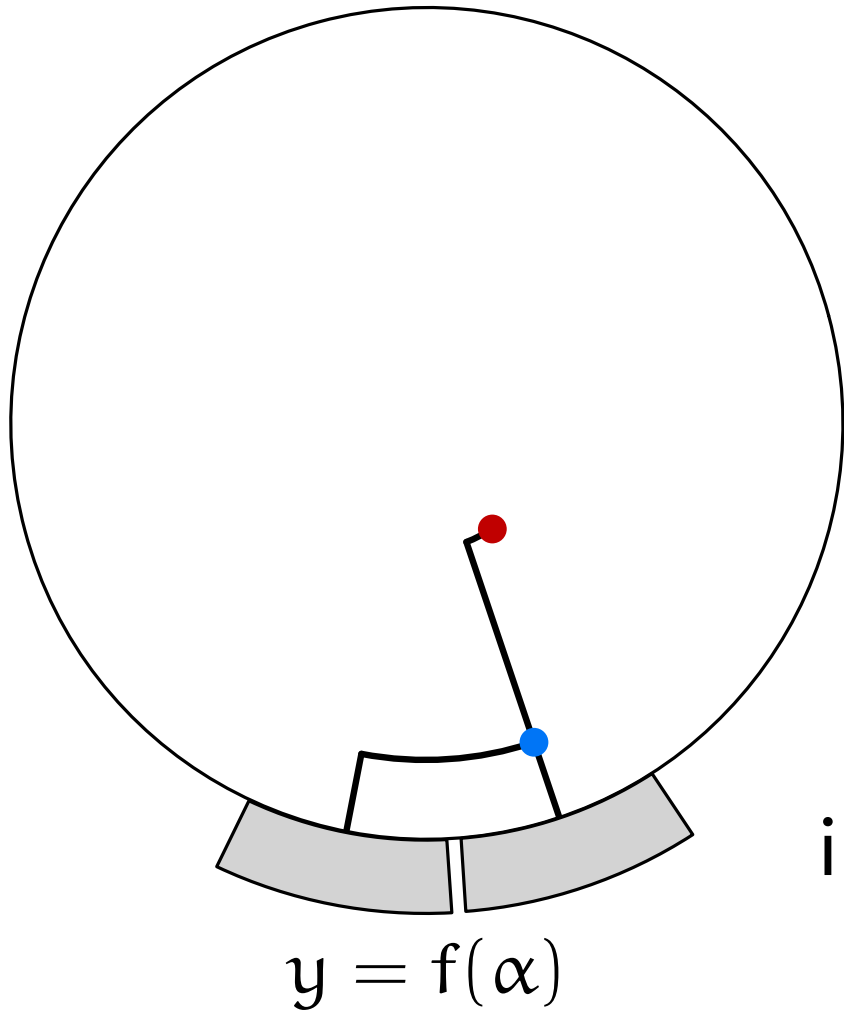
innermost:

- always shorter path

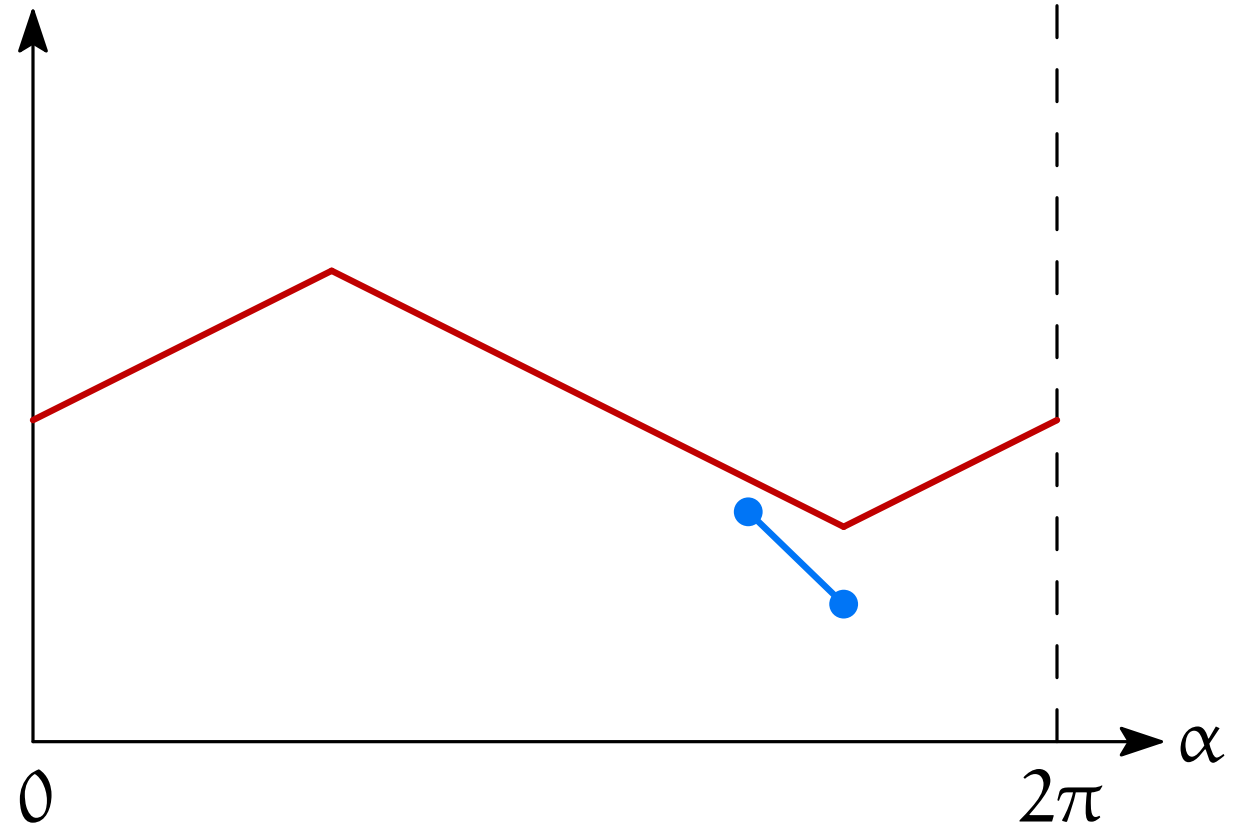
leader length



Rotating the labels



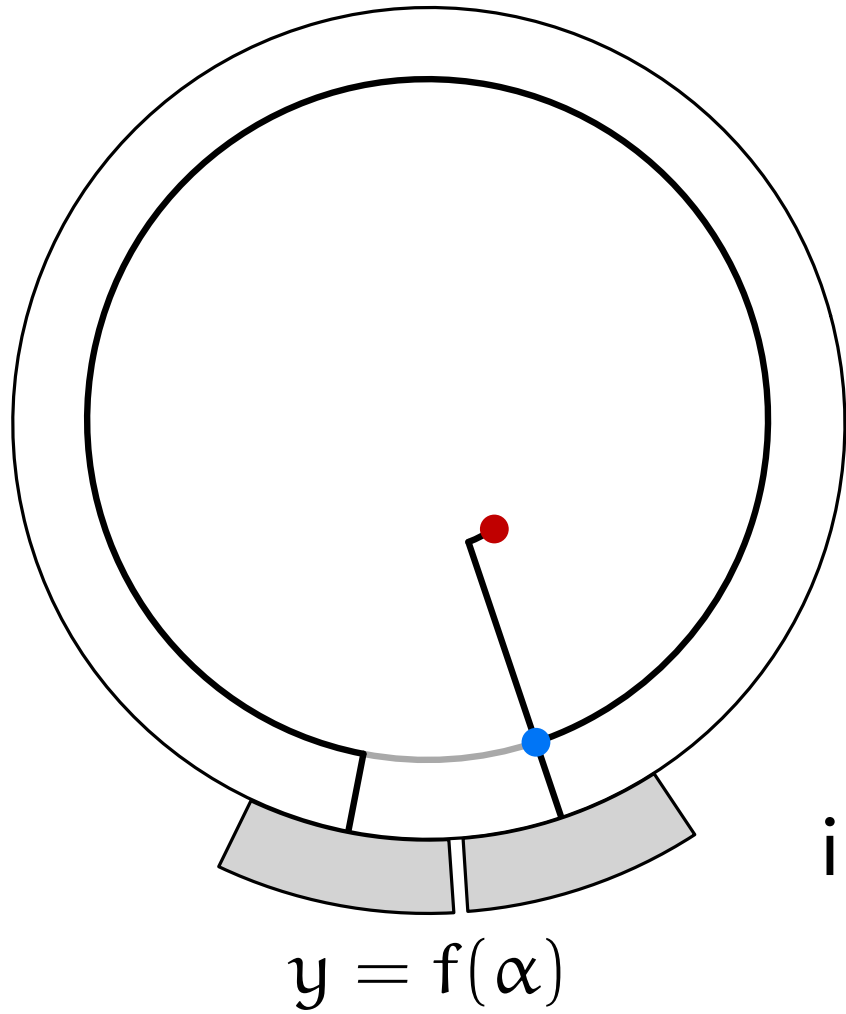
leader length



innermost:

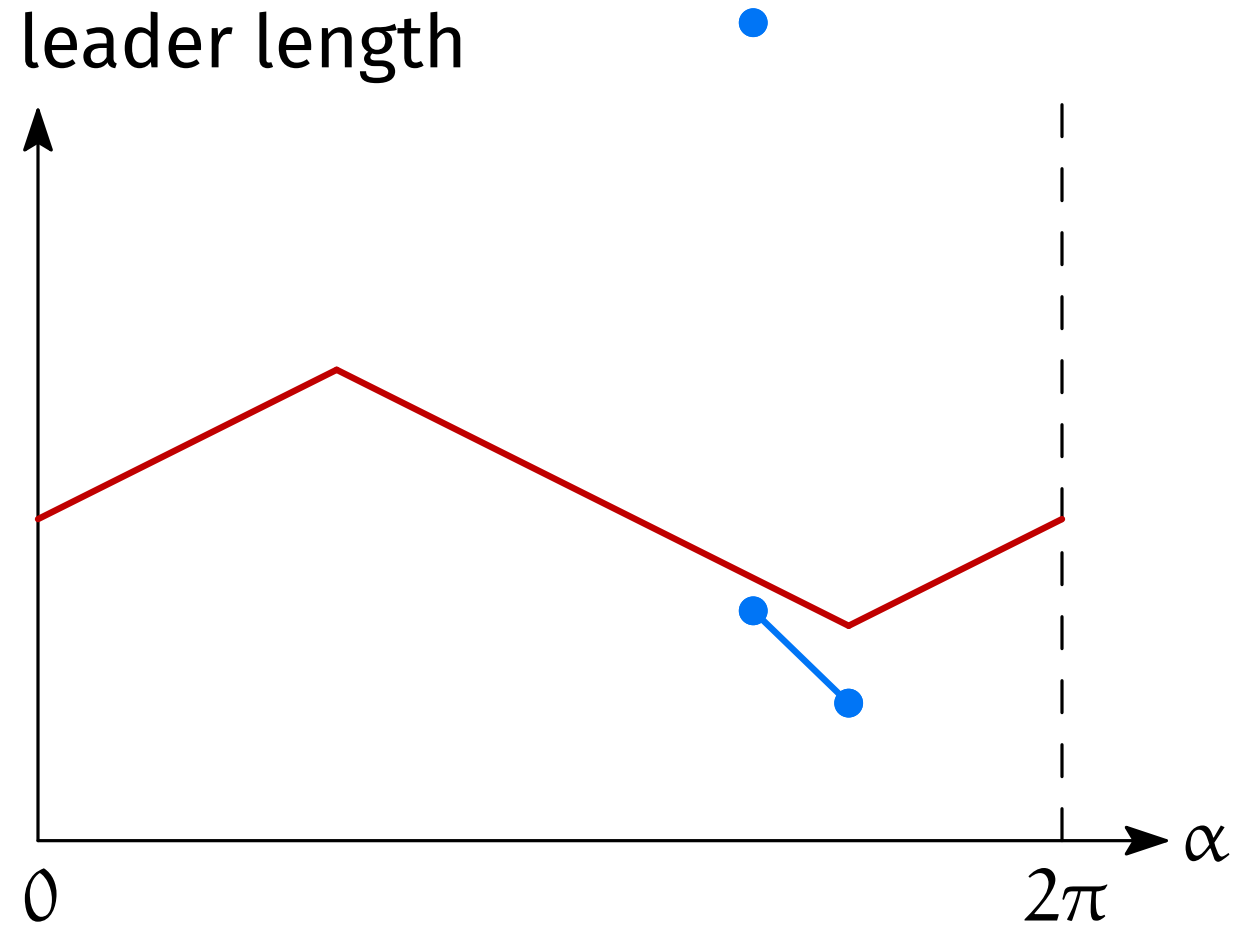
- always shorter path

Rotating the labels

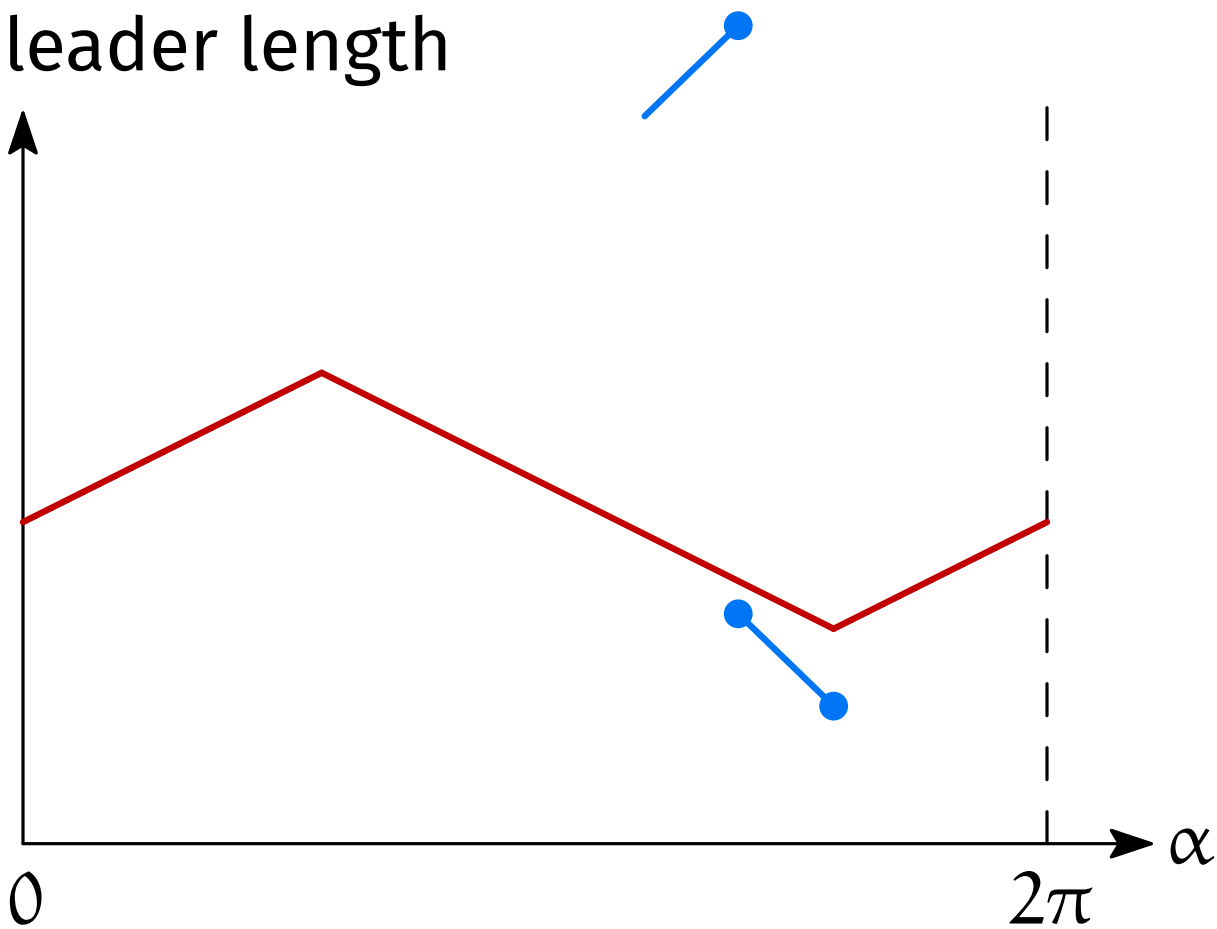
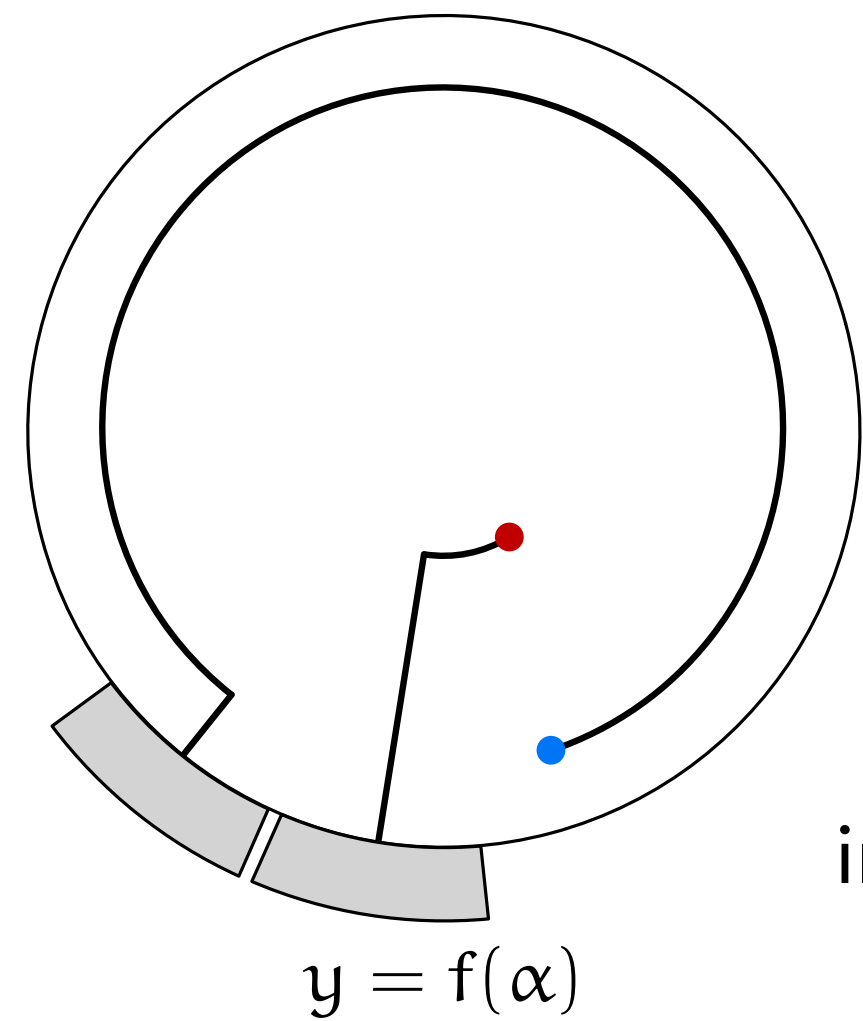


