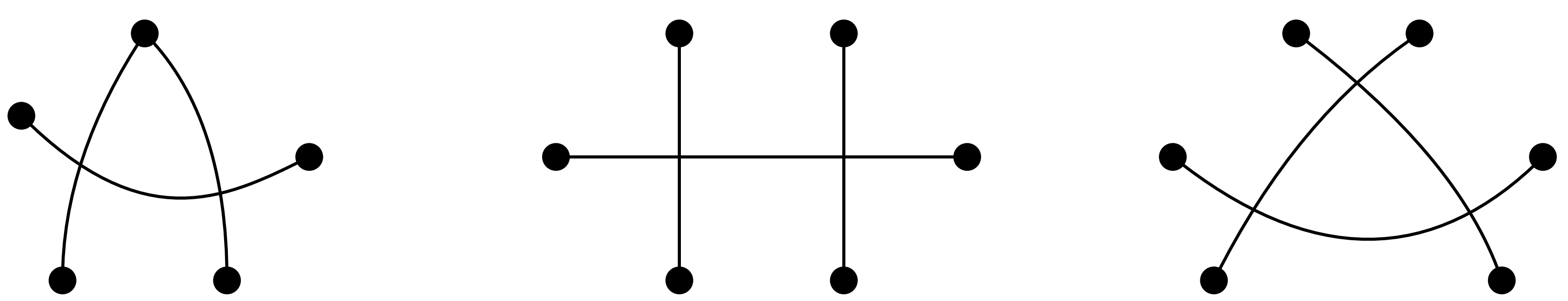
Universität Passau

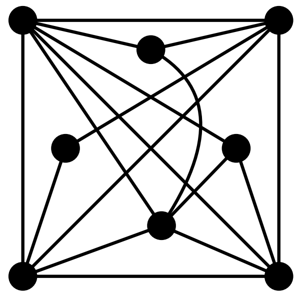
Parameterized Algorithms for Beyond Planar Crossing Numbers

Miriam Münch, Ignaz Rutter

18. September 2024



(Beyond-Planar) Crossing Numbers



crossing number of G = min. # crossings in any drawing of G

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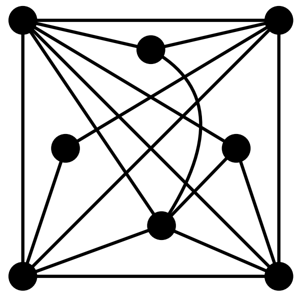

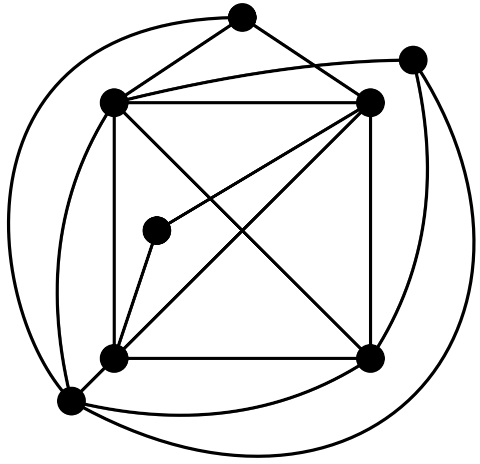

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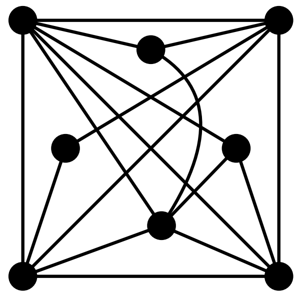

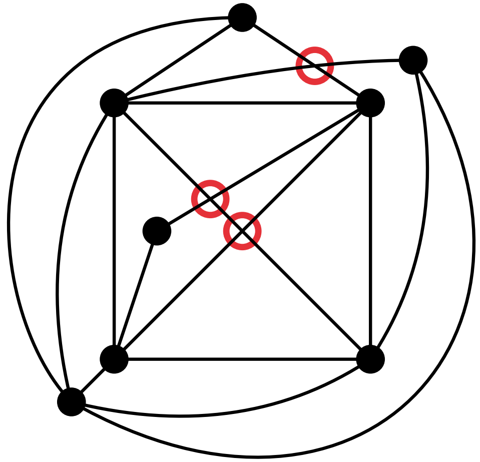

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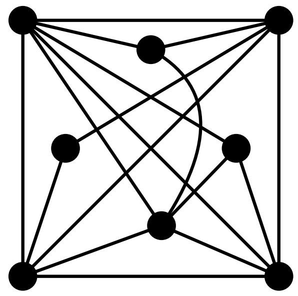

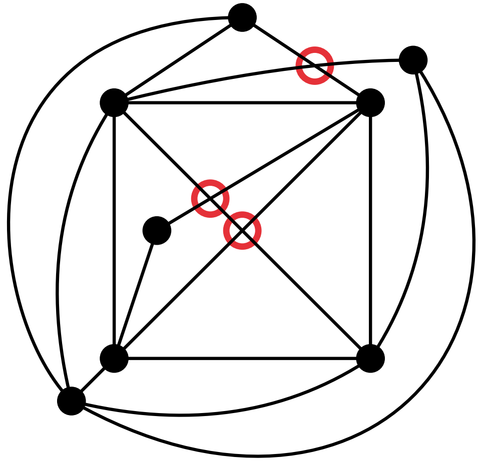

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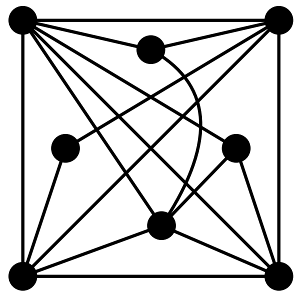

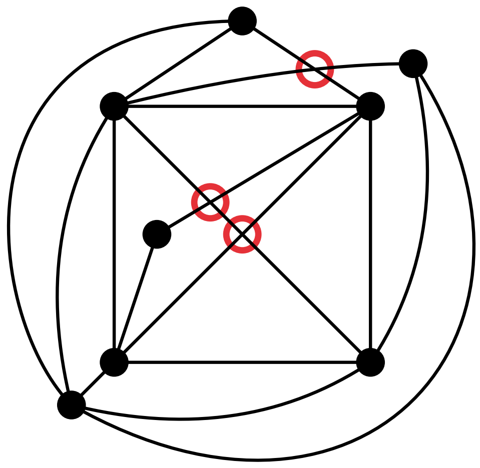

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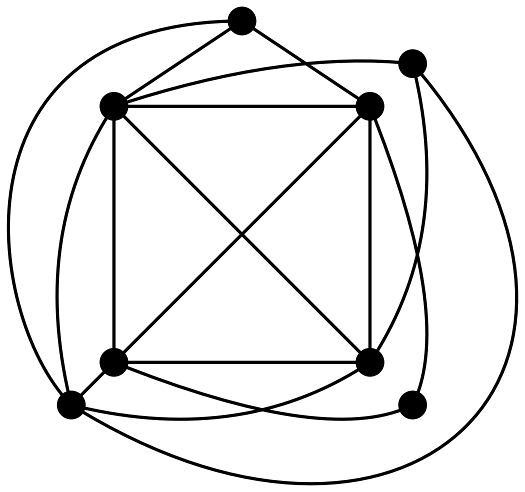

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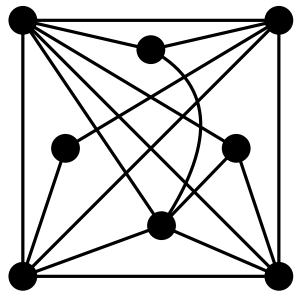

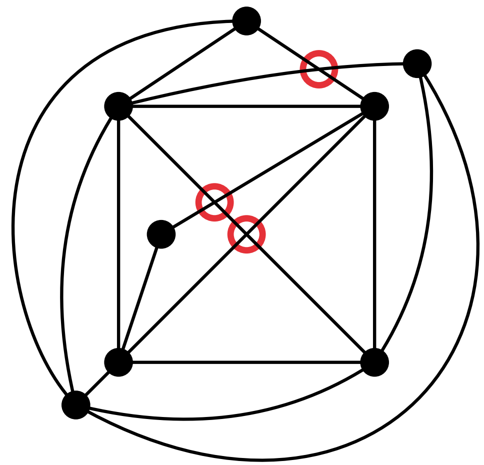

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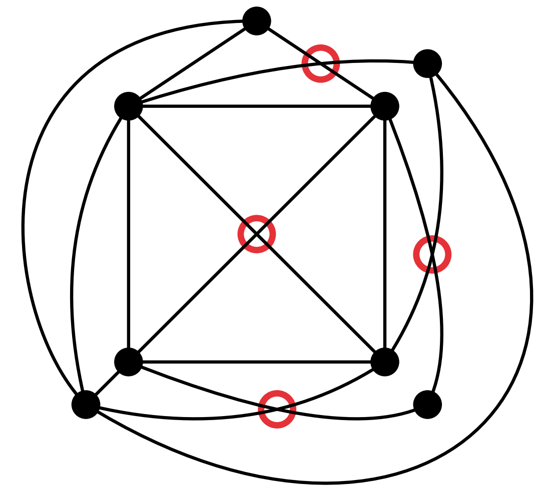

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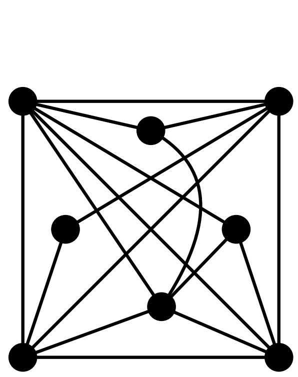

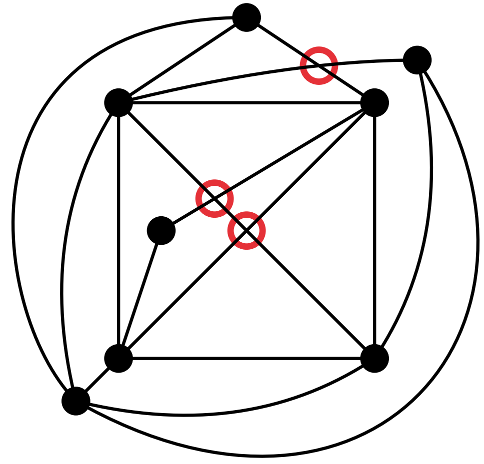

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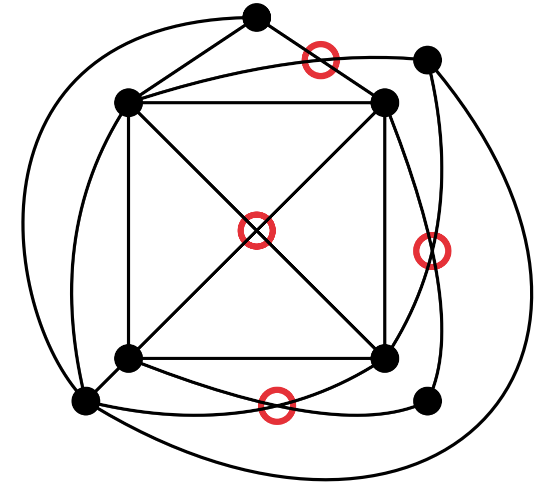

1-planar

beyond-planar drawing style

$\mathcal{D}$- crossing number of G = min. # crossings in any $\mathcal{D}$-drawing of G

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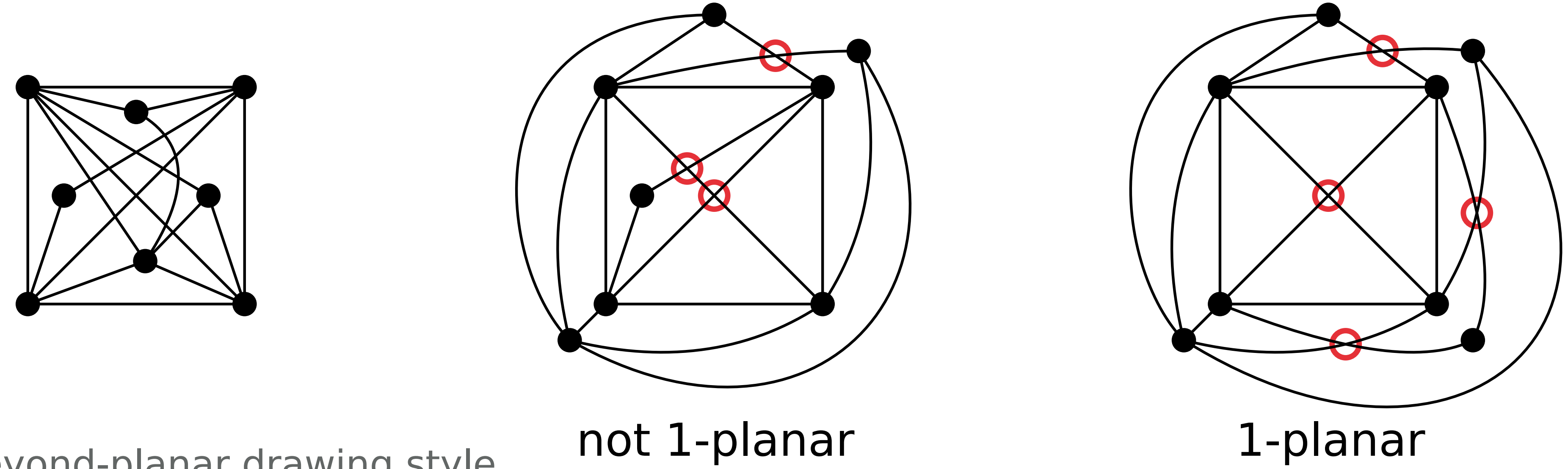

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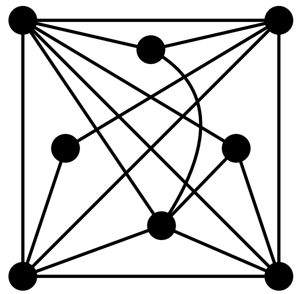

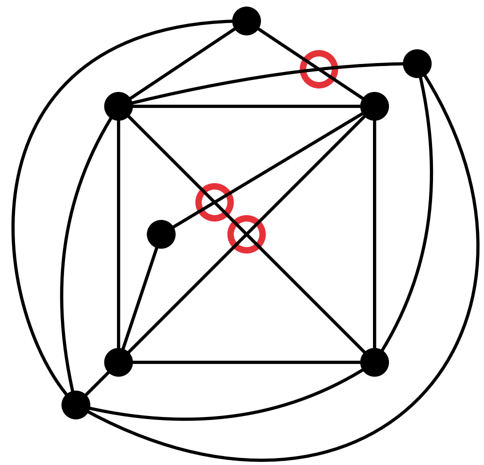

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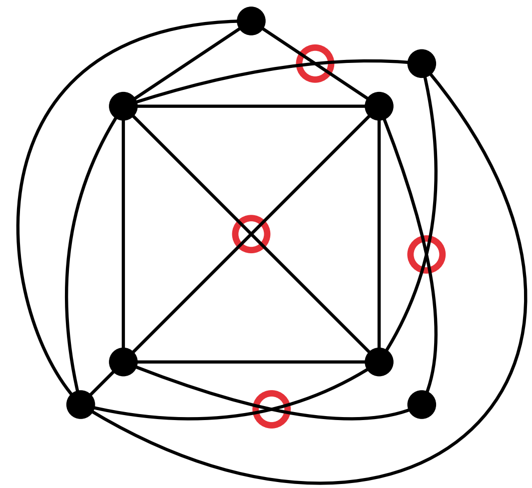

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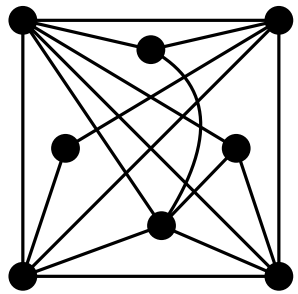

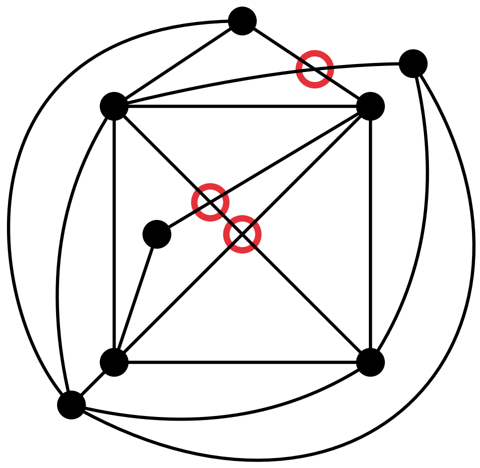

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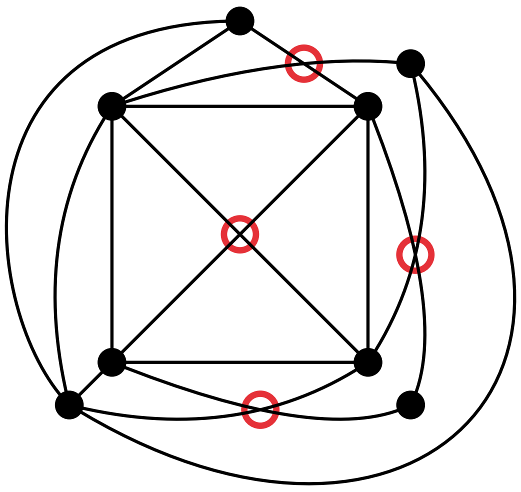

