The Parameterized Complexity of Extending Stack Layouts

Thomas Depian, Simon D. Fink,
Robert Ganian, Martin Nöllenburg

18. – 20. September · GD 2024

TU WIEN
ac ALGORITHMS AND COMPLEXITY GROUP

Stack Layouts

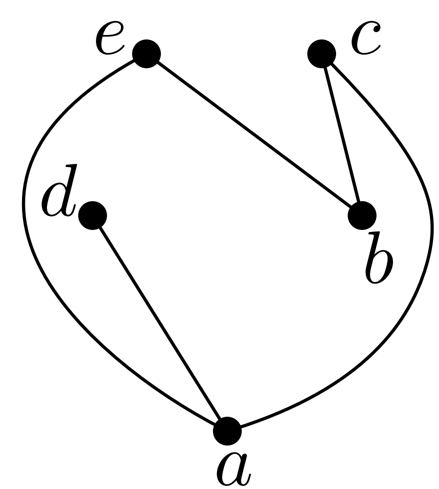
Stack Layout

Given:

Integer $\ell > 0$

Graph G

$\ell = 2$



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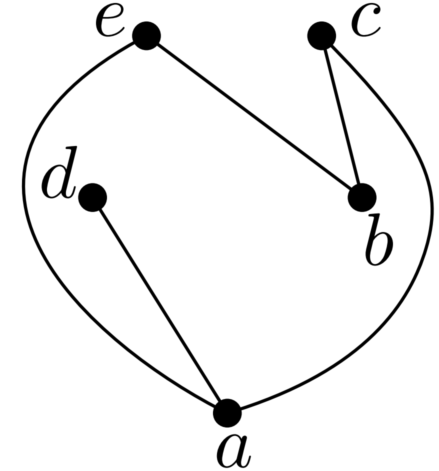

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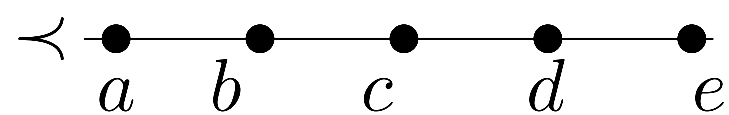
Linear order $\prec$ of vertices $V(G)$



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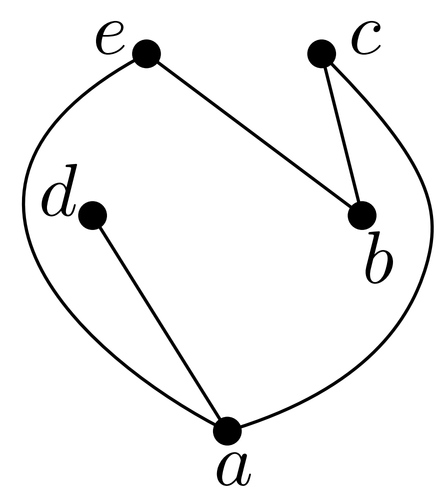

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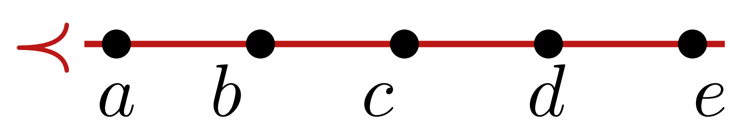

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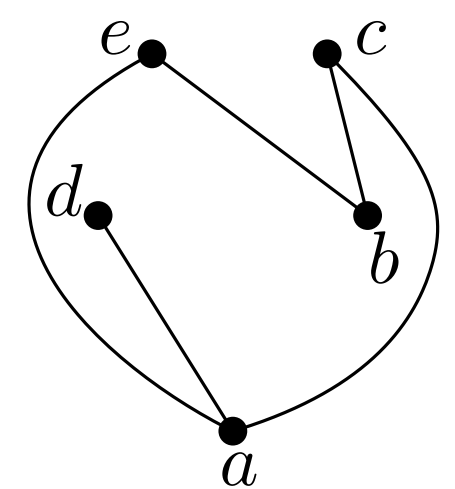

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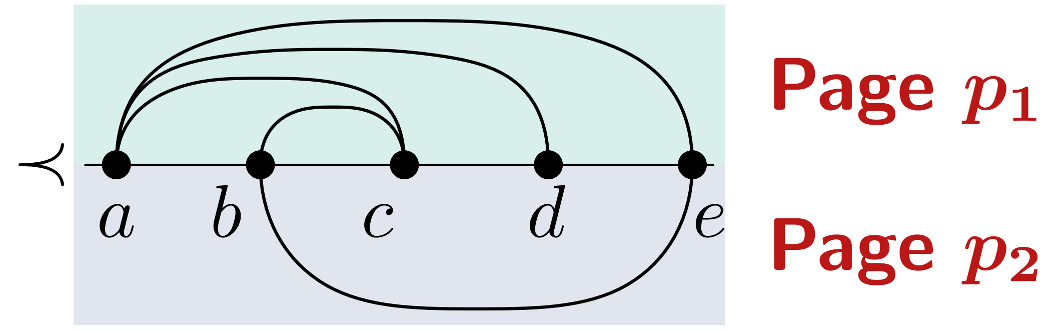

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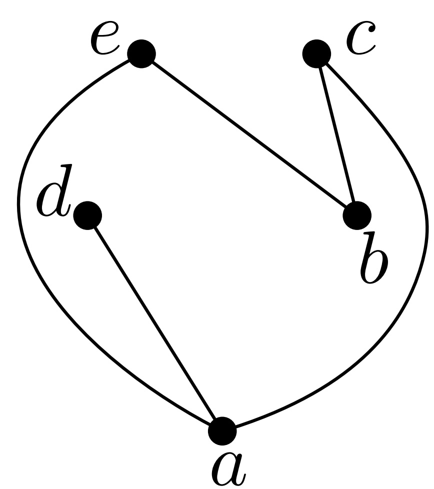

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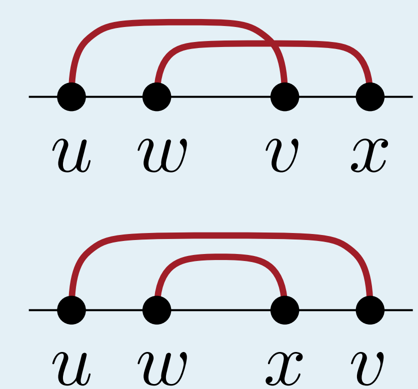
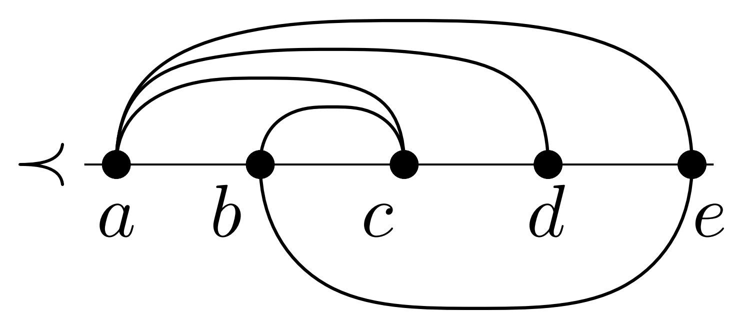


endpoints of no two edges on the same page alternate



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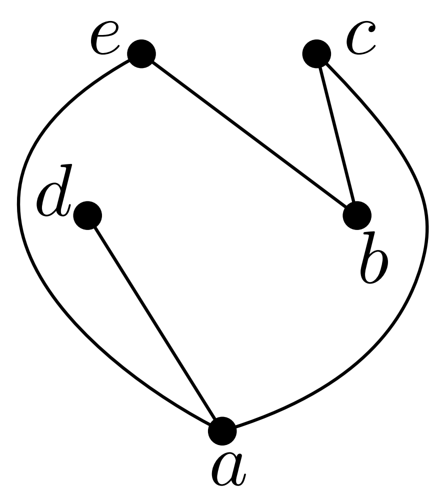

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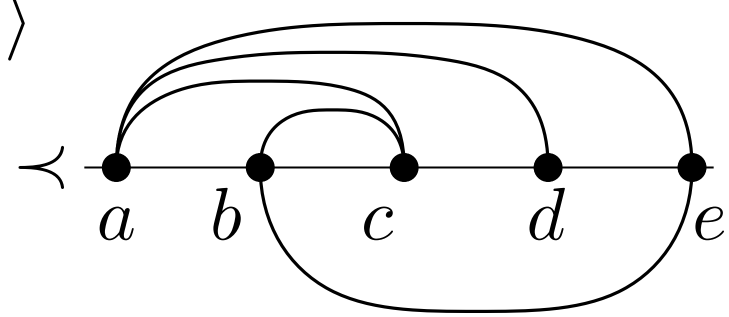

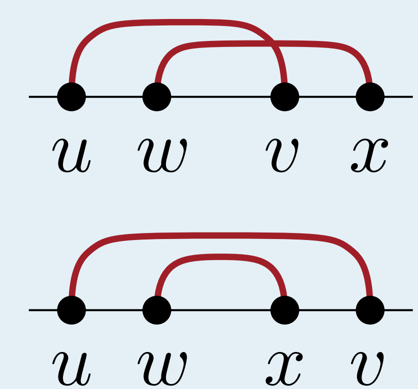

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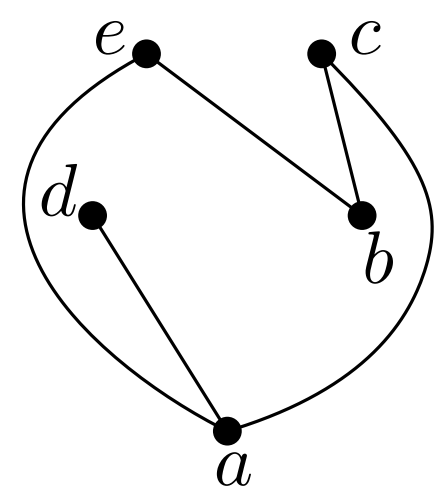

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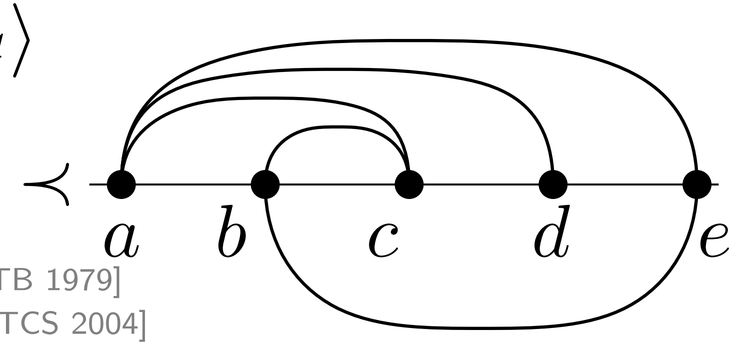

Well-studied field [Bernhard and Kainen, JCTB 1979] [Dujmović and Wood, DMTCS 2004]

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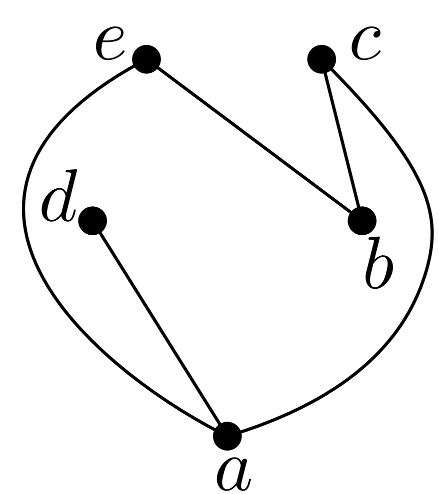

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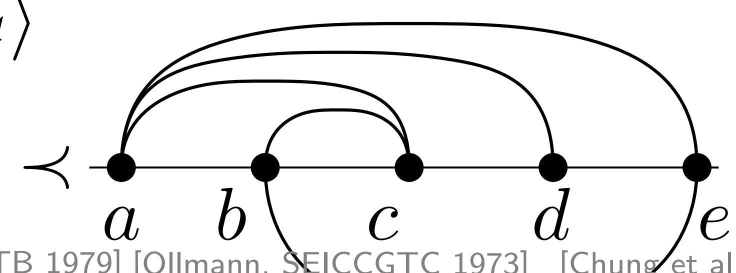

[Yannakakis, JCSS 1998] [Bekos et al., JOCG 2020]

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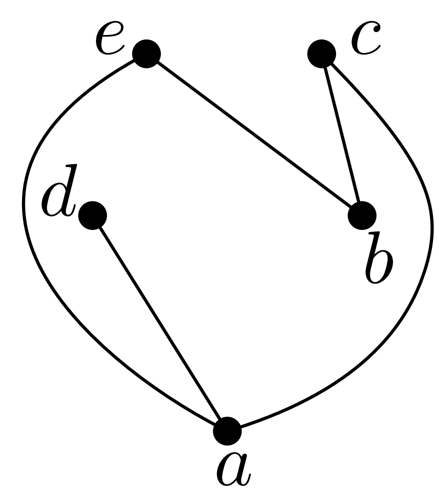

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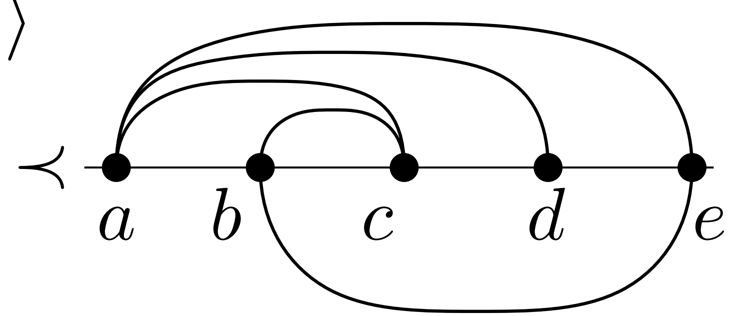

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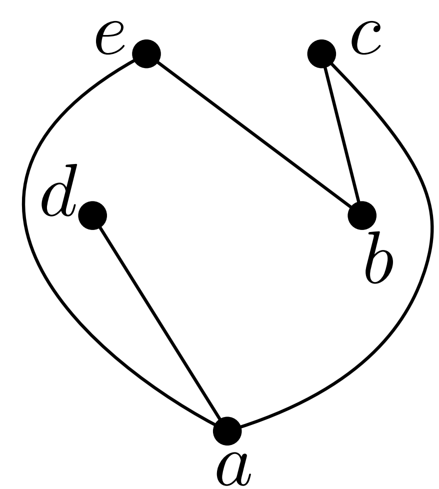

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(even if $\prec$ is given & $\ell = 4$)

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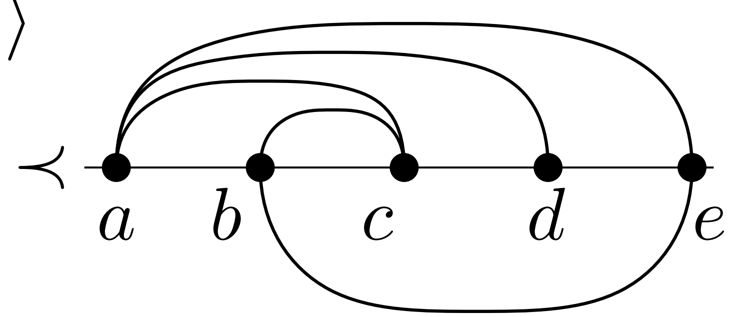

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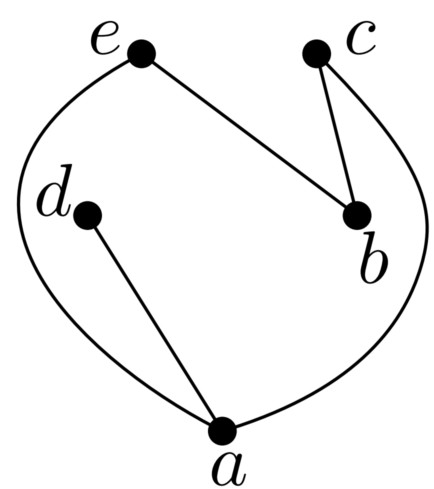

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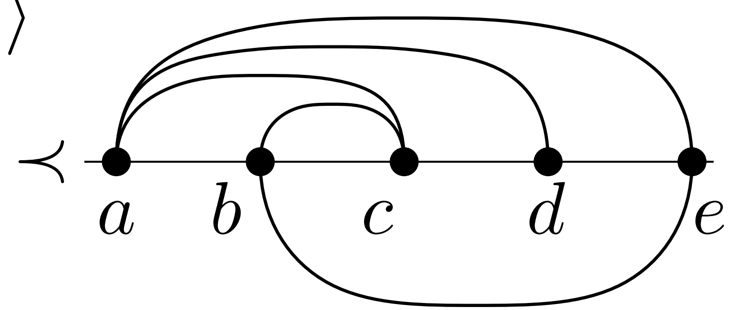

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Stack Layout Extension

STACK LAYOUT EXTENSION (SLE)

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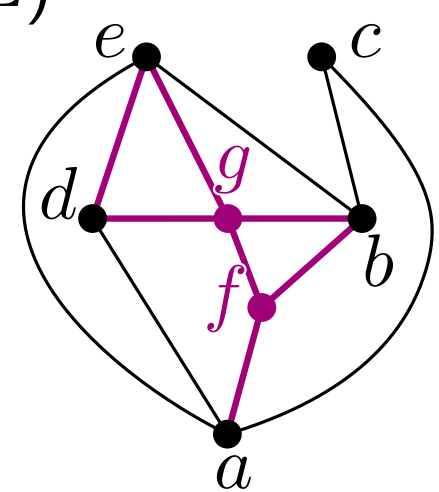

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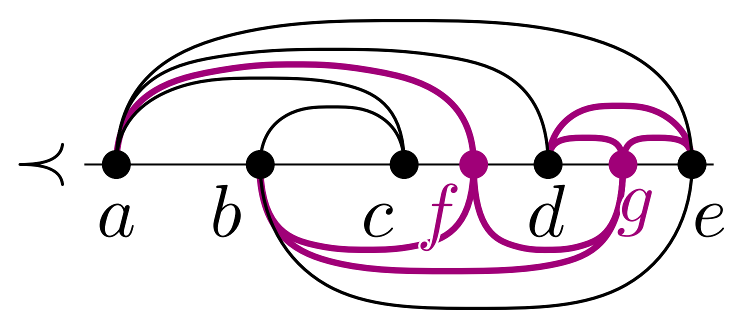

Stack Layout Extension

STACK LAYOUT EXTENSION (SLE)

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of subgraph $H \subseteq G$

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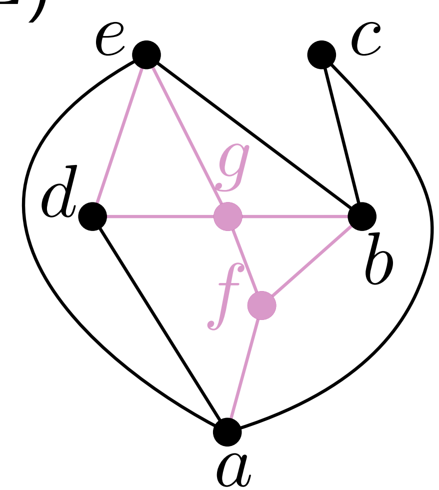

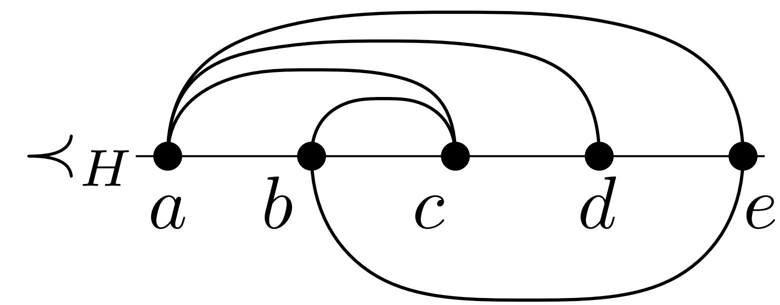

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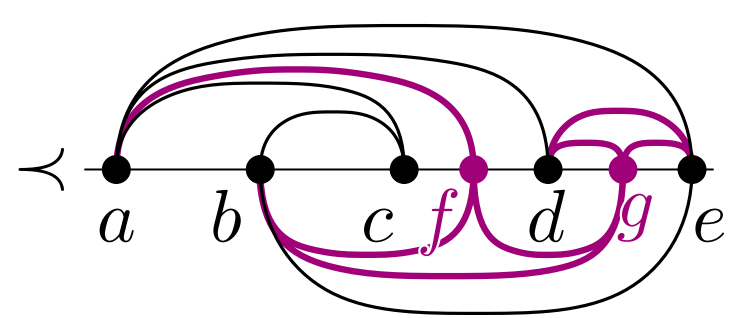

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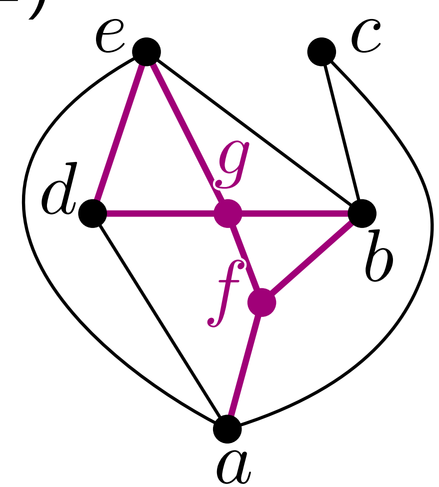

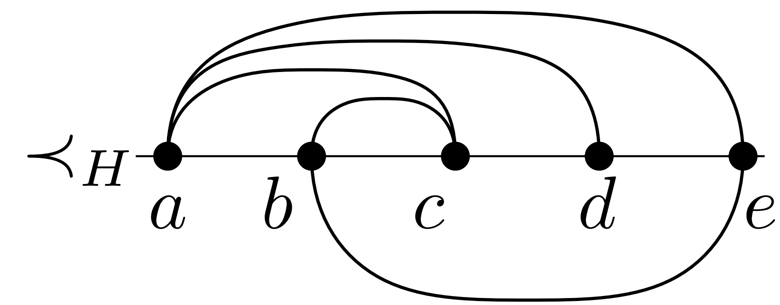

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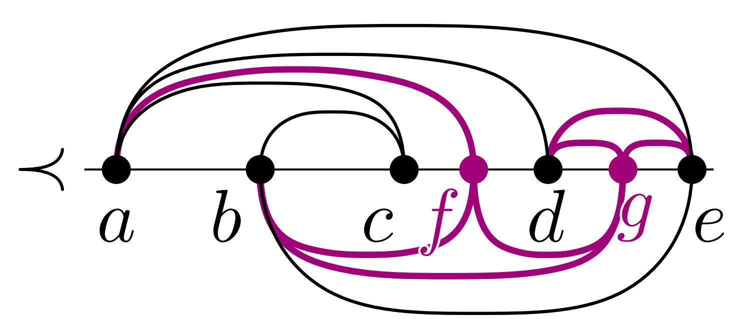

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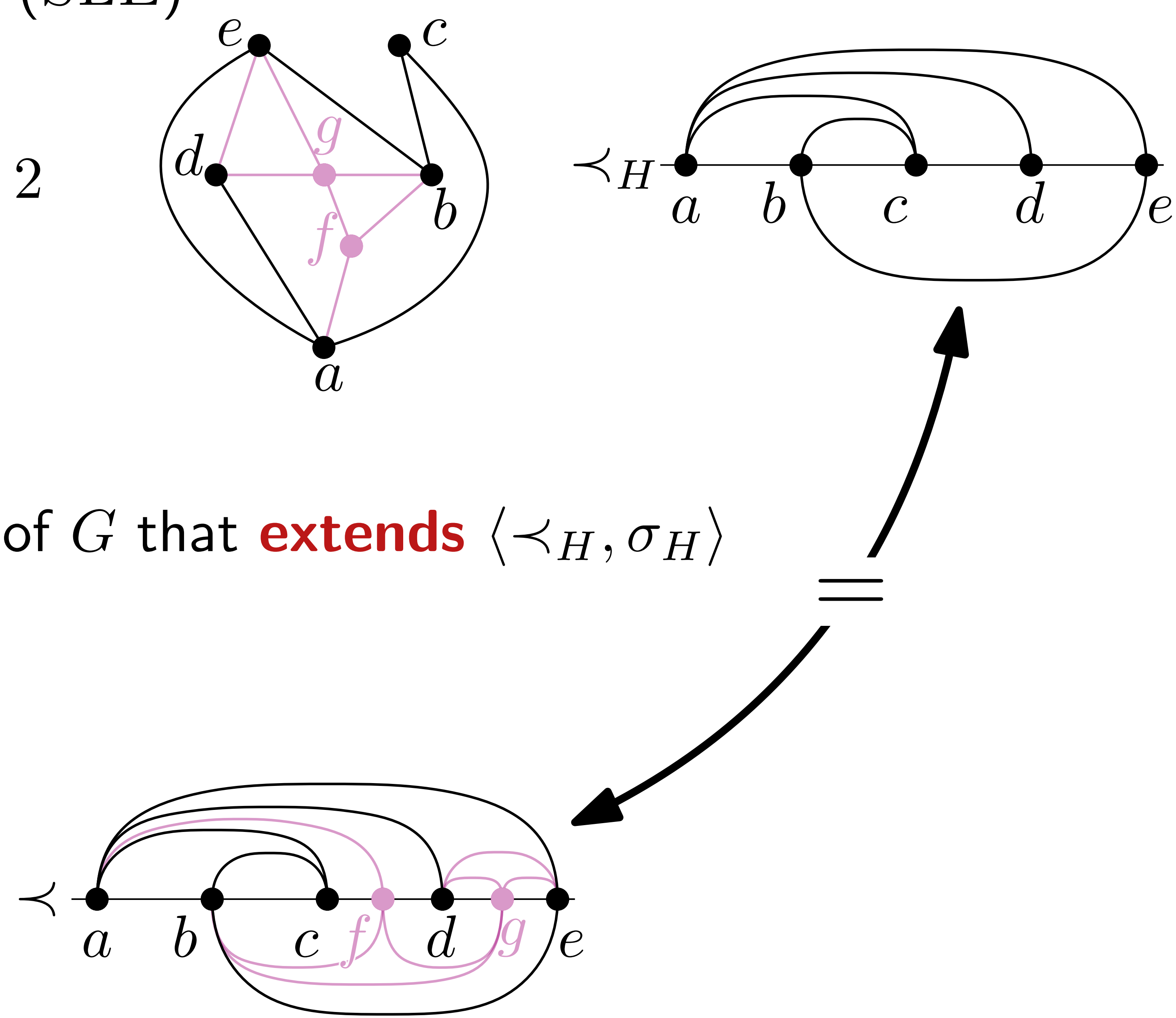

Our Results

NP-complete

$\emptyset$

[Chung et al., JADM 1987]

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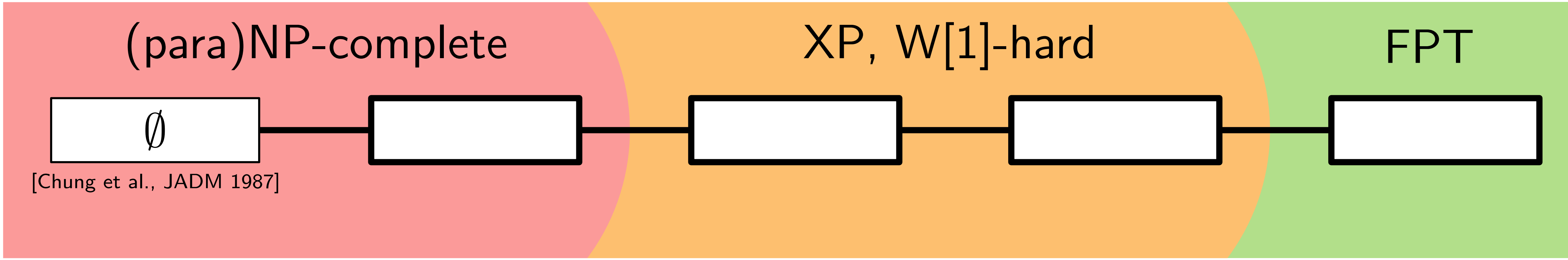