innermost:

- always shorter path



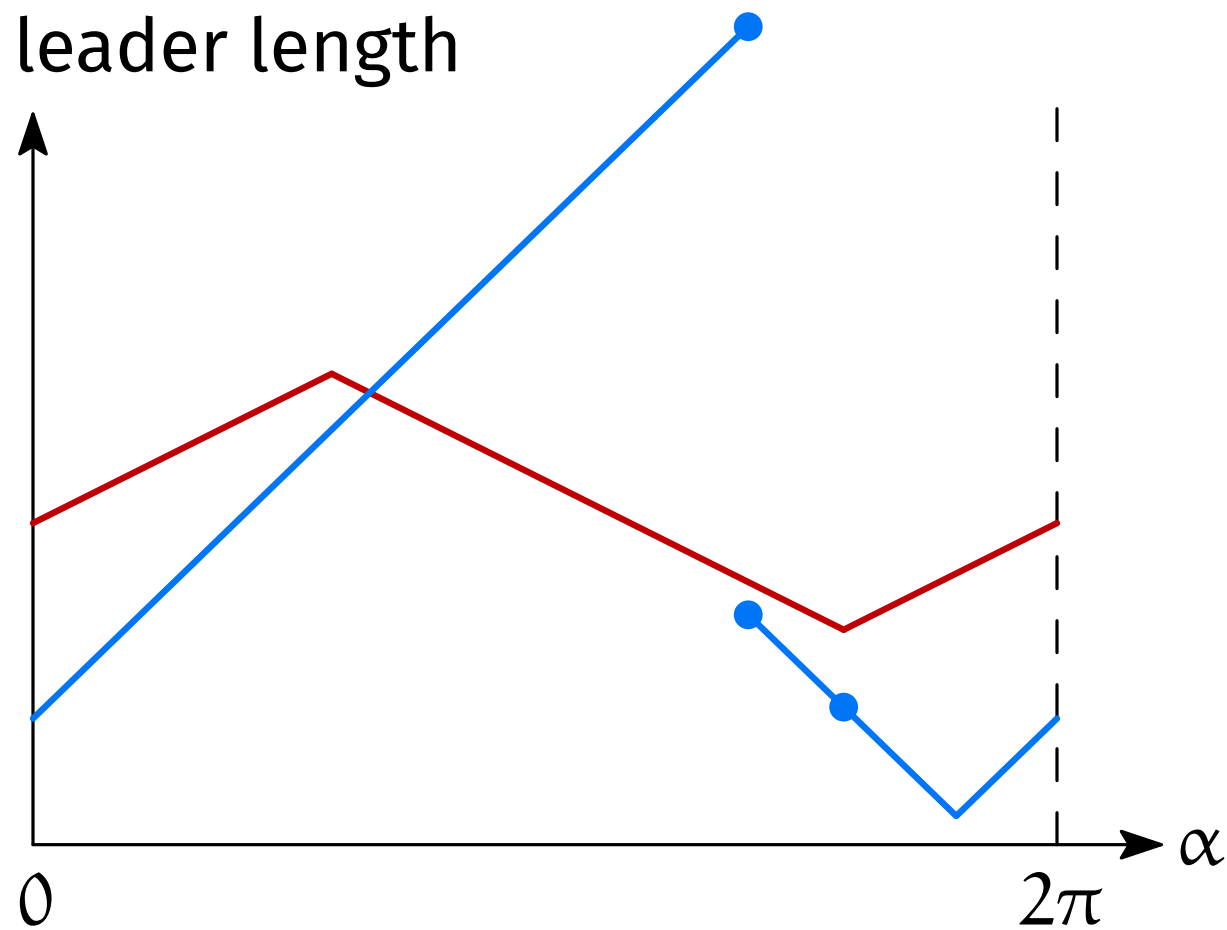
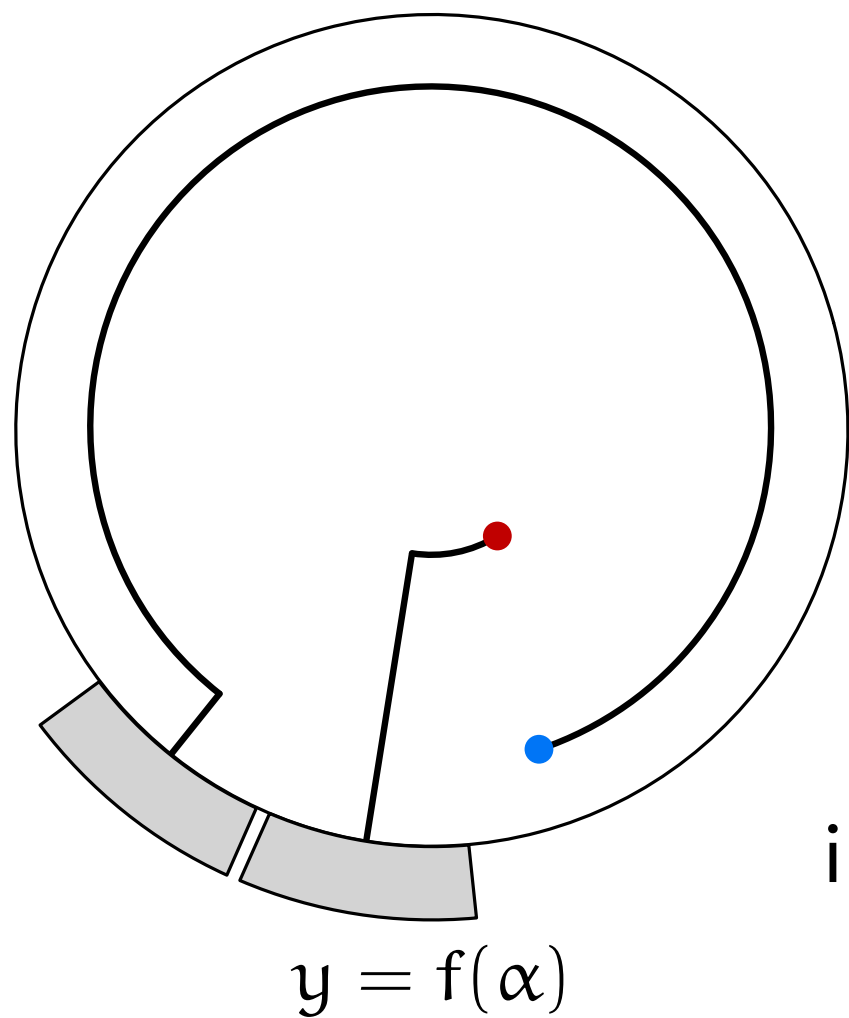
Rotating the labels



innermost:

- always shorter path

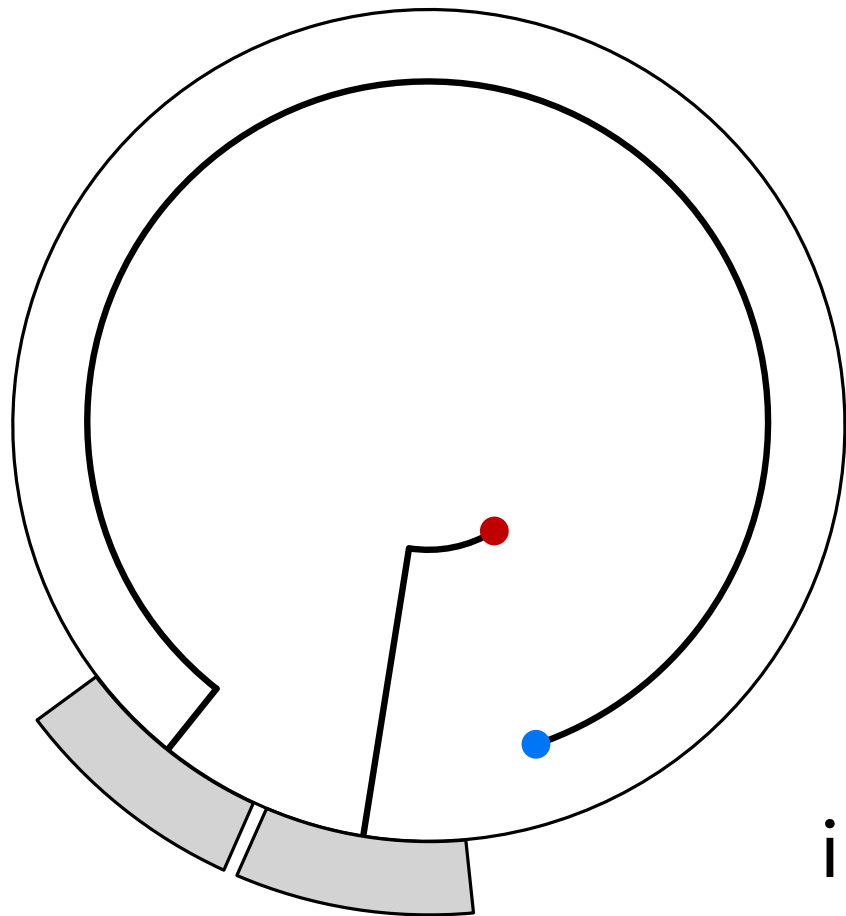
Rotating the labels



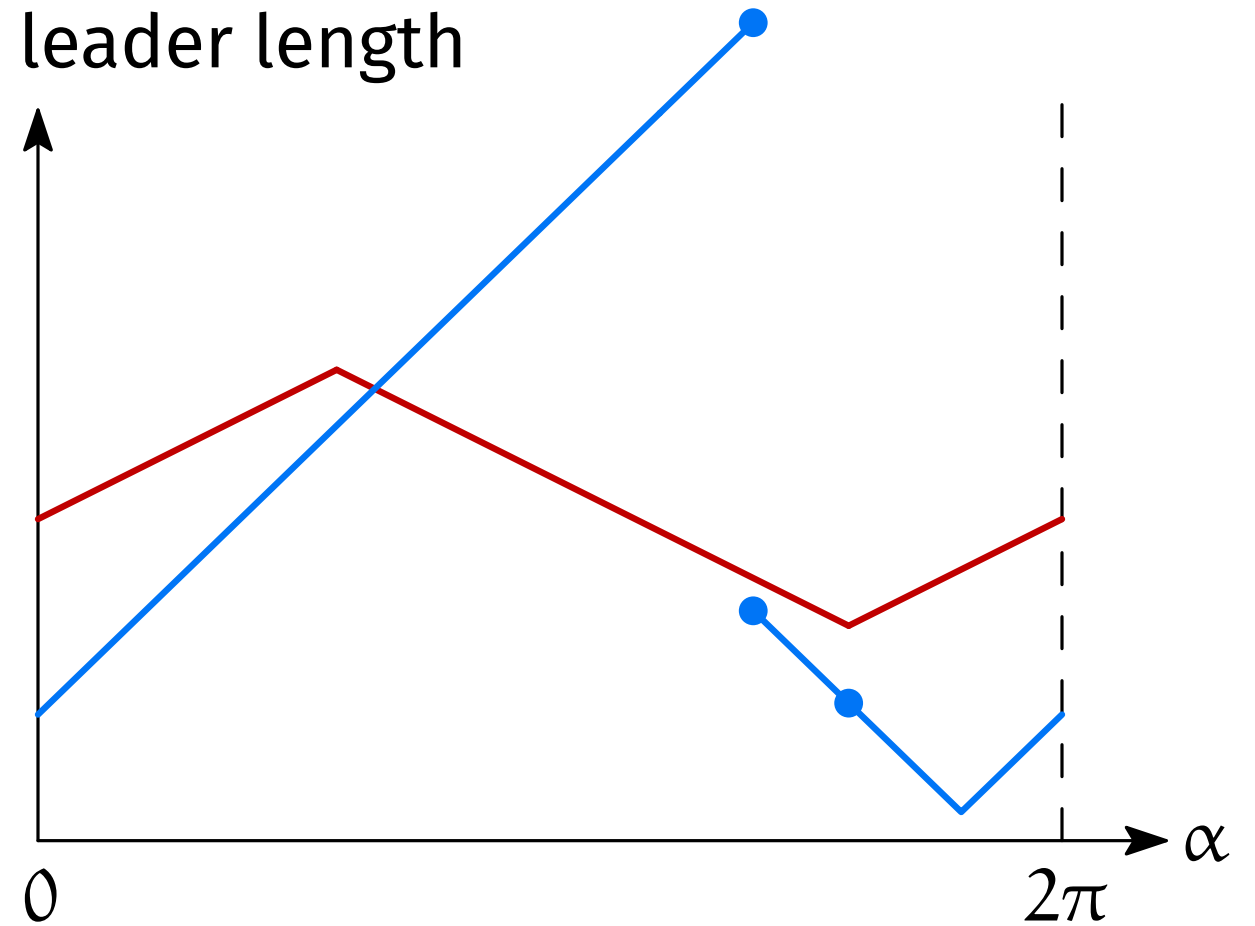
innermost:

- always shorter path

Rotating the labels



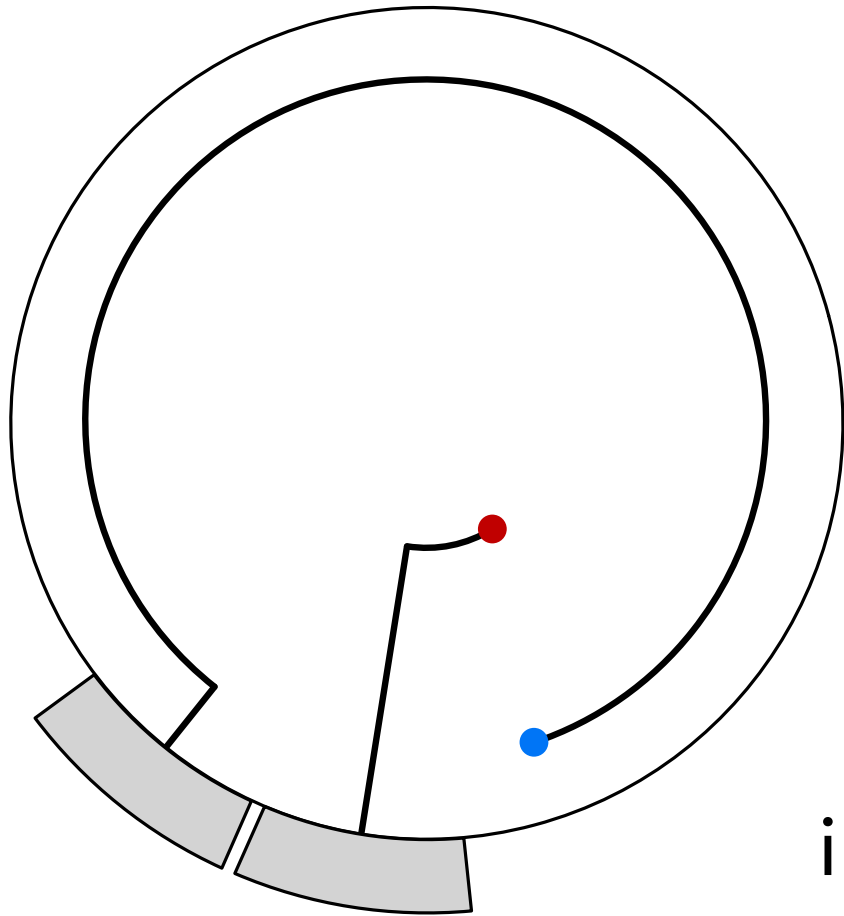
$$y = f(\alpha)$$



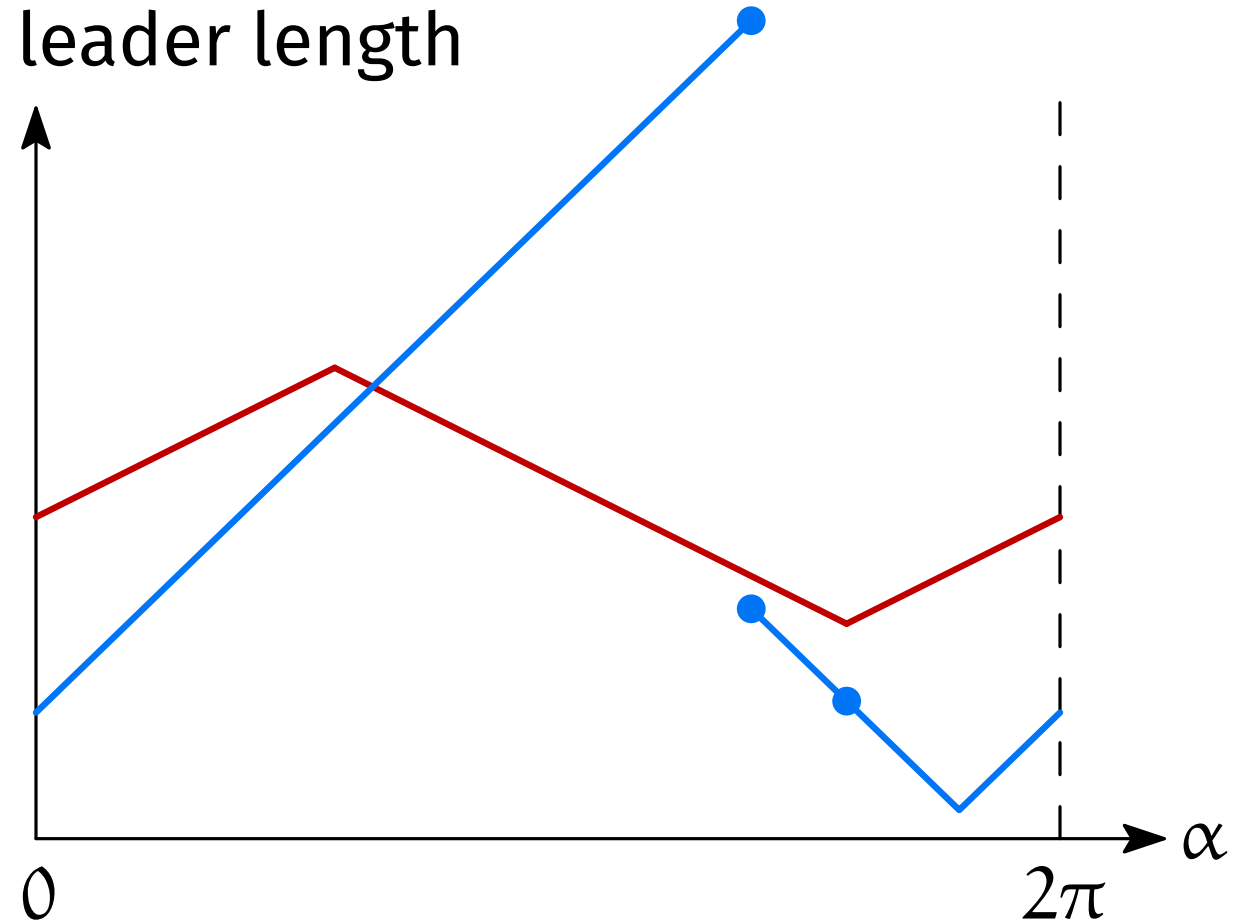
innermost:

- always shorter path
- decides all other leader directions

Rotating the labels



$$y = f(\alpha)$$

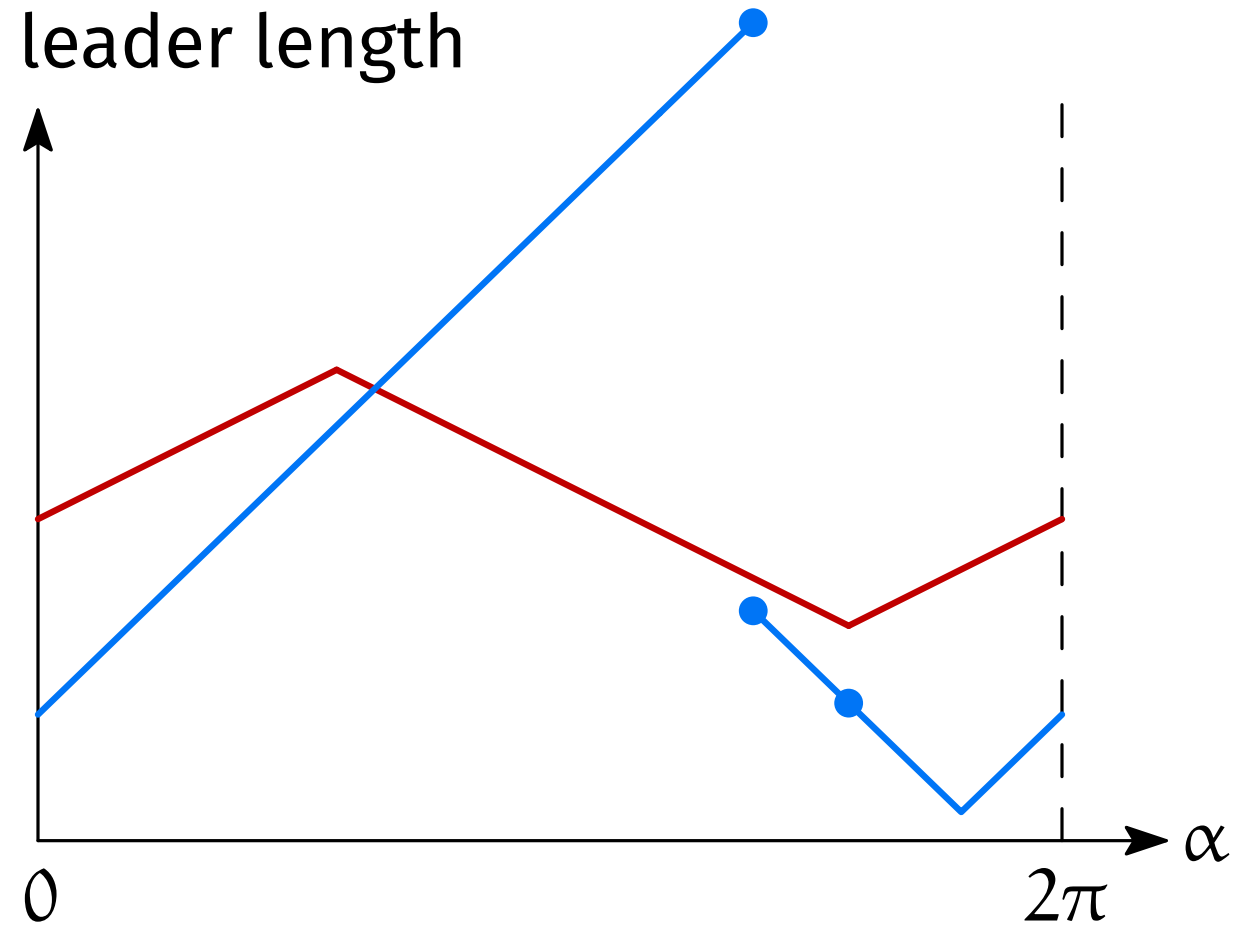
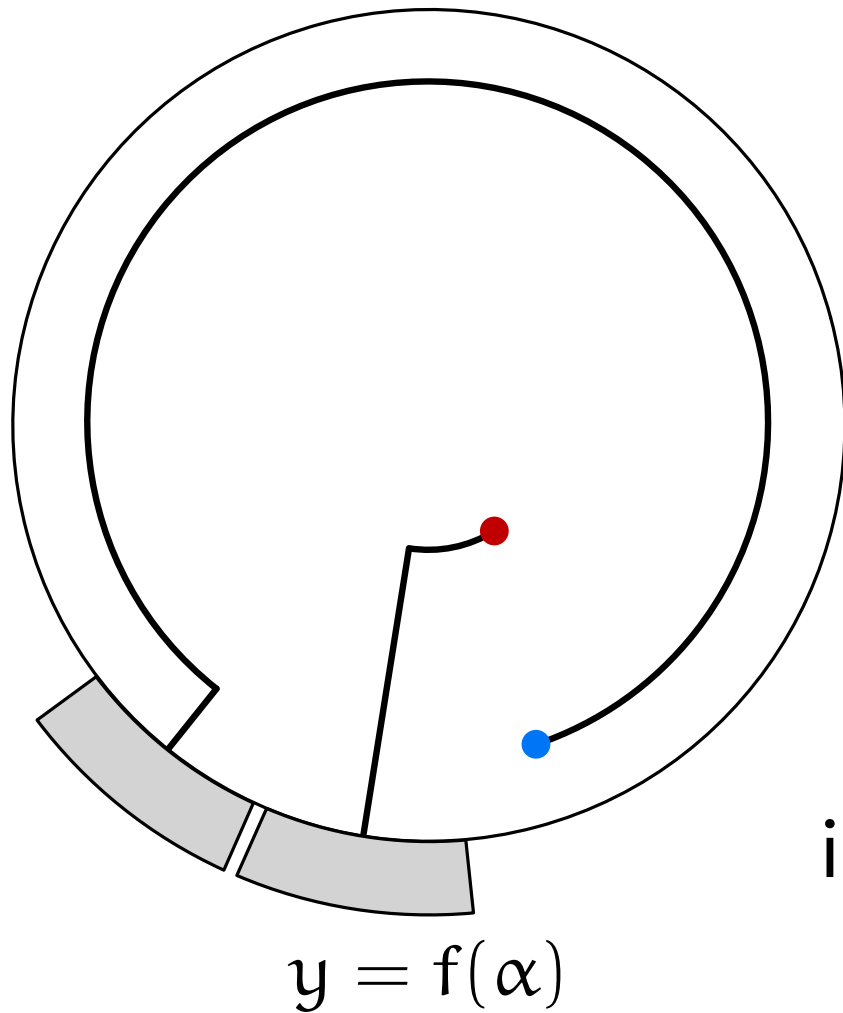


innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$

Rotating the labels



innermost:

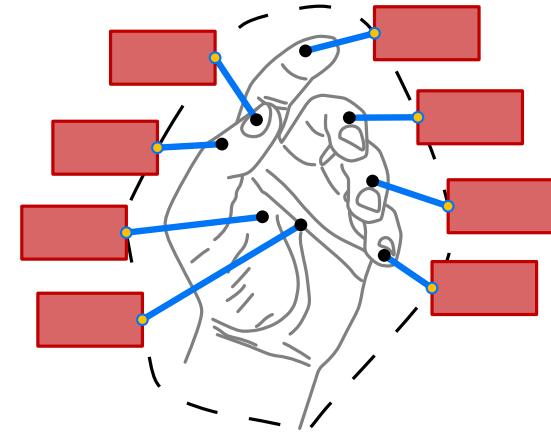
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$

Find minimum of **sum of functions** ($2n$ piece-wise linear parts) in $O(n)$

External Labeling [Bekos, Niedermann & Nöllenburg, '21]

Contour labeling



External Labeling [Bekos, Niedermann & Nöllenburg, '21]

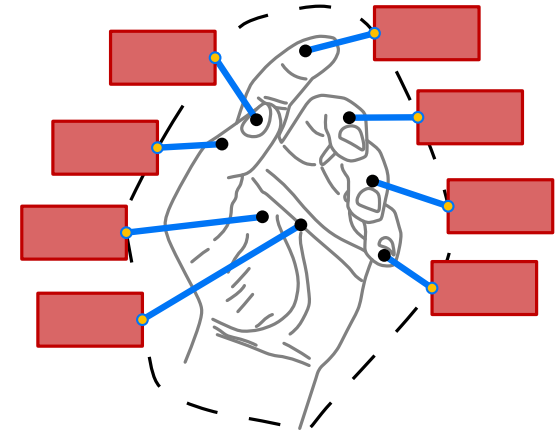
Contour labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

Generalized optimization function

[Benkert, Haverkort, Kroll, Nöllenburg, '09]



External Labeling [Bekos, Niedermann & Nöllenburg, '21]

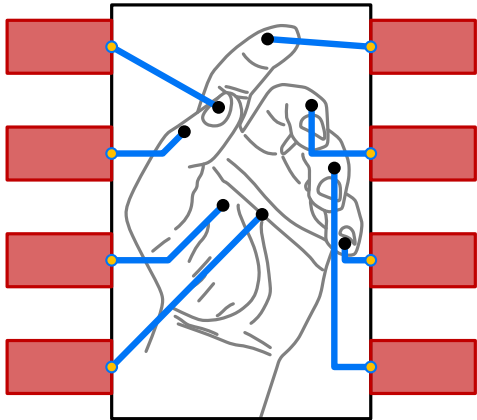
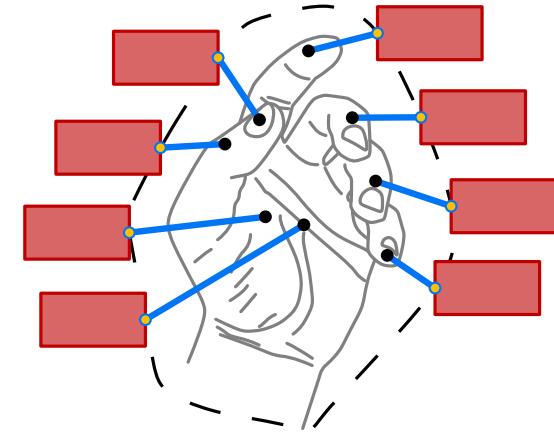
Contour labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

Generalized optimization function

[Benkert, Haverkort, Kroll, Nöllenburg, '09]



Boundary labeling

External Labeling [Bekos, Niedermann & Nöllenburg, '21]

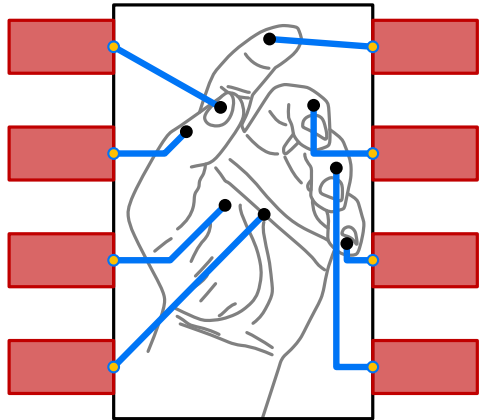
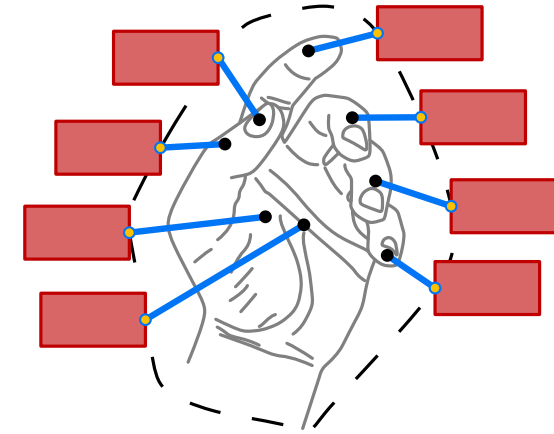
Contour labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

Generalized optimization function

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Boundary labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

Readability of different leader styles

[Barth, Gemsä, Niedermann, Nöllenburg, '19]

External Labeling [Bekos, Niedermann & Nöllenburg, '21]

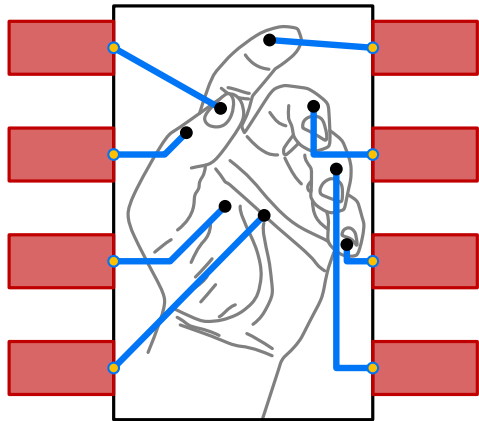
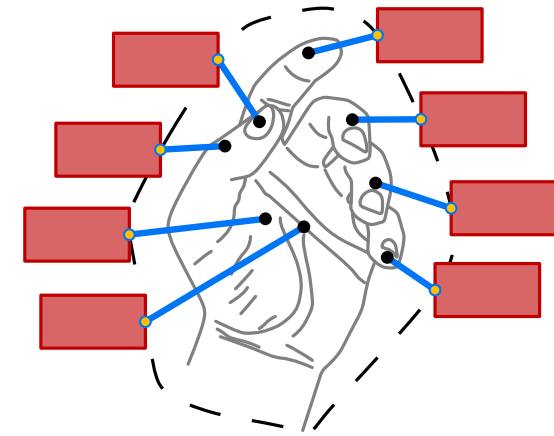
Contour labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