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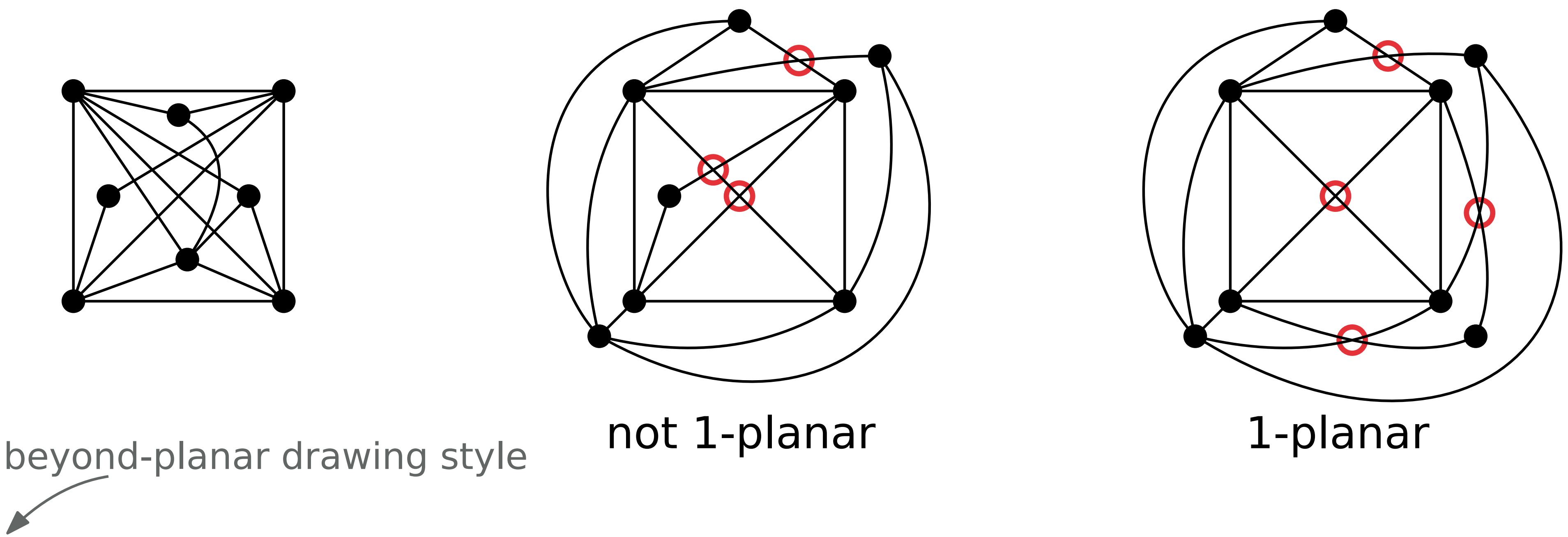

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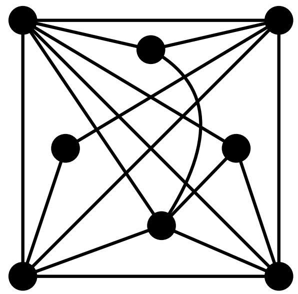

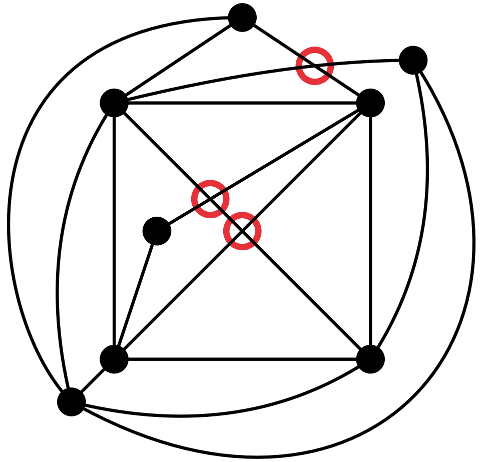

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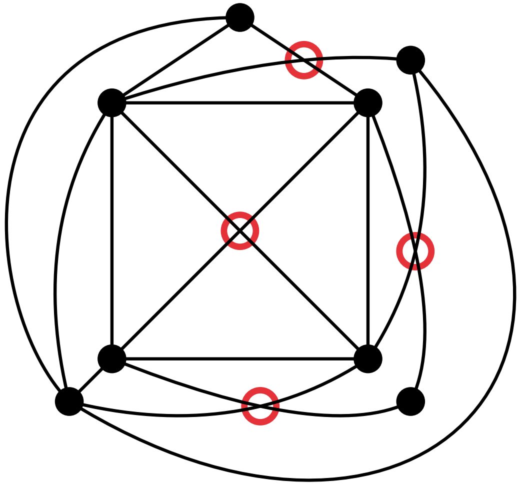

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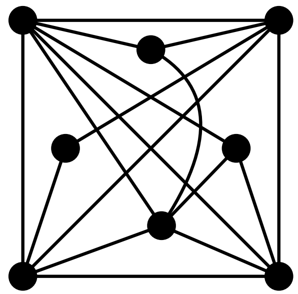

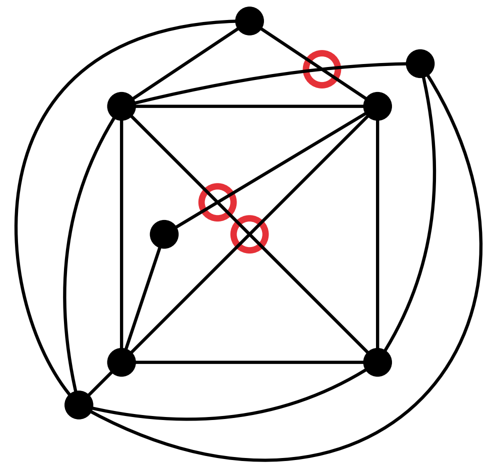

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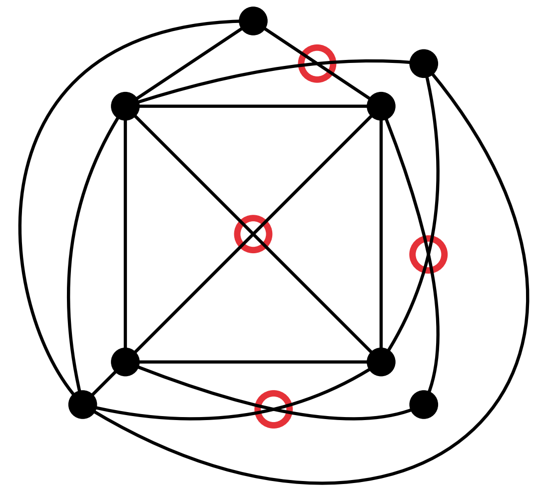

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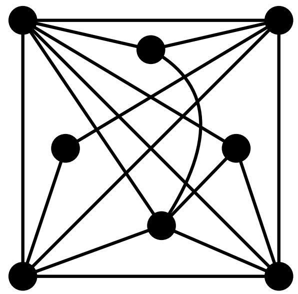

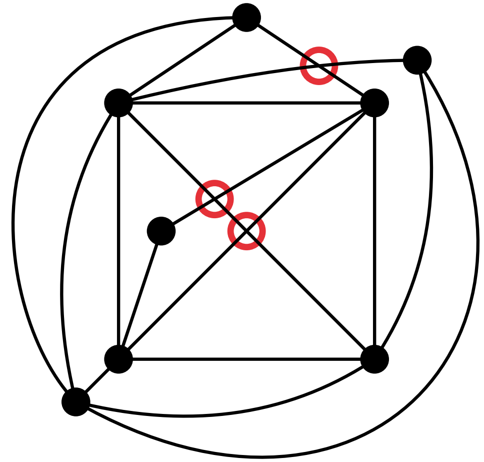

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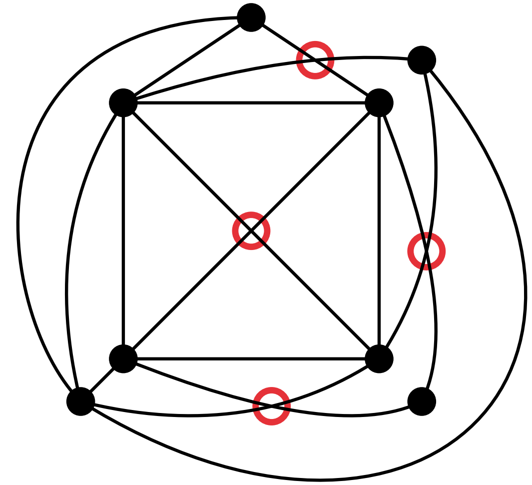

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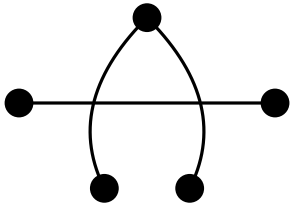
Can we decide $\mathcal{D}$-crn$(G) \leq c$ in time $f(c)n^{O(1)}$?

Main Result

Beyond-planar graph classes usually defined via forbidden patterns



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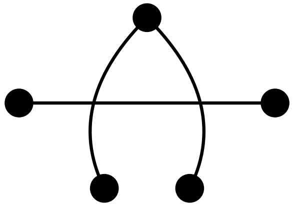

Goal: Combinatorial formalization

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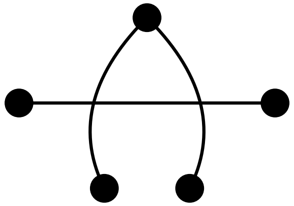

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For any set $\mathcal{F}$ of crossing patterns, testing whether a graph G admits drawing with $\leq c$ crossings that avoids all patterns in $\mathcal{F}$ is FPT w.r.t. c.

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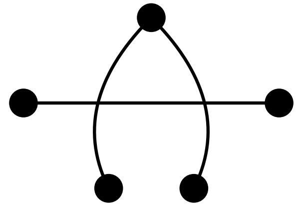

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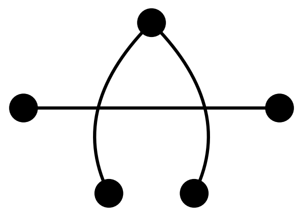

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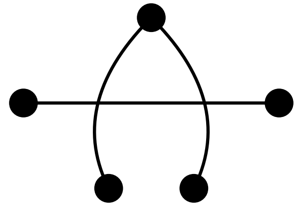

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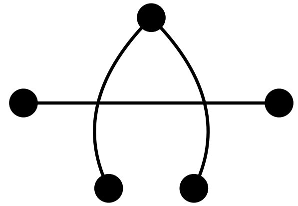

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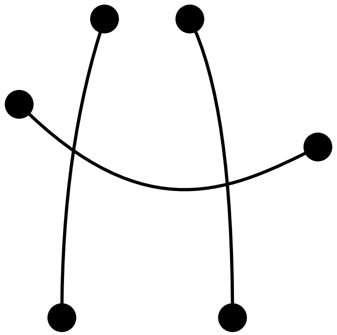
Phase 2: Solve problem on graphs of bounded treewidth via Courcelle

Crossing Patterns



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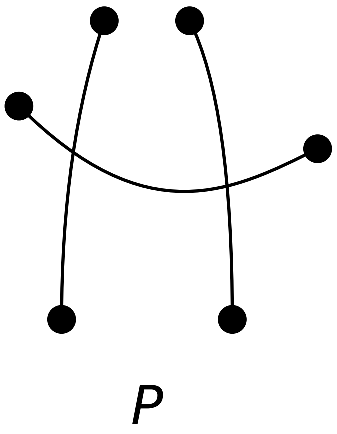

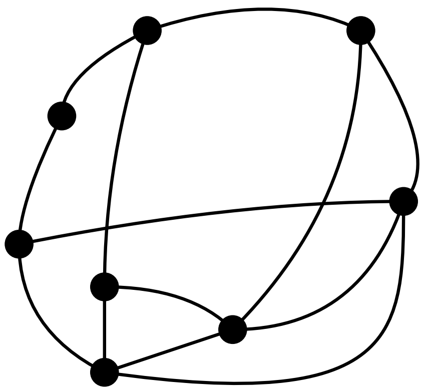

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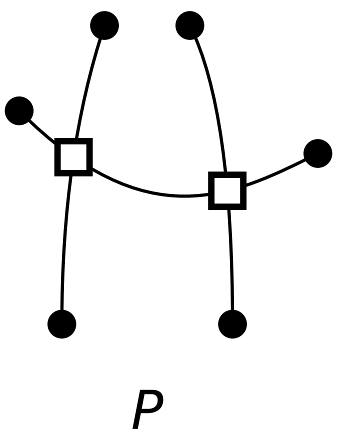

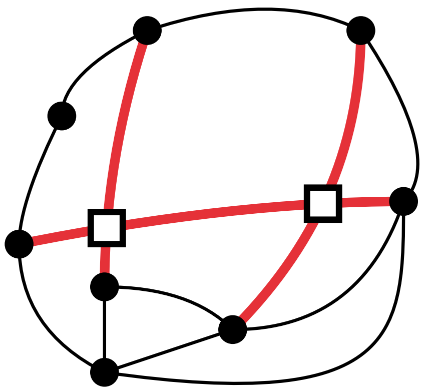

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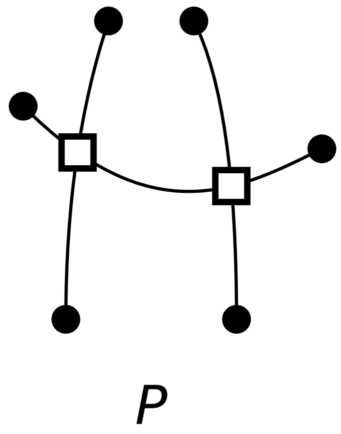
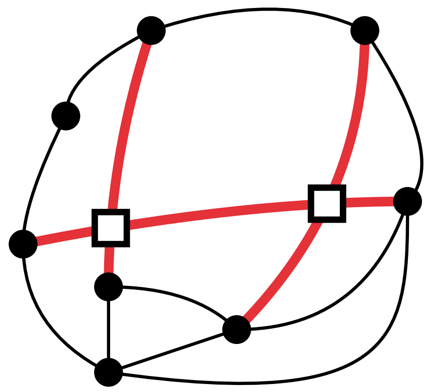
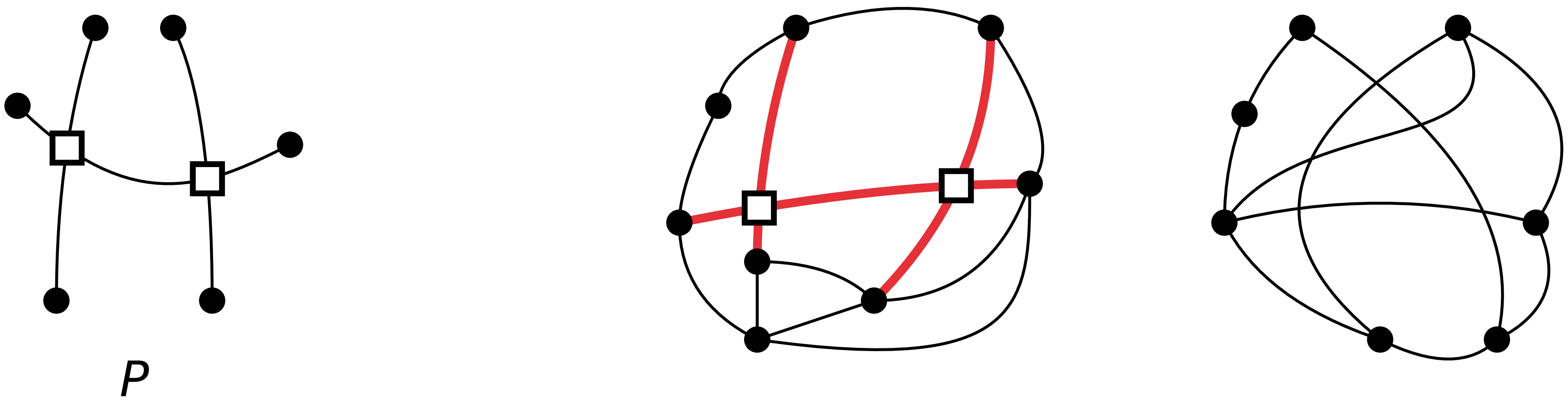




- crossing in $P \rightarrow$ crossing in drawing required

Crossing Patterns



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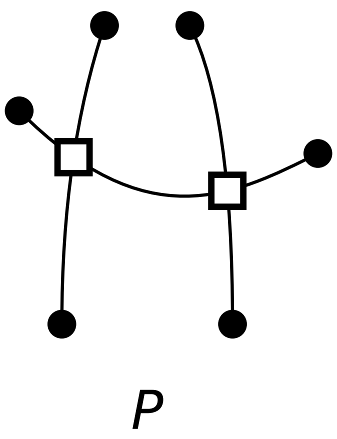

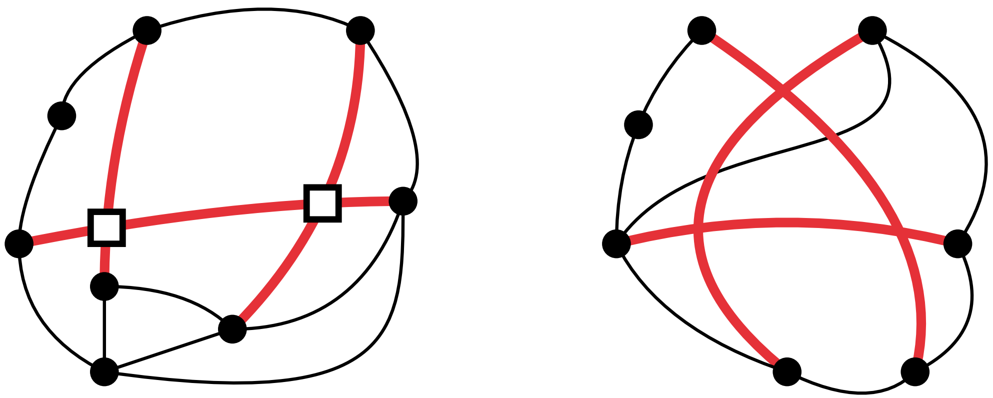

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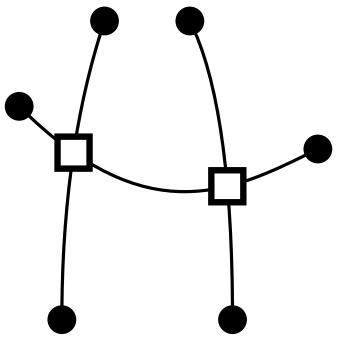