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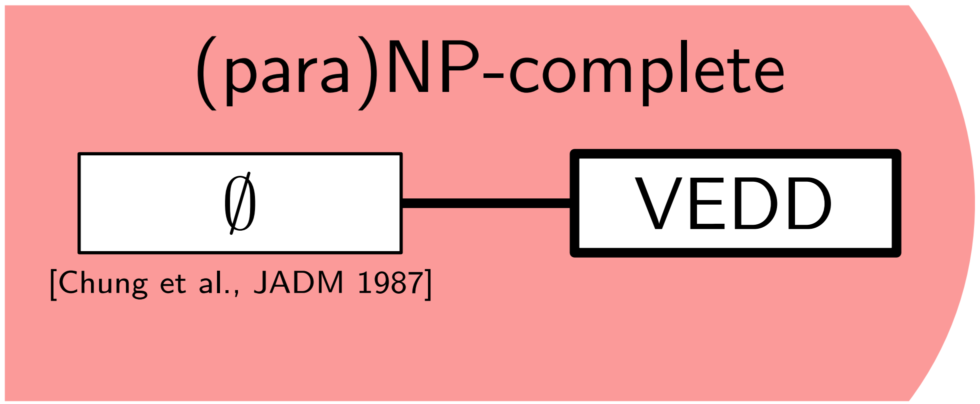
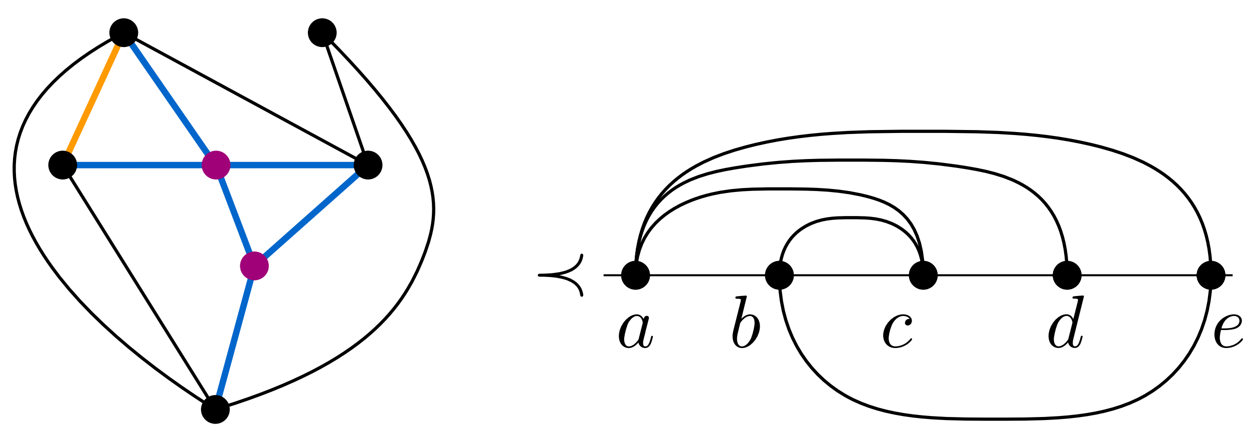


VEDD: #vertices (inc. **incident edges**) & edges to delete from G to obtain H



Our Results

(para)NP-complete

∅ — **VEDD**

[Chung et al., JADM 1987]

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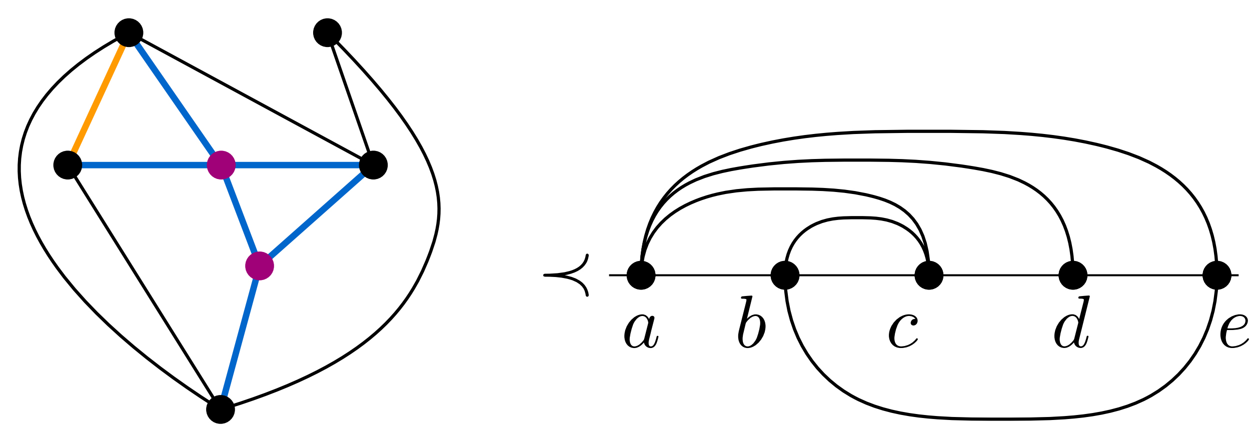
$\text{VEDD} = \mathbf{2} + \mathbf{1} = 3$



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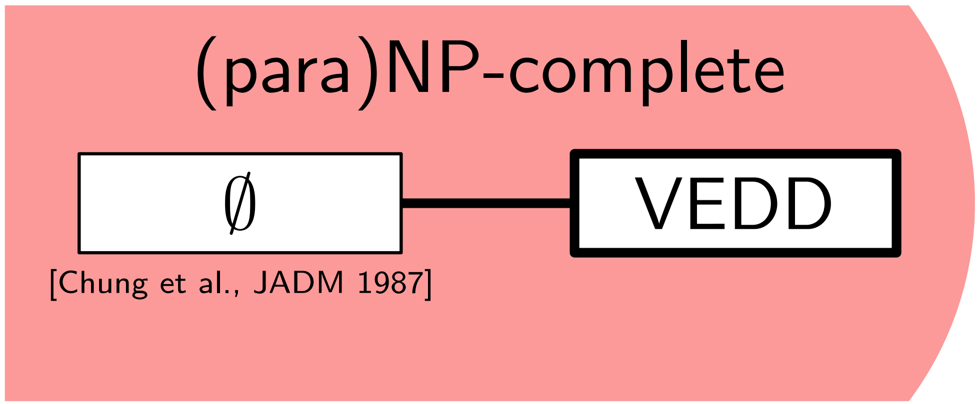

VEDD was successfully used in other drawing extension problems

[Bhore et al., SoCG 2020] [Eiben et al., MFCS 2020]
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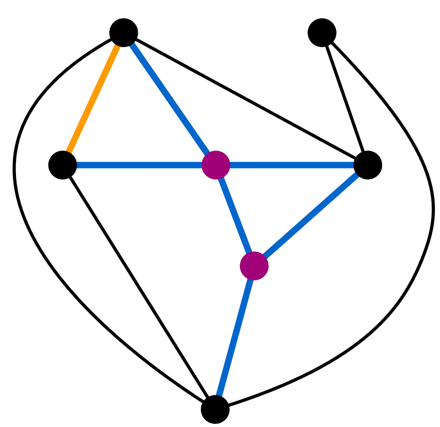

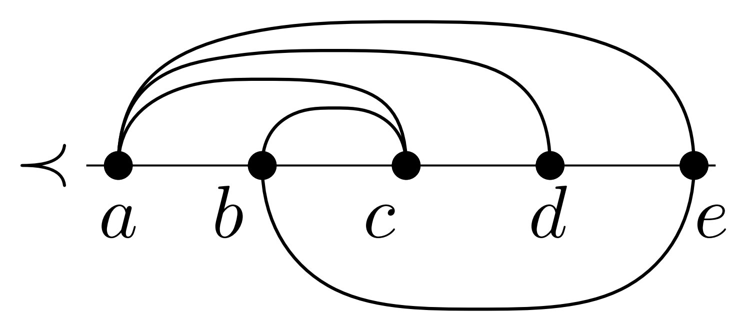

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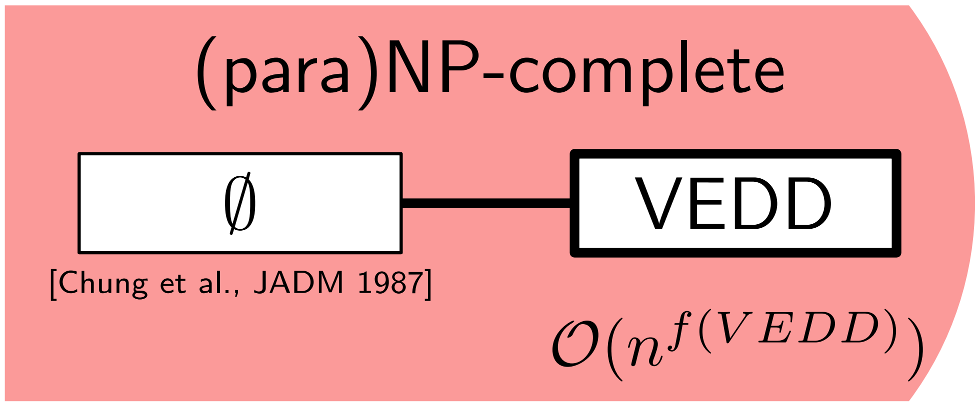

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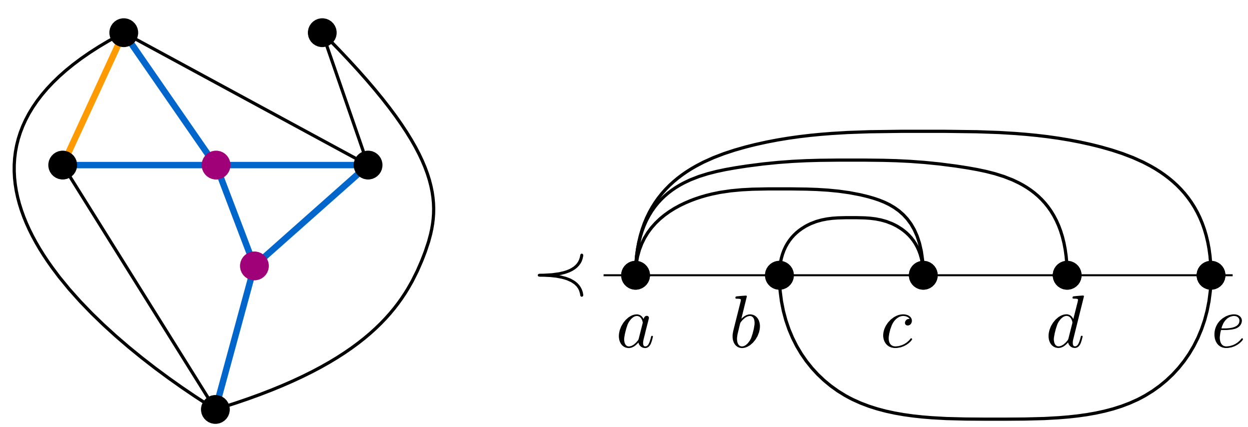

SLE With Two Missing Vertices is NP-complete

Reduction from 3-Sat

$$\varphi = (x_1 \vee \neg x_2 \vee x_3) \wedge \dots$$

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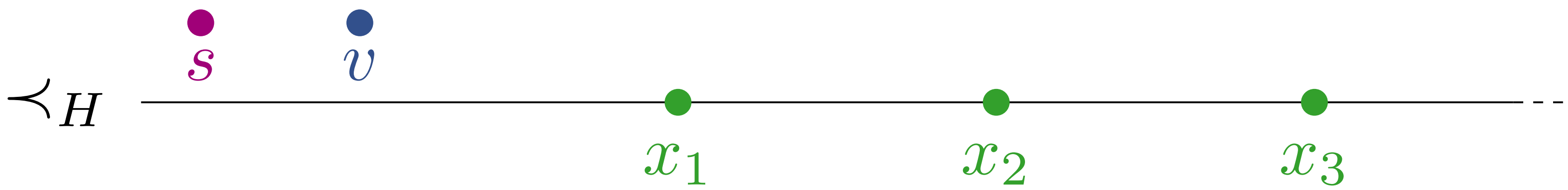

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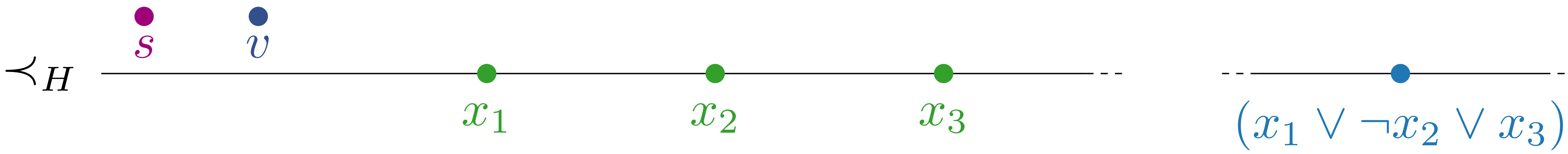

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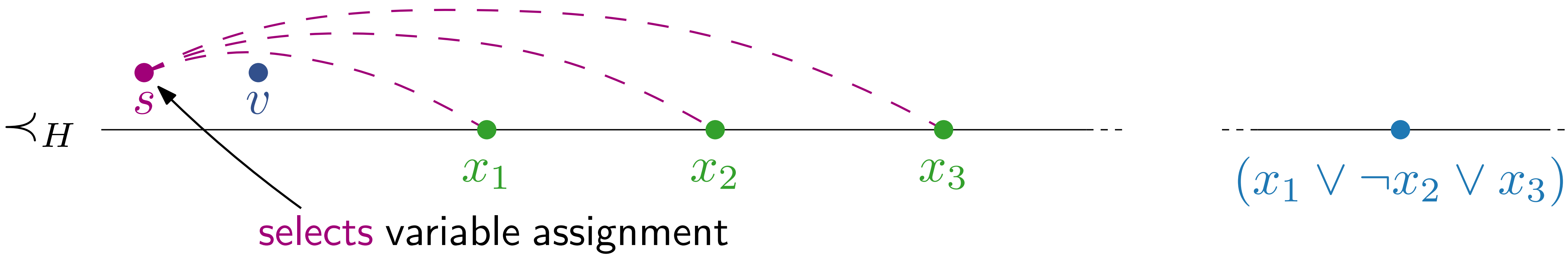

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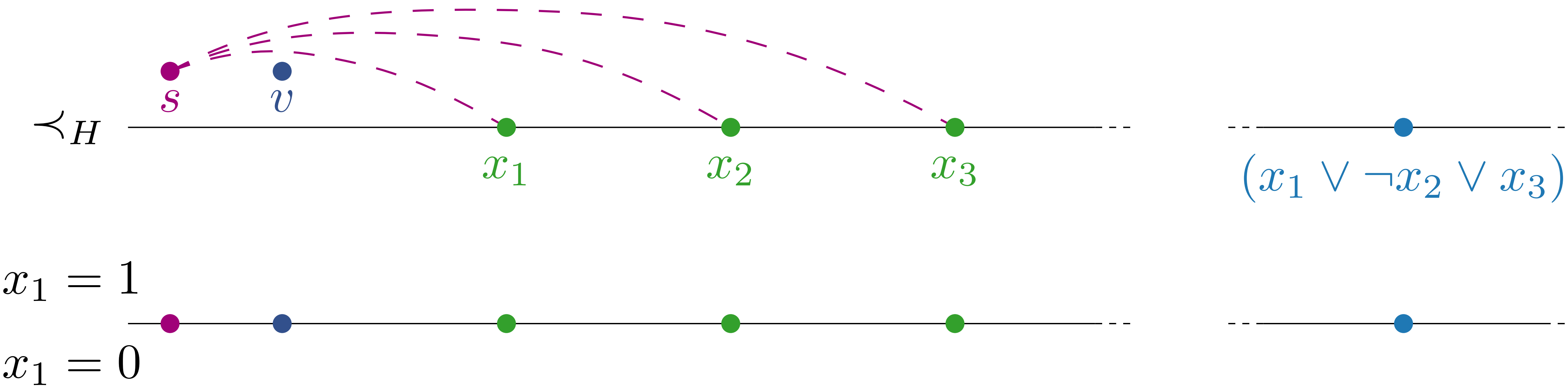

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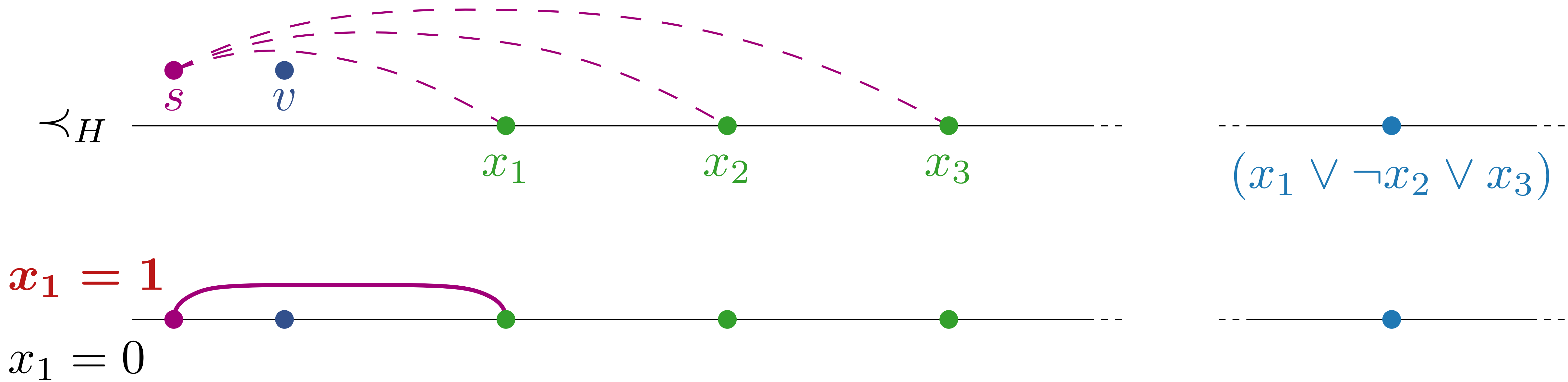

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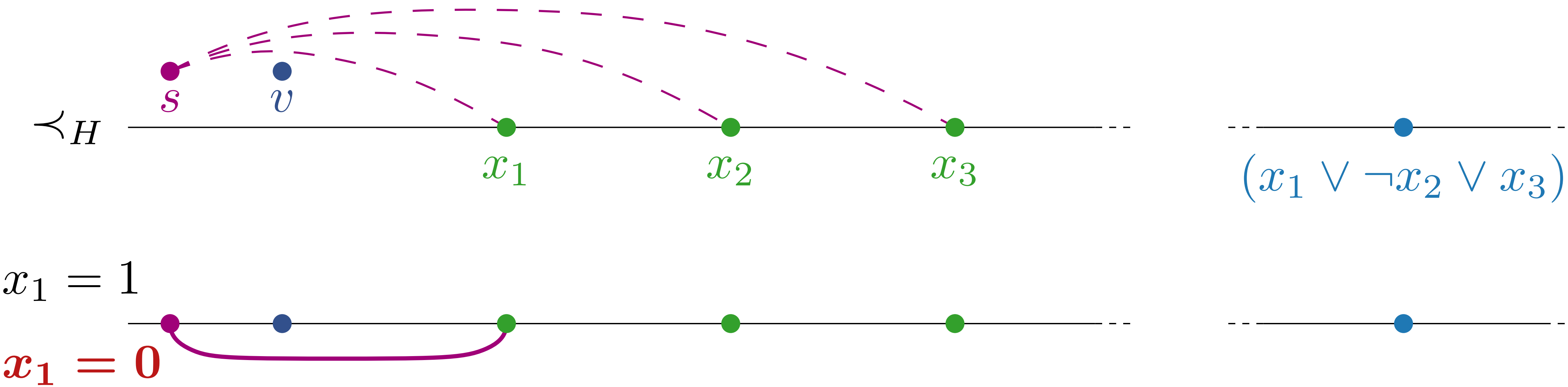

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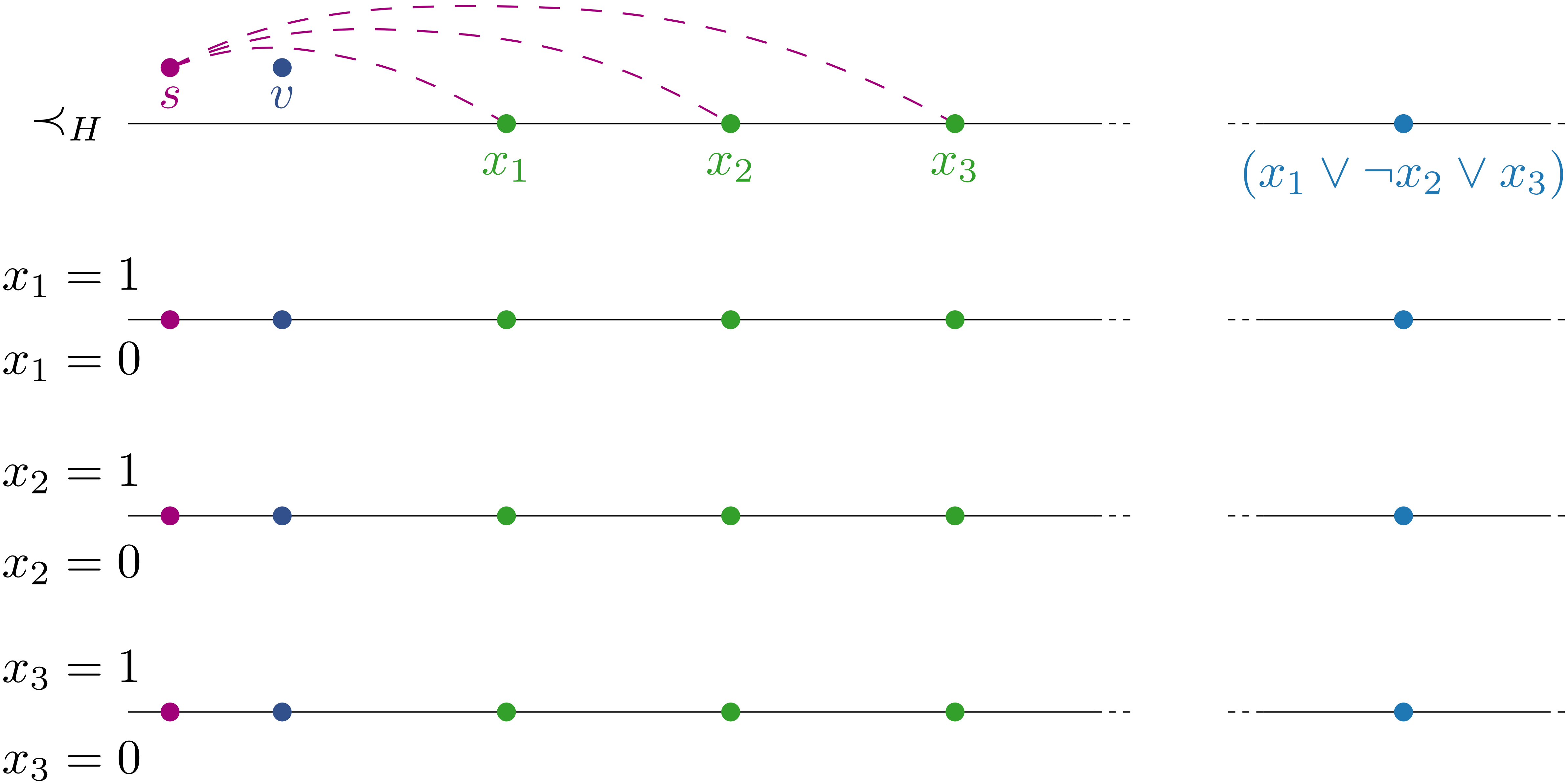

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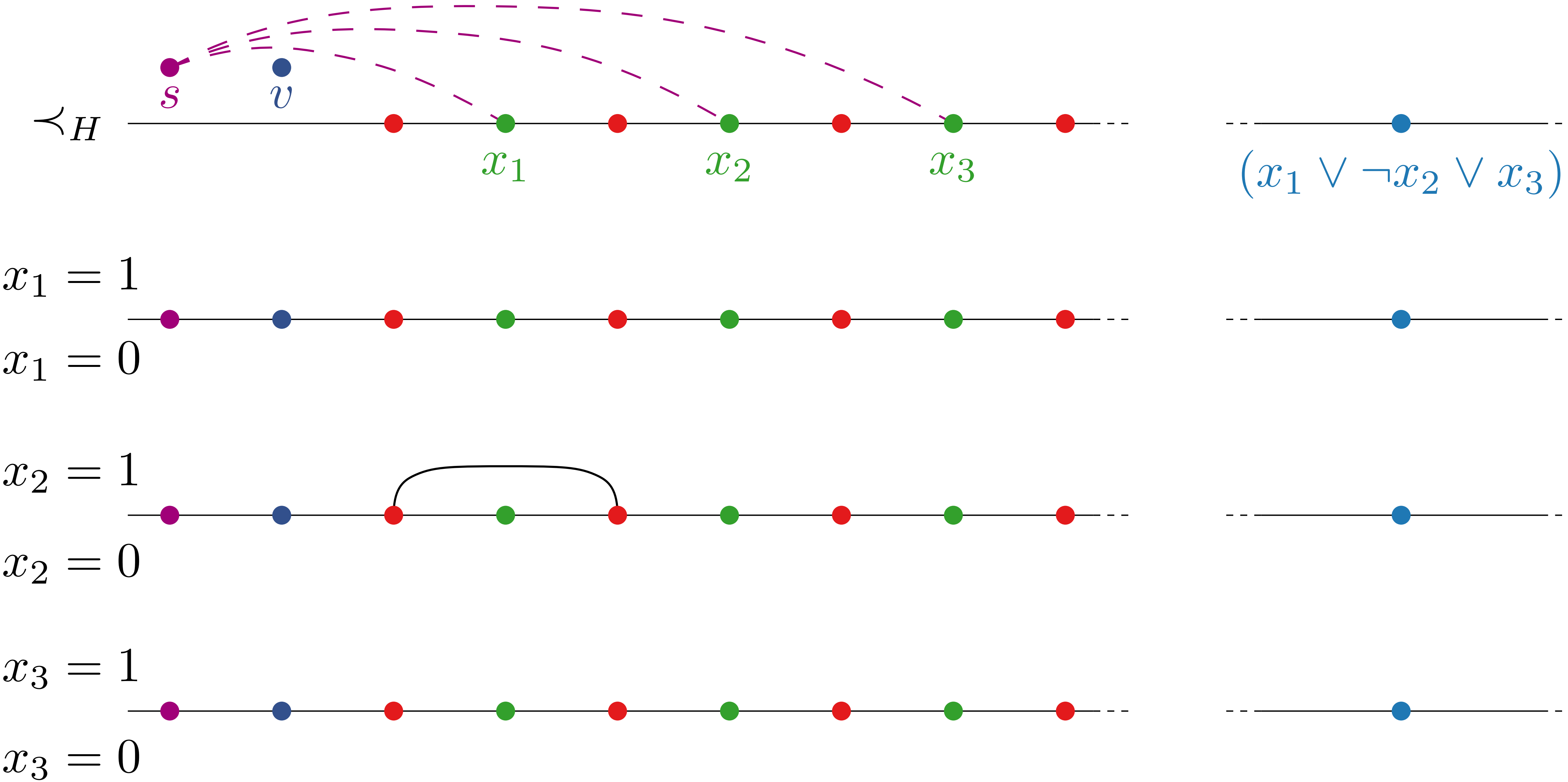

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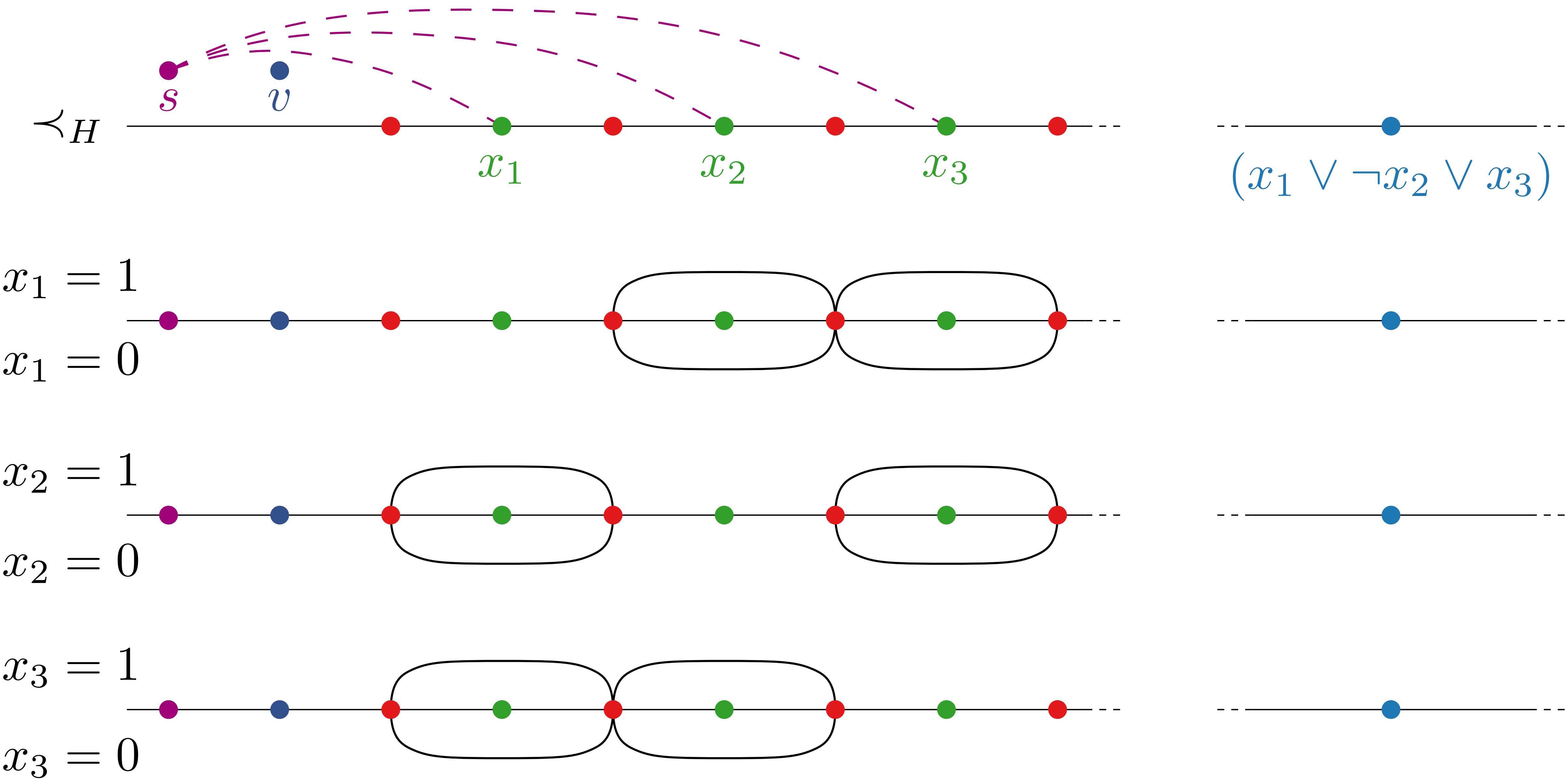