Generalized optimization function

[Benkert, Haverkort, Kroll, Nöllenburg, '09]



Boundary labeling

Minimum leader length labeling

[Bekos, Kaufmann, Symvonis, Wolff, '07]

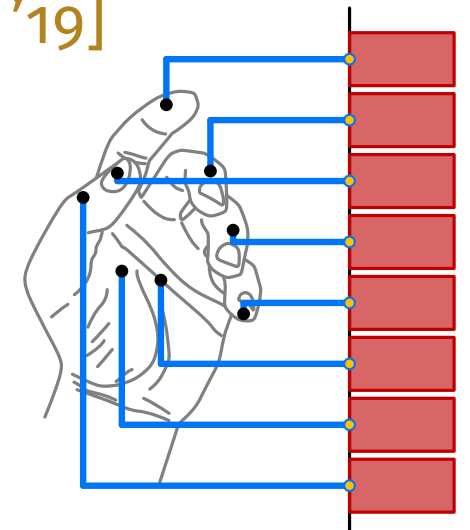
Readability of different leader styles

[Barth, Gemsa, Niedermann, Nöllenburg, '19]

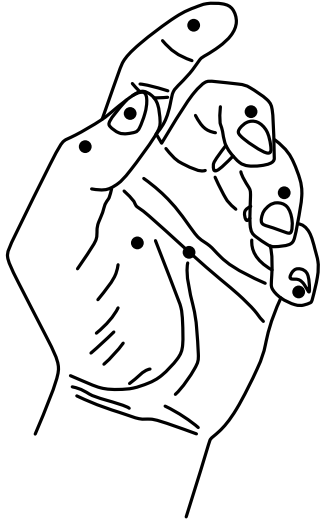
Boundary labeling (1-sided, po-leaders)

Generalized optimization function, $O(n^3)$ algorithm

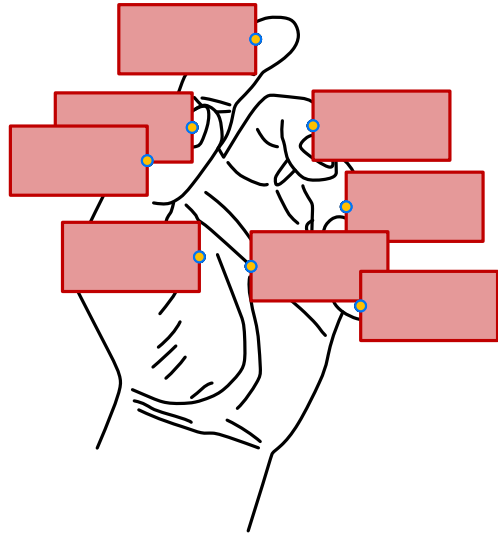
[Benkert, Haverkort, Kroll, Nöllenburg, '09]



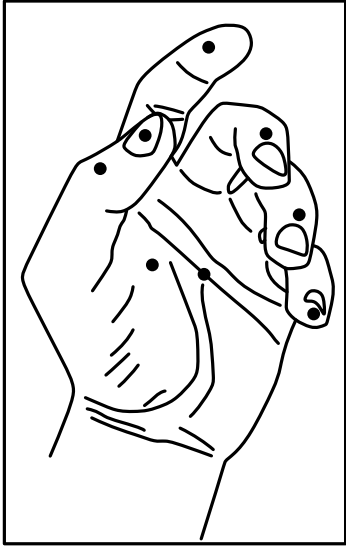
External Labeling [Bekos, Niedermann & Nöllenburg, '21]



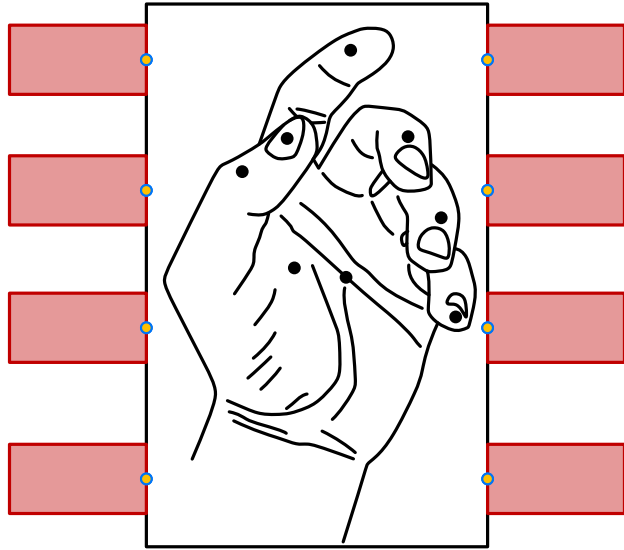
External Labeling [Bekos, Niedermann & Nöllenburg, '21]



External Labeling [Bekos, Niedermann & Nöllenburg, '21]

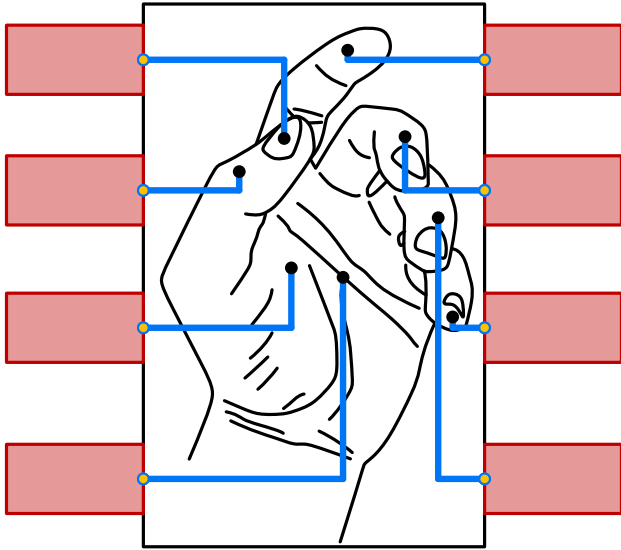


External Labeling [Bekos, Niedermann & Nöllenburg, '21]



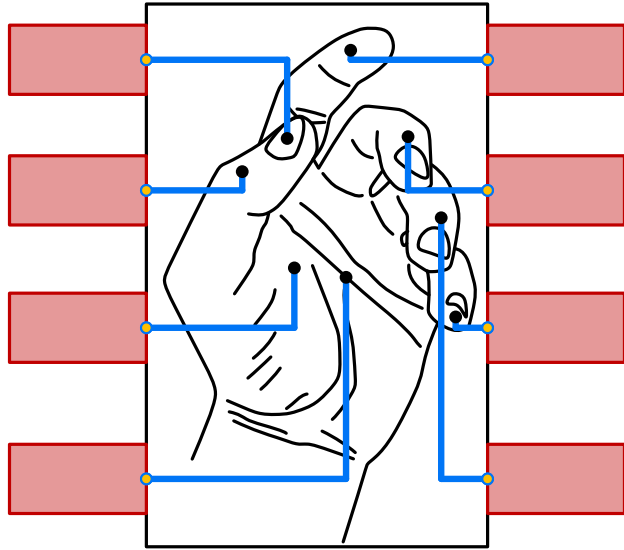
External Labeling [Bekos, Niedermann & Nöllenburg, '21]