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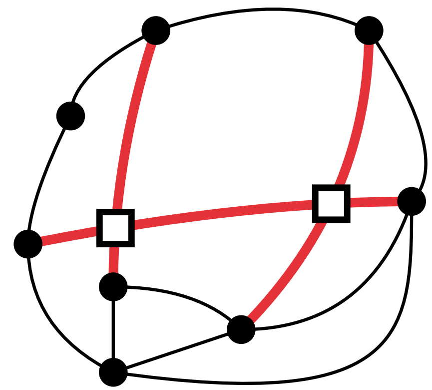

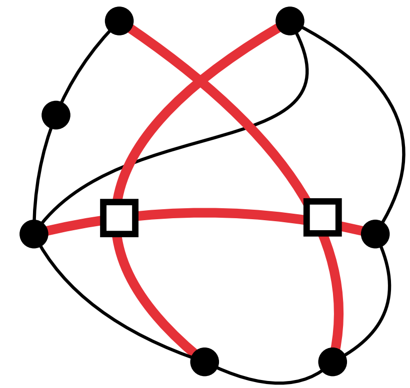

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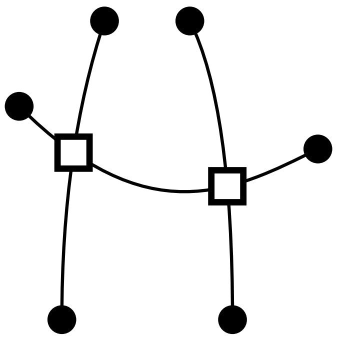

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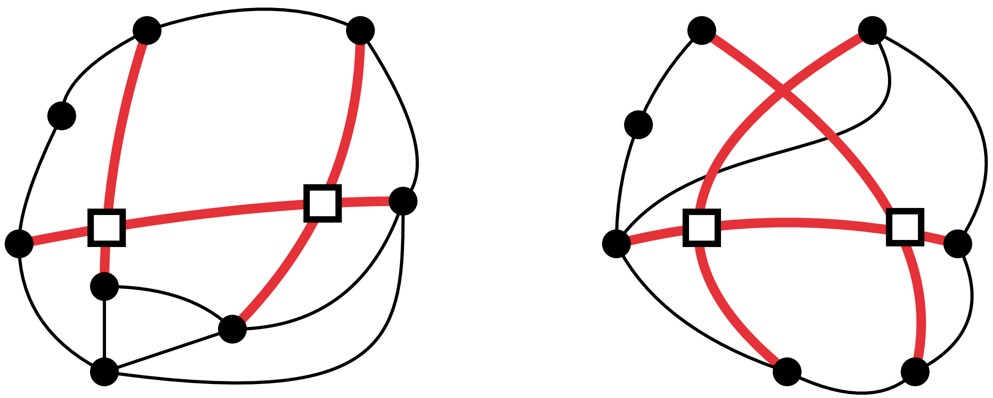

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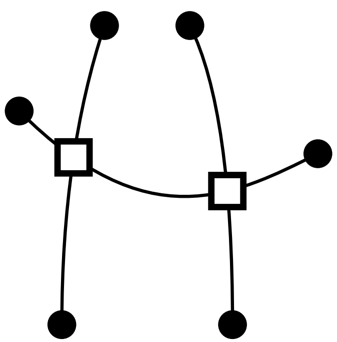

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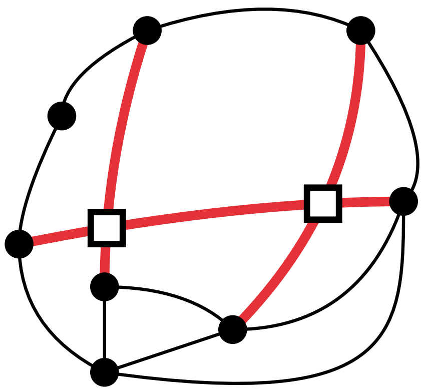

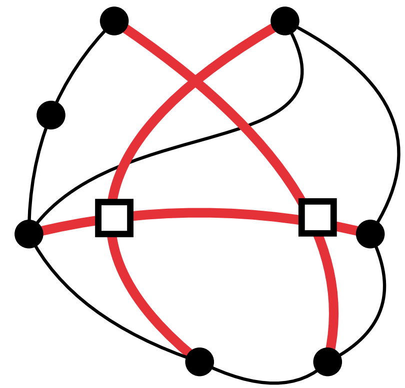

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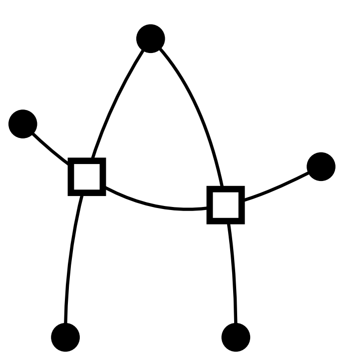
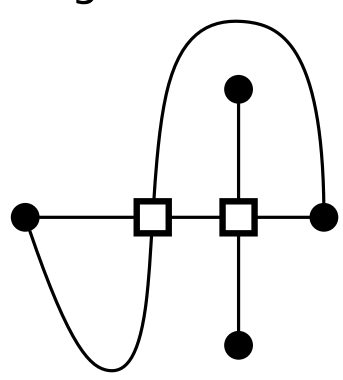
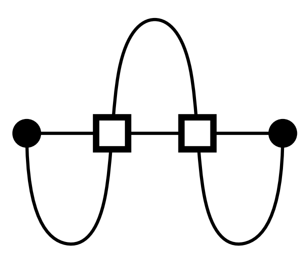
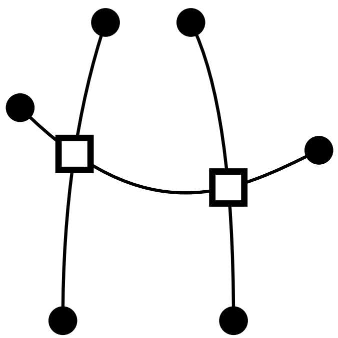
Forbidden in 1-planar drawings:



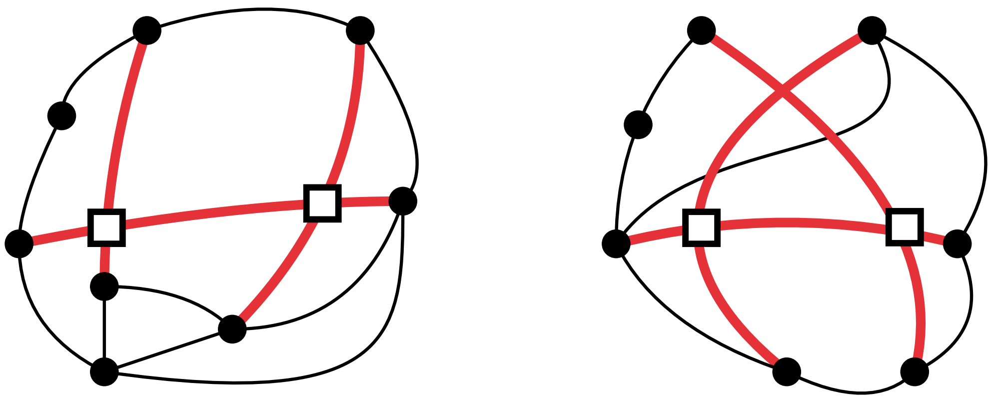




Crossing Patterns



P



- crossing in $P \rightarrow$ crossing in drawing required
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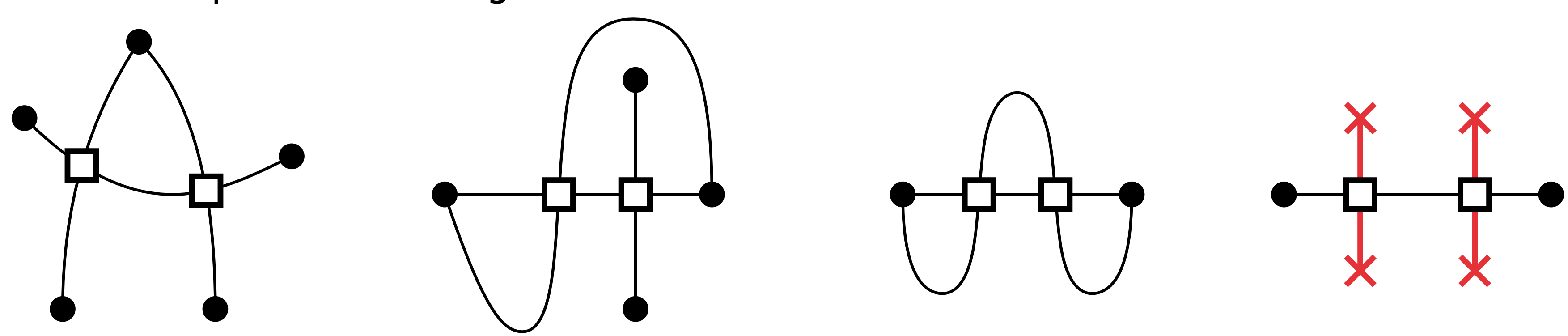

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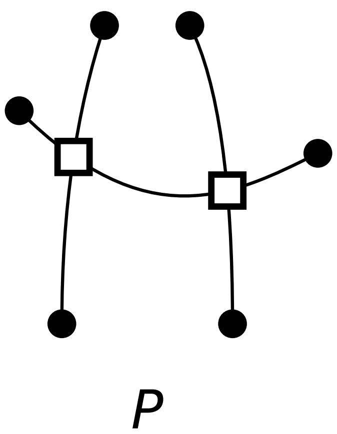

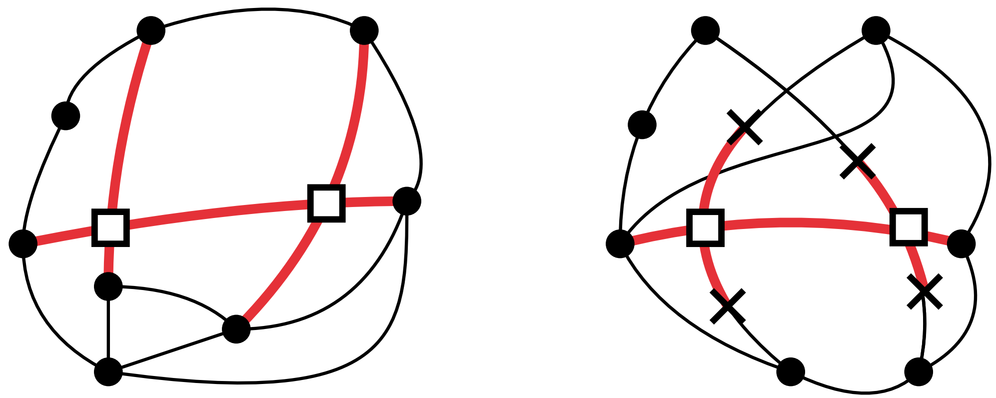

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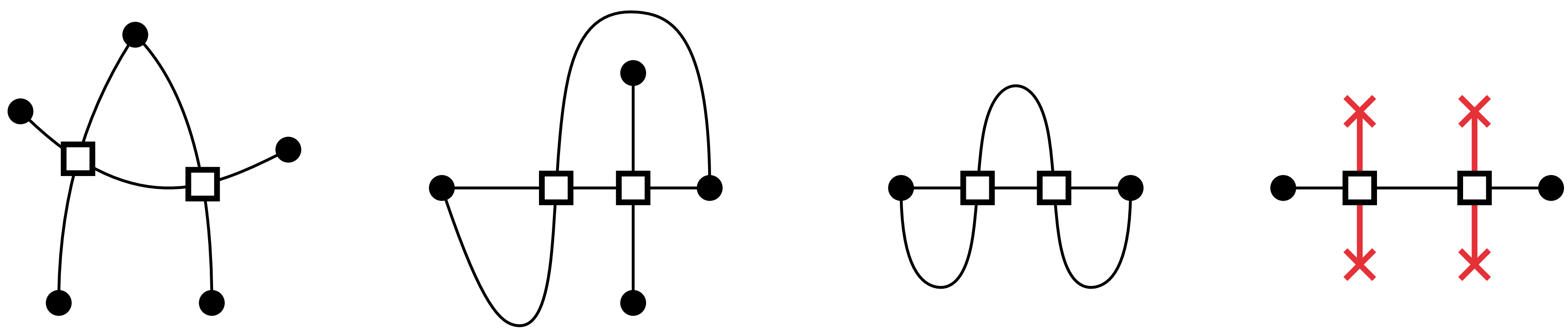

Crossing Patterns – Definition

real / crossing / subdivision

A **crossing pattern** is a graph $P = (V_P, E_P)$ with $V_P = R \cup C \cup S$ s.t.

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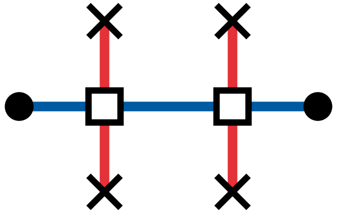

1-planar

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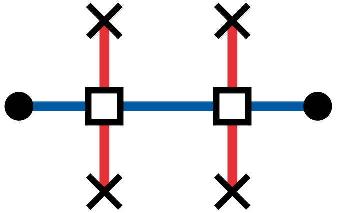

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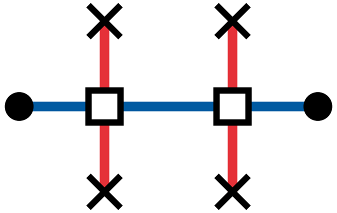

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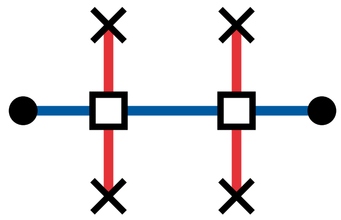
Forbidden Patterns



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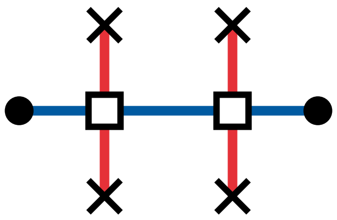

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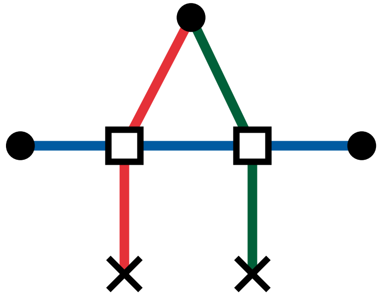

fan-crossing free

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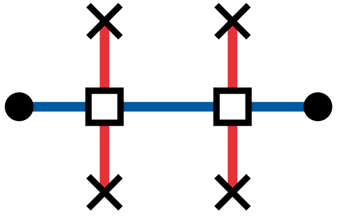

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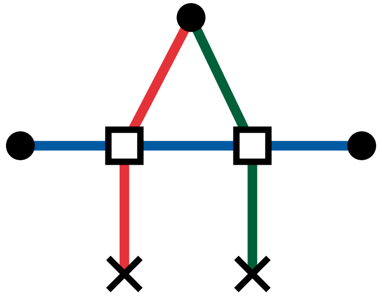

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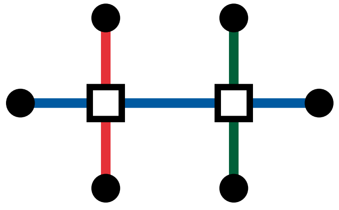

fan-crossing

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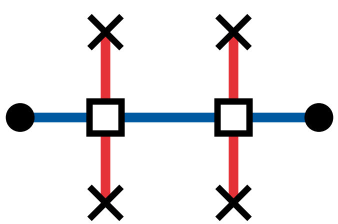

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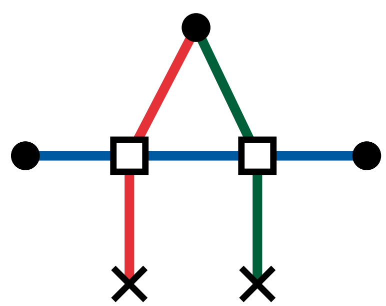
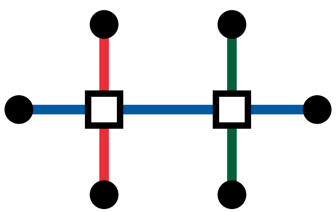
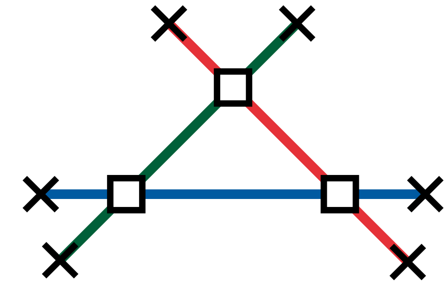


fan-crossing free



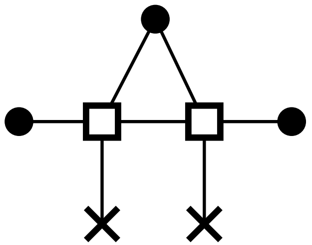
fan-crossing

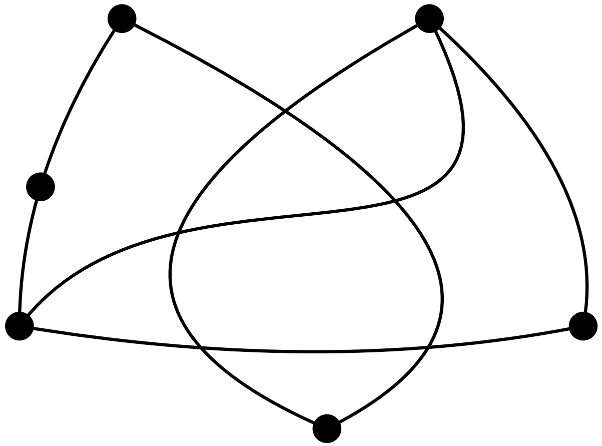


quasi-planar

Containing a Pattern

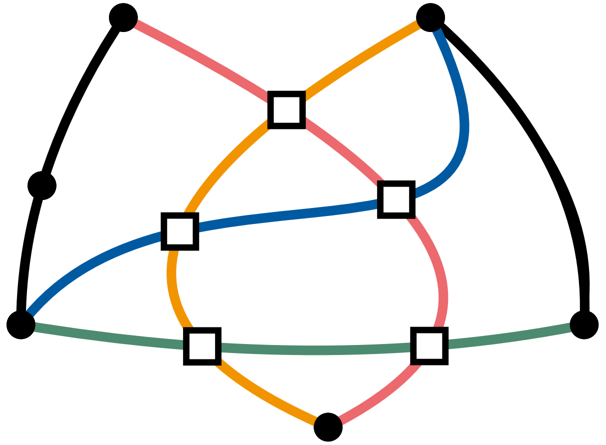
Fan-crossing free?