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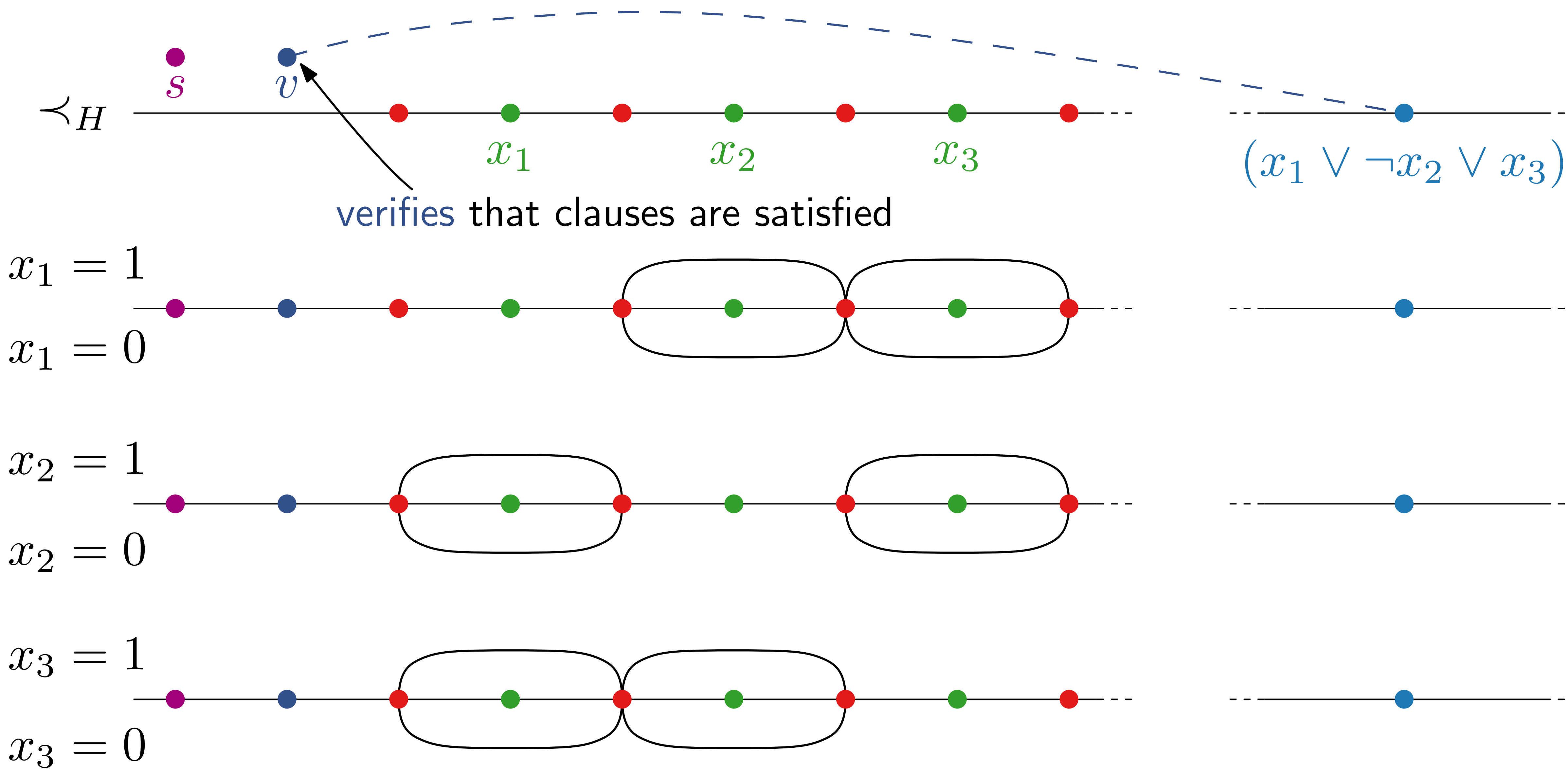

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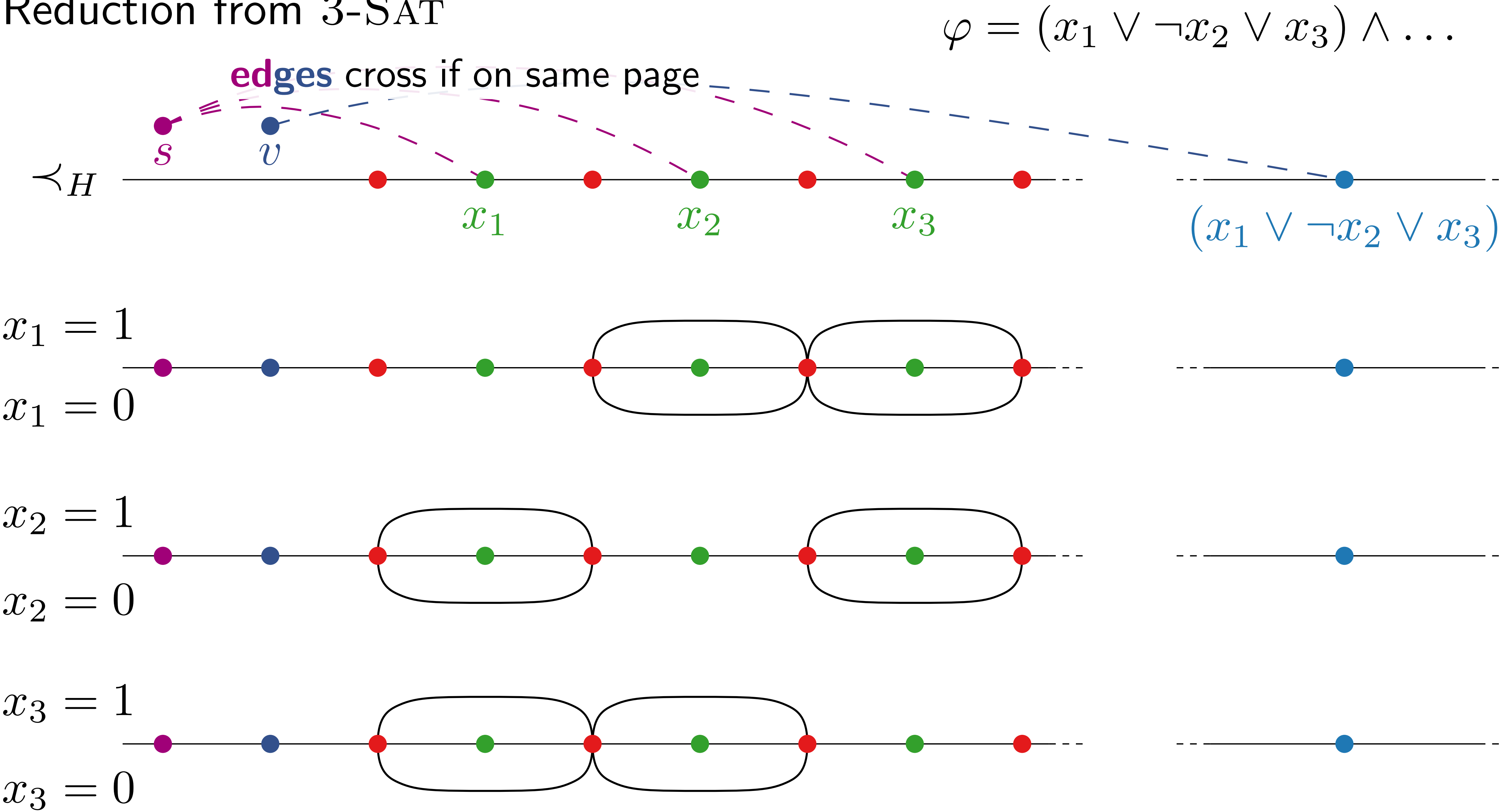

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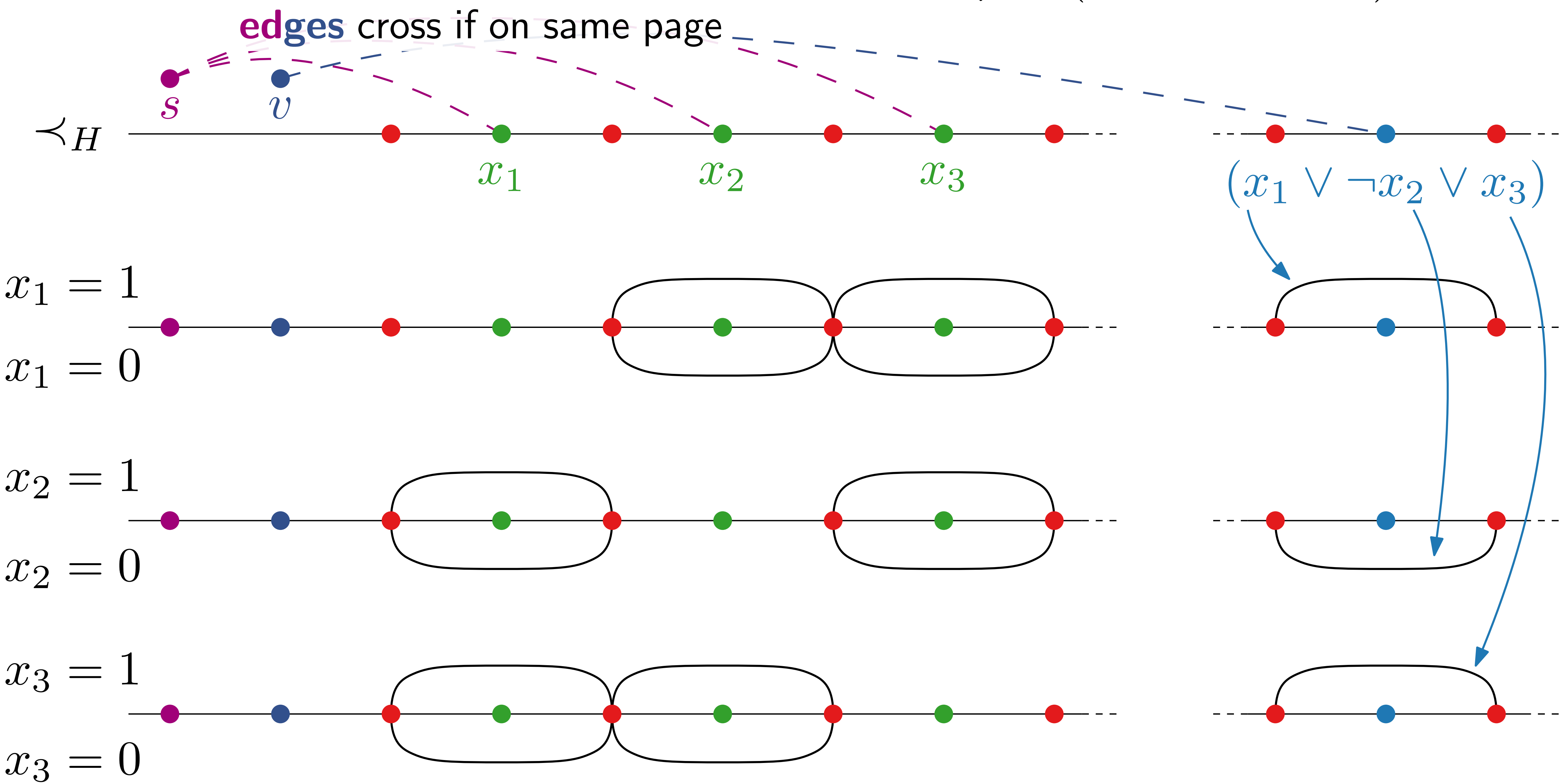

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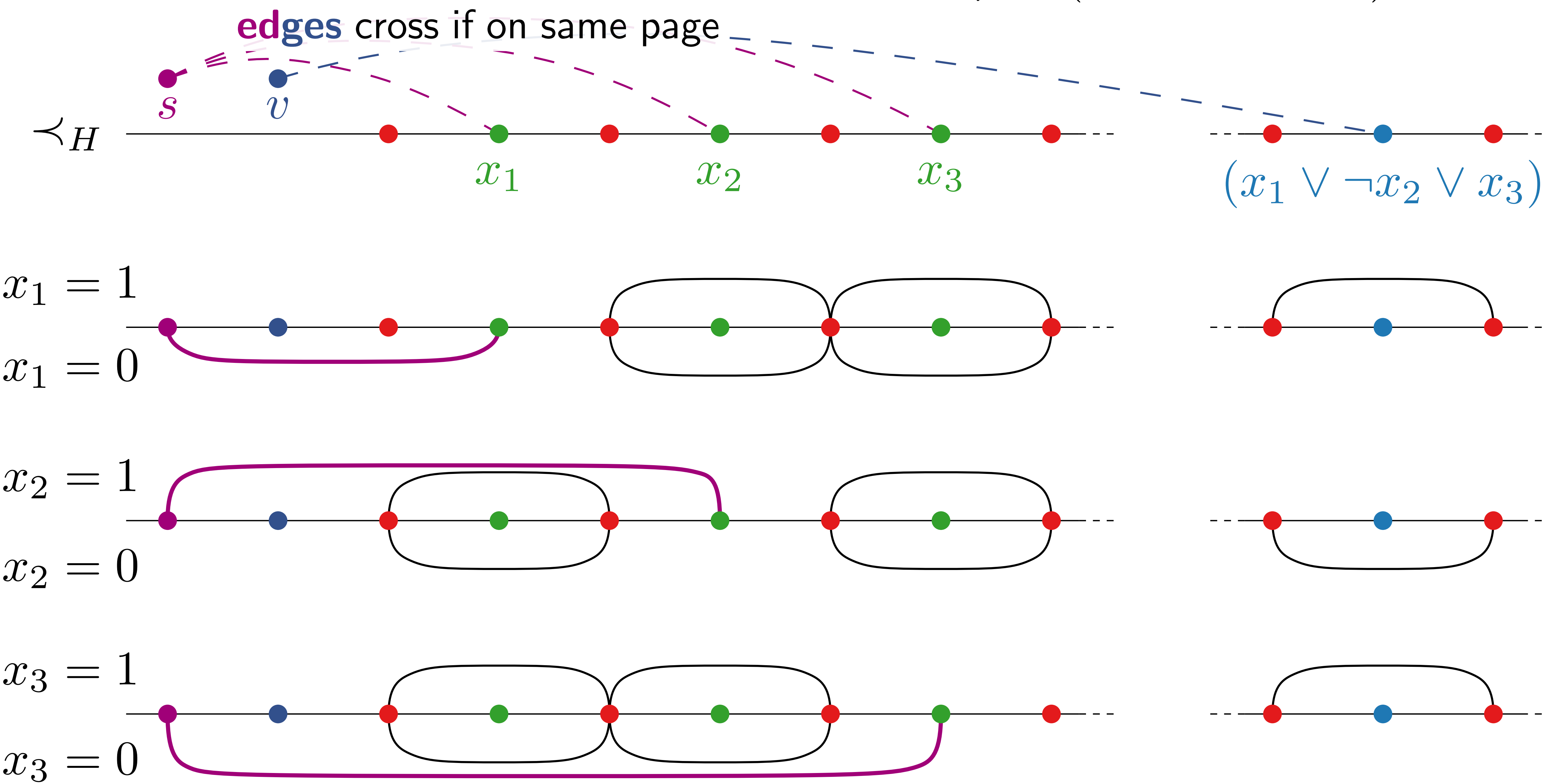

SLE With Two Missing Vertices is NP-complete

Reduction from 3-SAT

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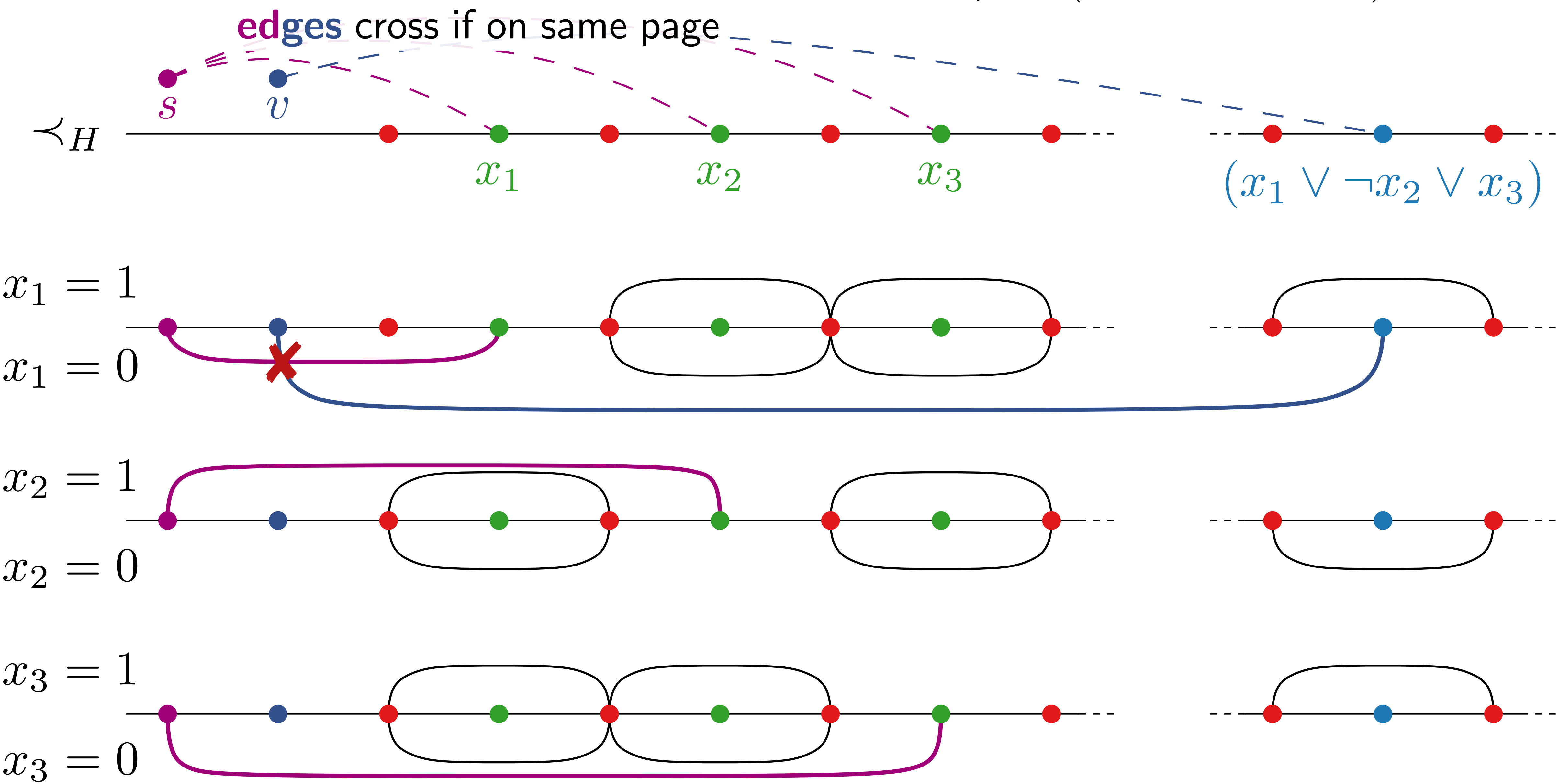

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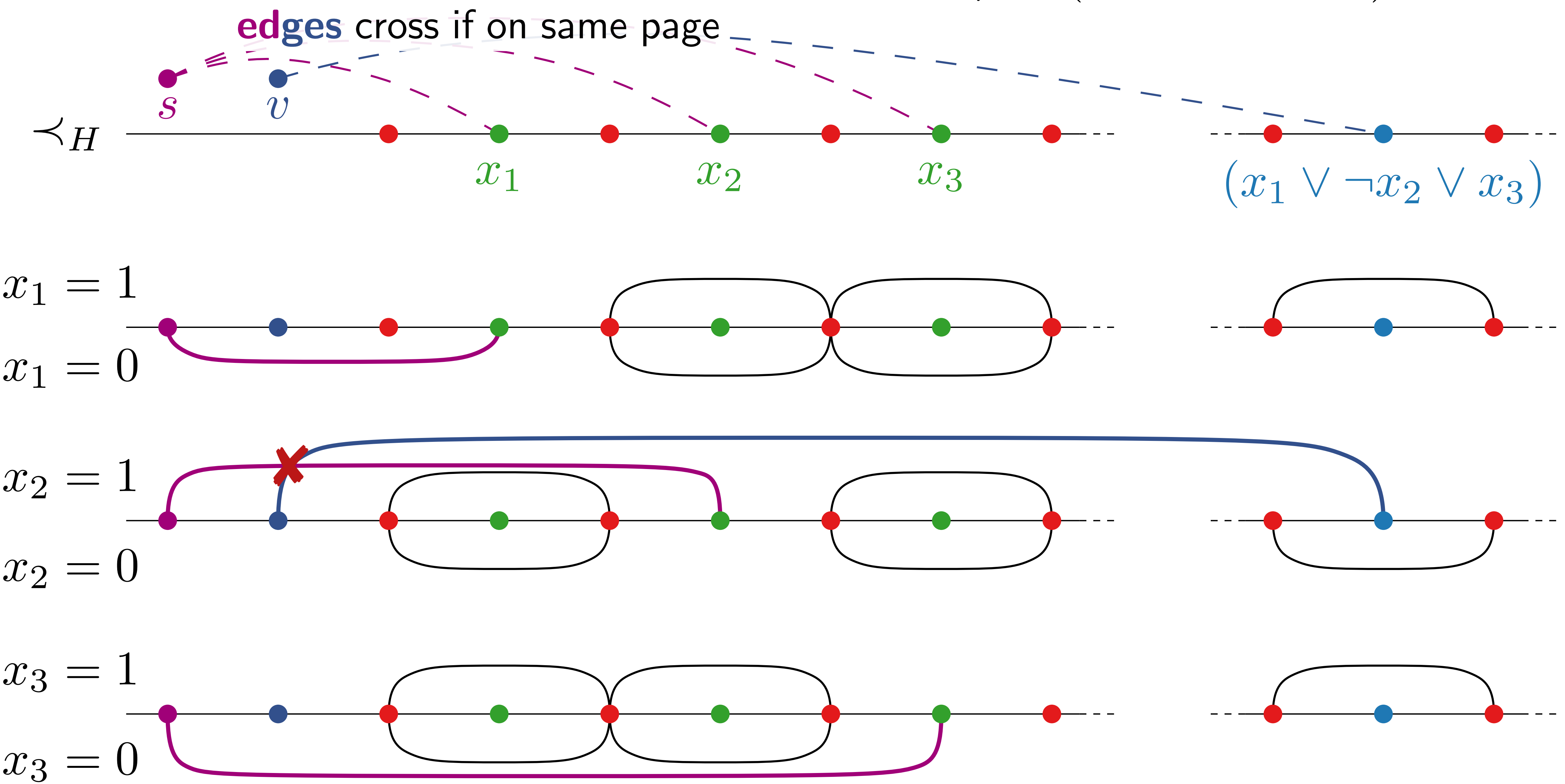

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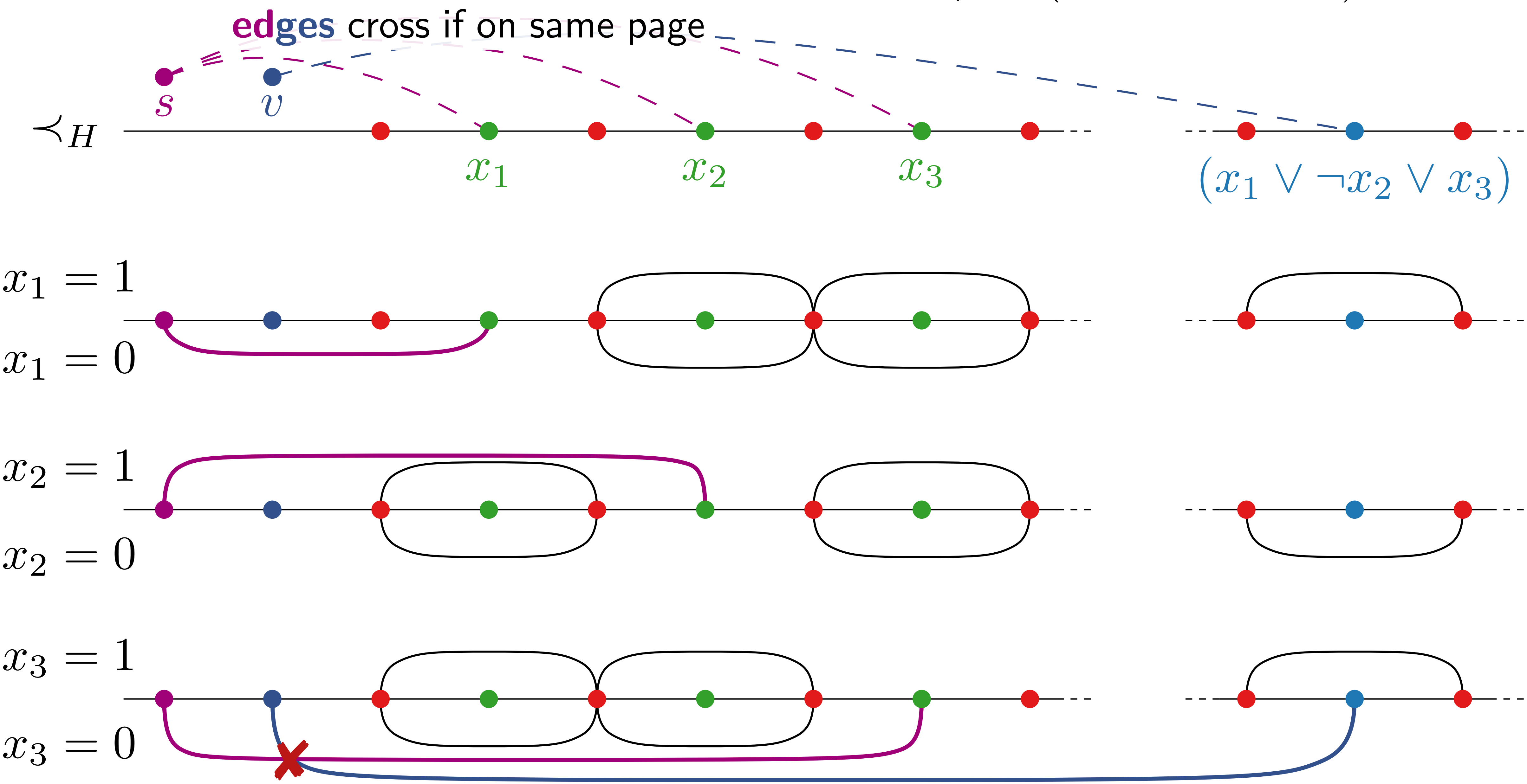