Boundary labeling

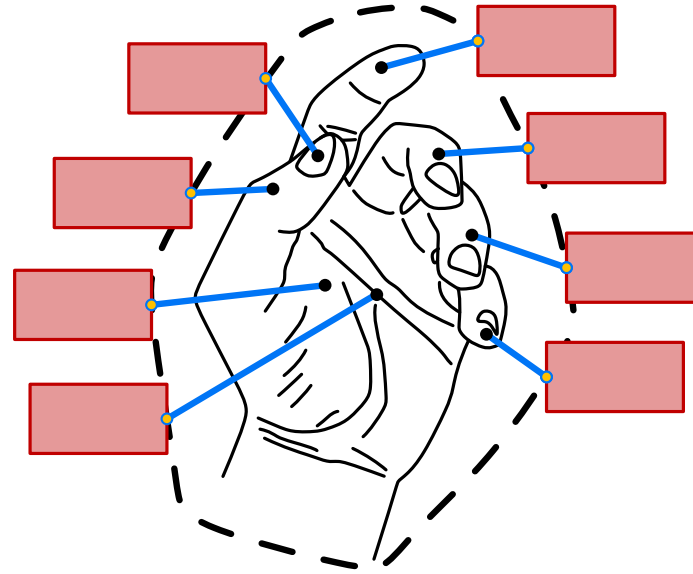


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

Boundary labeling

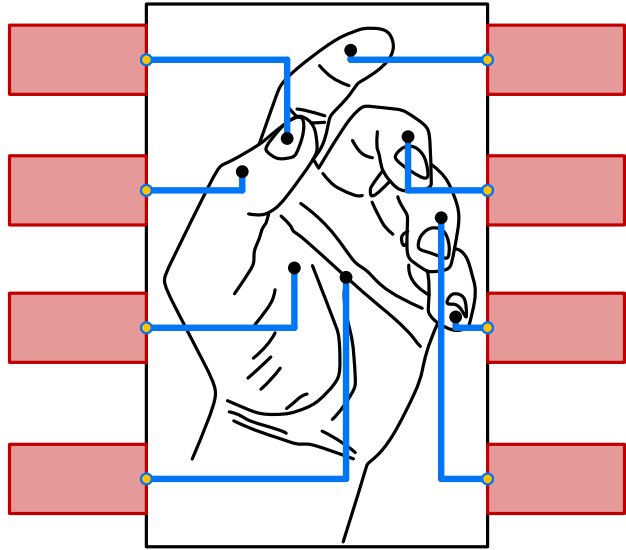


Contour labeling

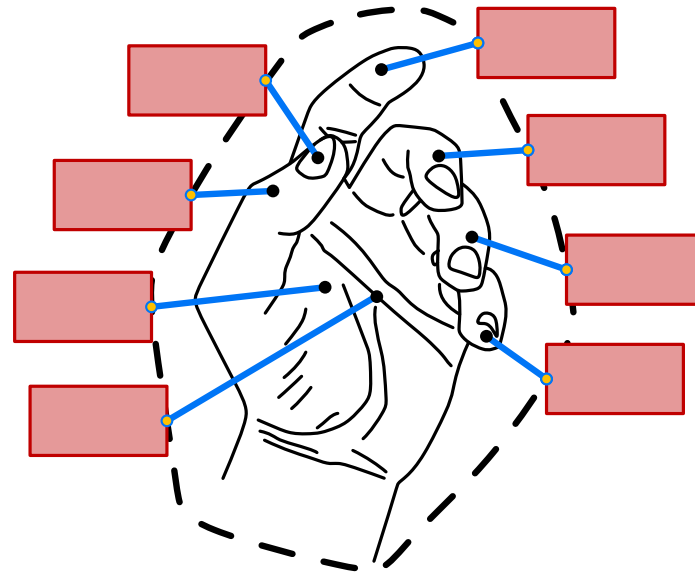


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

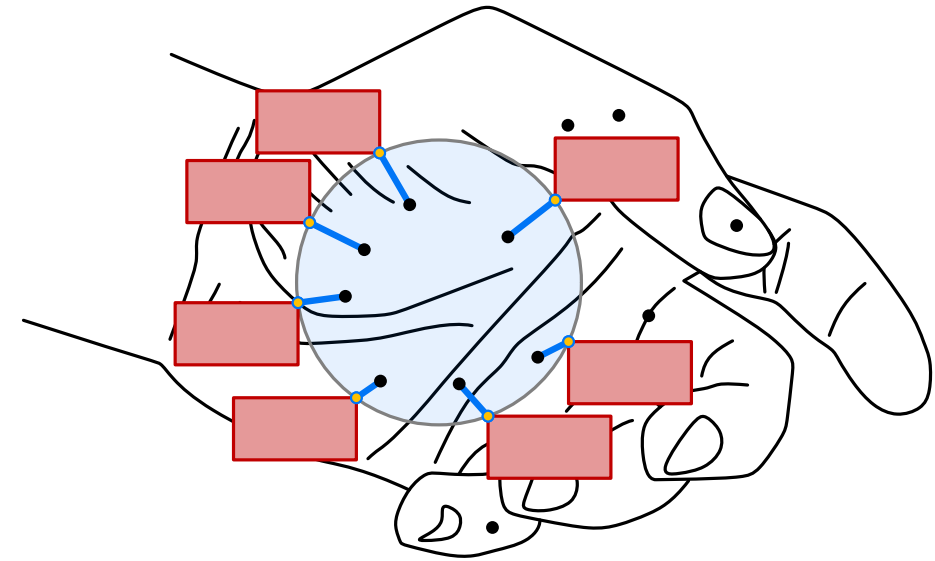
Boundary labeling



Contour labeling

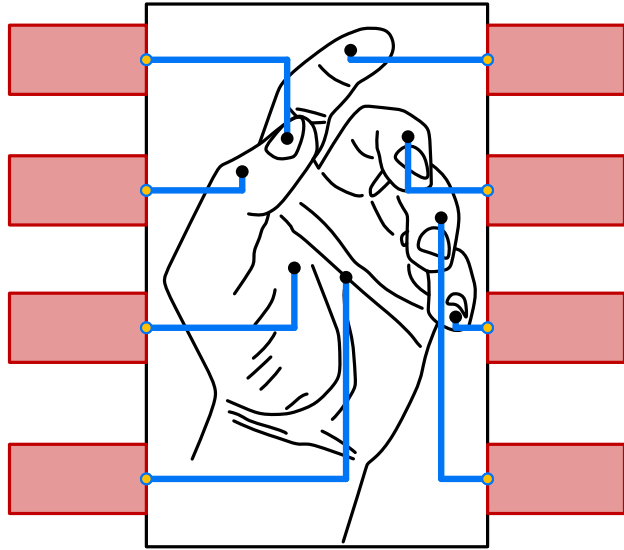


Eccentric labeling

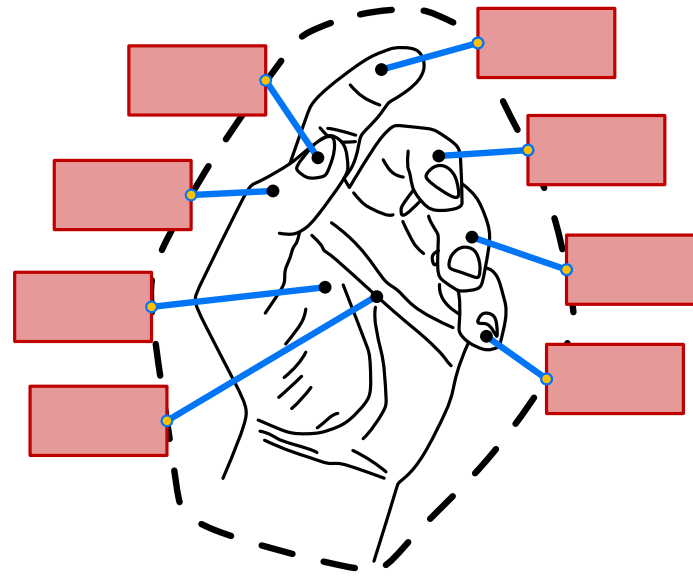


External Labeling [Bekos, Niedermann & Nöllenburg, '21]

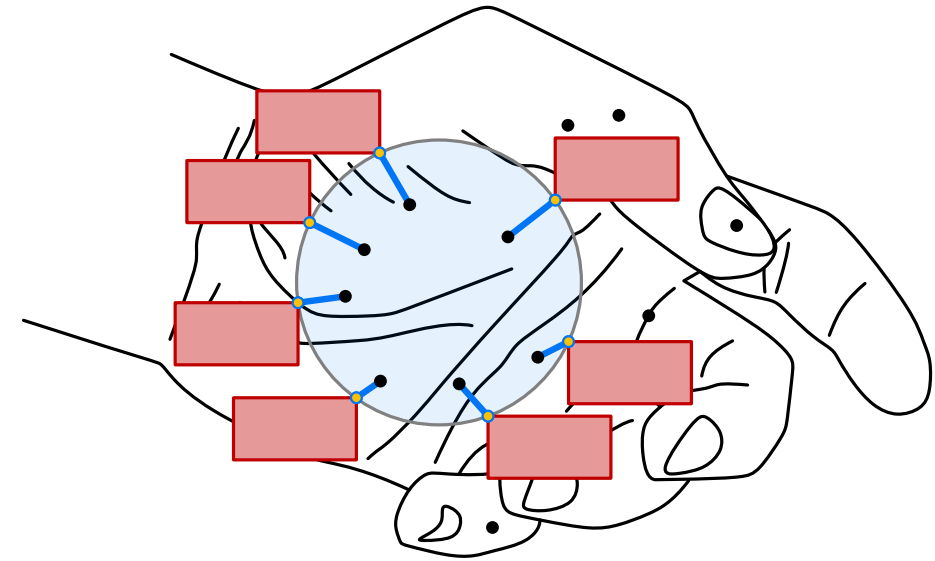
Boundary labeling



Contour labeling



Eccentric labeling



Boundary labeling (1-sided, po-leaders)

Generalized optimization function

Dynamic program $O(n^3)$

[Benkert, Haverkort, Kroll, Nöllenburg, '09]