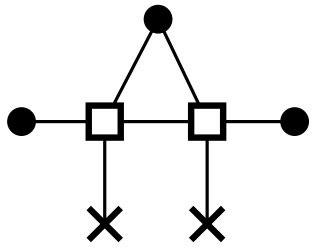




Containing a Pattern

Fan-crossing free?





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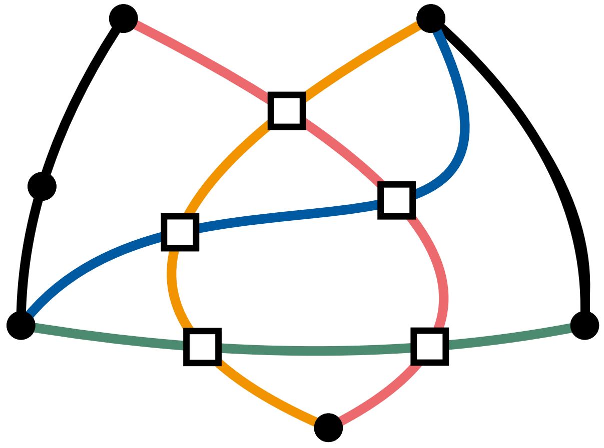
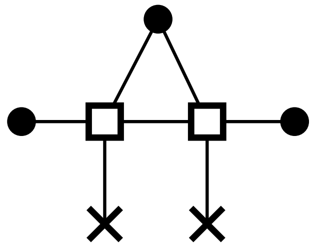
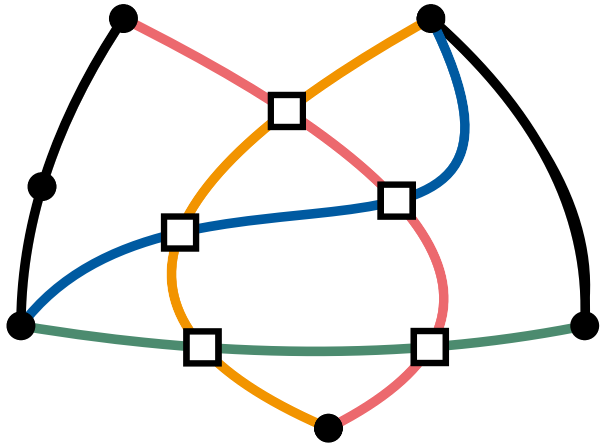


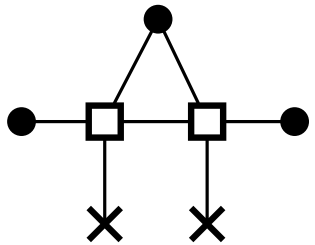


Allowed Operations:

(*i*) delete isolated vertices

Fan-crossing free?





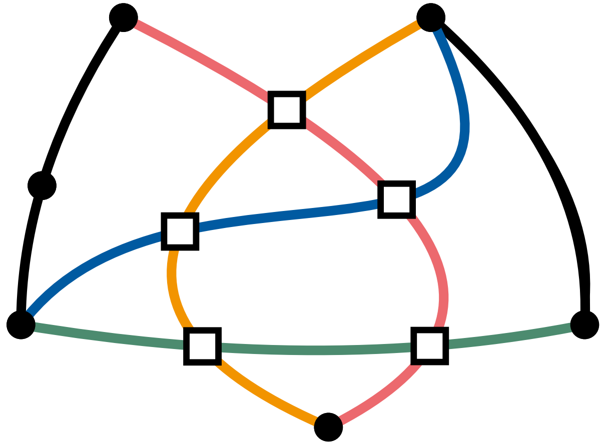
Allowed Operations:

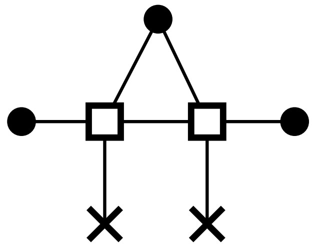
(*i*) delete isolated vertices

(*ii*) subdivide edge by introducing subdivision vertex

Containing a Pattern

Fan-crossing free?





Allowed Operations:

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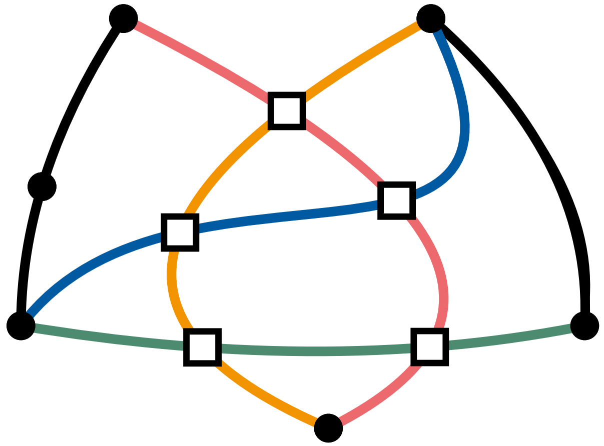

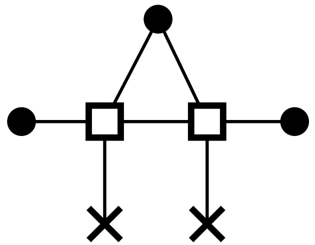

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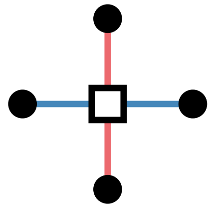

Containing a Pattern

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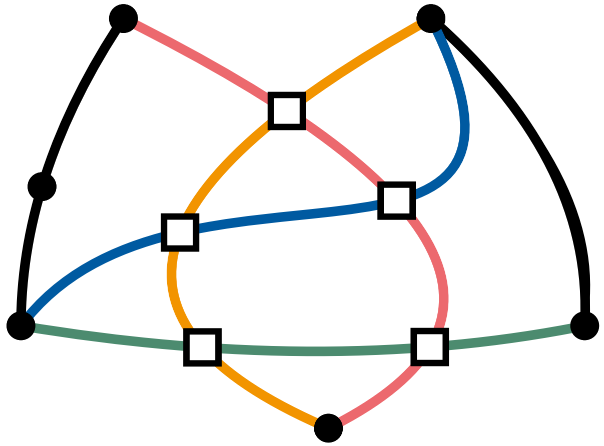

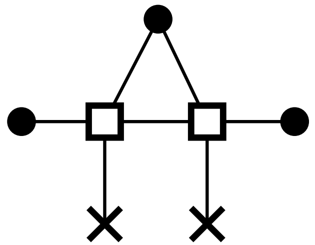

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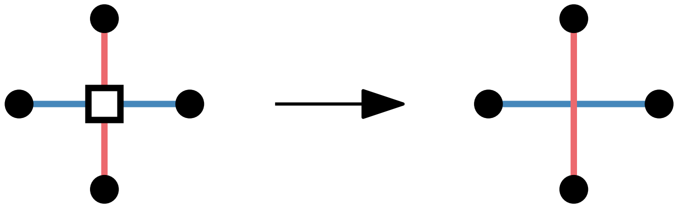

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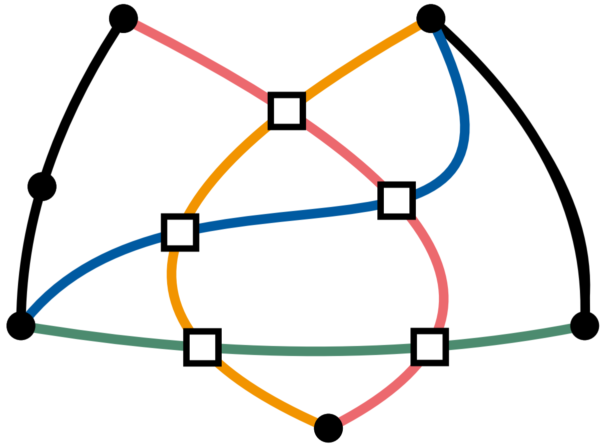

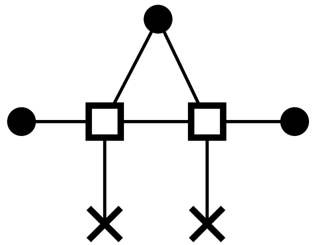

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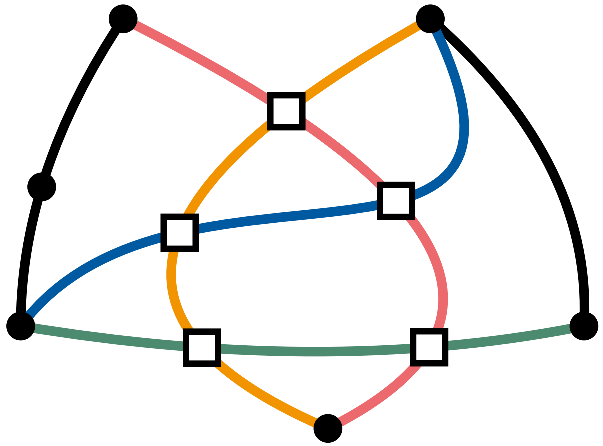

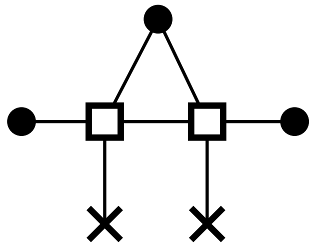

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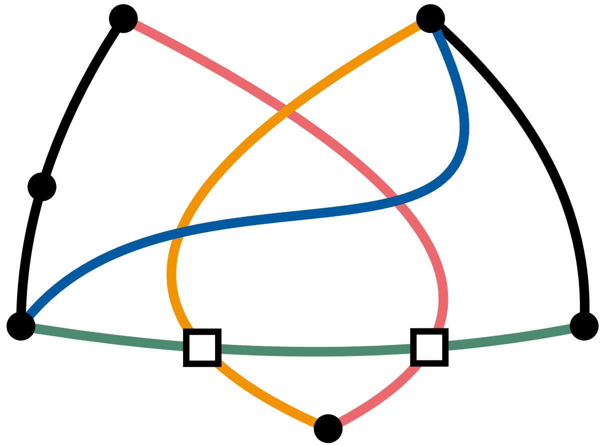

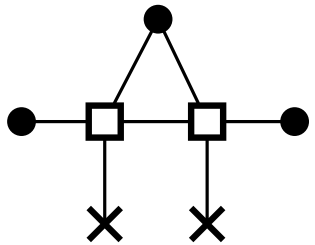

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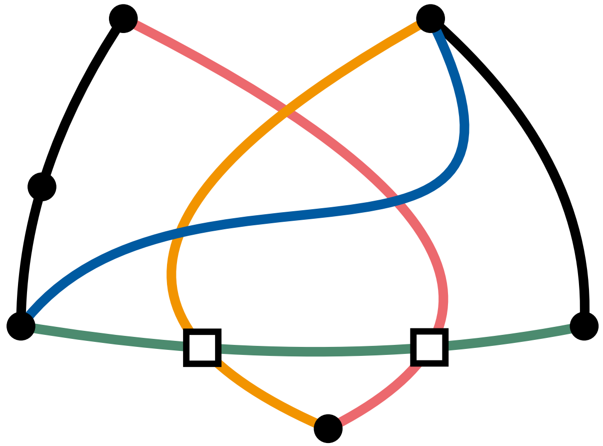

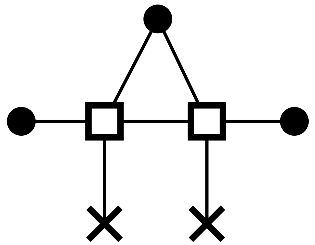

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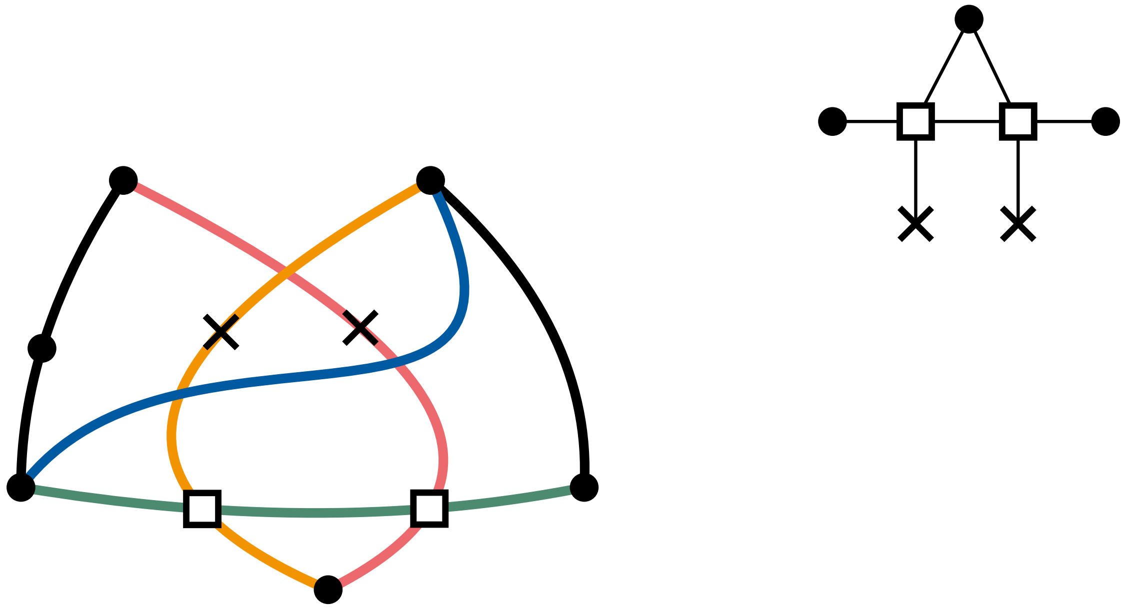

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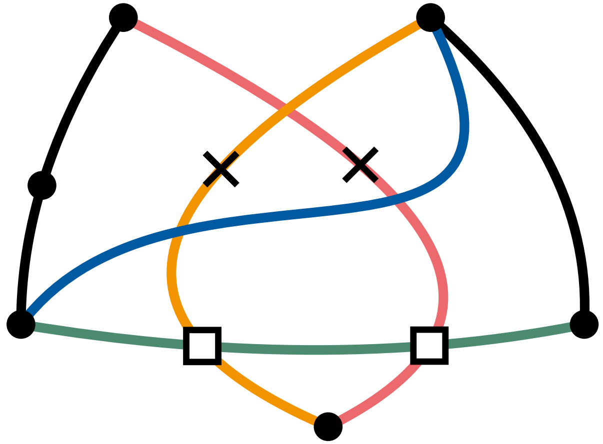

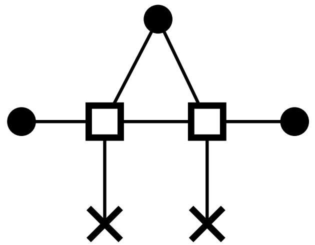

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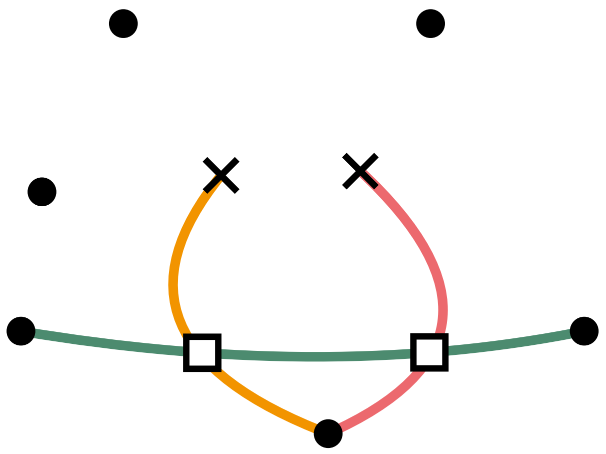

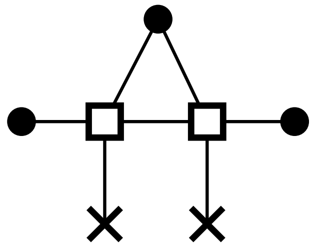

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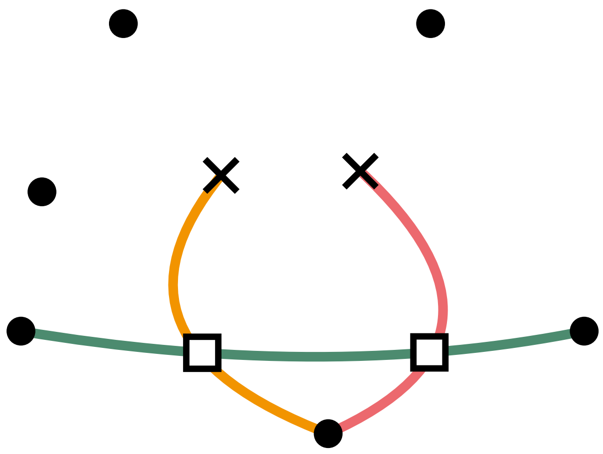

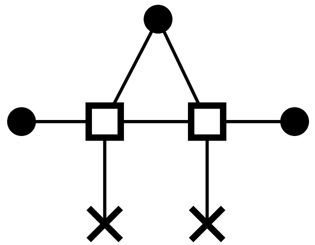

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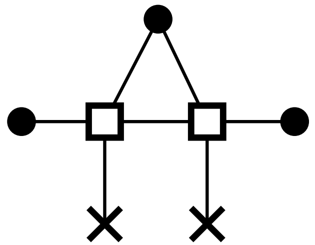

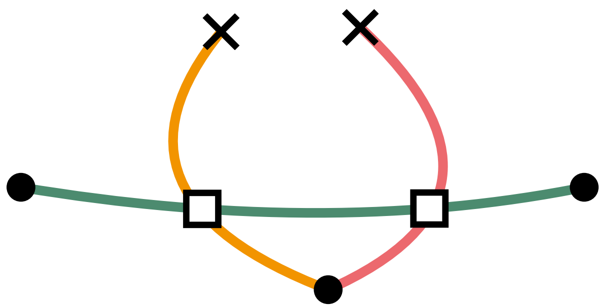

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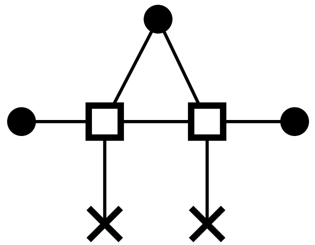
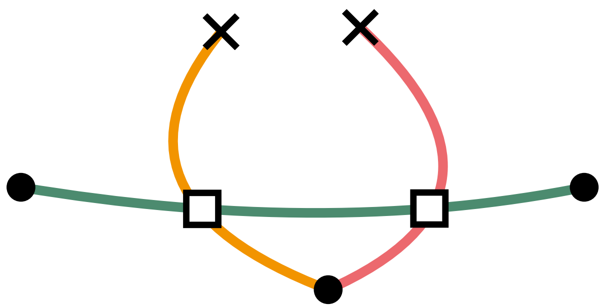
Containing a Pattern

Fan-crossing free? No!