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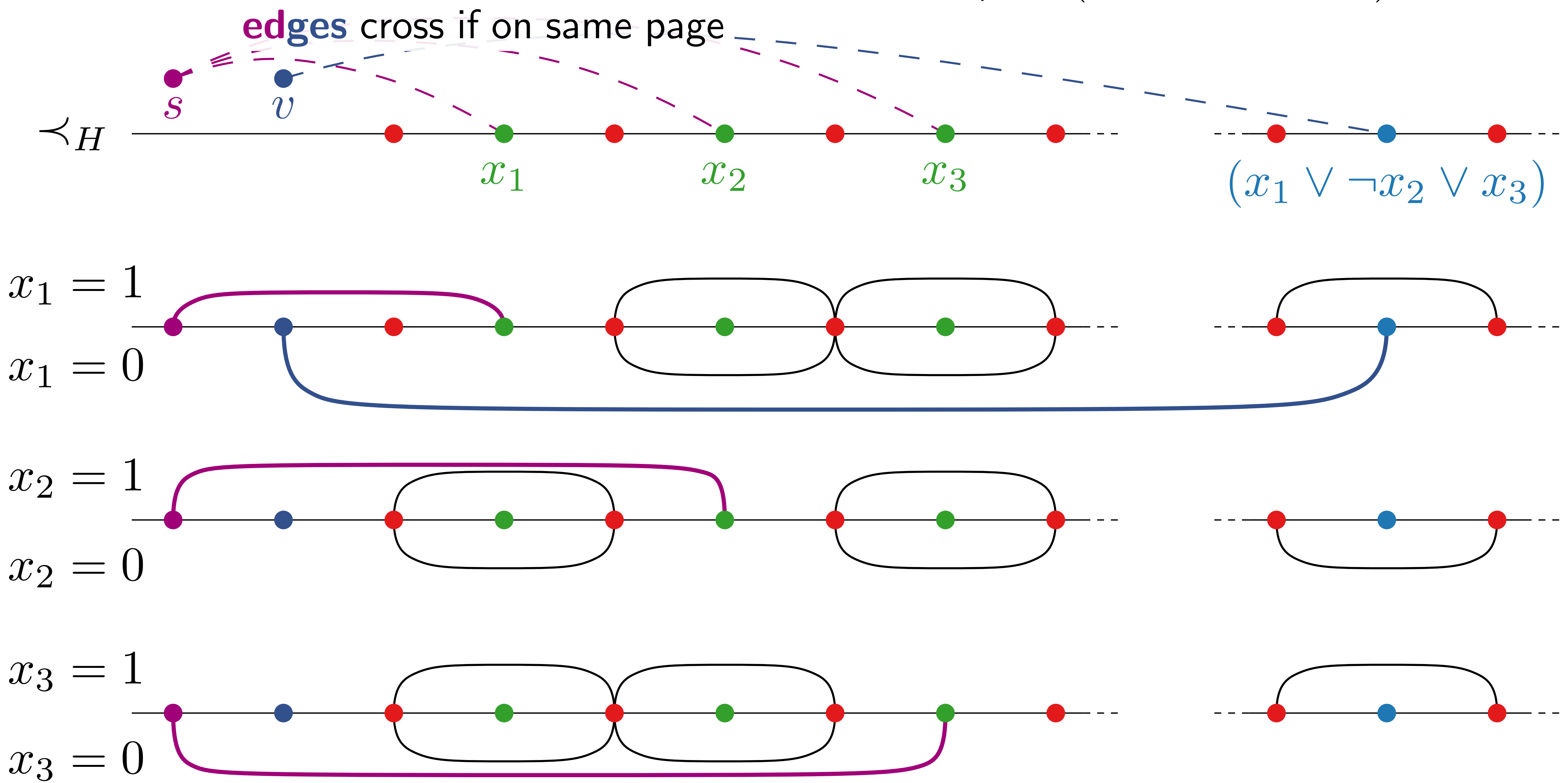

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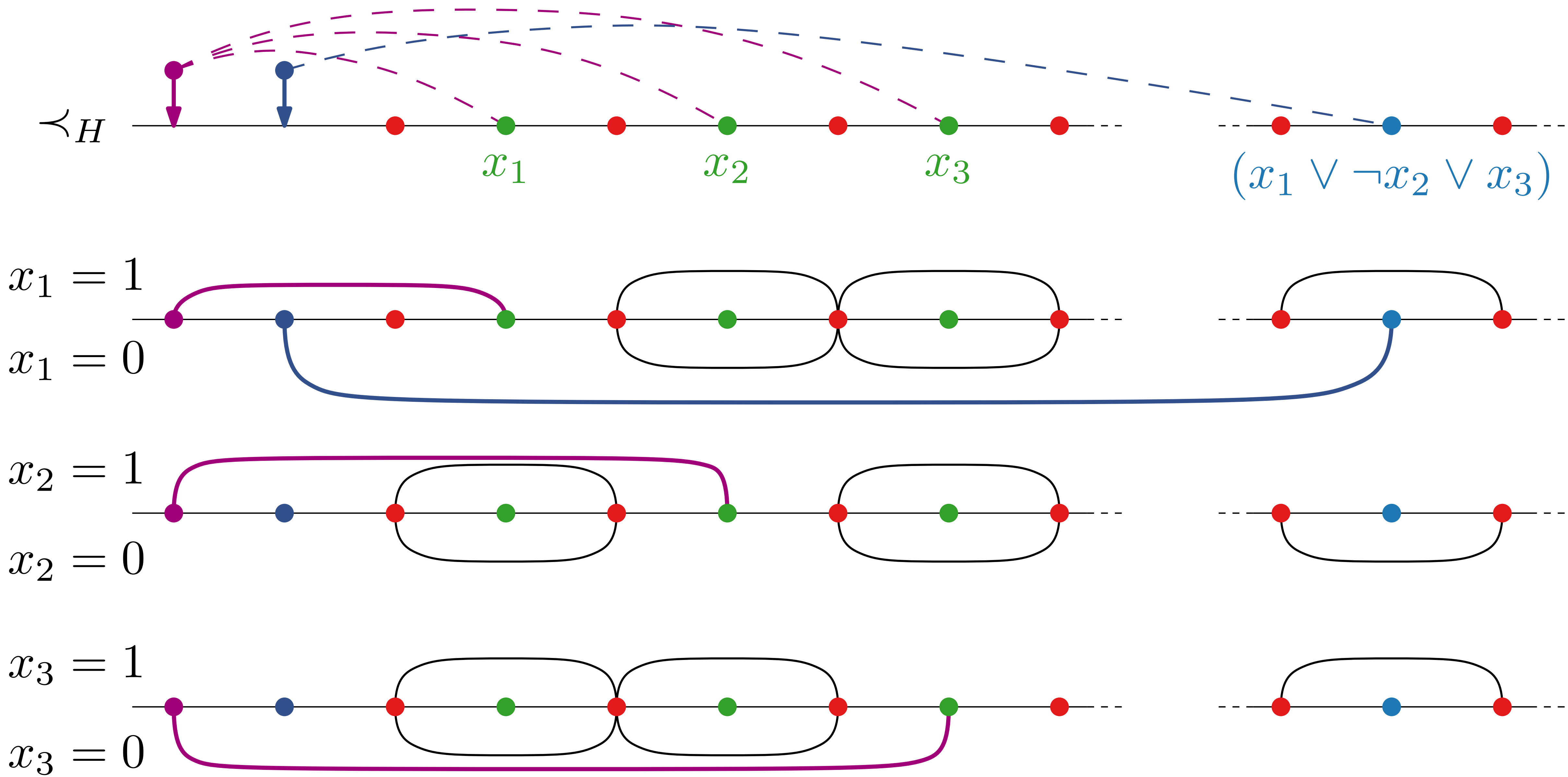

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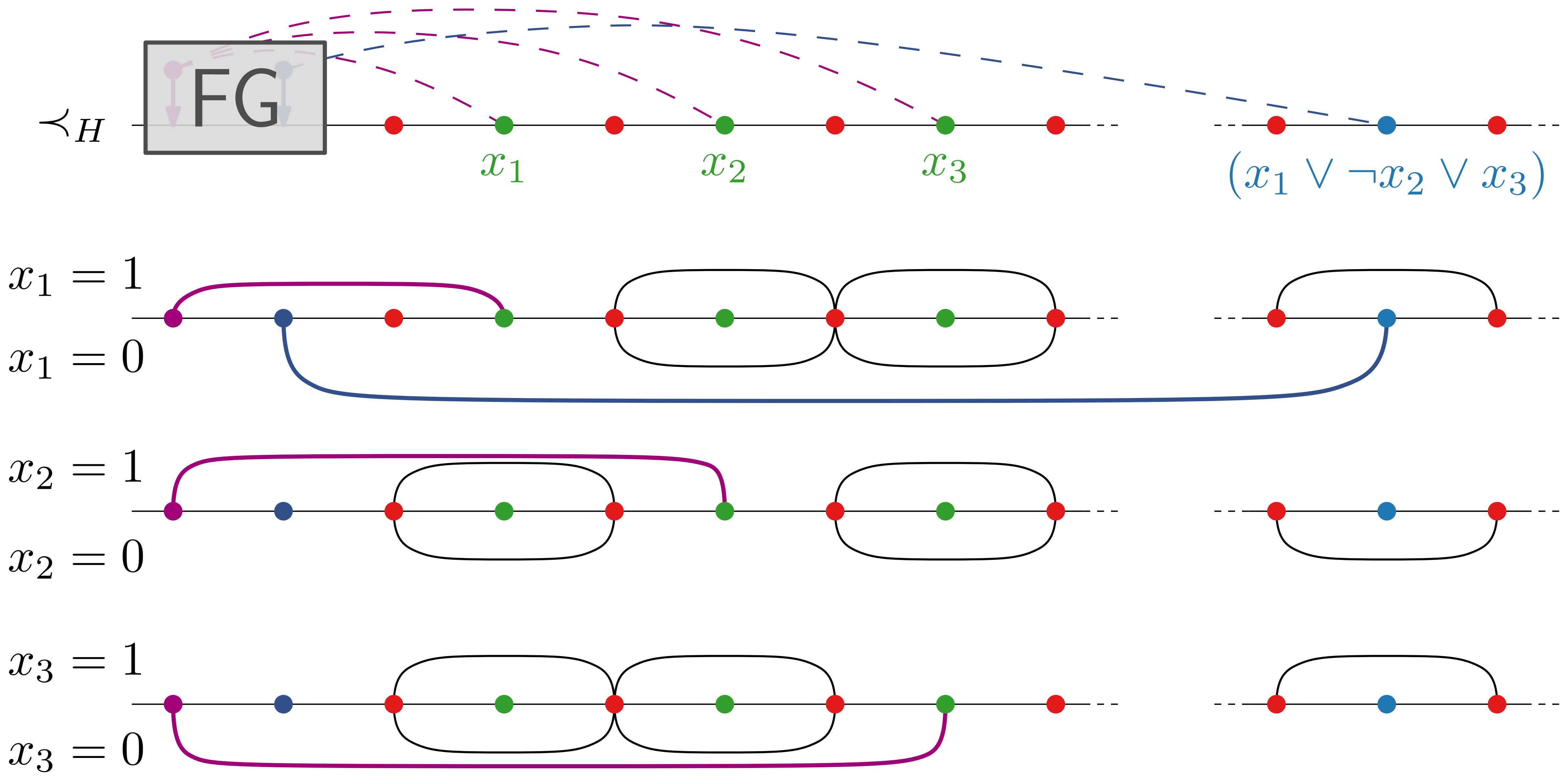
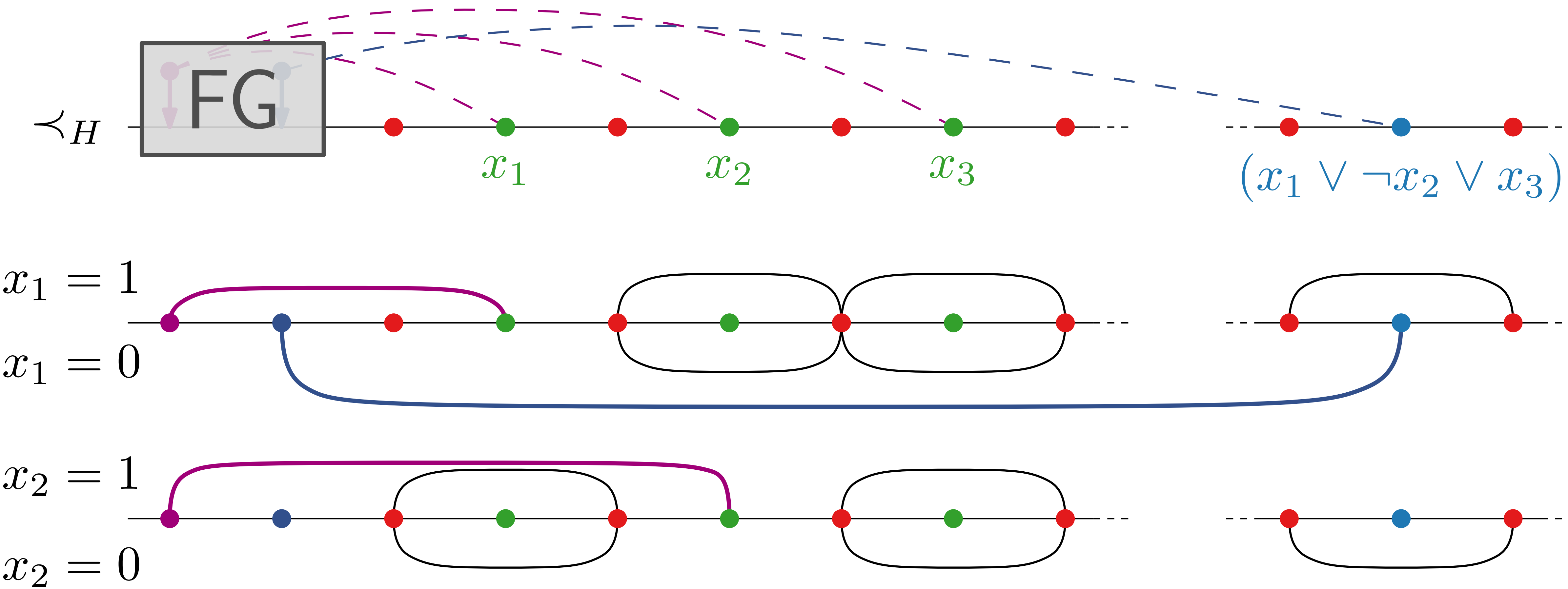


SLE With Two Missing Vertices is NP-complete

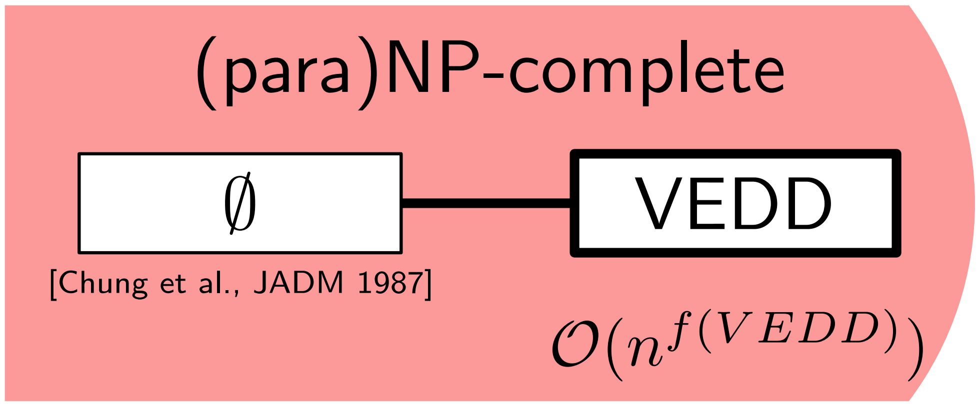
Reduction from 3-SAT

$$\varphi = (x_1 \vee \neg x_2 \vee x_3) \wedge \dots$$



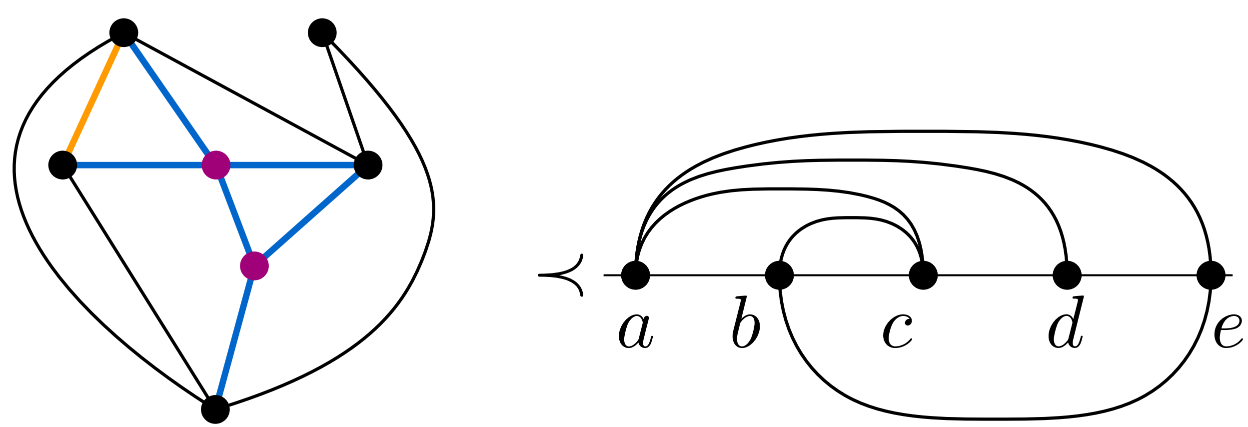
Theorem:
SLE is NP-complete, even for just **two** missing vertices to which all missing edges are incident to.

Our Results



VEDD: #vertices (inc. **incident edges**) & edges to delete from G to obtain H

VEDD $= 2 + 1 = 3$



Our Results

(para)NP-complete

∅ — VEDD

[Chung et al., JADM 1987]

~~$\mathcal{O}(n^{f(VEDD)})$~~

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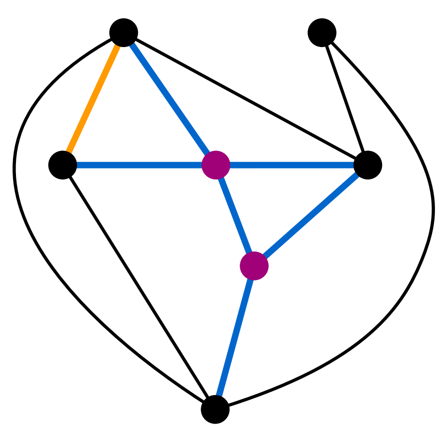
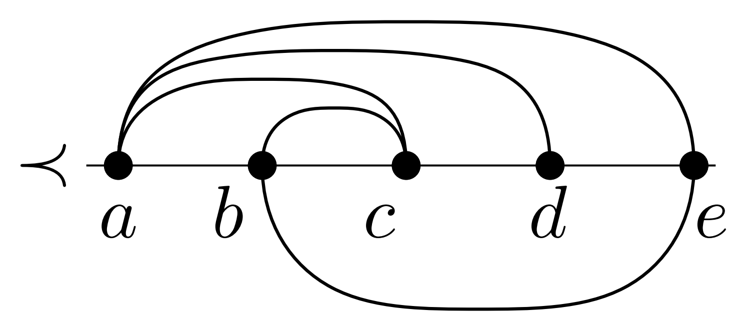
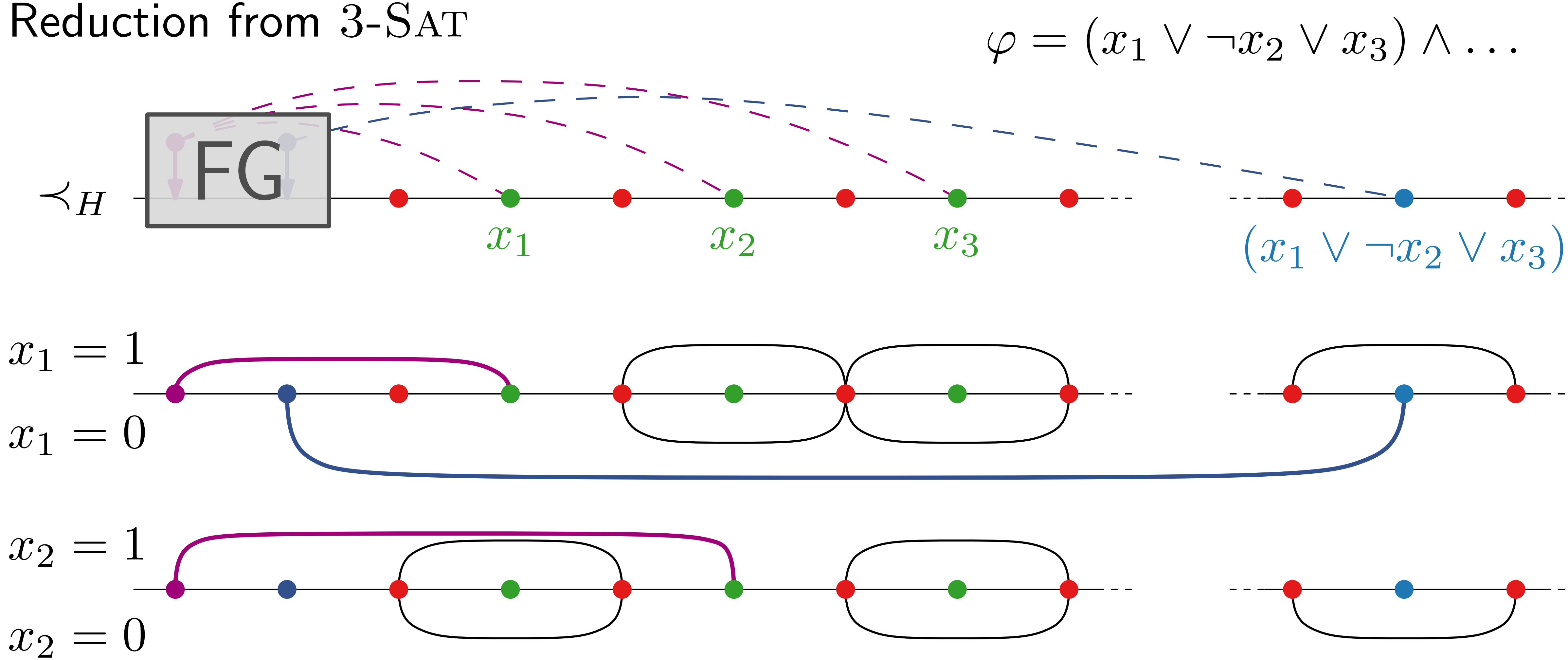




Rewind: SLE With Two Missing Vertices is NP-complete

Reduction from 3-SAT



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Rewind: SLE With Two Missing Vertices is NP-complete

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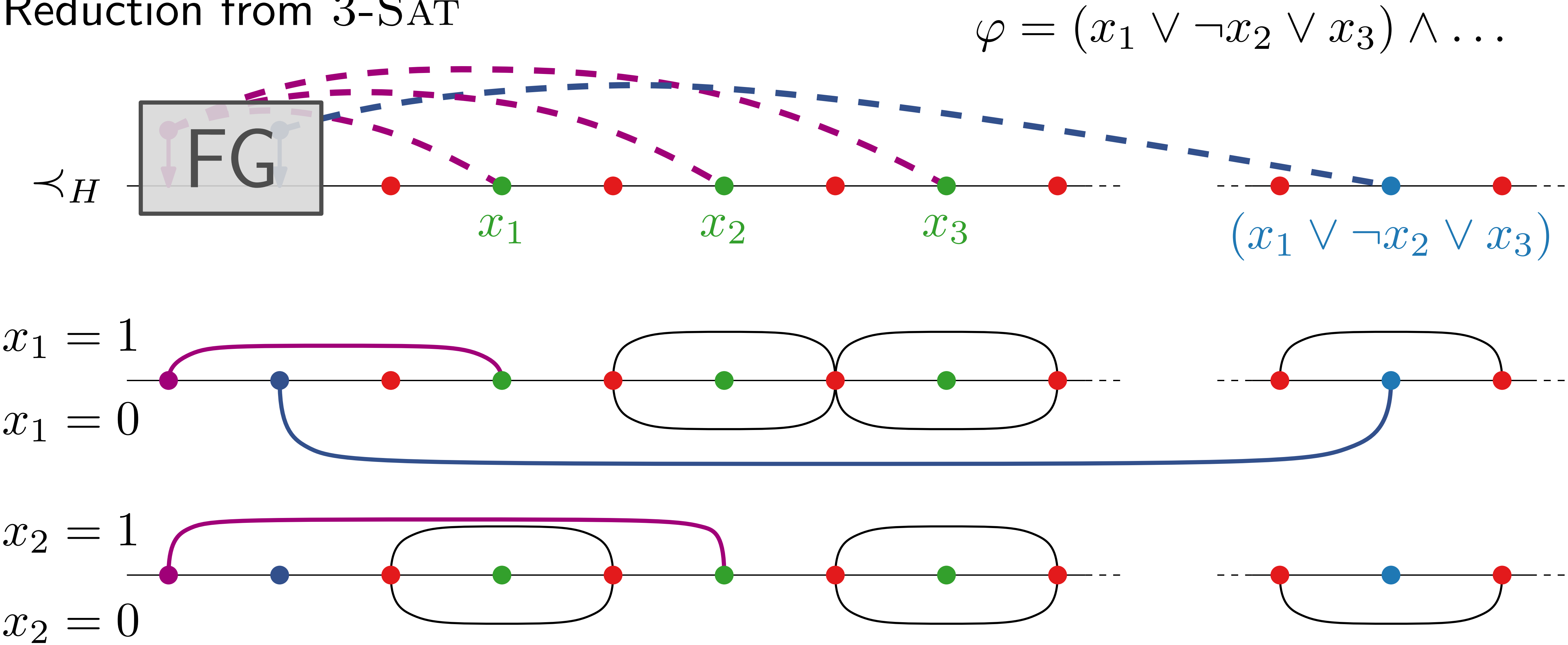

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Our Results

(para)NP-complete

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[Chung et al., JADM 1987]

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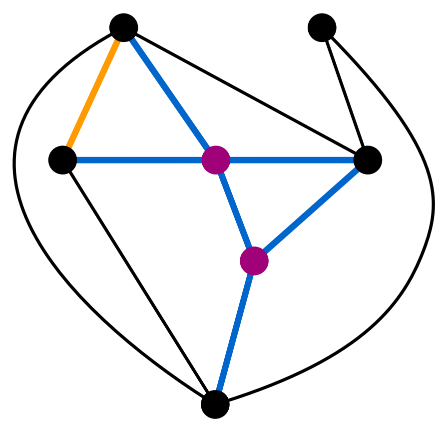
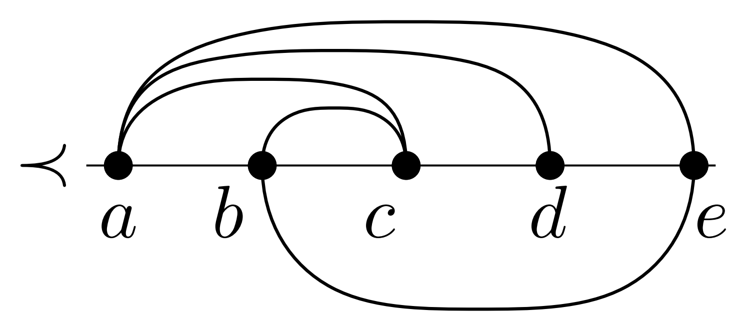
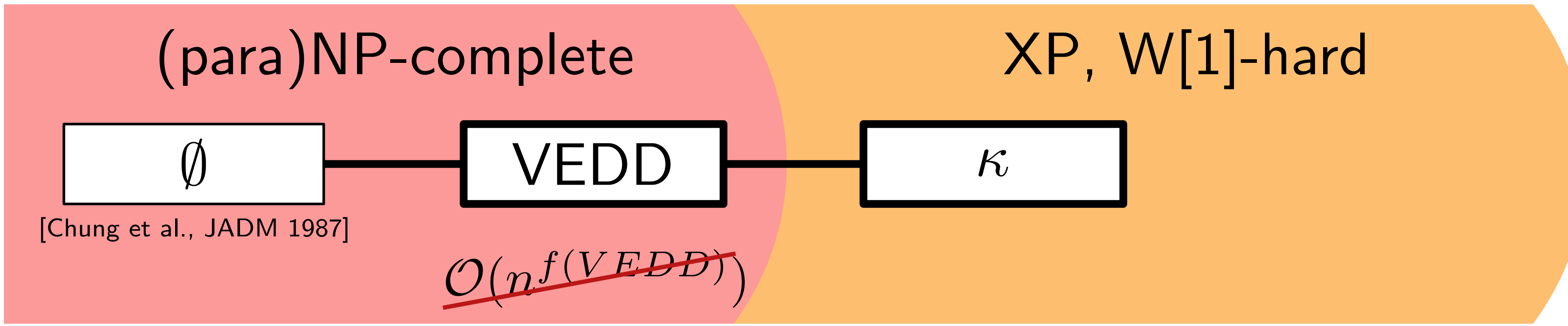