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Bounding Treewidth

Goal:

For any fixed set $\mathcal{F}$ of crossing patterns, testing whether a graph G admits drawing with at most c crossings that avoids all patterns in $\mathcal{F}$ is FPT with respect to c.

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There is a linear-time algo that, given a graph G, either

- recognizes: every $\mathcal{F}$-free drawing has $> c$ crossings,

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Lemma [Grohe]

There is a linear-time algo that, given a graph G, either

- recognizes: every $\mathcal{F}$-free drawing has $> c$ crossings, reject!
- recognizes $\mathrm{tw}(G) \leq w$ or move to Phase 2

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For any fixed set $\mathcal{F}$ of crossing patterns, testing whether a graph G admits drawing with at most c crossings that avoids all patterns in $\mathcal{F}$ is FPT with respect to c.

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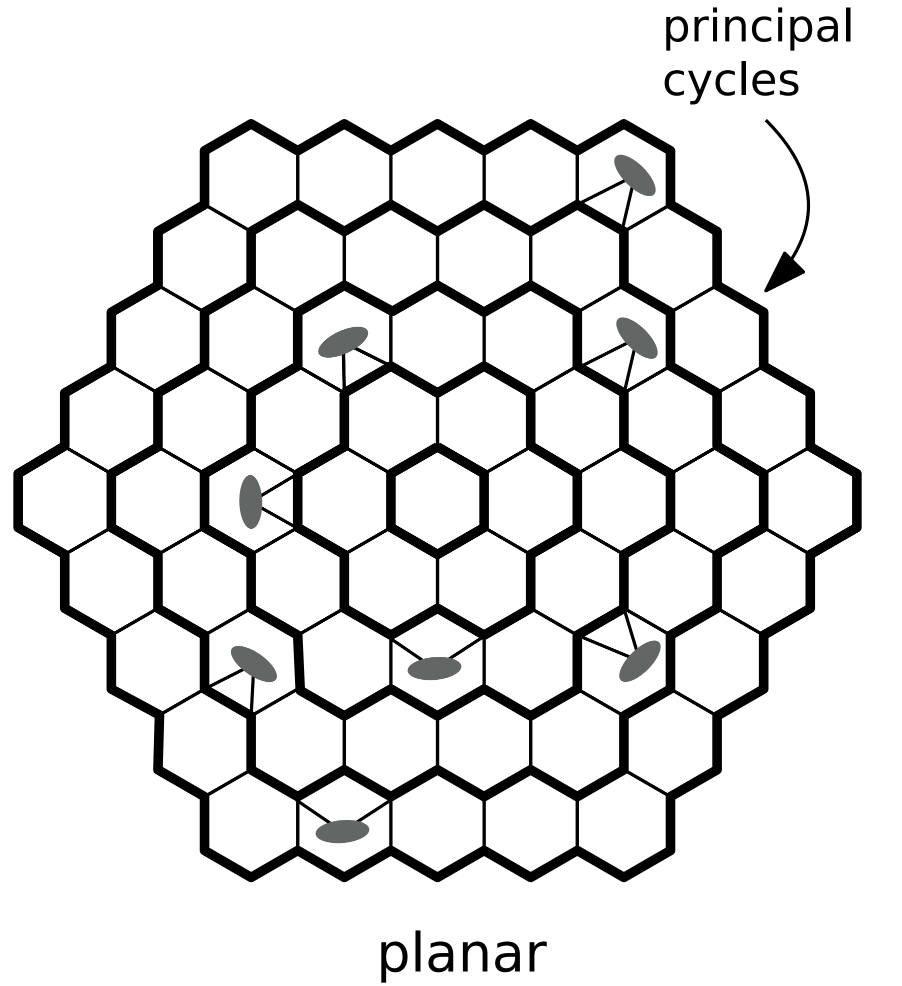
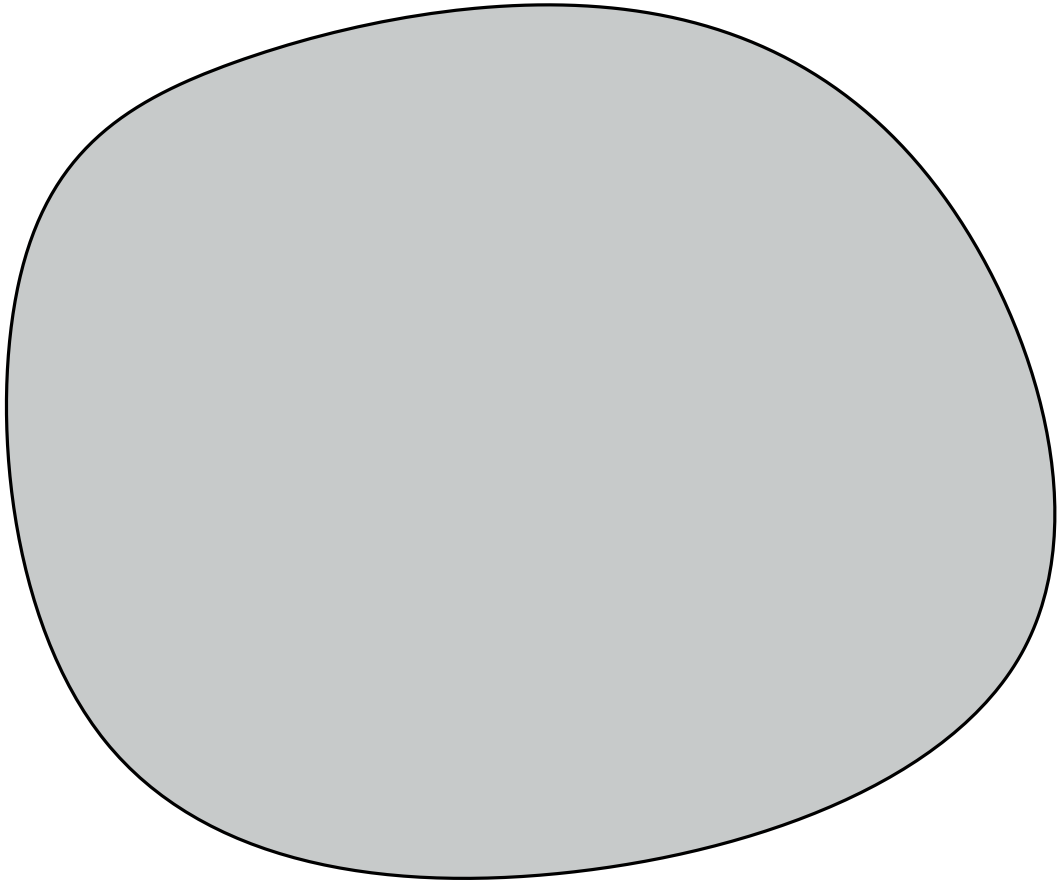
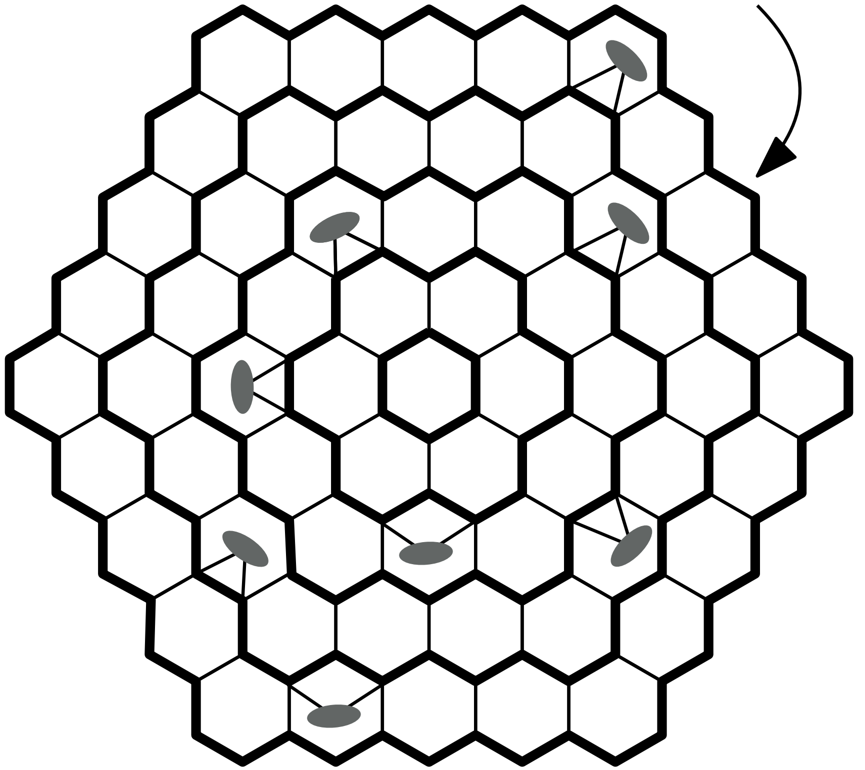
Bounding Treewidth



drawing of G with $\leq c$ crossings



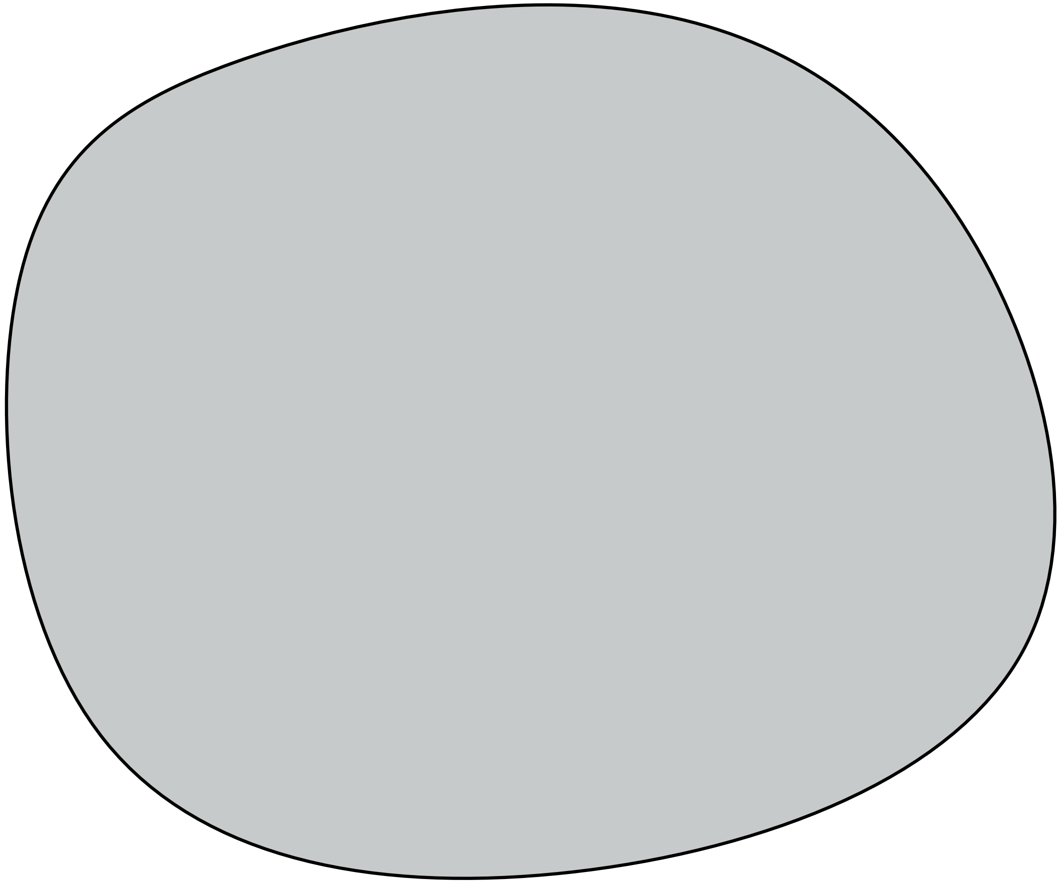
principal cycles

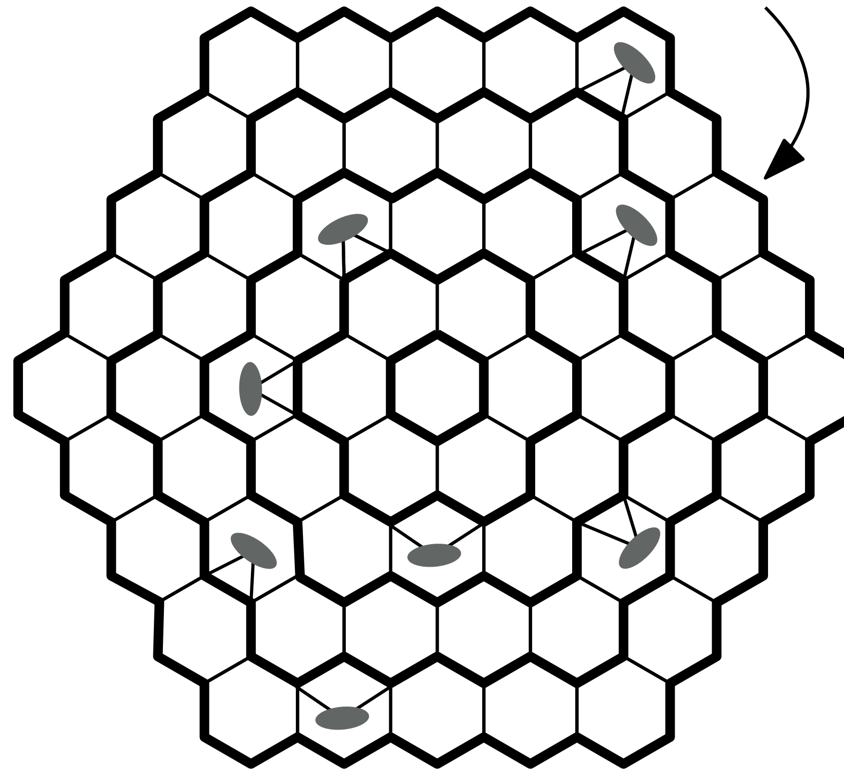


planar

Bounding Treewidth

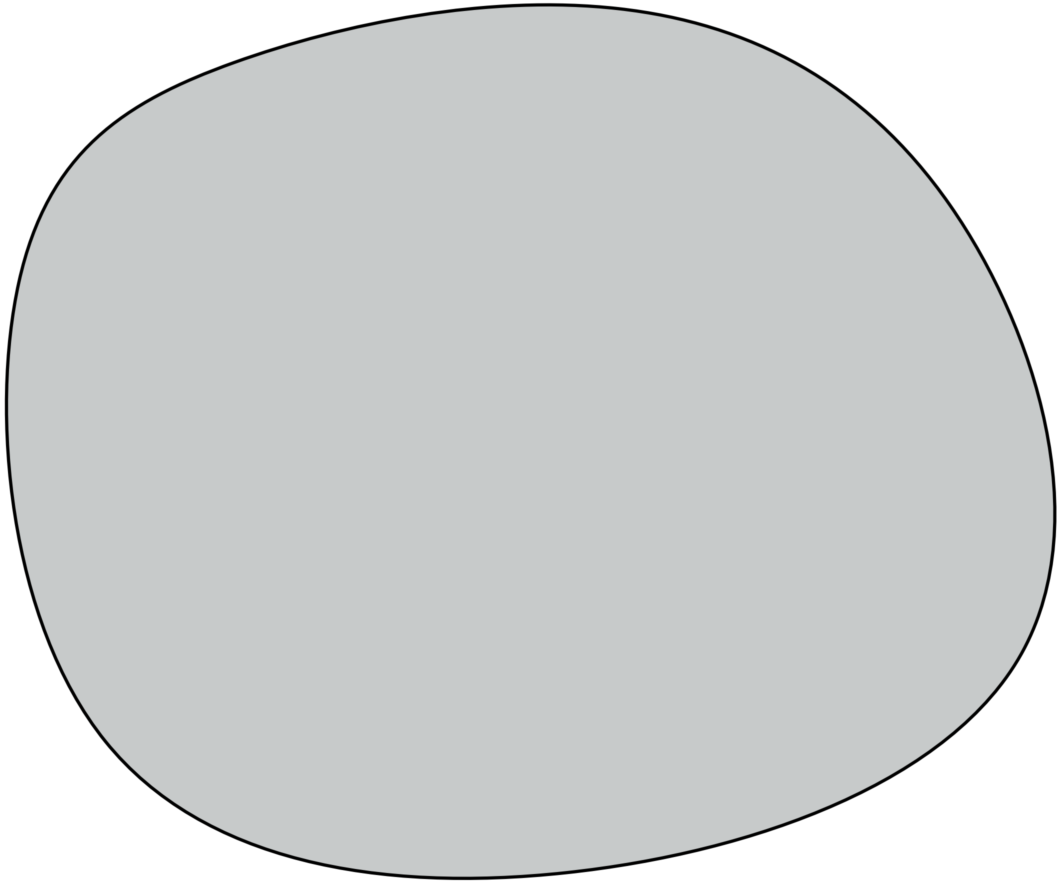
drawing of G with $\leq c$ crossings

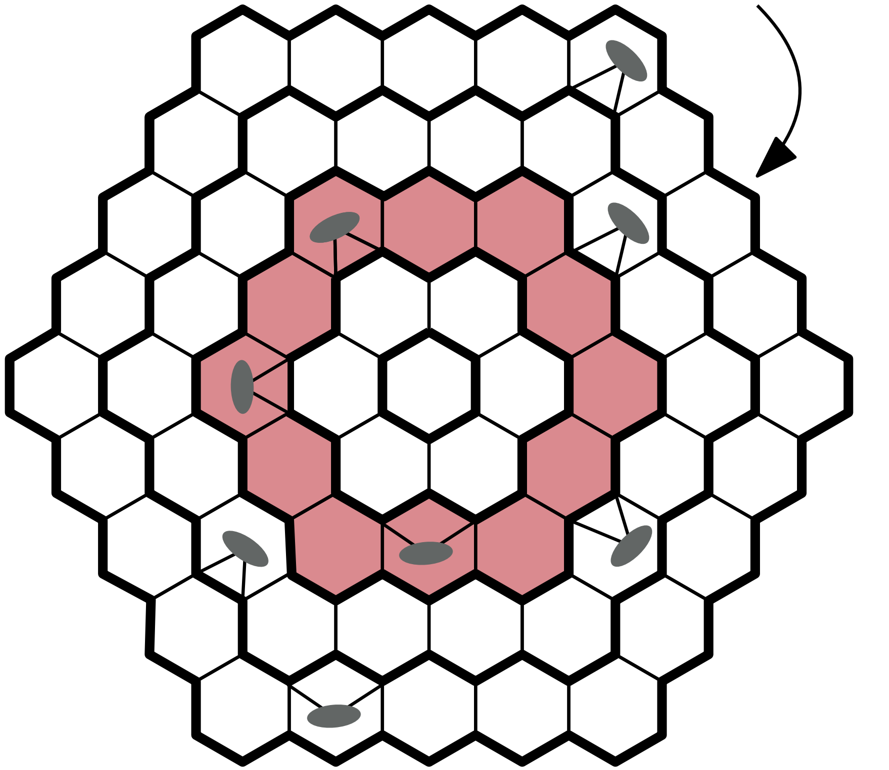


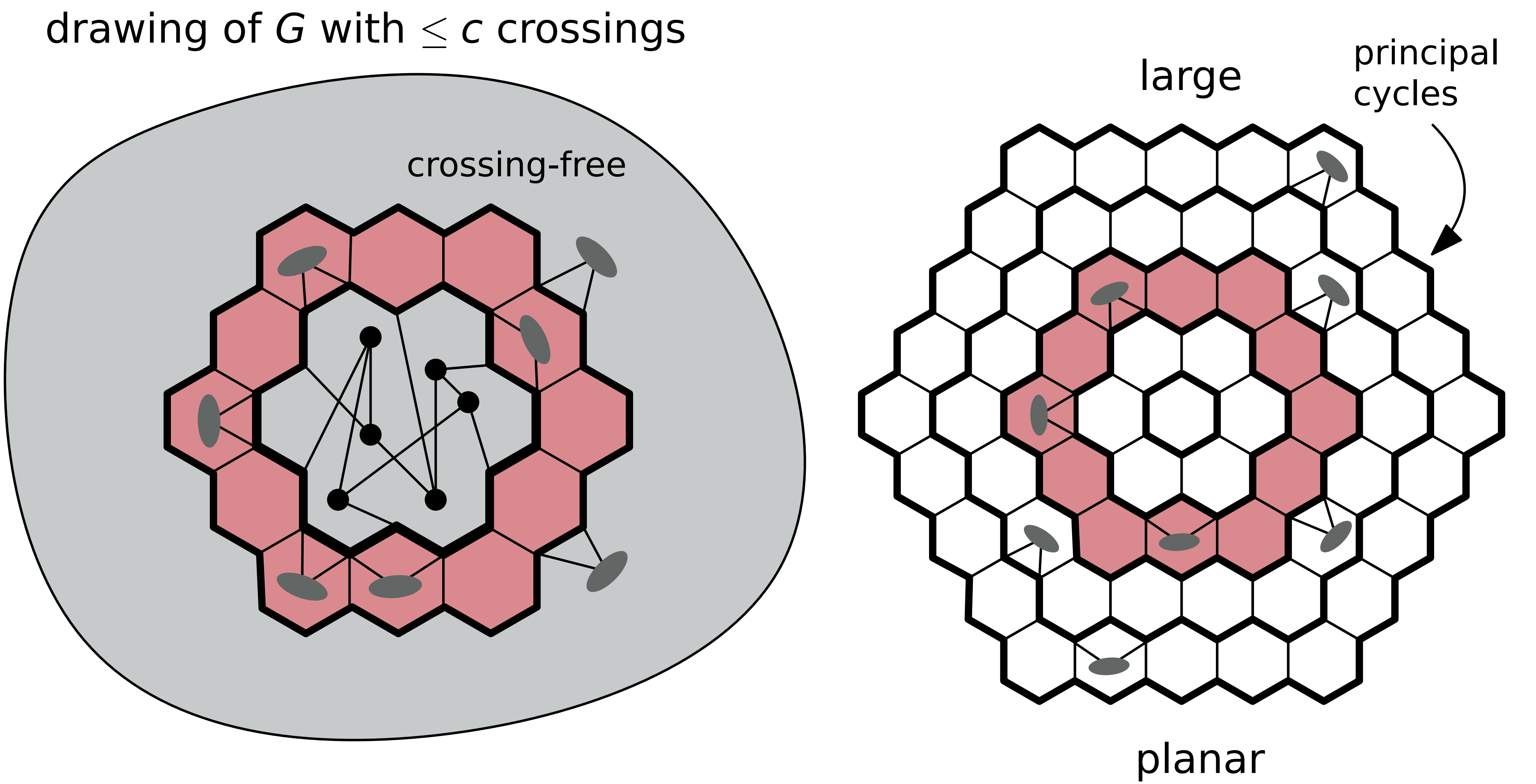


Bounding Treewidth

drawing of G with $\leq c$ crossings

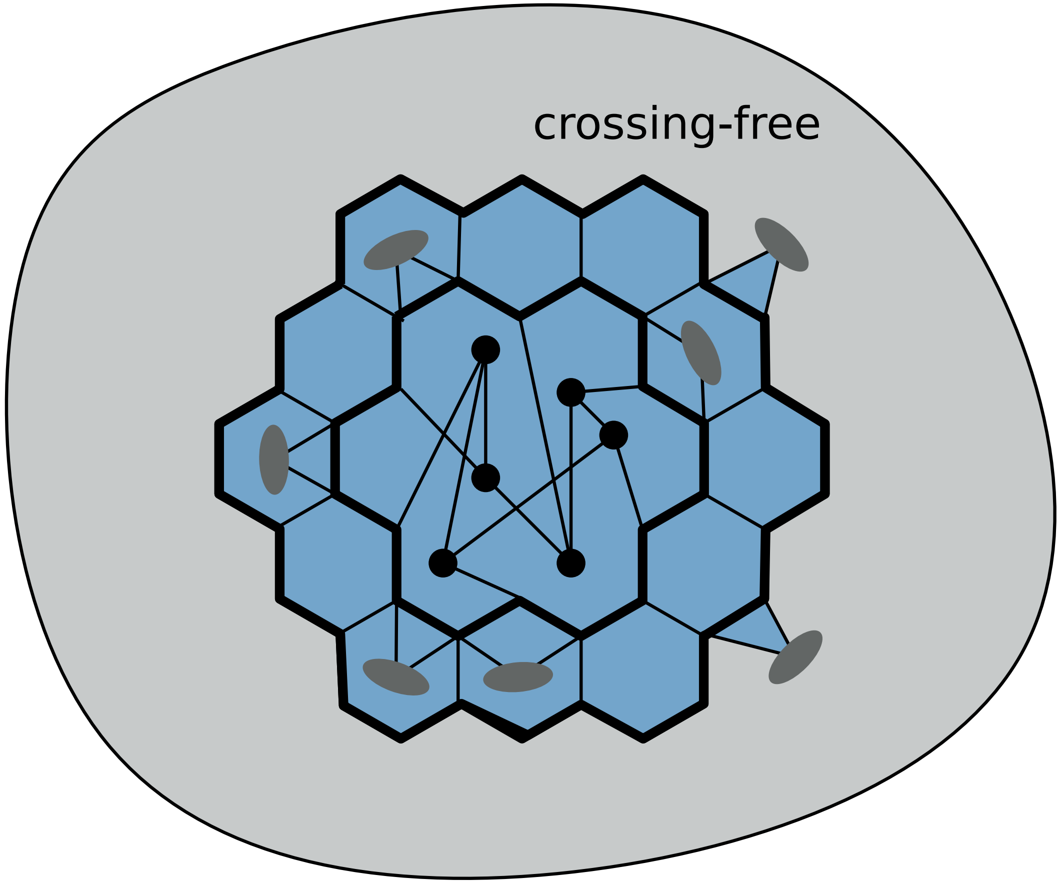


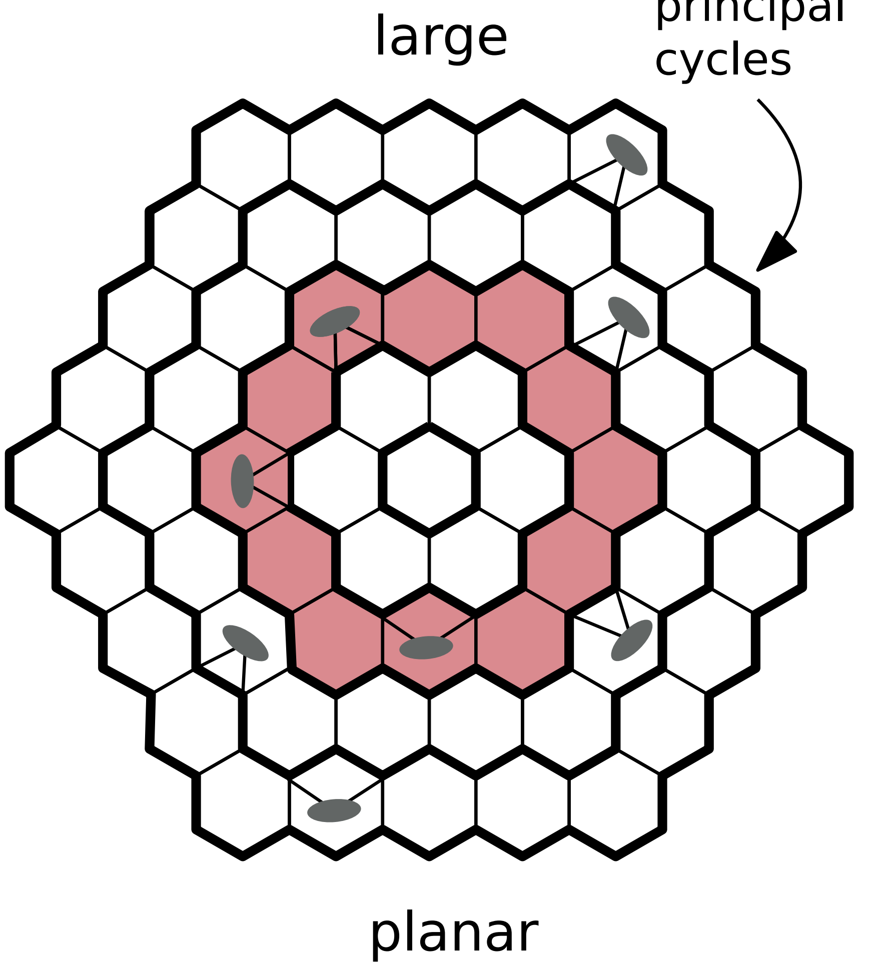



drawing of G with $\leq c$ crossings
crossing-free
large
principal cycles
planar

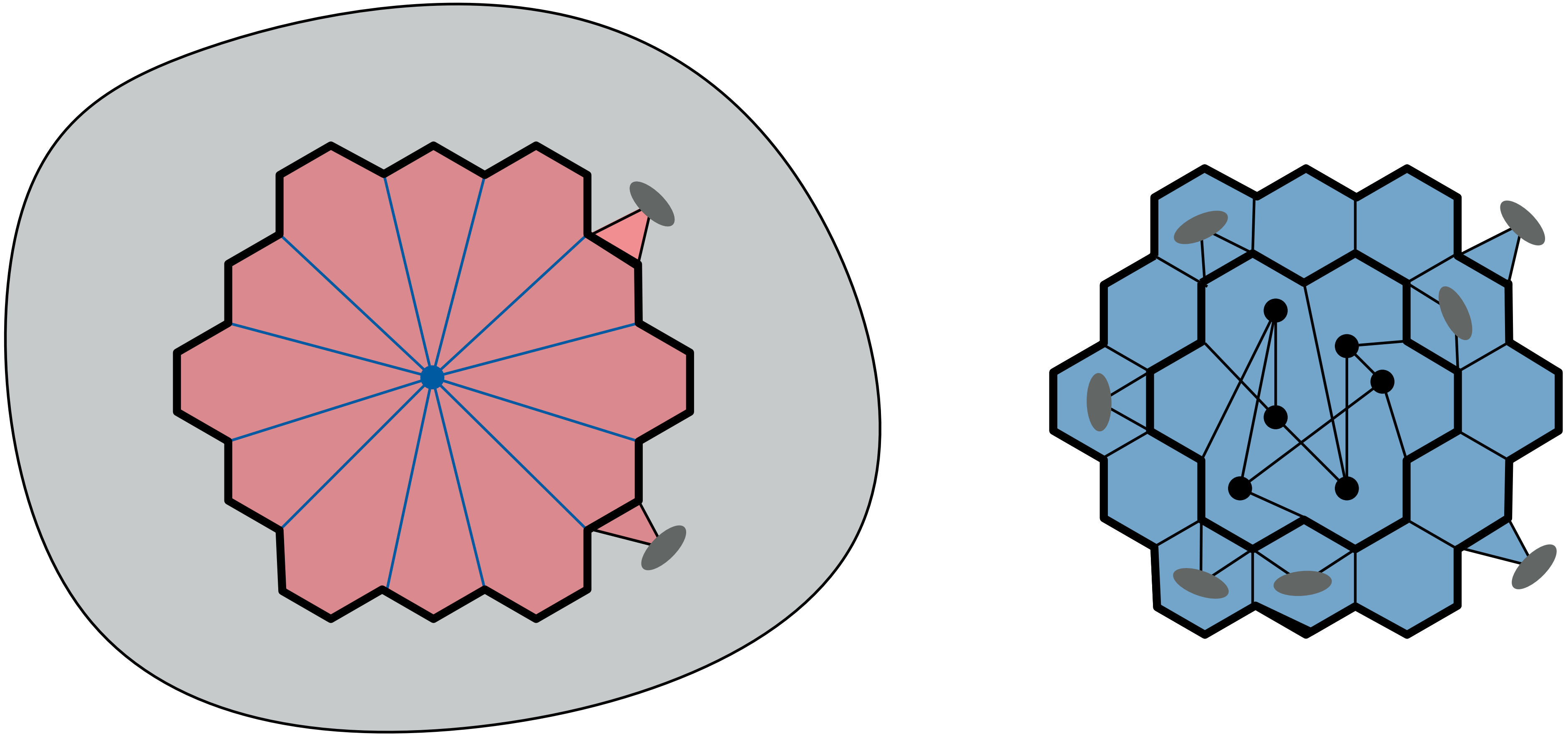
Bounding Treewidth

drawing of G with $\leq c$ crossings



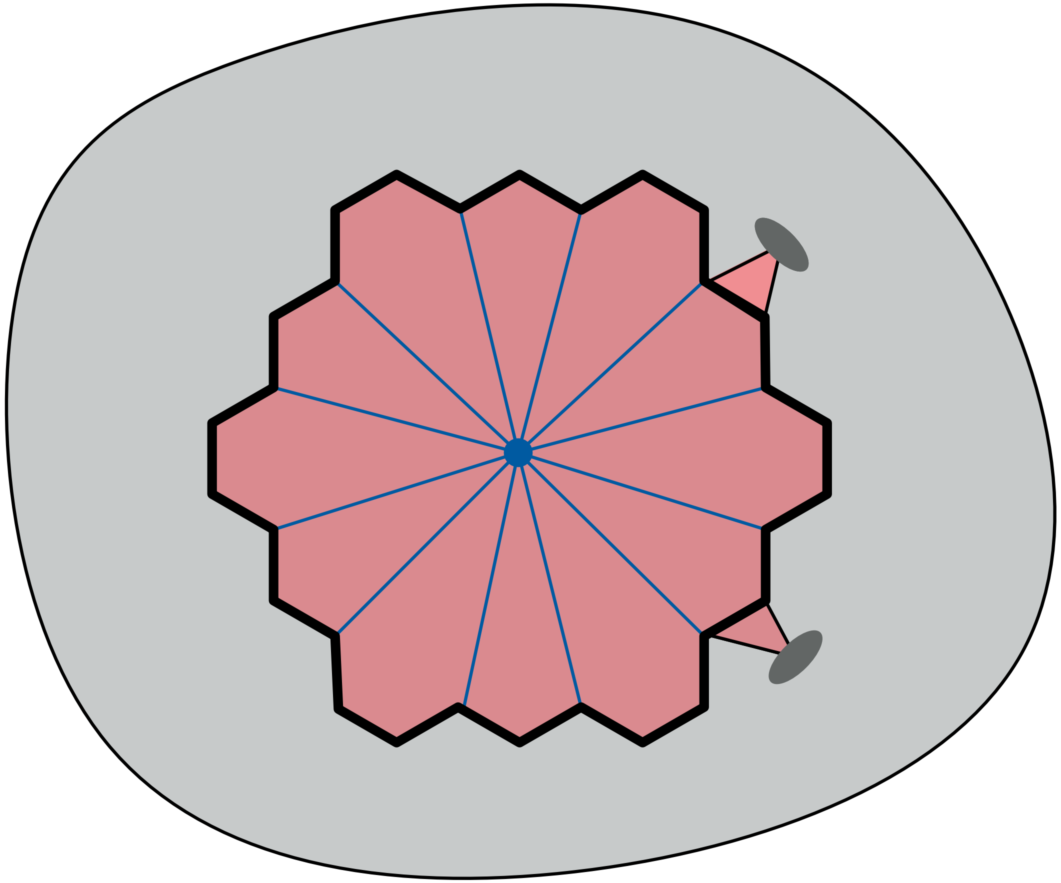


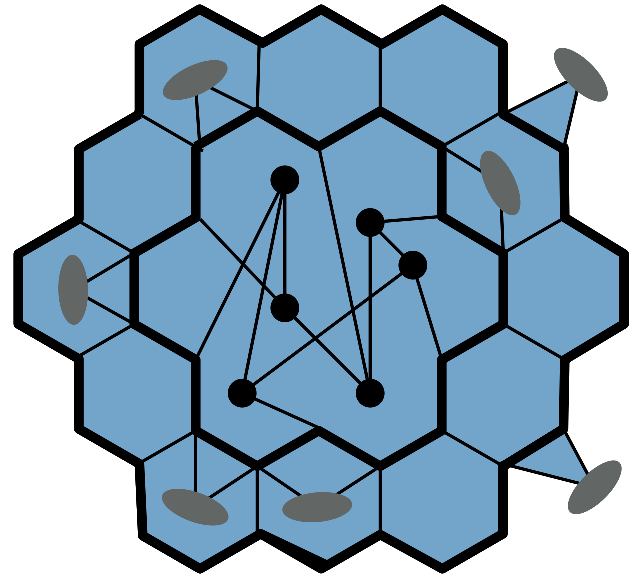
drawing of G' with $\leq c$ crossings



Bounding Treewidth

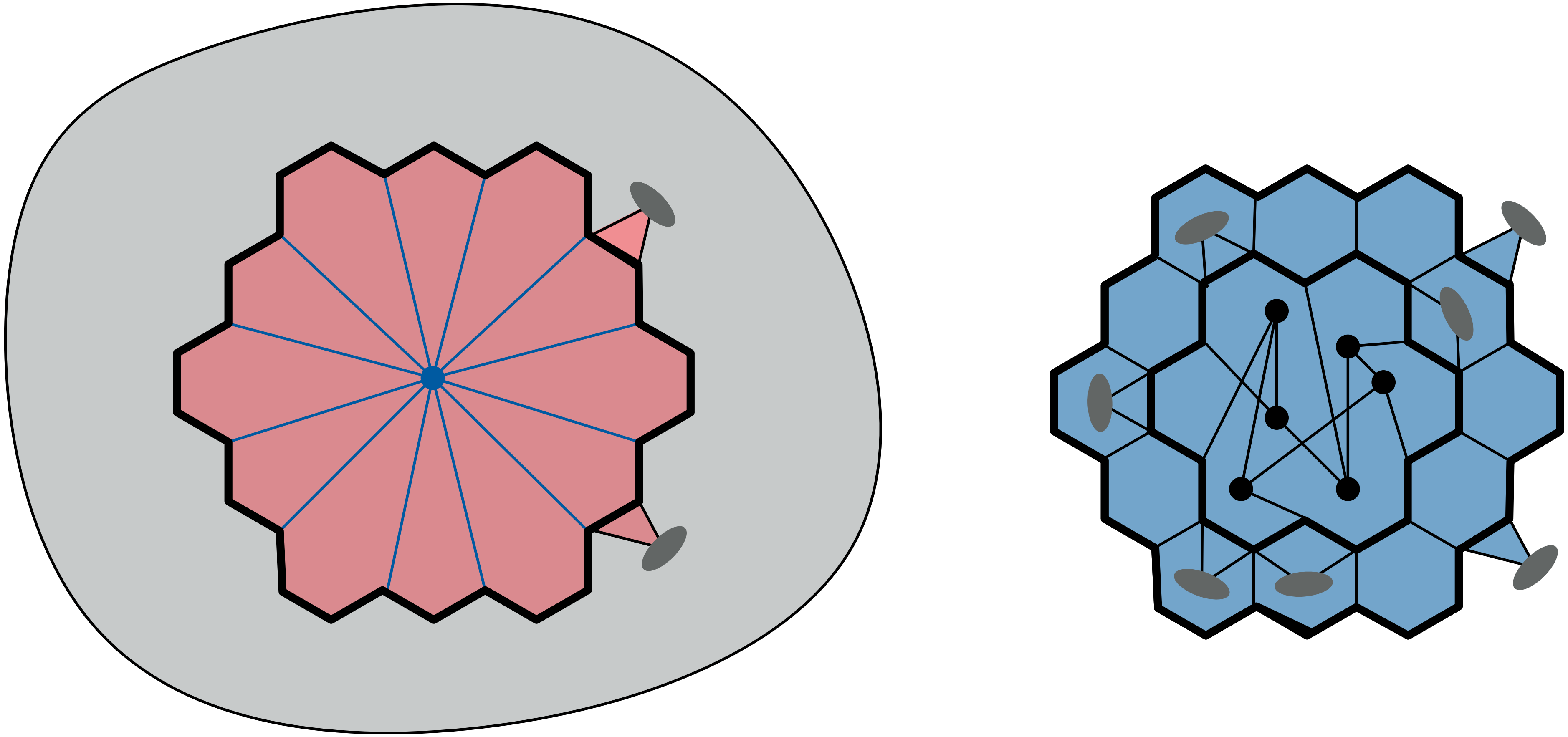