Our Results



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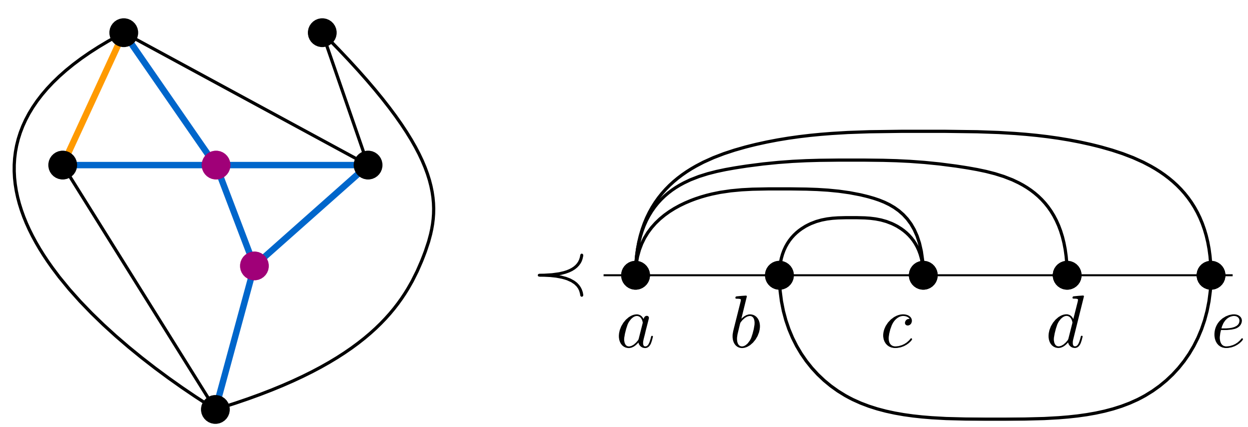

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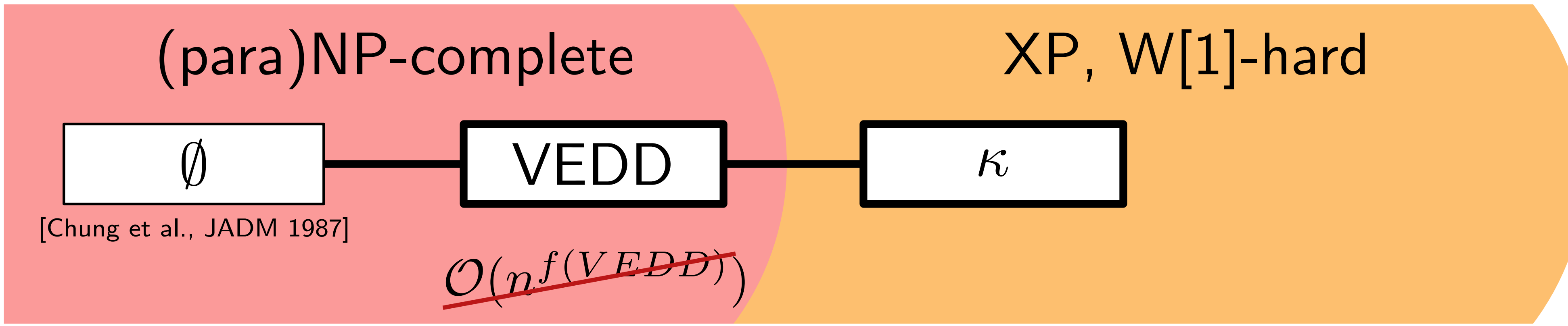

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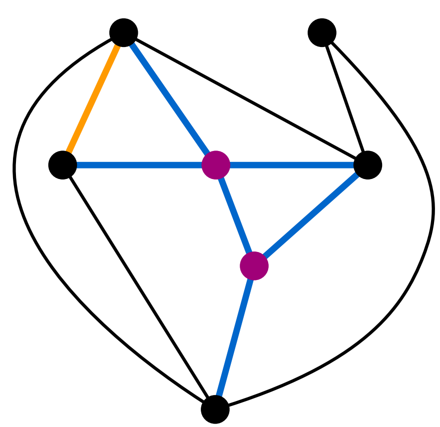

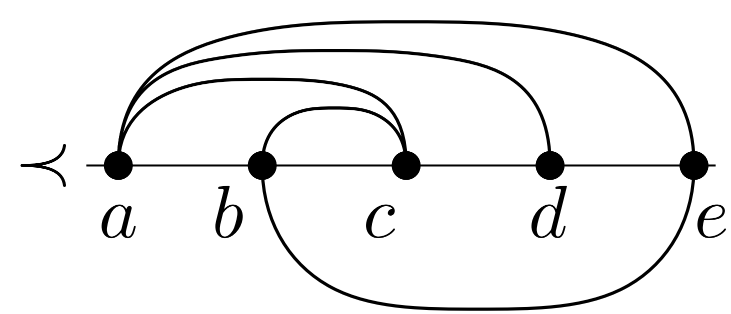

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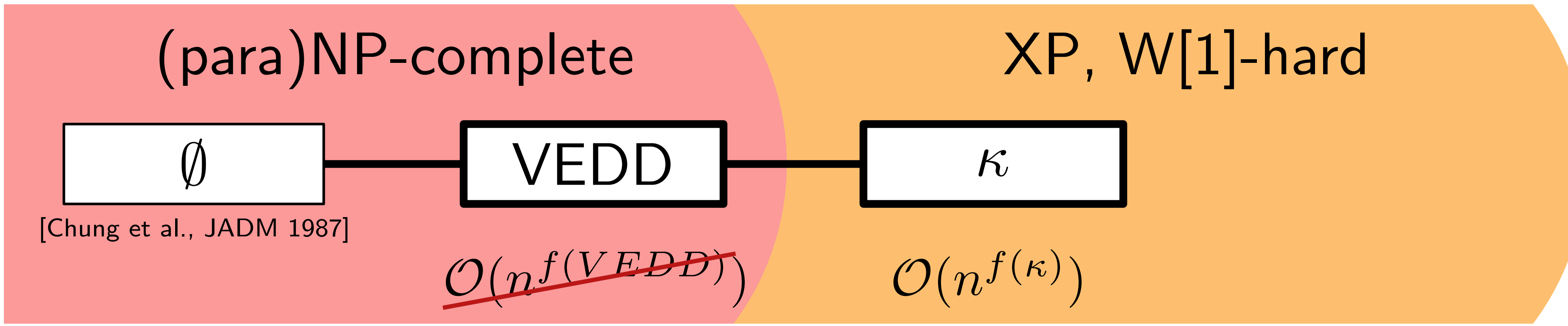

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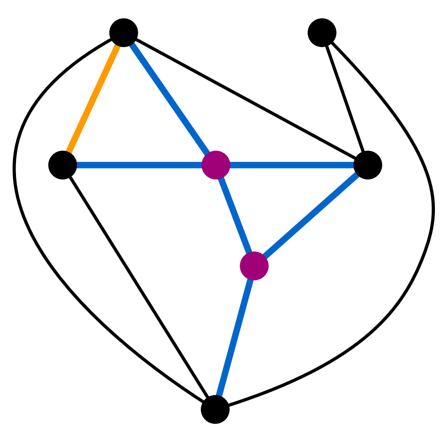
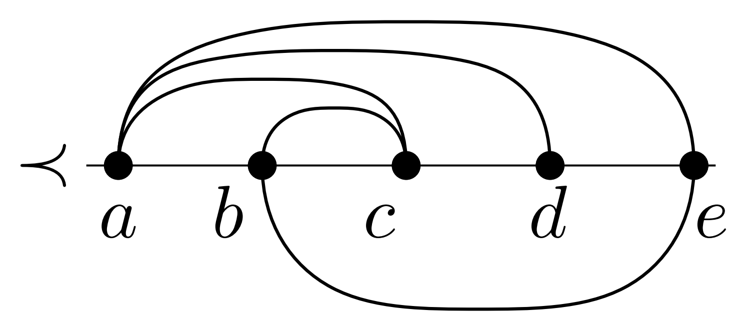
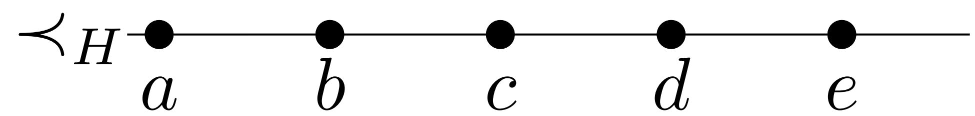




SLE Parameterized by κ is in XP

Step 1: Guess the extended spine order $\prec_G$



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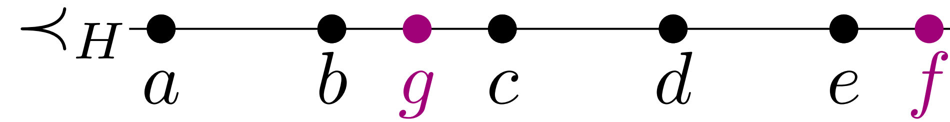

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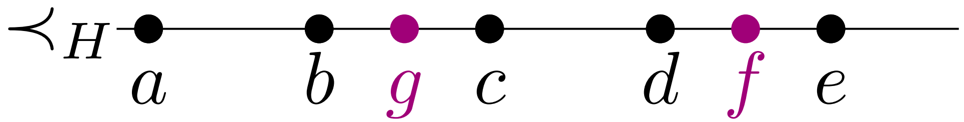

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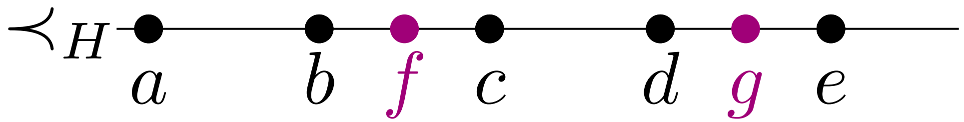

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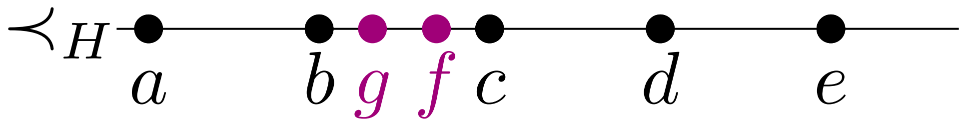

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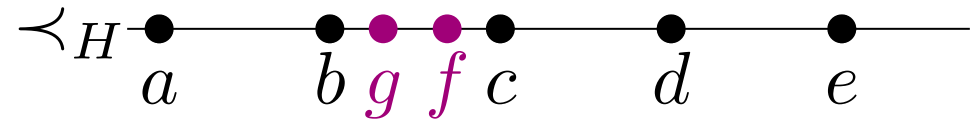

Step 2: Solve special instance of SLE where only edges are missing

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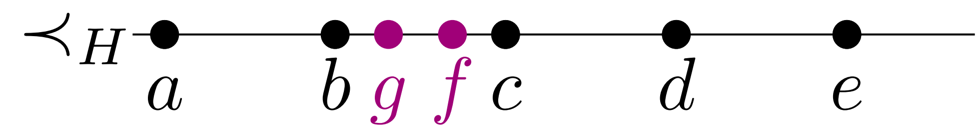

Step 2: Solve special instance of SLE where only edges are missing

$\mathcal{O}({m_{add}}^{m_{add}} \cdot |\mathcal{I}|)$ time

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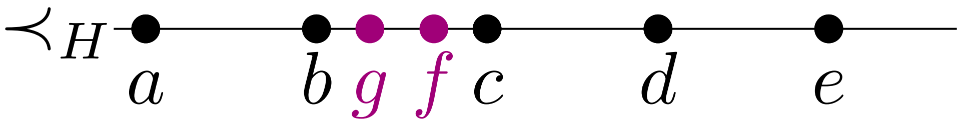

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SLE can be solved in $\mathcal{O}(|\mathcal{I}|^{n_{\text{add}}+1} \cdot {m_{\text{add}}}^{m_{\text{add}}})$ time.

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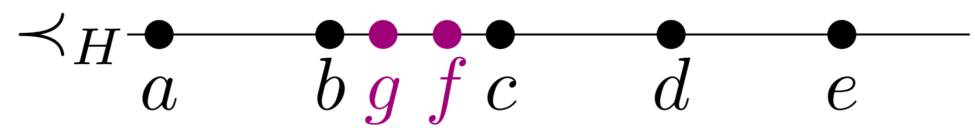

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Size of instance

#missing vertices

#missing edges

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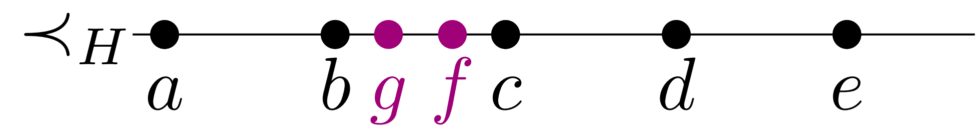

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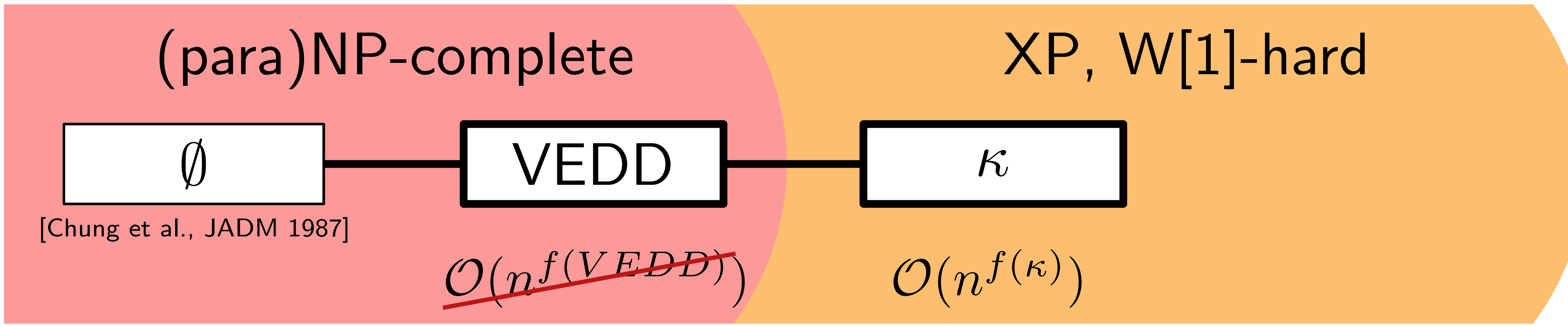
Our Results



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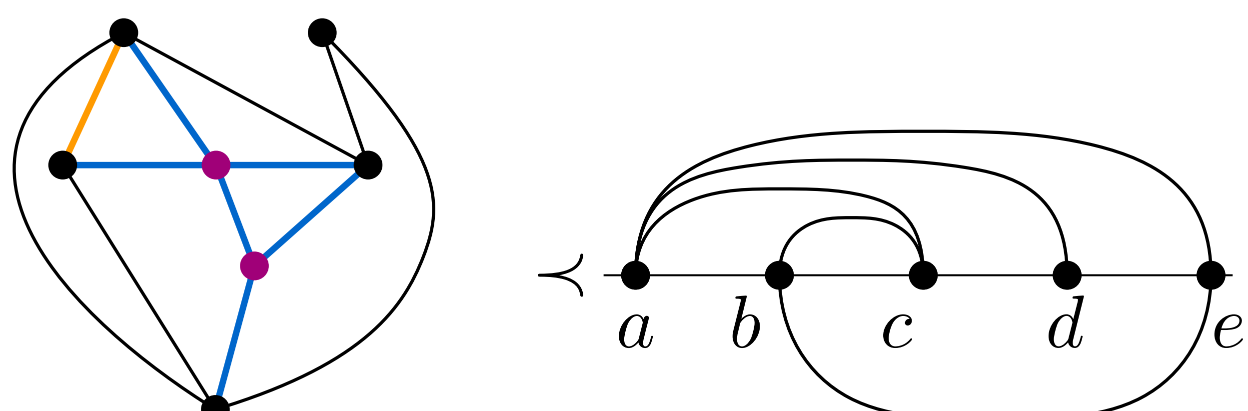
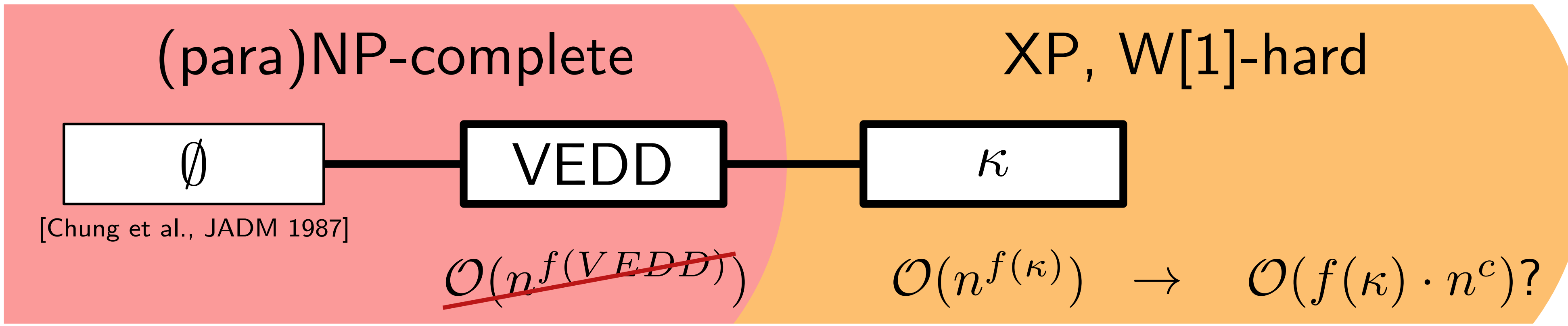


Our Results



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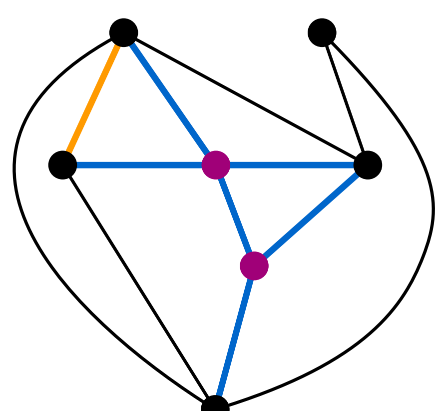
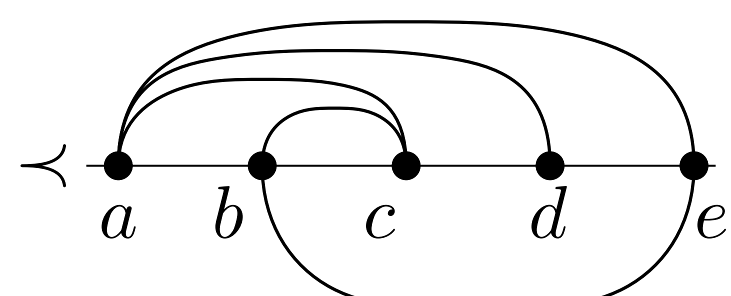
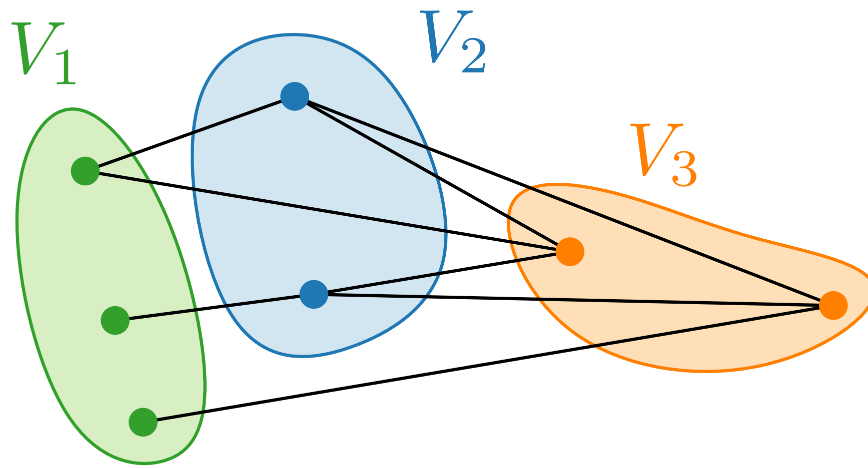




SLE Parameterized by κ is W[1]-hard

Reduction from Multi-Colored Clique (McC)



Parameter k = size of clique

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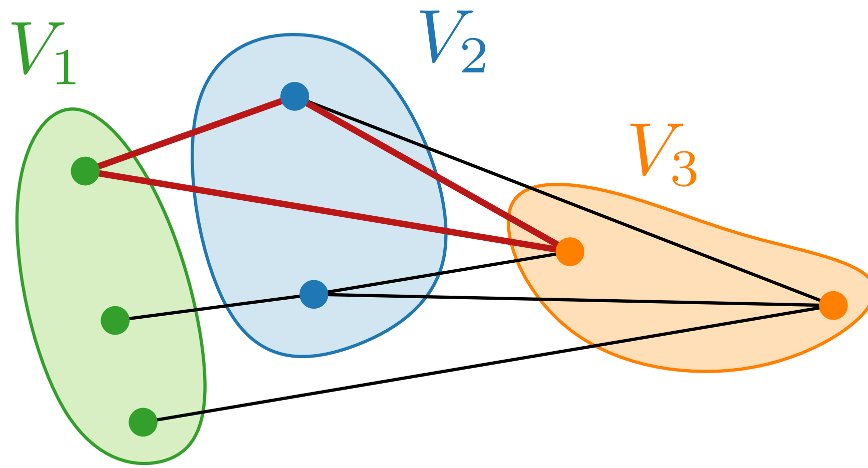

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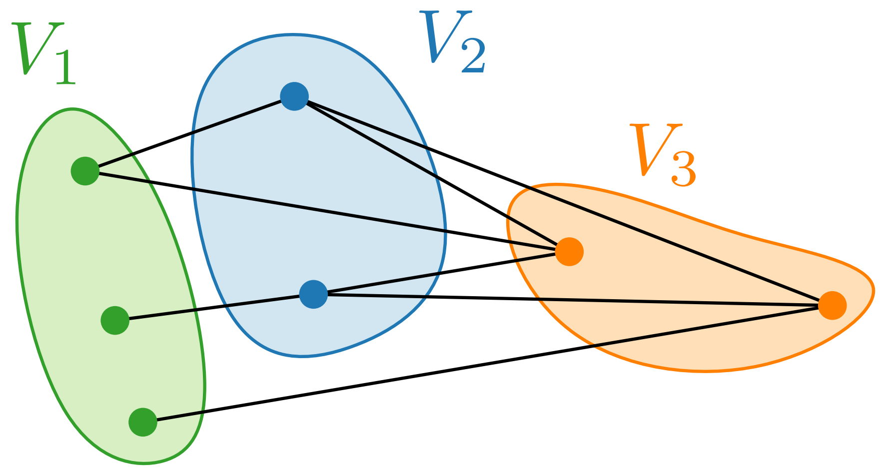

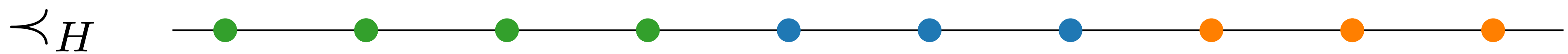

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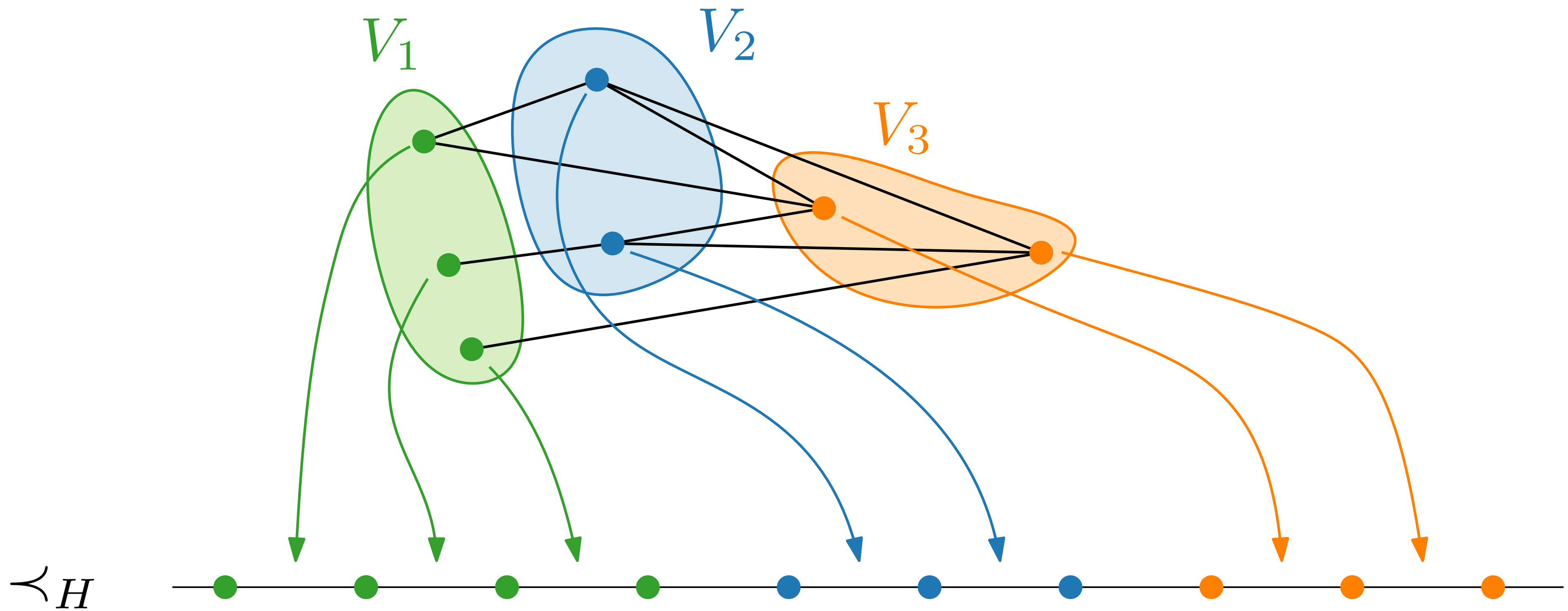

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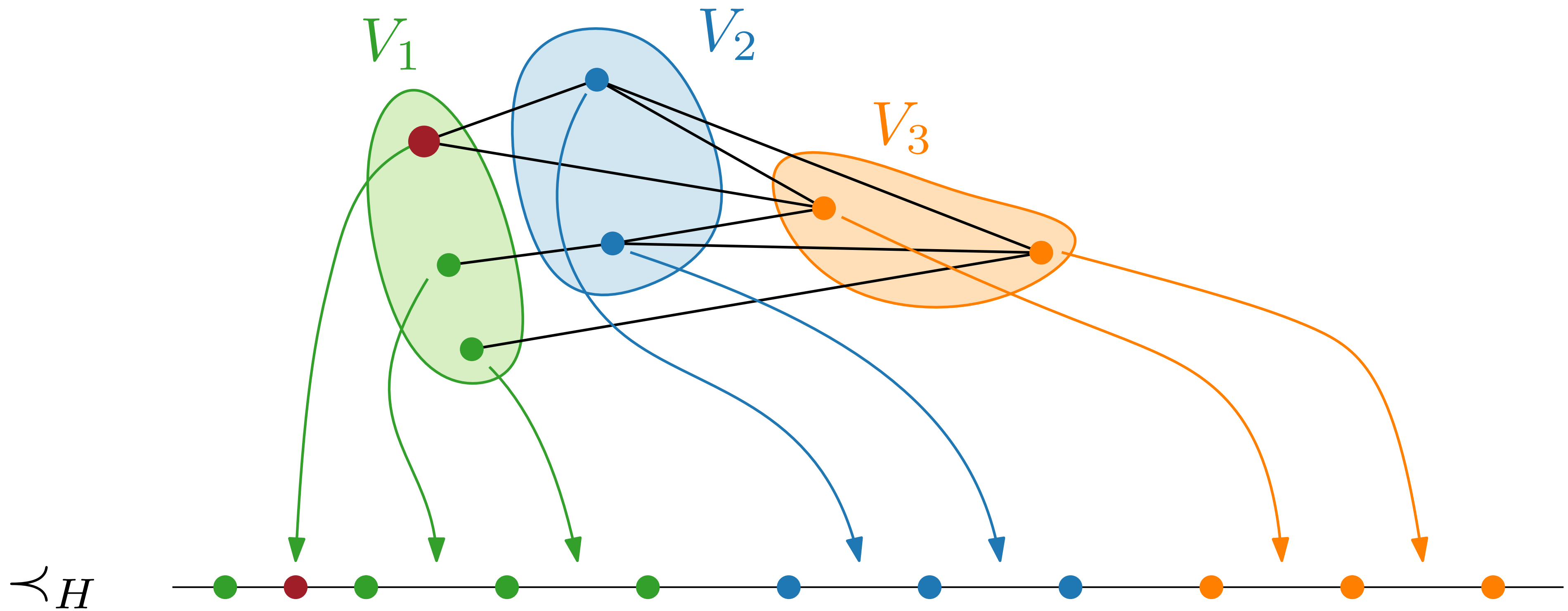