drawing of G' with $\leq c$ crossings





- $|V(G')| < |V(G)|$

drawing of G' with $\leq c$ crossings



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G admits drawing with $\leq$ c crossings that avoids all patterns in $\mathcal{F}$

Bounding Treewidth

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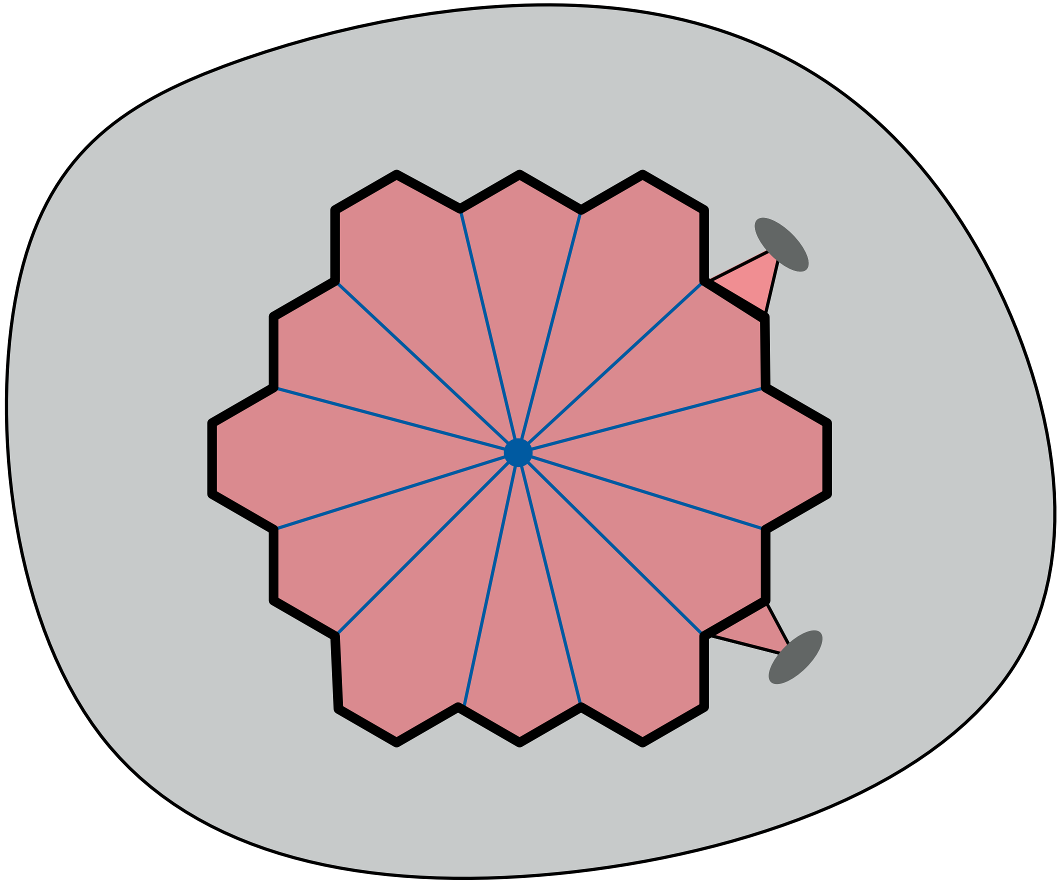

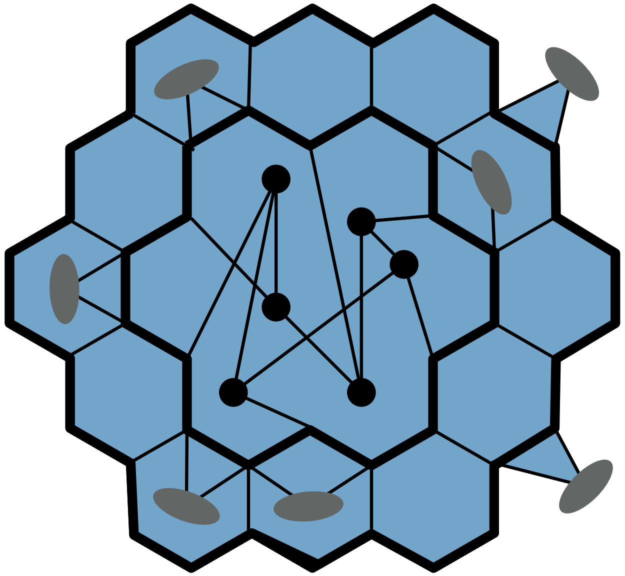

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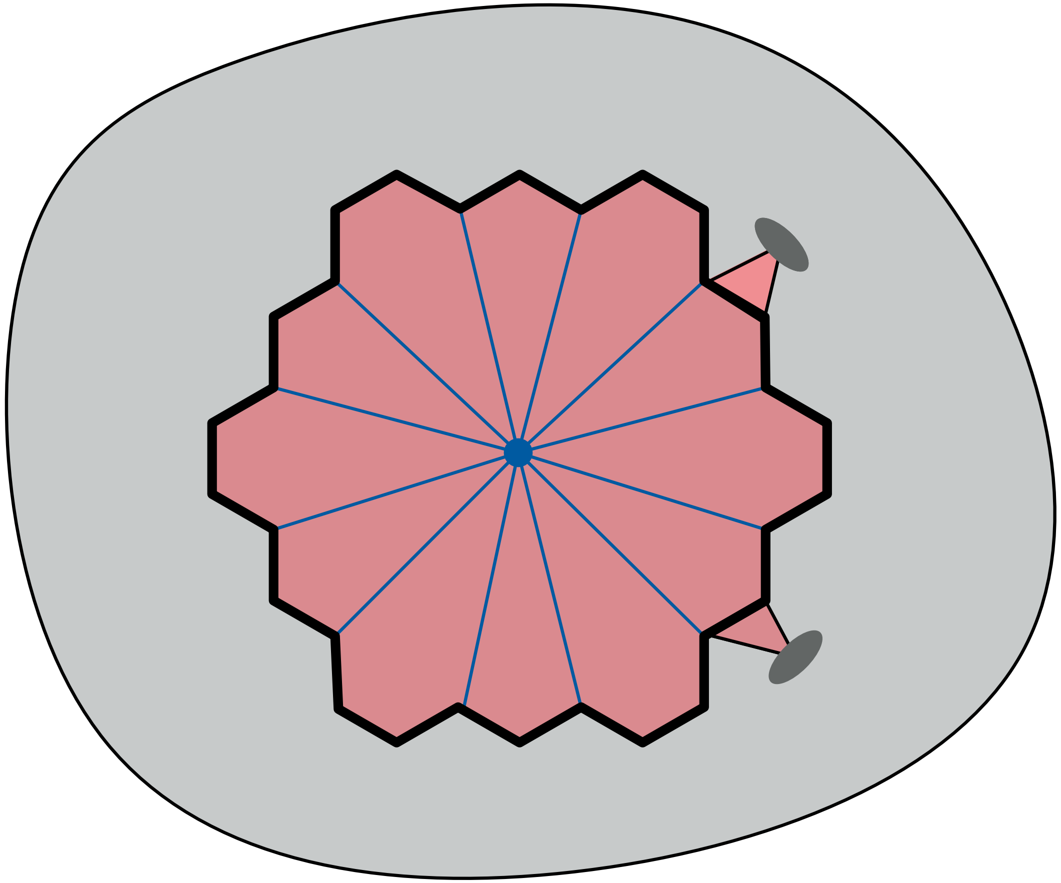
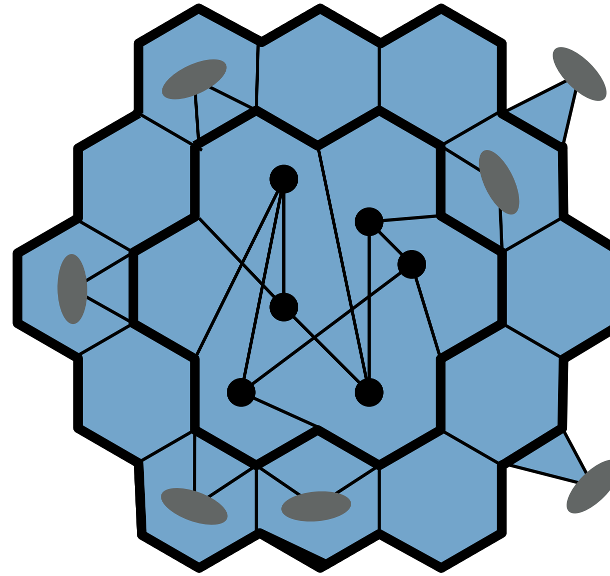
G admits drawing with $\leq$ c crossings that avoids all patterns in $\mathcal{F}$

$\Rightarrow$ G' admits drawing with $\leq c$ crossings that avoids all patterns in $\mathcal{F}$

drawing of G' with $\leq c$ crossings





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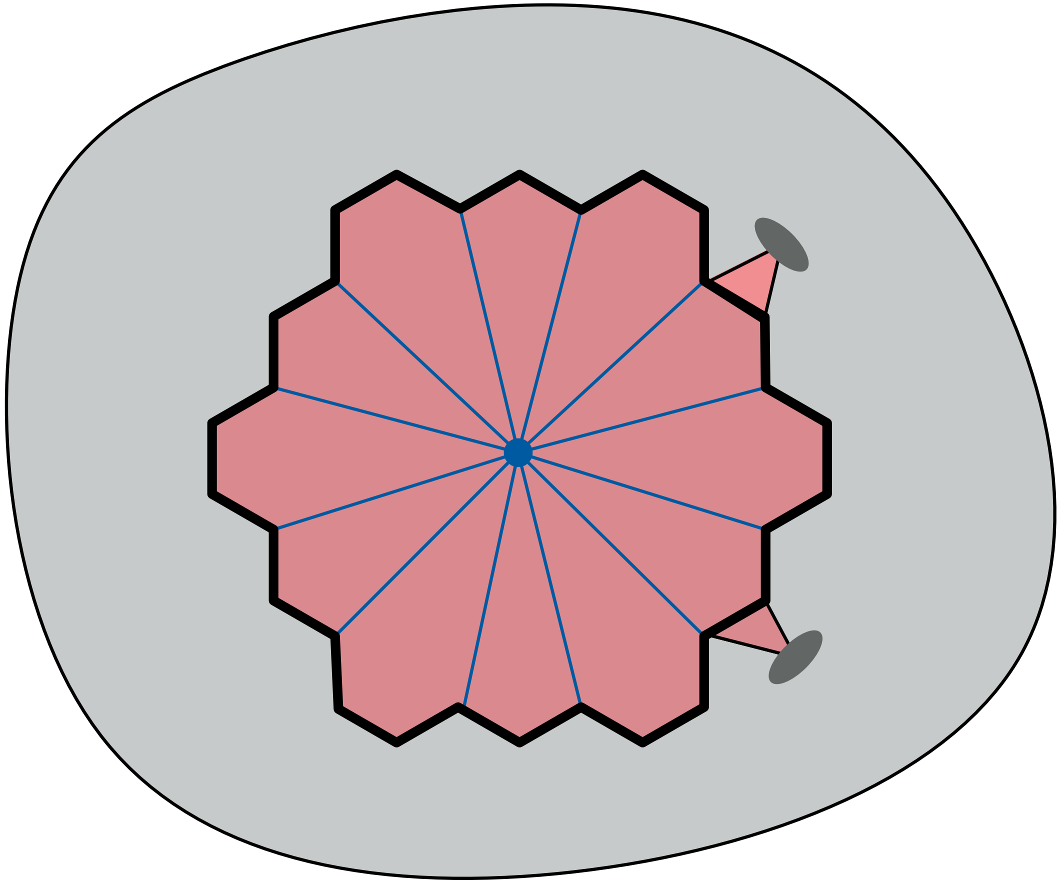