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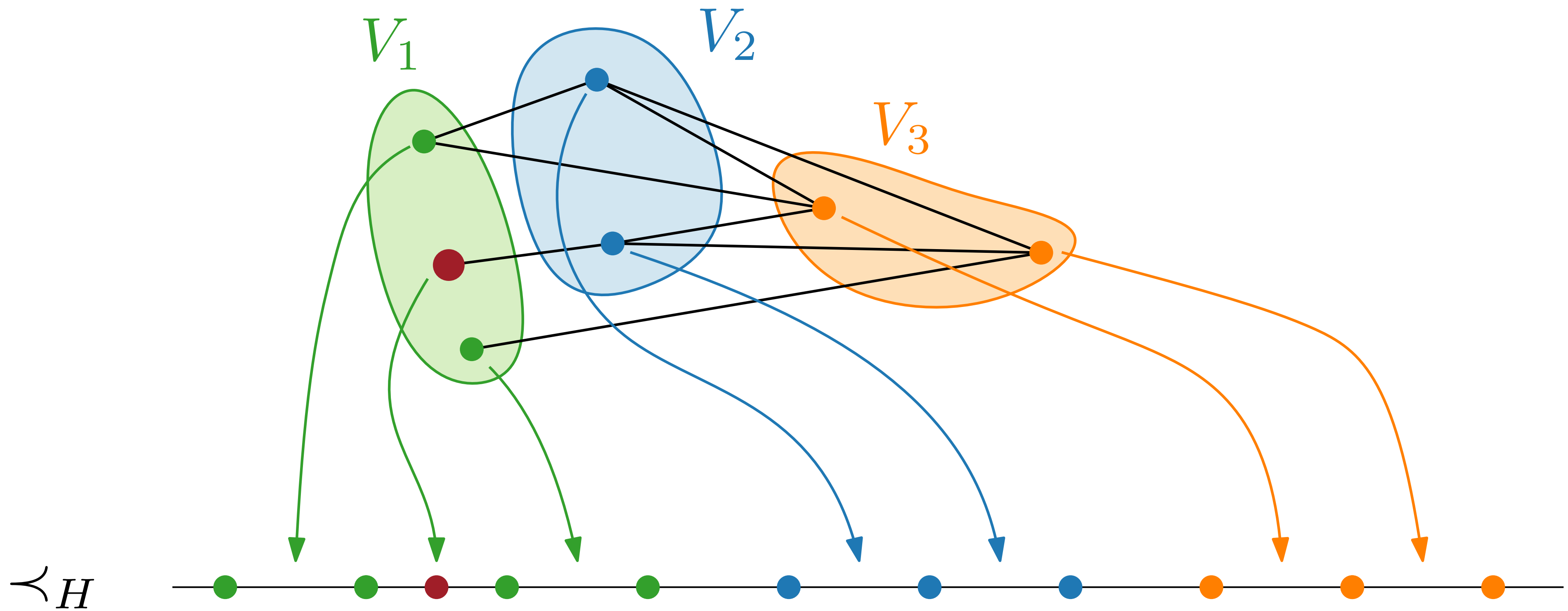

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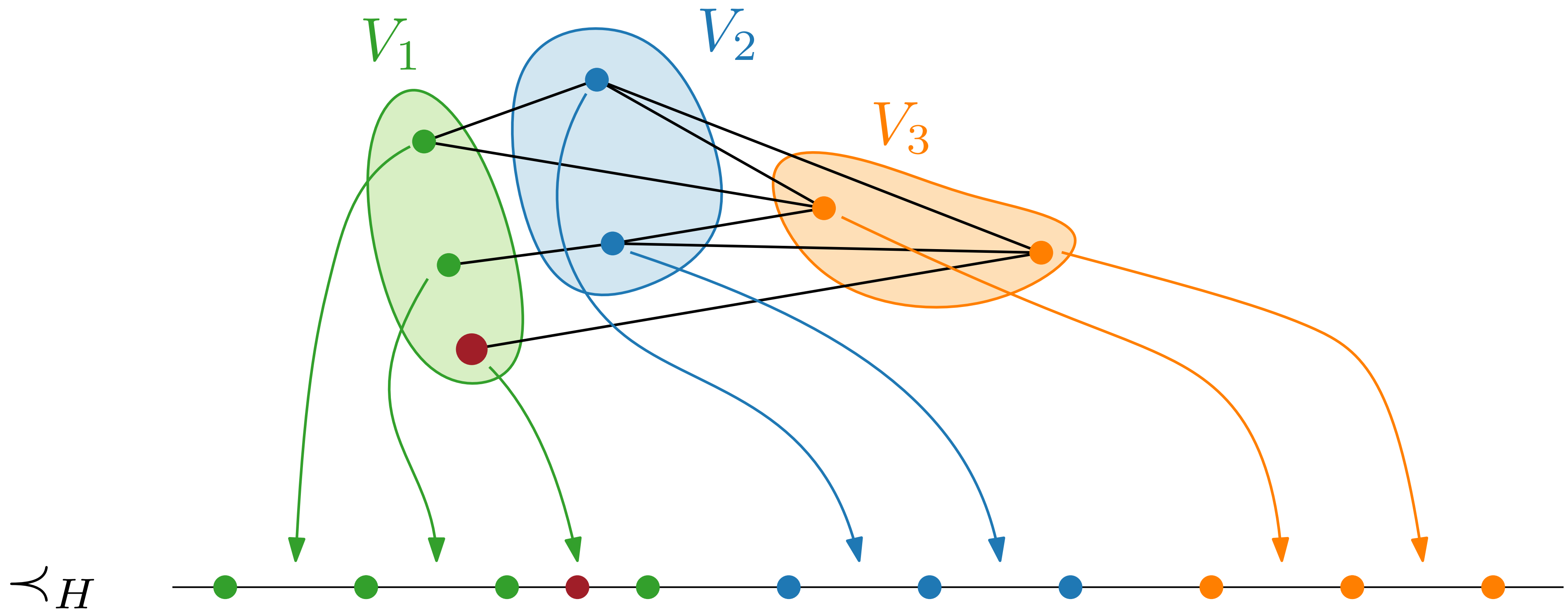
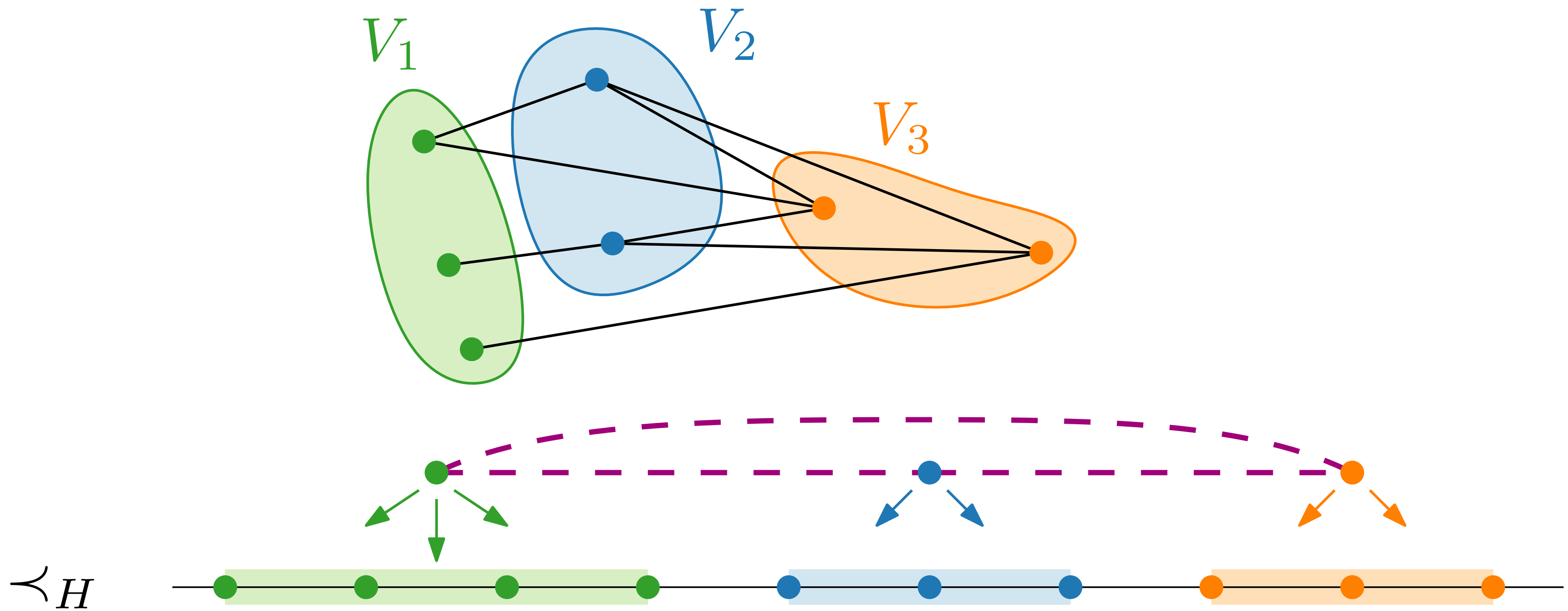


SLE Parameterized by κ is W[1]-hard

Reduction from MULTI-COLORED CLIQUE (McC)



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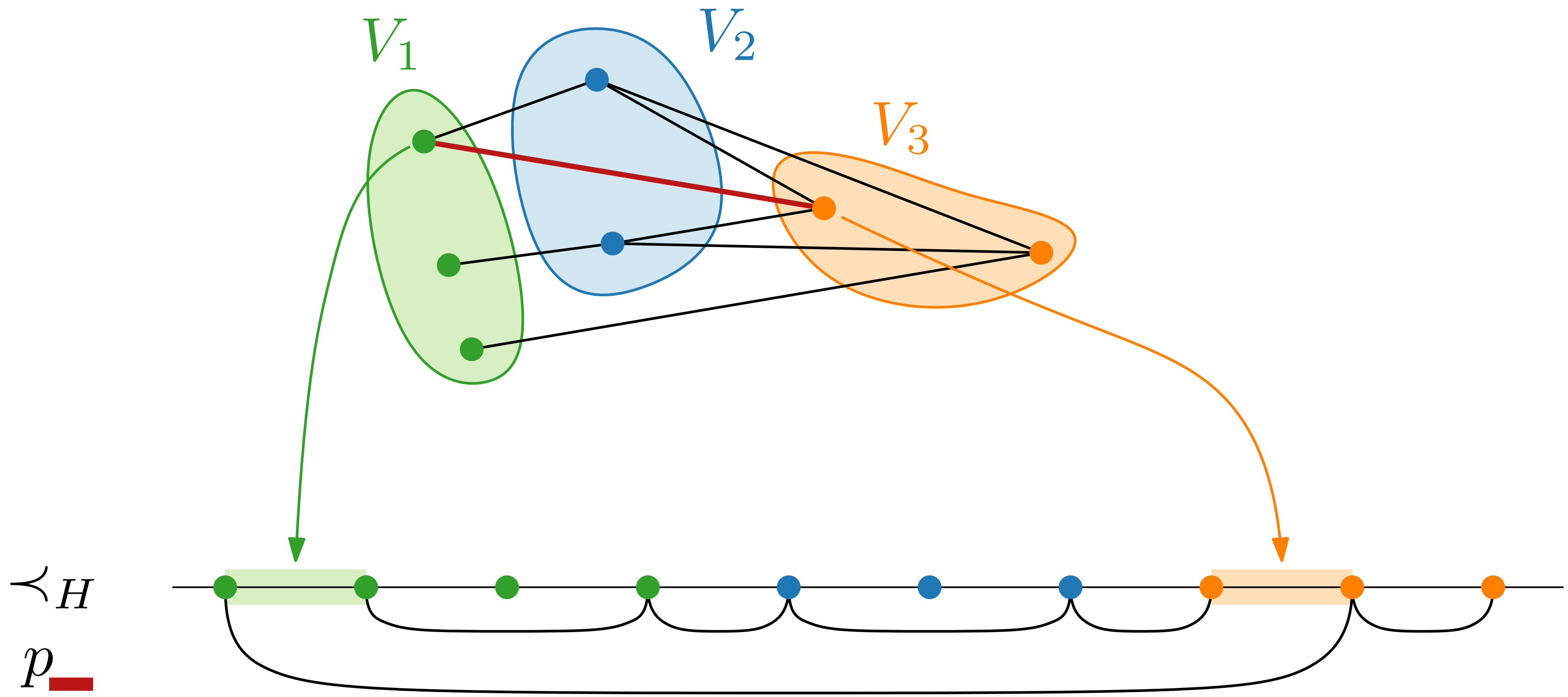
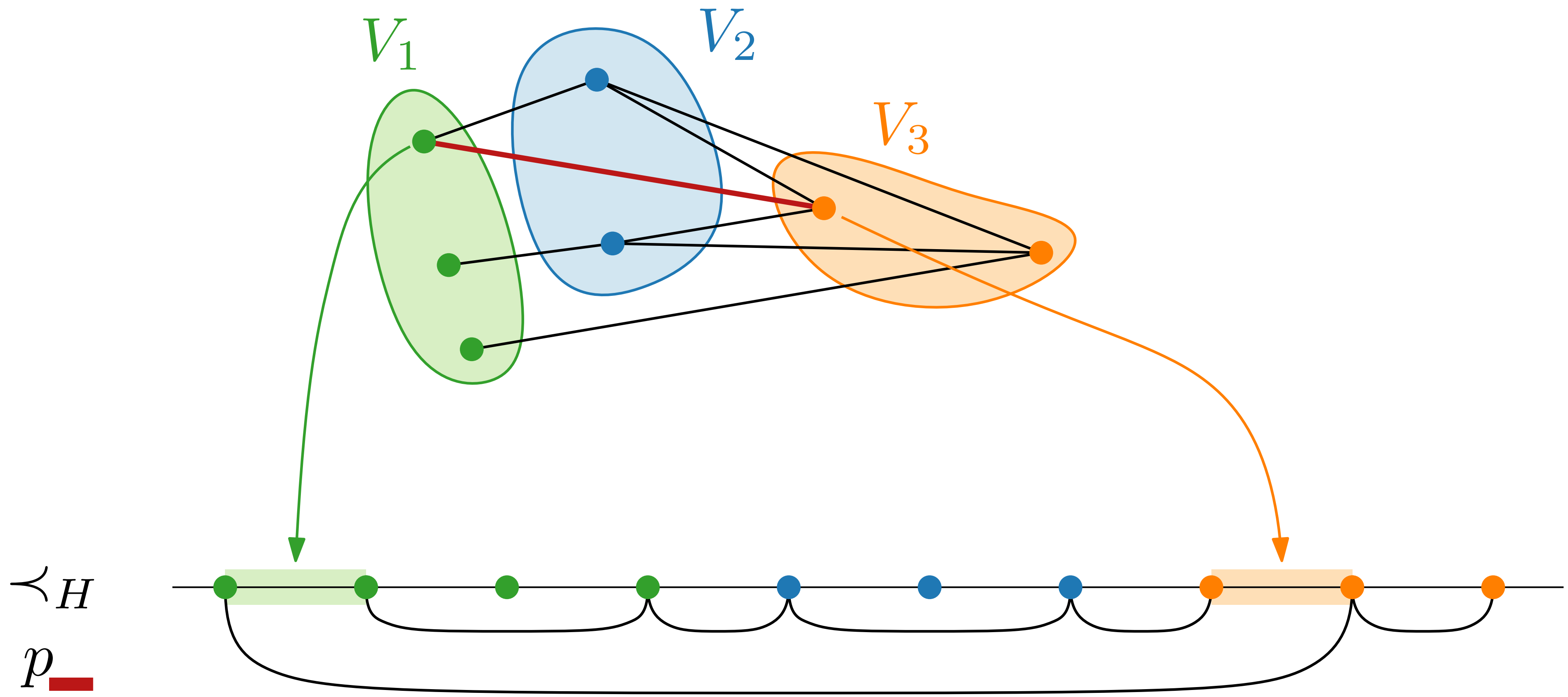


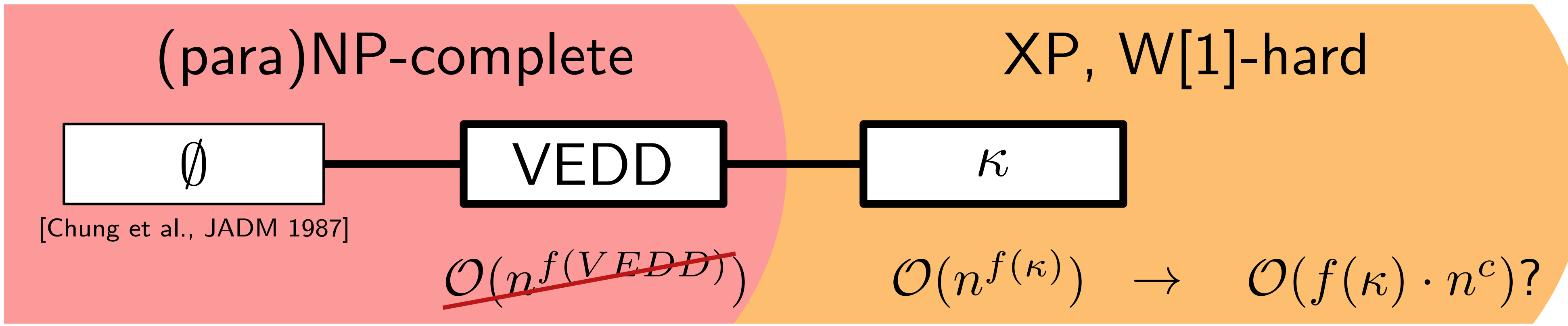
SLE Parameterized by κ is W[1]-hard

Reduction from MULTI-COLORED CLIQUE (MCC)



Theorem:
SLE parameterized by κ, i.e., the number of missing vertices and edges, is W[1]-hard.

Our Results



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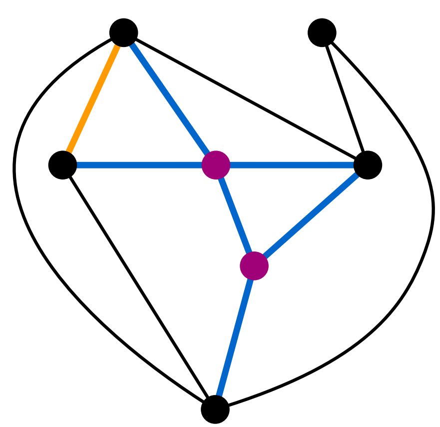

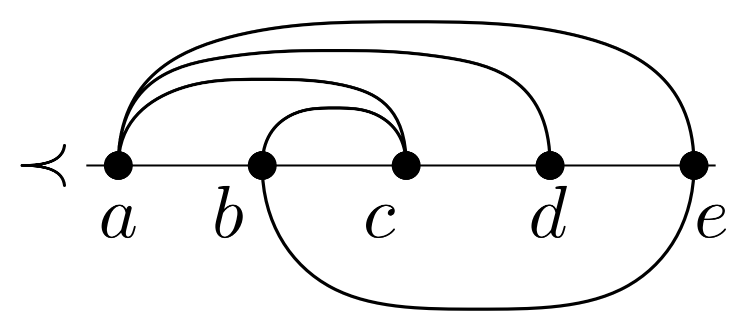

Our Results

(para)NP-complete | XP, W[1]-hard

$\emptyset$ — VEDD — κ

[Chung et al., JADM 1987]

~~$\mathcal{O}(n^{f(VEDD)})$~~ $\mathcal{O}(n^{f(\kappa)}) \rightarrow$ ~~$\mathcal{O}(f(\kappa) \cdot n^{c})$~~?

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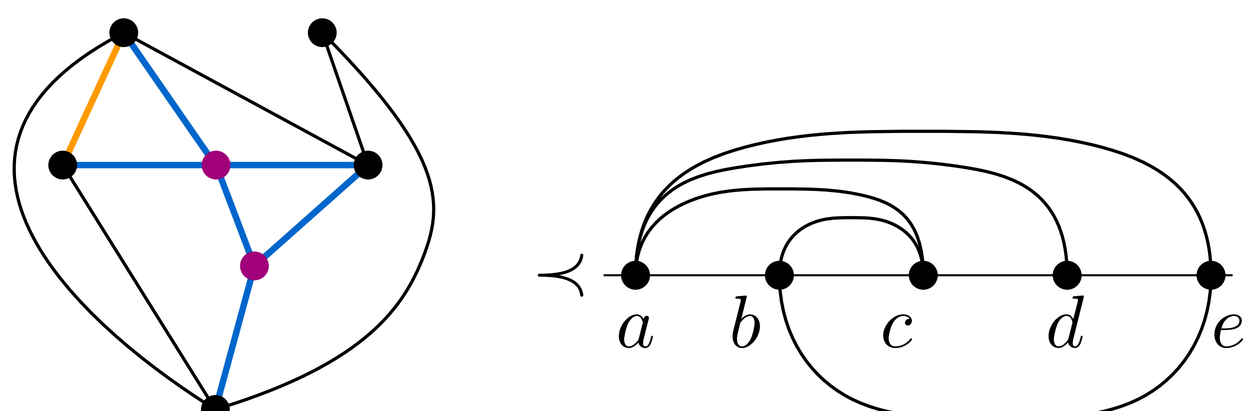
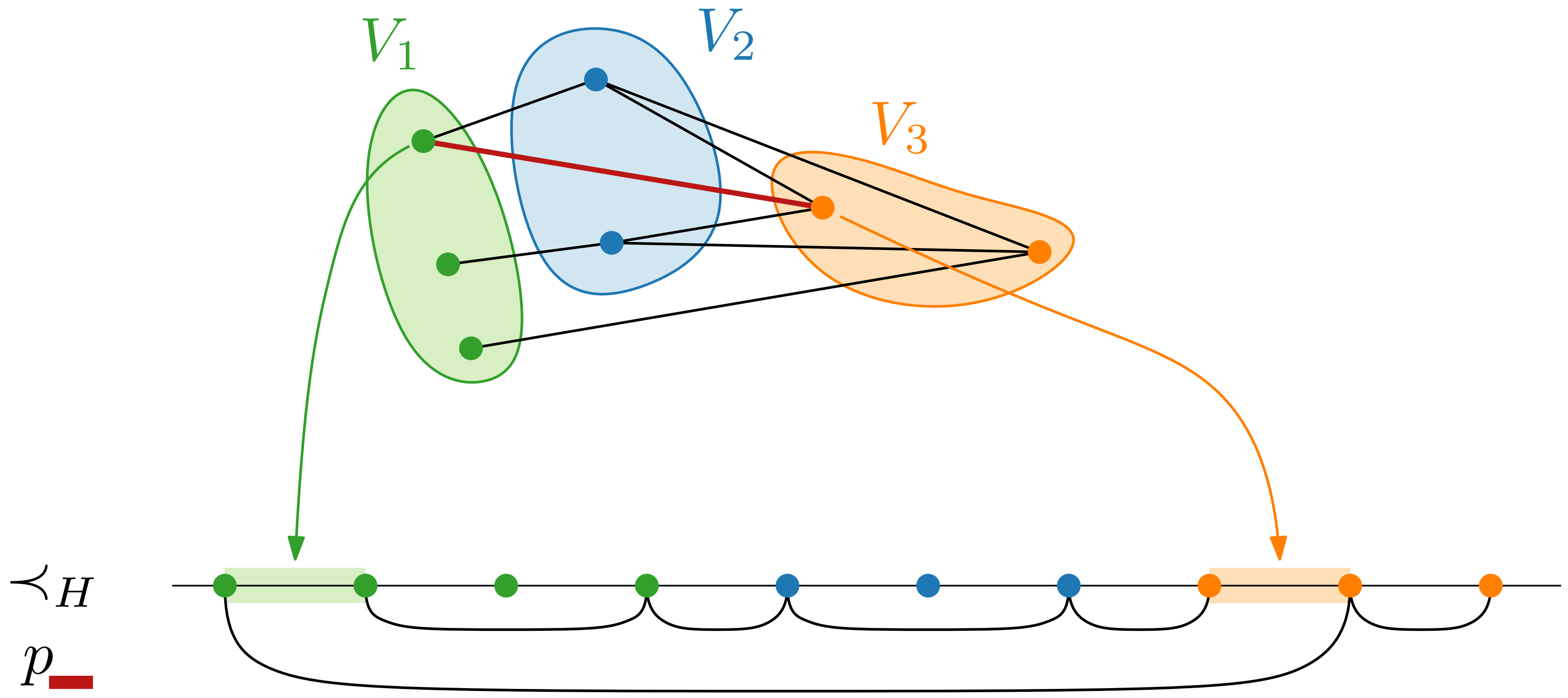


Rewind: SLE Parameterized by κ is W[1]-hard

Reduction from MULTI-COLORED CLIQUE (MCC)



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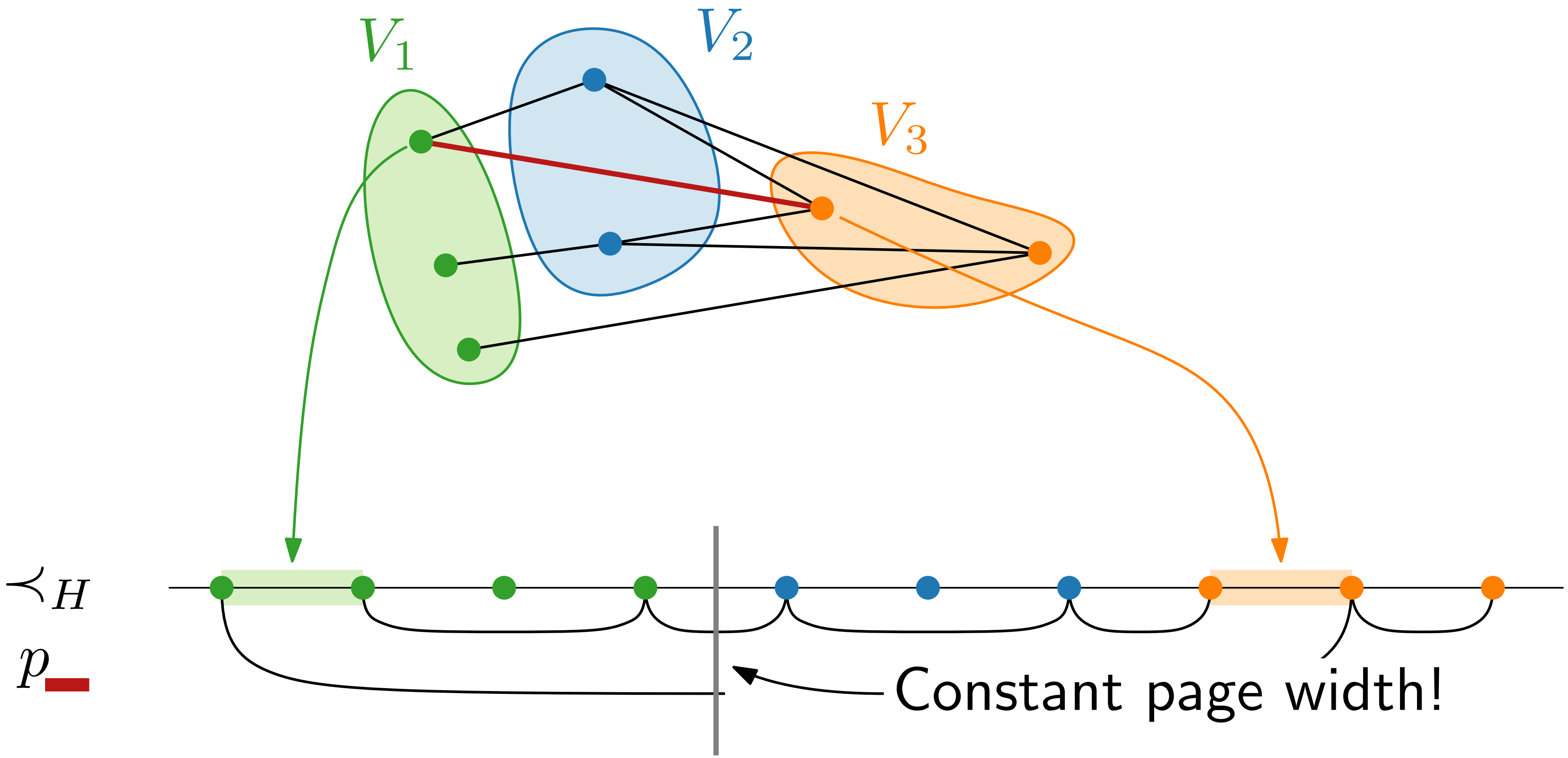
Rewind: SLE Parameterized by κ is W[1]-hard

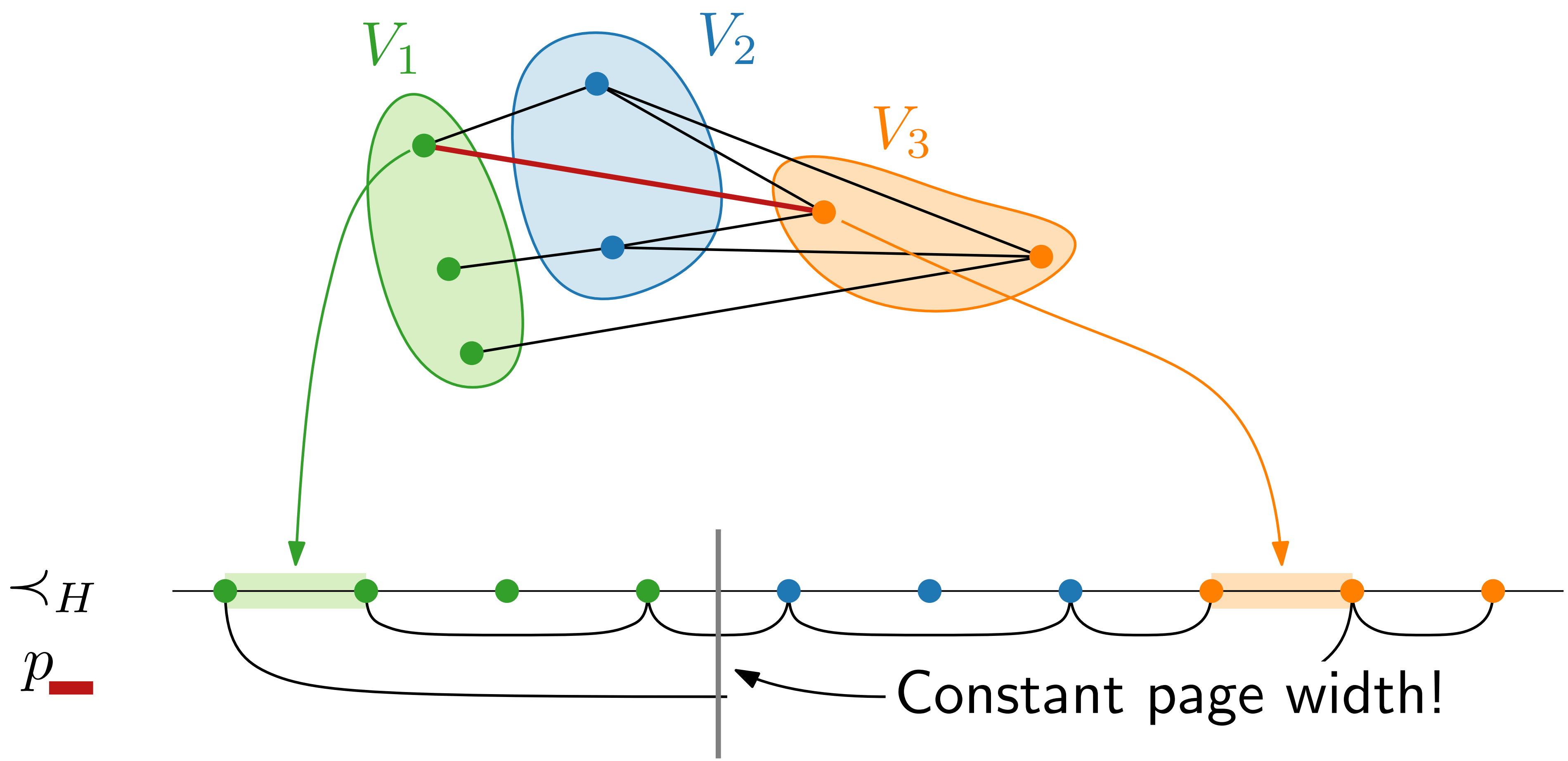
Reduction from MULTI-COLORED CLIQUE (MCC)



Theorem:
SLE parameterized by κ, i.e., the number of missing vertices and edges, is W[1]-hard.

Rewind: SLE Parameterized by κ is W[1]-hard

Reduction from Multi-Colored Clique (McC)



Theorem:
SLE parameterized by κ, i.e., the number of missing vertices and edges, and the **page width ω of the given layout** is W[1]-hard.

Our Results

(para)NP-complete | XP, W[1]-hard

$\emptyset$ — VEDD — κ

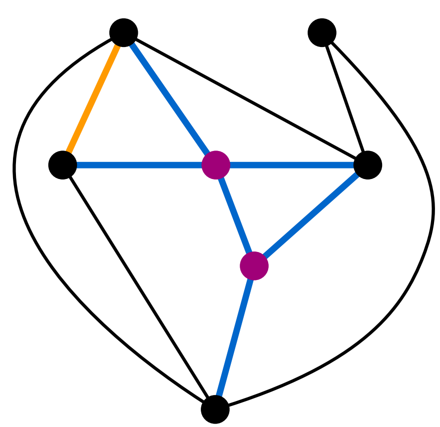
[Chung et al., JADM 1987]

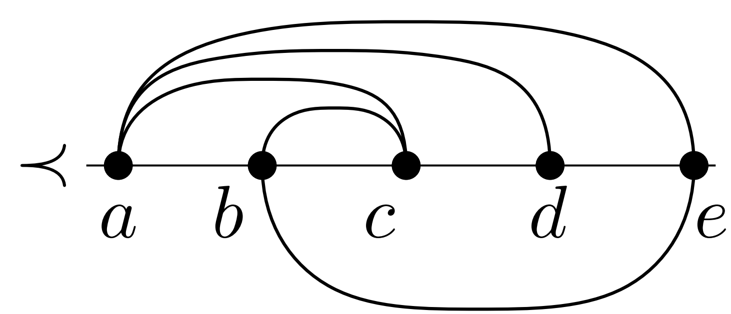
~~$\mathcal{O}(n^{f(VEDD)})$~~ $\mathcal{O}(n^{f(\kappa)}) \rightarrow$ ~~$\mathcal{O}(f(\kappa) \cdot n^c)$~~?

VEDD: #vertices (inc. **incident edges**) & edges to delete from G to obtain H
$\boldsymbol{\kappa}$**:** #missing vertices **plus** #missing edges

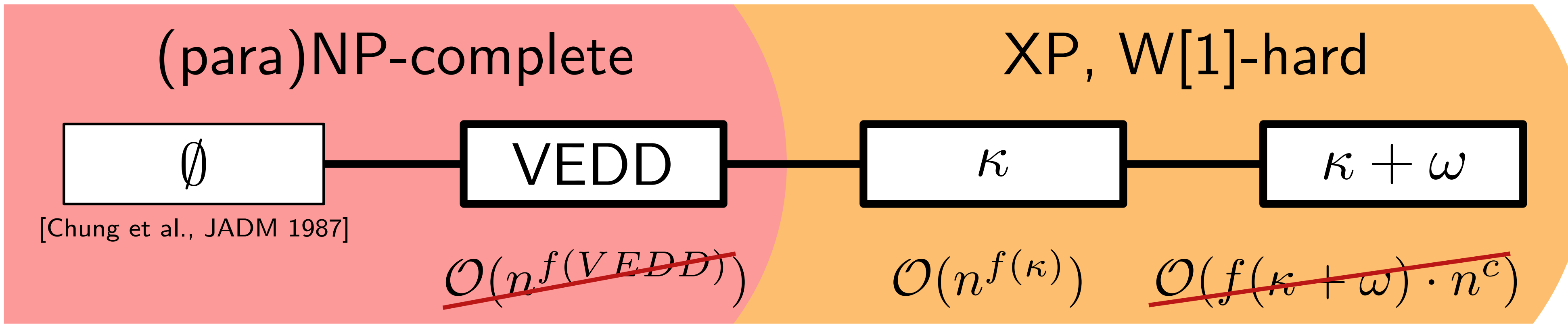
VEDD $=$ **2** $+$ **1** $= 3$

$\kappa =$ **2** $+$ **1** $+$ **6** $= 9$





Our Results



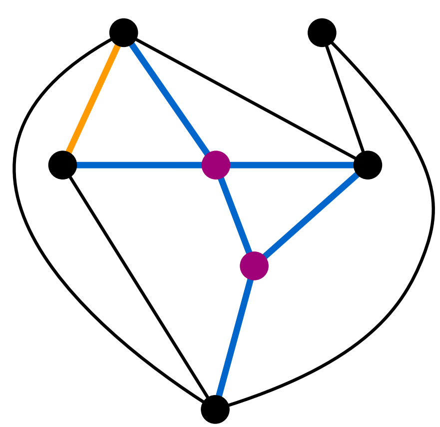
VEDD: #vertices (inc. **incident edges**) & edges to delete from G to obtain H

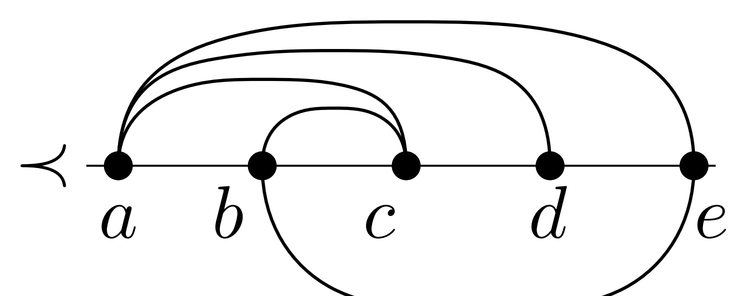
$\boldsymbol{\kappa}$**:** #missing vertices **plus** #missing edges

$\boldsymbol{\omega}$**:** page width of $\langle \prec_H, \sigma_H \rangle$

VEDD = **2** + **1** = 3

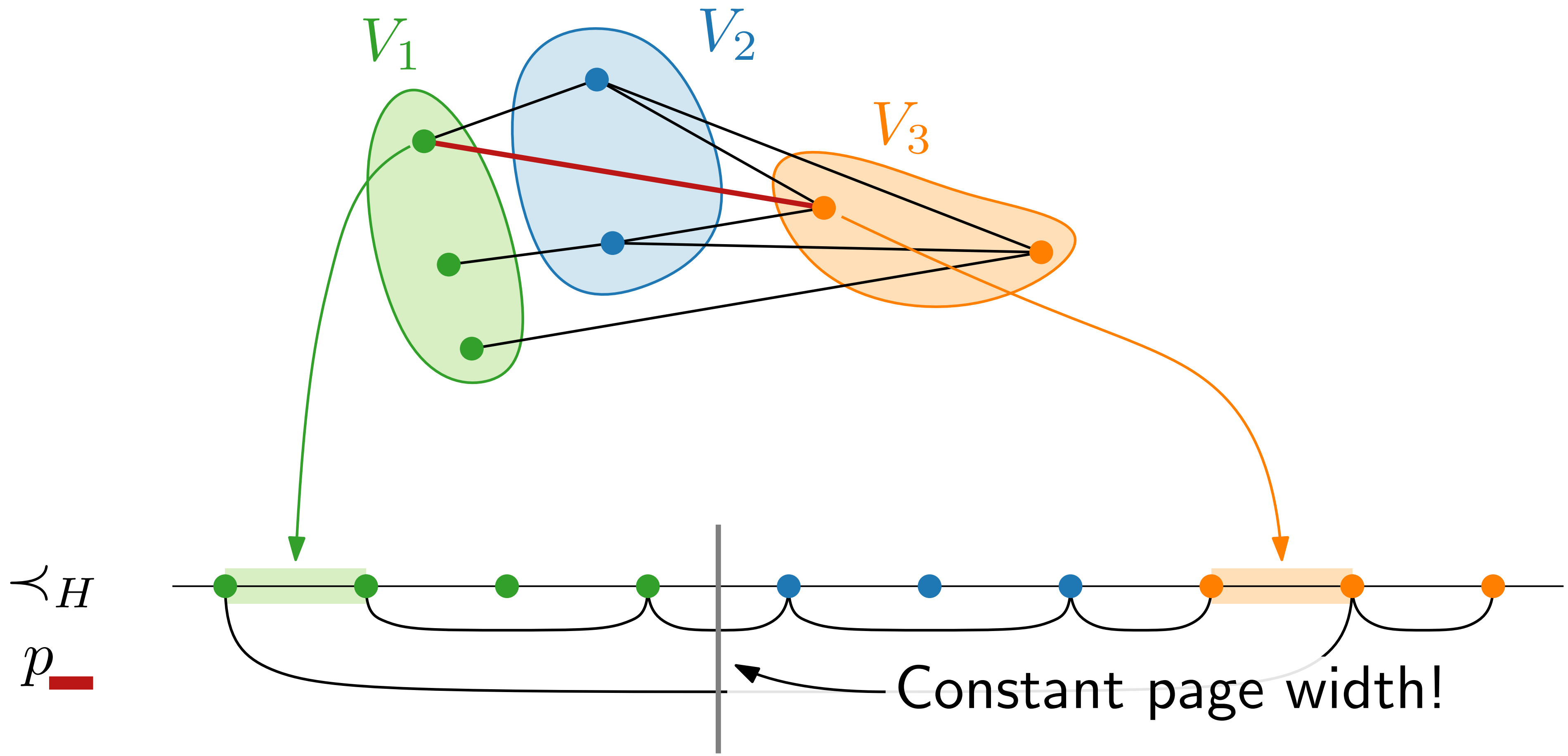
κ = **2** + **1** + **6** = 9





Rewind II: SLE Parameterized by κ is W[1]-hard

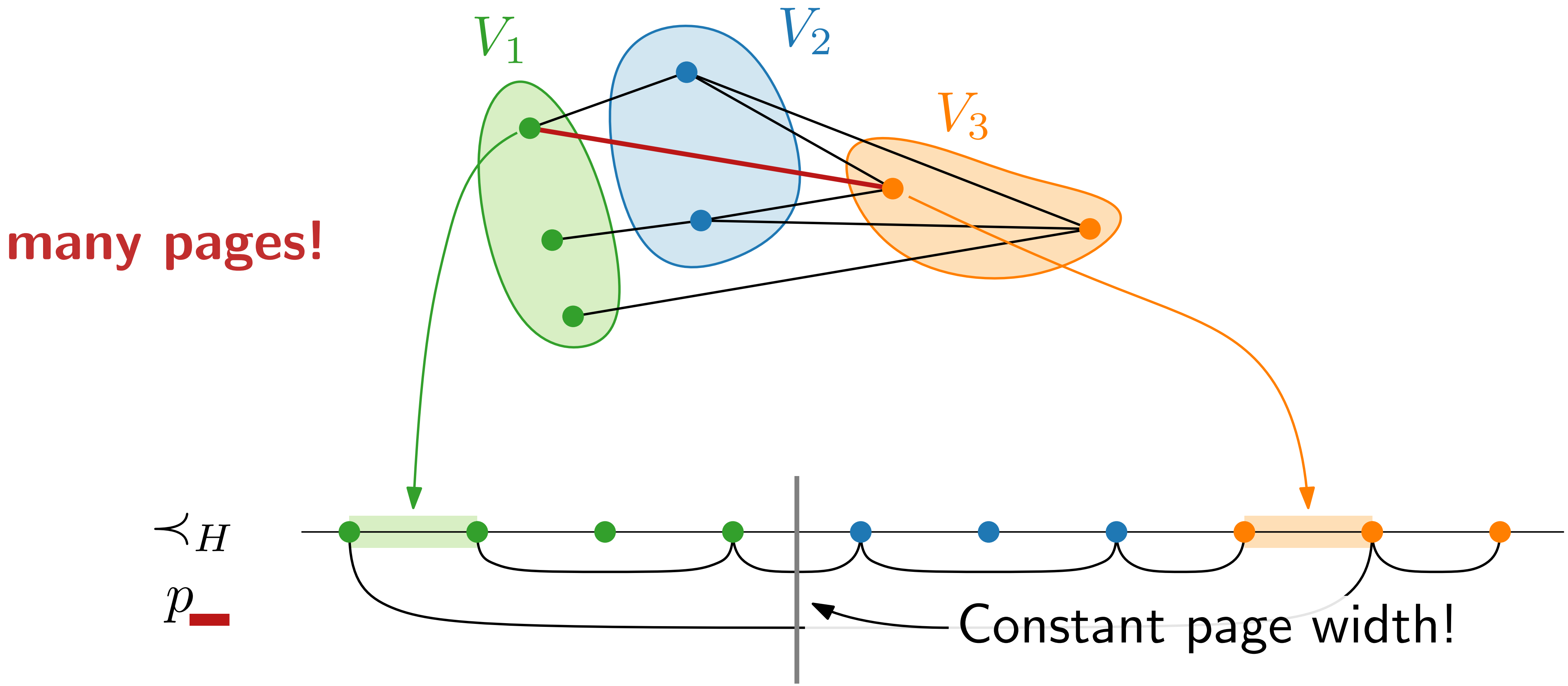
Reduction from Multi-Colored Clique (McC)



Theorem:
SLE parameterized by κ, i.e., the number of missing vertices and edges, and the **page width ω of the given layout** is W[1]-hard.

Rewind II: SLE Parameterized by κ is W[1]-hard

Reduction from MULTI-COLORED CLIQUE (MCC)



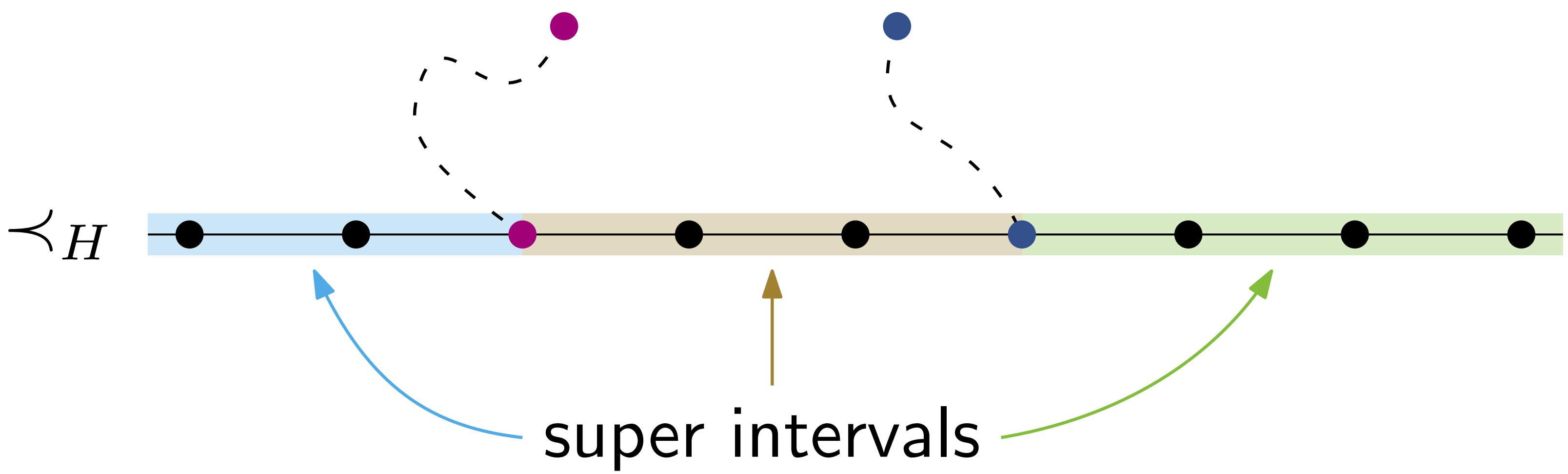
Theorem:
SLE parameterized by κ, i.e., the number of missing vertices and edges, and the **page width ω of the given layout** is W[1]-hard.

An FPT-Algorithm Parameterized by κ, ω, and ℓ

Step 1: Guess the final page assignment σ_G

Step 2: Guess the relative order among missing vertices

Step 3: Guess the assignment of missing vertices to *super-intervals*



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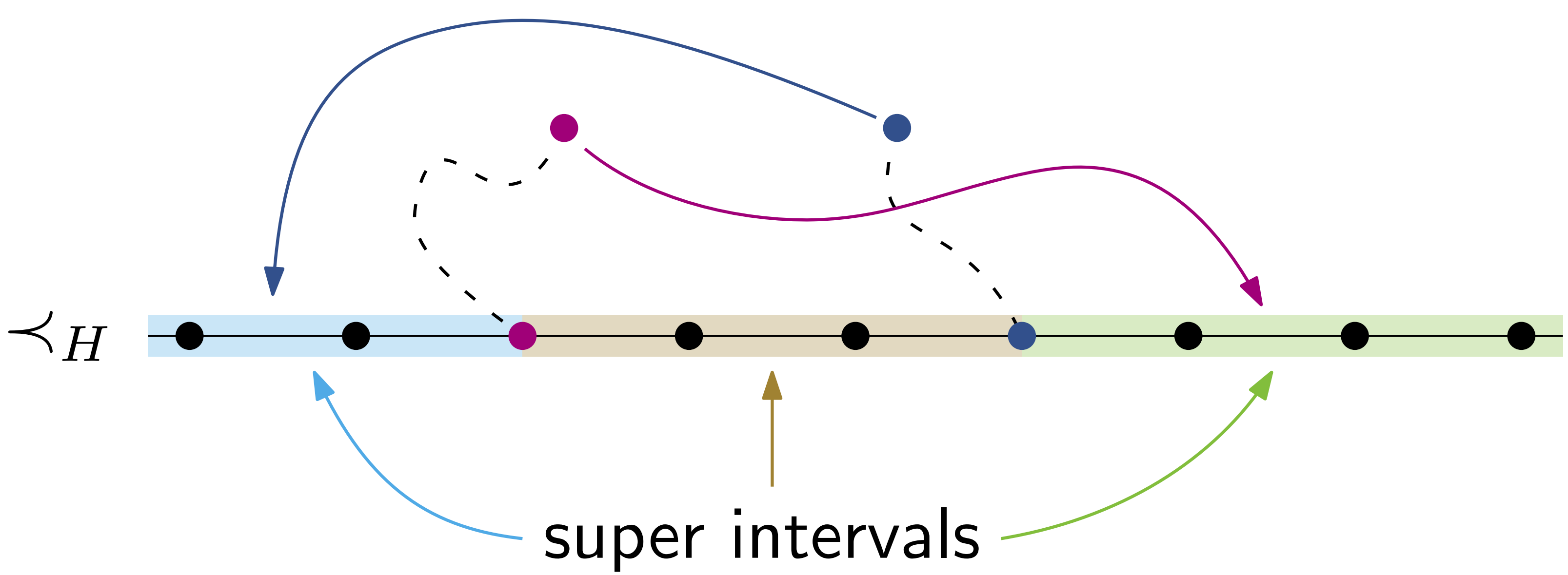

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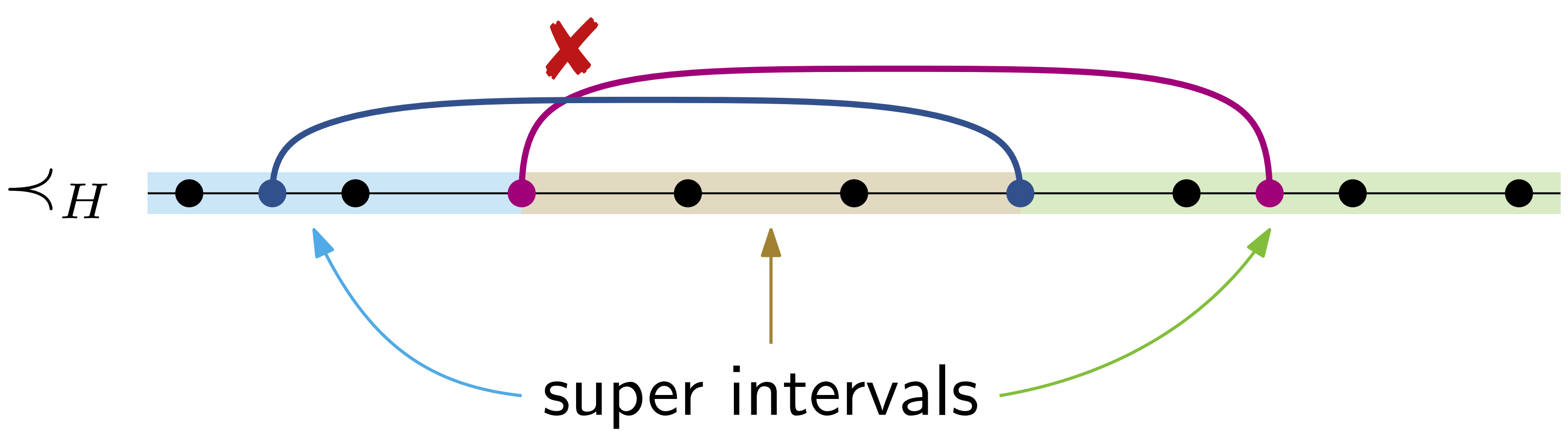
$\Rightarrow$ Steps 1 – 3: $\mathcal{O}(f(\kappa + \ell))$ branches



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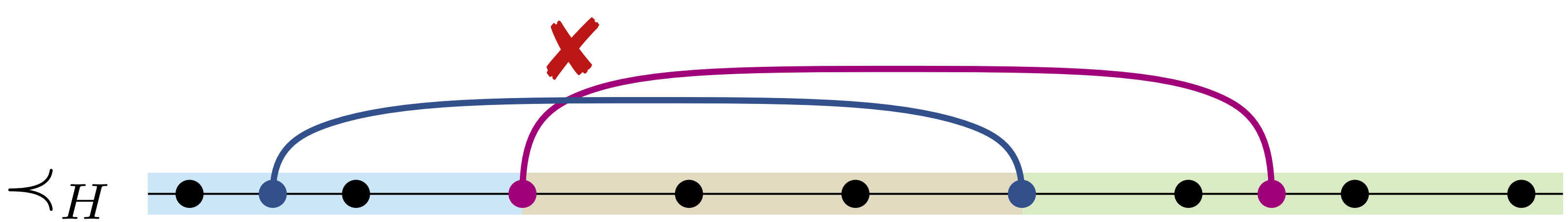
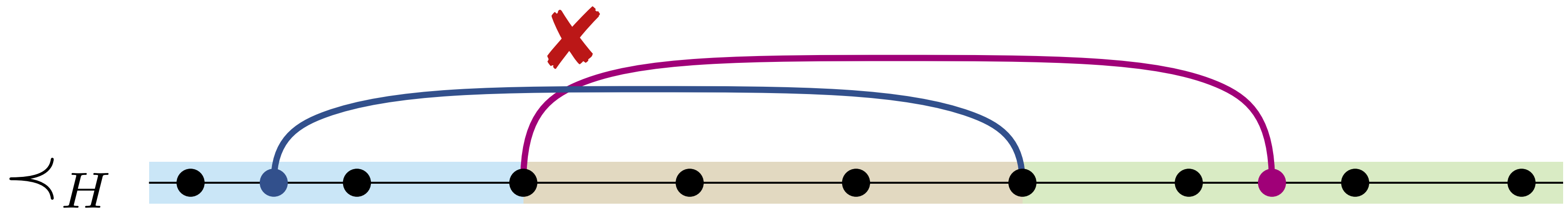
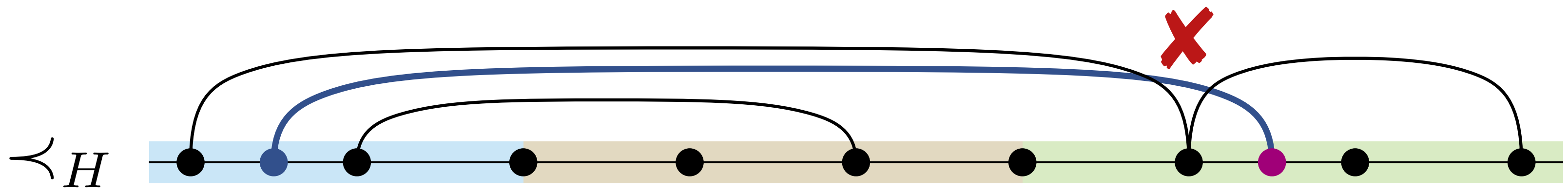


So are we done?

A Dynamic Program To Prevent Remaining Crossings



A Dynamic Program To Prevent Remaining Crossings



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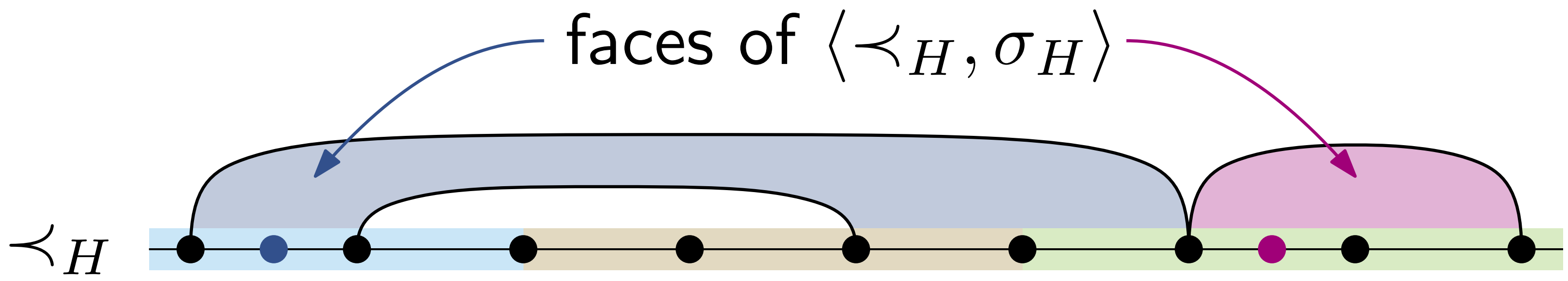

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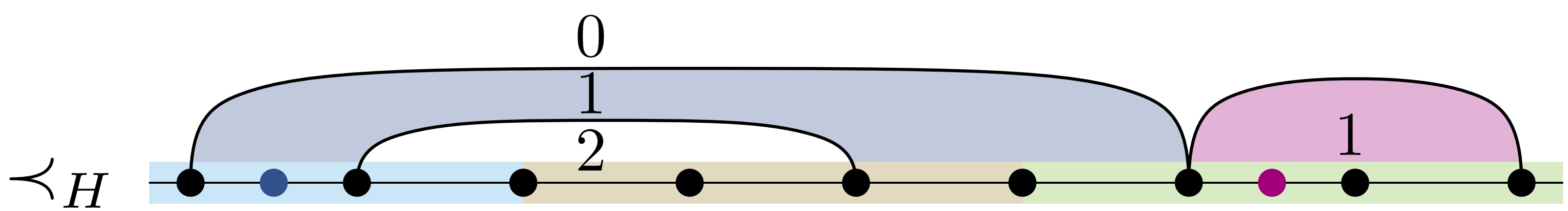
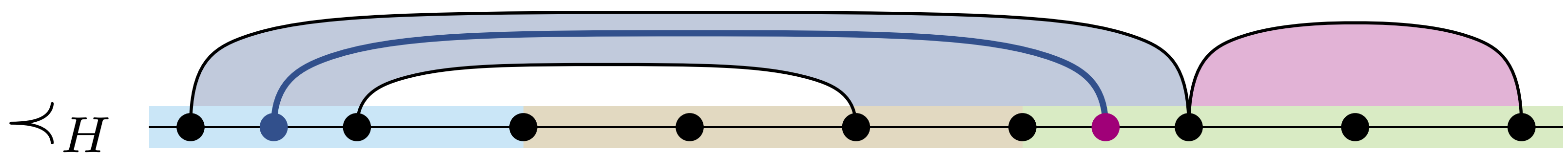


Step 4: Guess for each missing edge its distance to the outer face

A Dynamic Program To Prevent Remaining Crossings



Step 4: Guess for each missing edge its distance to the outer face

Step 5: Apply a DP in each branch to check if extended stack layout exists

$\mathcal{O}(n_{\mathsf{add}} m_{\mathsf{add}} |\mathcal{I}|)$ time per branch

Finally FPT!

Step 1: Guess the final page assignment σ_G

Step 2: Guess the relative order among missing vertices

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Theorem:
$\textsc{SLE}$ can be solved in $\mathcal{O}(\ell^{m_{\mathsf{add}}} \cdot n_{\mathsf{add}}! \cdot {m_{\mathsf{add}}}^{n_{\mathsf{add}}} \cdot \omega^{m_{\mathsf{add}}} \cdot n_{\mathsf{add}} m_{\mathsf{add}} |\mathcal{I}|)$ time.

Finally FPT!

Step 1: Guess the final page assignment σ_G

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#missing edges

Size of instance

#pages

#missing vertices

page width

Theorem:
SLE can be solved in $\mathcal{O}(\ell^{m_{\text{add}}} \cdot n_{\text{add}}! \cdot {m_{\text{add}}}^{n_{\text{add}}} \cdot \omega^{m_{\text{add}}} \cdot n_{\text{add}} m_{\text{add}} \, |\mathcal{I}|)$ time.

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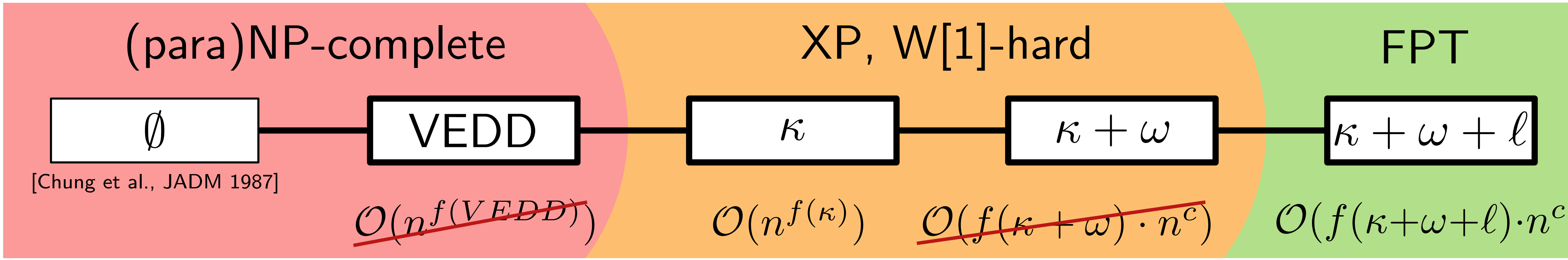
Theorem:
SLE can be solved in $\mathcal{O}(f(\kappa + \omega + \ell) |\mathcal{I}|)$ time.

Conclusion


(para)NP-complete
XP, W[1]-hard
FPT
$\emptyset$
VEDD
κ
$\kappa + \omega$
$\kappa + \omega + \ell$
[Chung et al., JADM 1987]
$\mathcal{O}(n^{f(VEDD)})$
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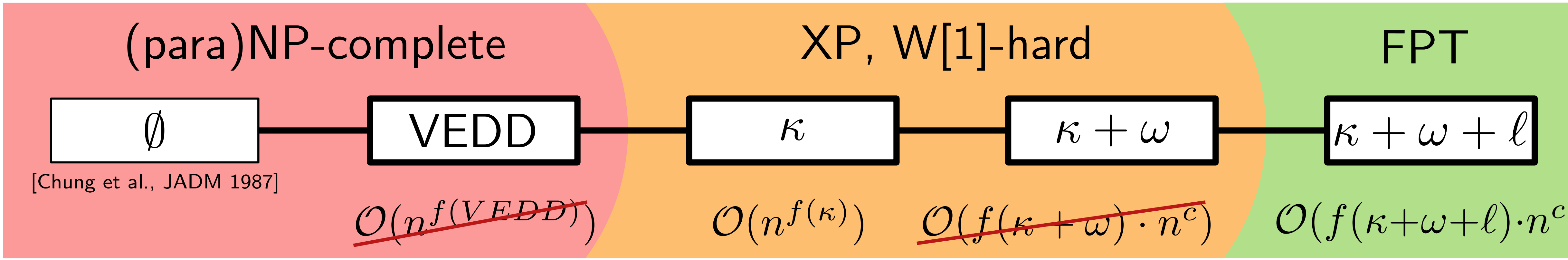
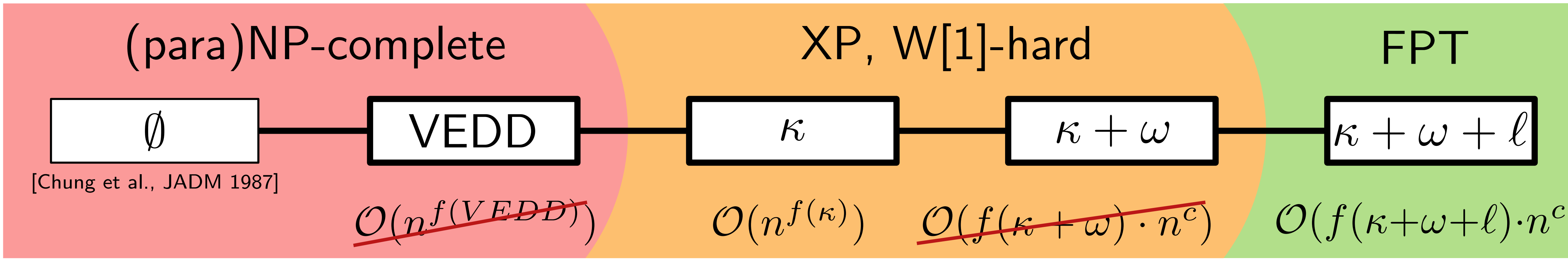
Conclusion



Future work:

Parameterization by $\kappa + \ell$: FPT or W[1]-hard?

Conclusion

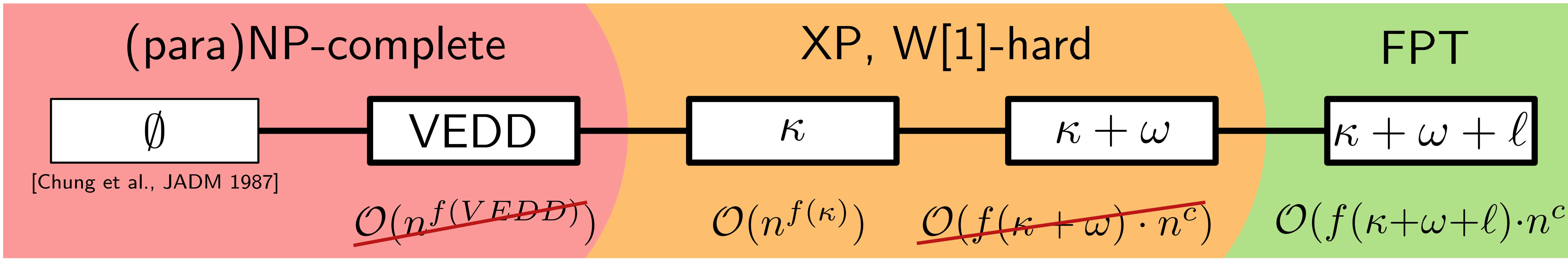


Future work:

Parameterization by $\kappa + \ell$: FPT or W[1]-hard?

Queue Layout Extension?

Conclusion



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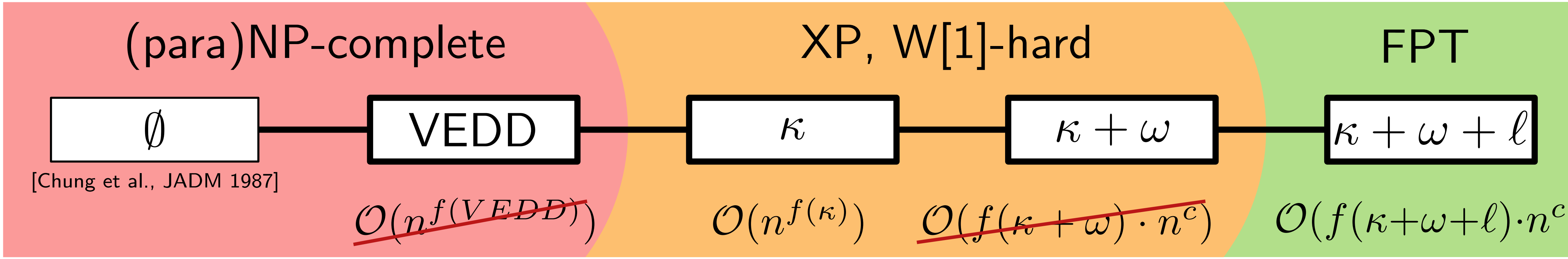
Parameterization by $\kappa + \ell$: FPT or W[1]-hard?

Queue Layout Extension?

Generalized notion of extension: Given spine order for some vertices and page assignment for some edges, does there exist a stack layout that extends both?

Conclusion



Future work:

Parameterization by $\kappa + \ell$: FPT or W[1]-hard?

Queue Layout Extension?

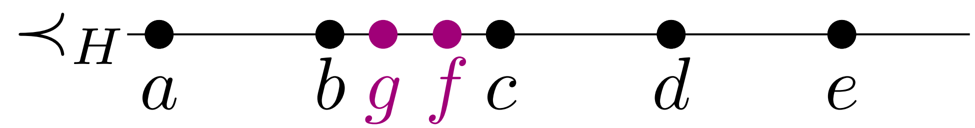
Generalized notion of extension: Given spine order for some vertices and page assignment for some edges, does there exist a stack layout that extends both?

Thank you for your attention!

App.: SLE Parameterized by κ is in XP

Step 1: Guess the extended spine order $\prec_G$

$\mathcal{O}(|\mathcal{I}|^{n_{add}})$ branches

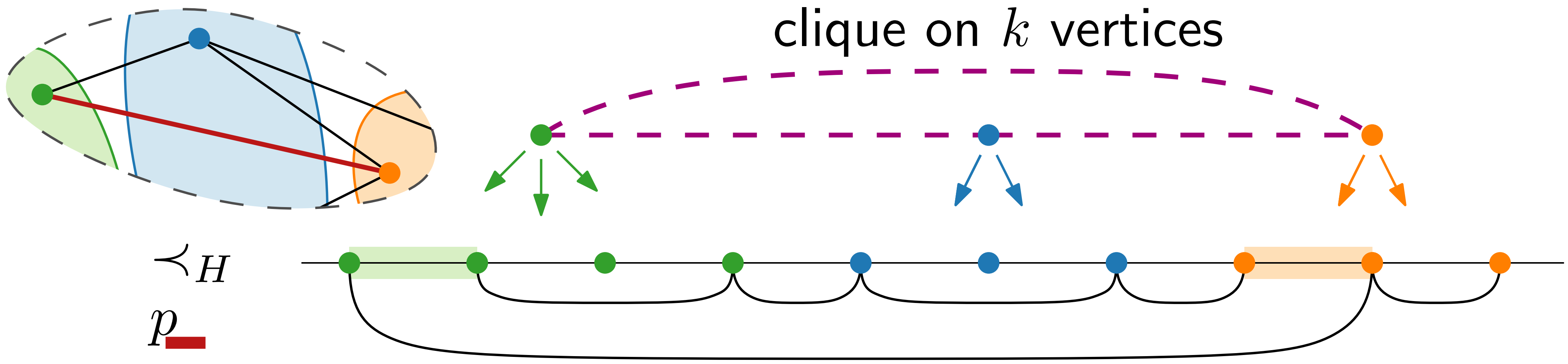


Step 2: Solve instance of SLE with only missing edges

$\mathcal{O}({m_{add}}^{m_{add}} \cdot |\mathcal{I}|)$ time per branch

Theorem:
SLE can be solved in $\mathcal{O}(|\mathcal{I}|^{n_{\text{add}}+1} \cdot {m_{\text{add}}}^{m_{\text{add}}})$ time.

App.: SLE Parameterized by κ is W[1]-hard – Tunnel



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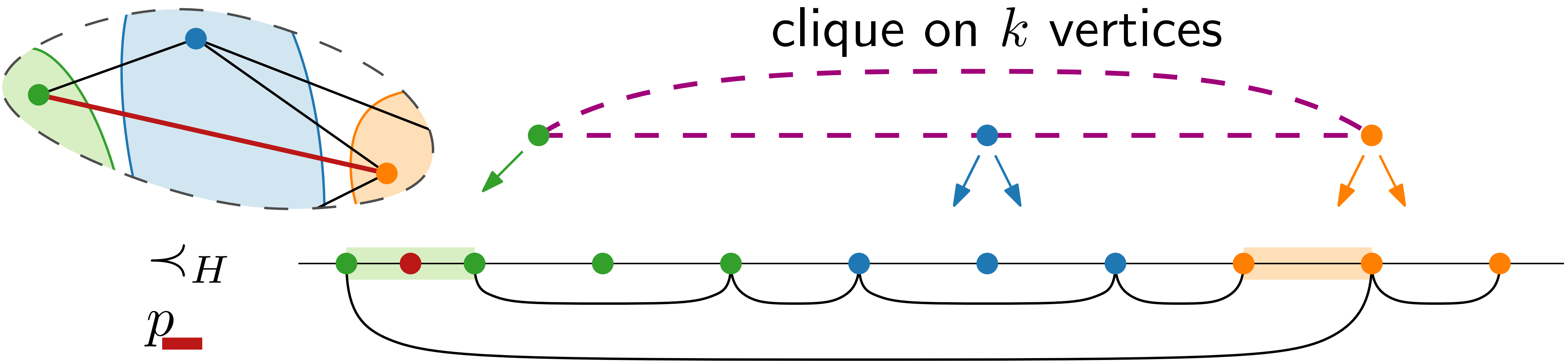

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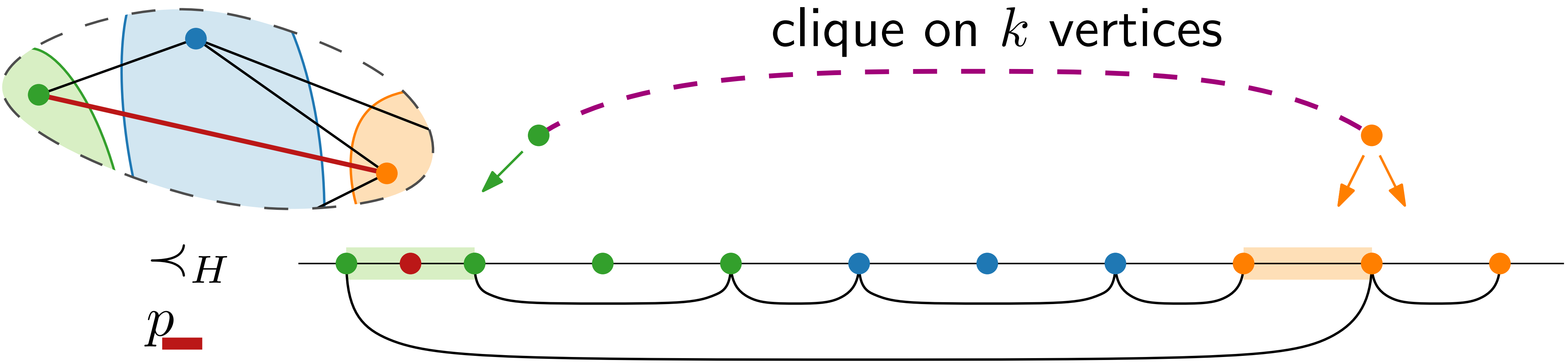

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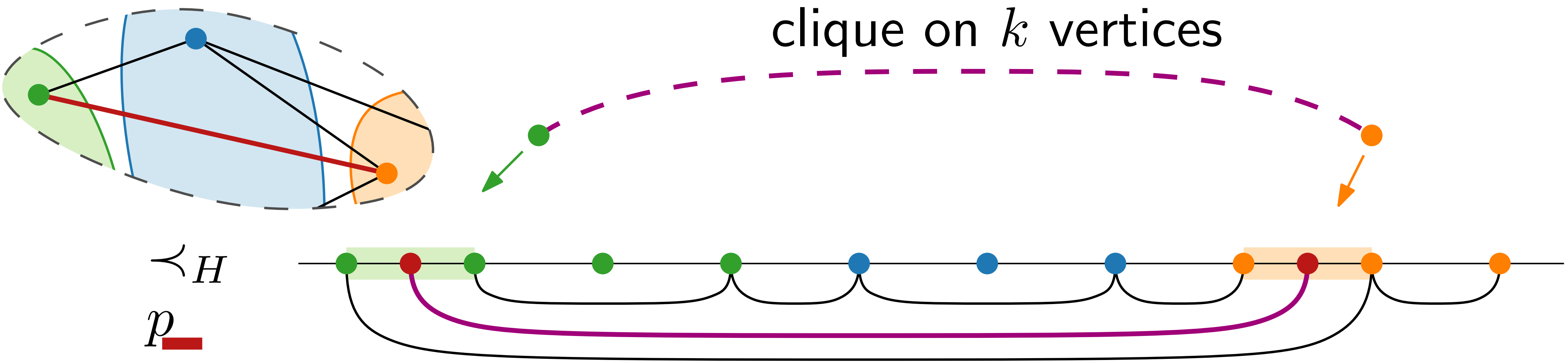
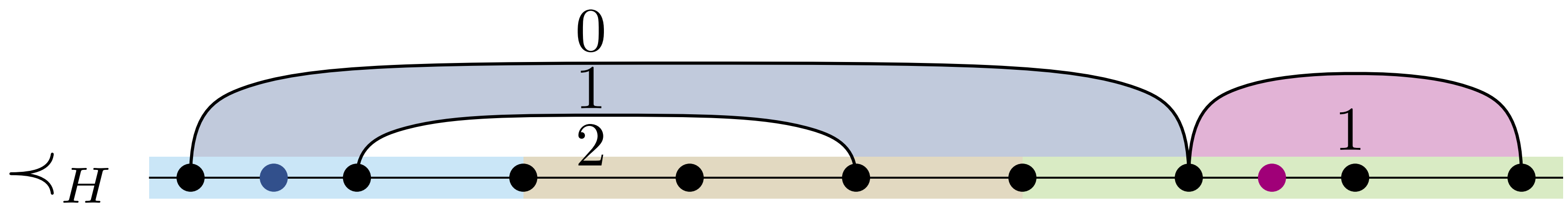


A Dynamic Program To Prevent the Remaining Crossings



Number faces from outside-in $\Rightarrow$ numbers bounded by ω

Step 4: Guess for each new edge its distance to the outerface

Step 5: Apply a DP in each branch to compute stack layout

$\mathcal{O}(n_{\text{add}} m_{\text{add}} |\mathcal{I}|)$ time per branch

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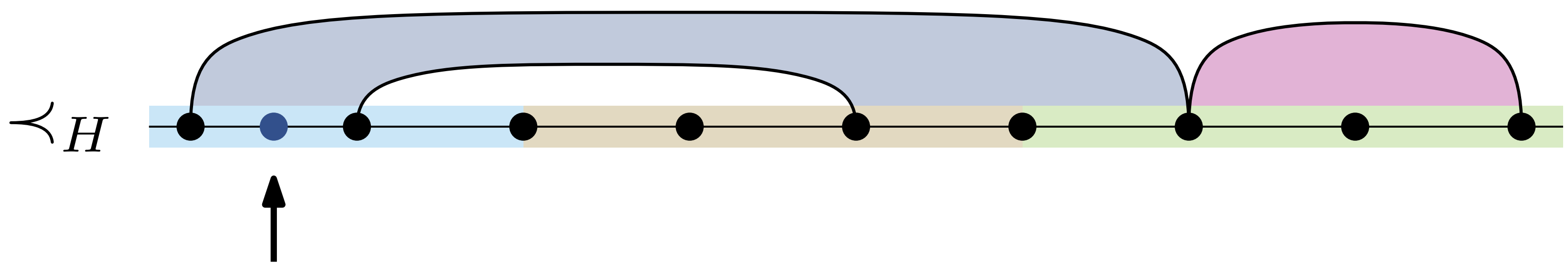

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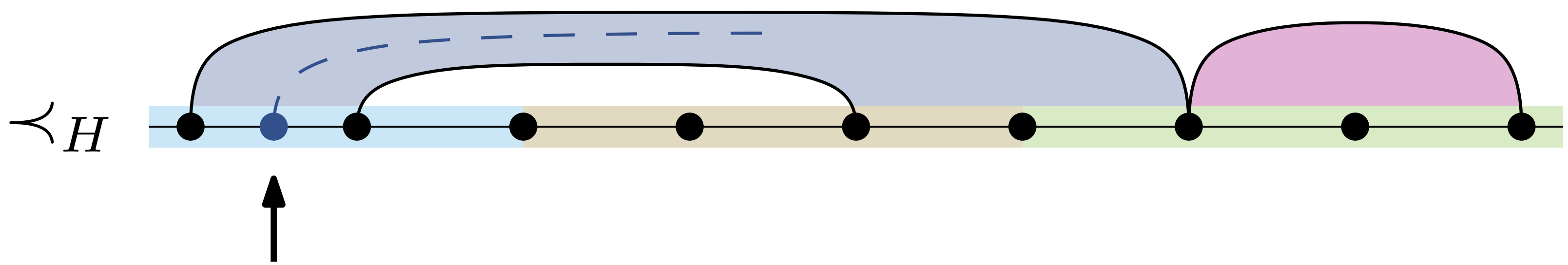

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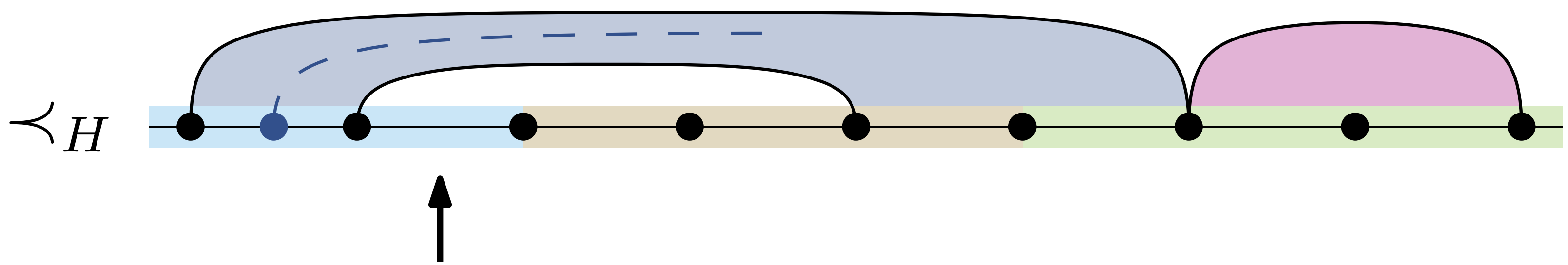

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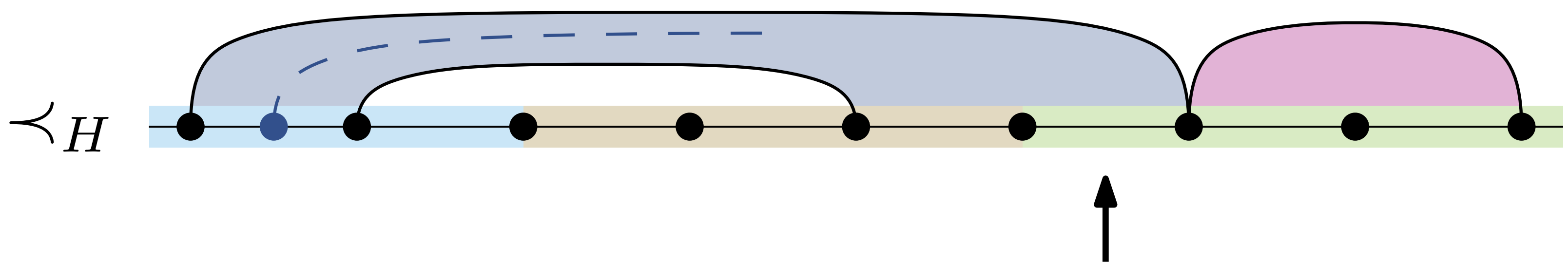

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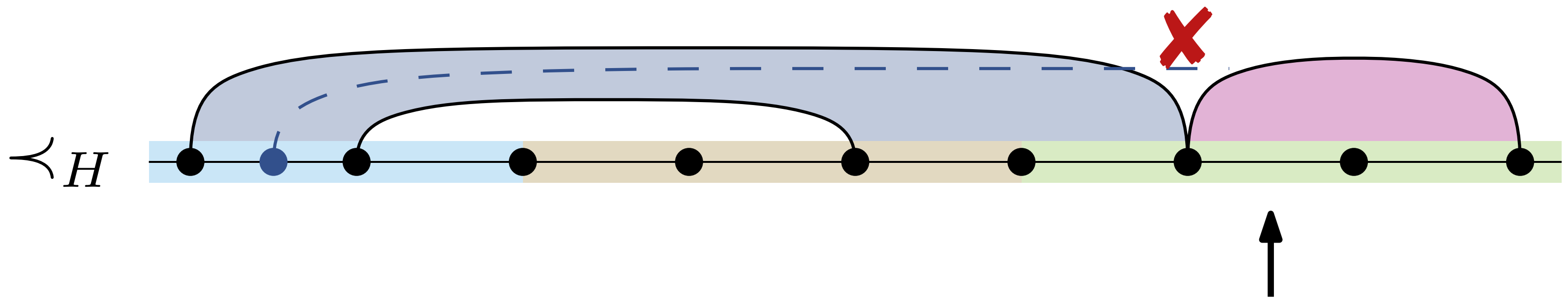

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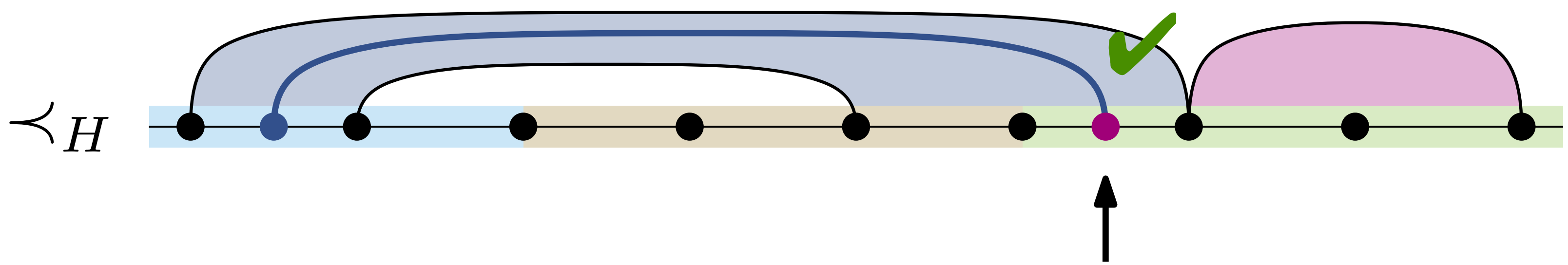

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