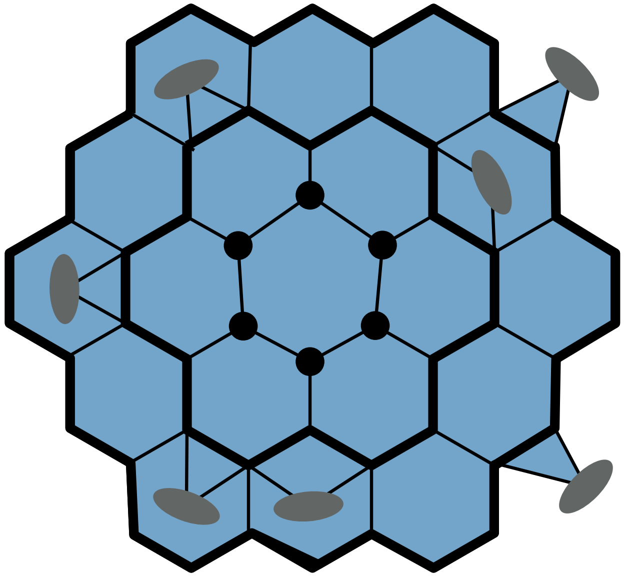

planar

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Bounding Treewidth

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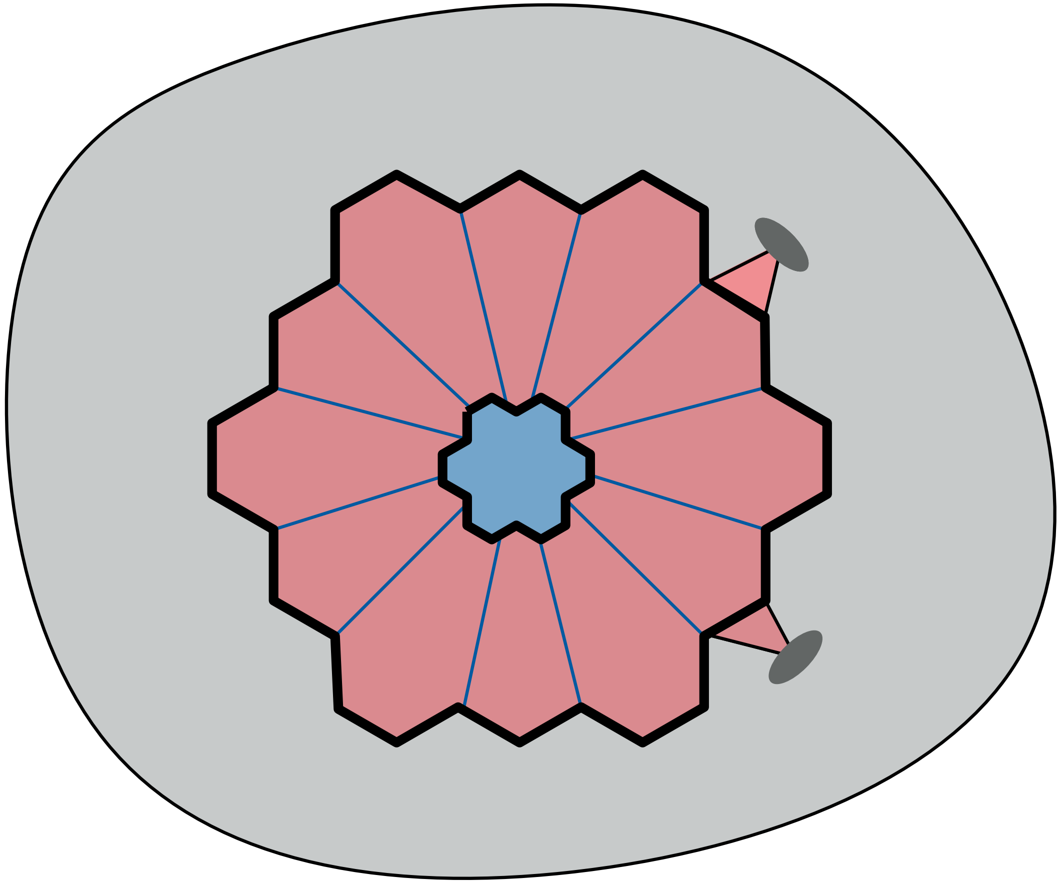

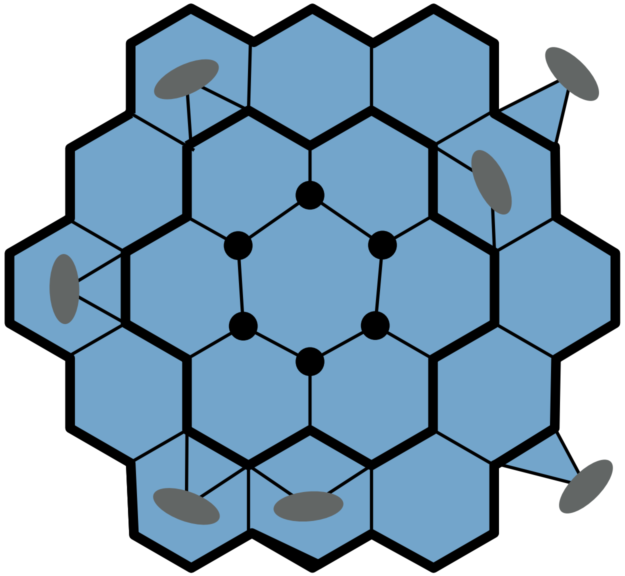

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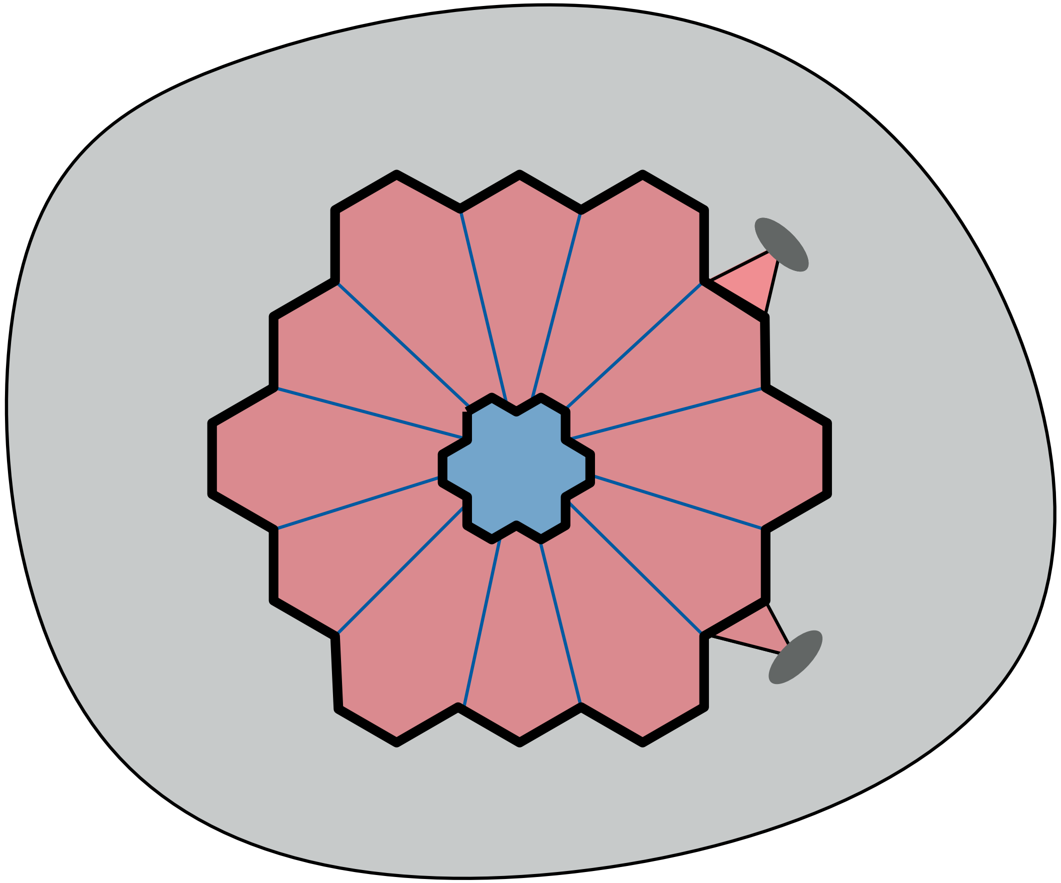

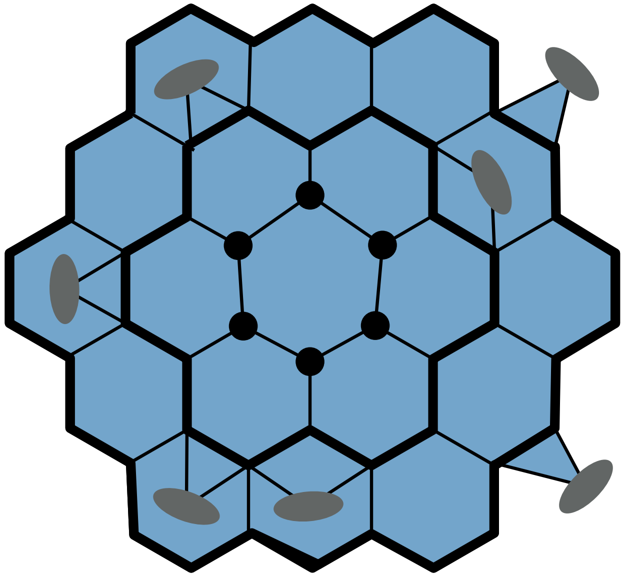

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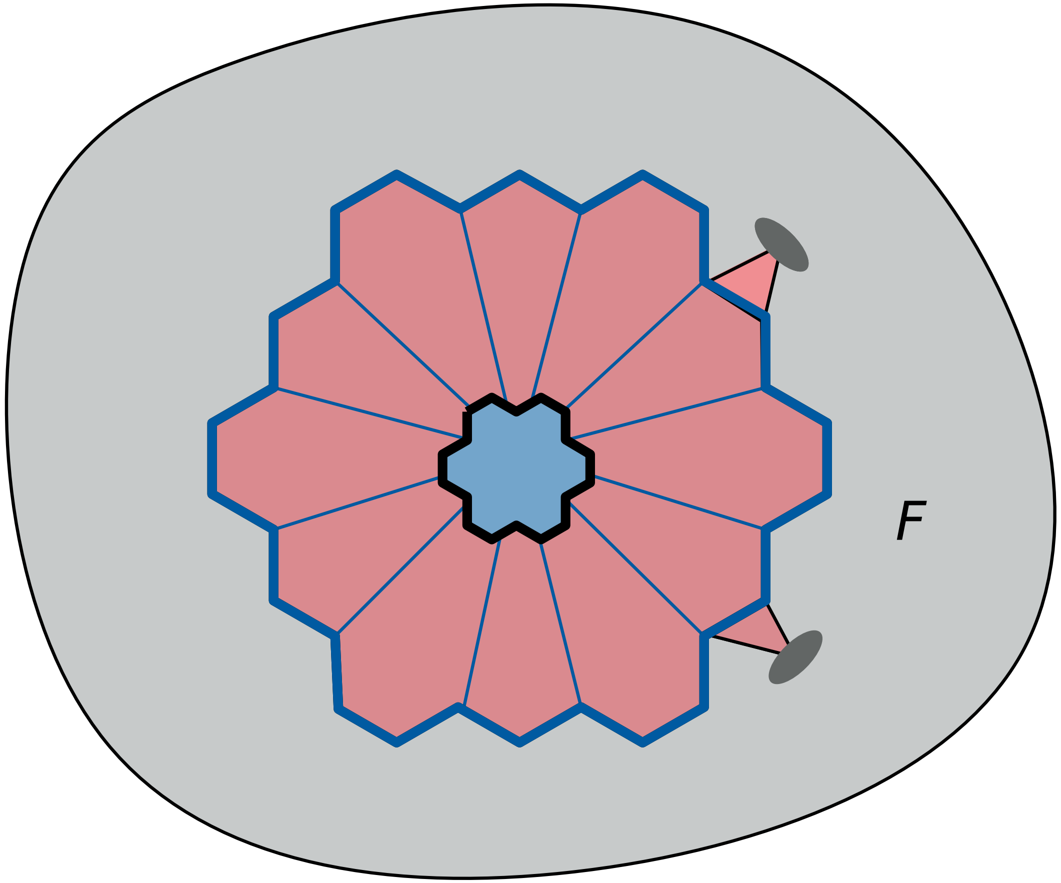
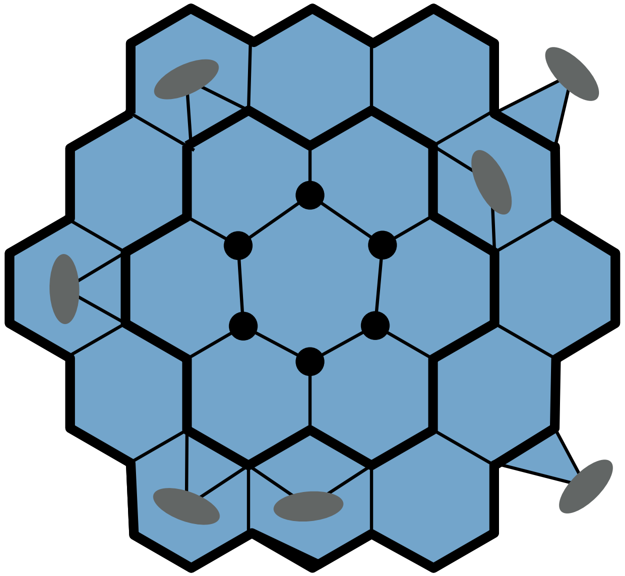
drawing of G' with $\leq c$ crossings





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all blue edges uncrossed!

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Formulation in MSO_2

Goal: $\varphi_{\mathcal{F}}(F) \vDash G \Leftrightarrow$ G has $(F, \mathcal{F}, c)$-good drawing

- no edge in $F \subseteq E$ is involved in a crossing
- no pattern in $\mathcal{F}$ contained
- $\leq c$ crossings

Formulation in MSO_2

Goal: $\varphi_{\mathcal{F}}(F) \vDash G \Leftrightarrow$ G has $(F, \mathcal{F}, c)$-good drawing

$\rightarrow$ express existence of suitable planarization

Formulation in MSO_2

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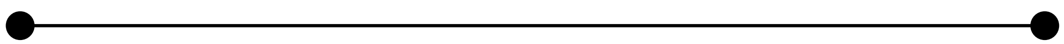
$e \in E \setminus F$



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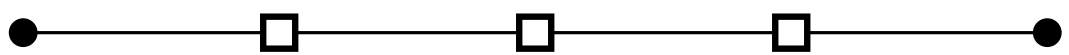

- c crossing dummies

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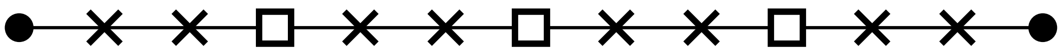

- c crossing dummies
- 2 subdivision dummies

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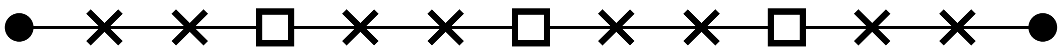

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Input: free variables $x_1, \ldots x_c$, $y_1, \ldots, y_c$

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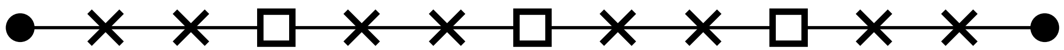

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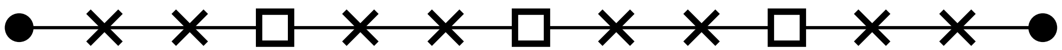

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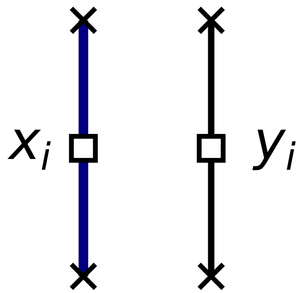

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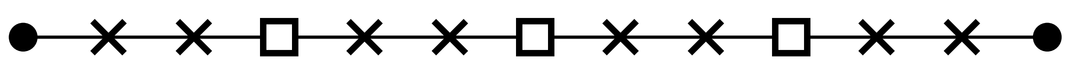

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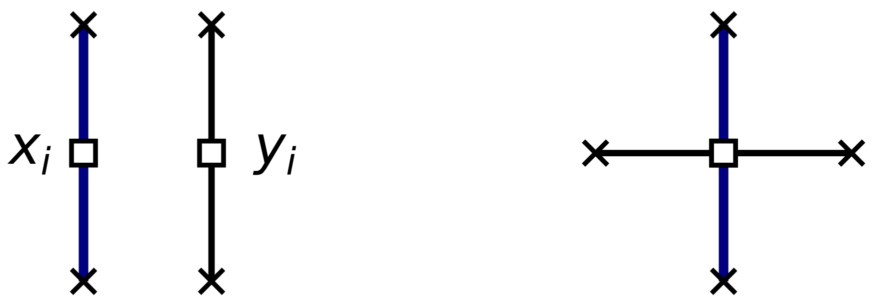

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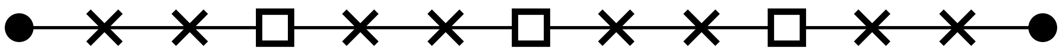

- c crossing dummies
- 2 subdivision dummies

Input: free variables $x_1, \ldots x_c$, $y_1, \ldots, y_c$ vertices in C, x_i identified with y_i

- pairwise distinct (except $x_i = y_i$)

Formulation in MSO_2

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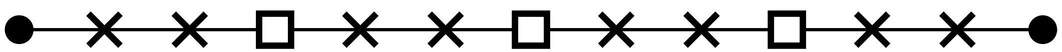
$\rightarrow$ express existence of suitable planarization



- c crossing dummies
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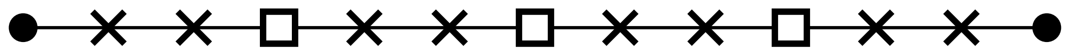
Input: free variables $x_1, \ldots x_c$, $y_1, \ldots, y_c$ vertices in C, x_i identified with y_i

- pairwise distinct (except $x_i = y_i$)
- no self-crossings

Formulation in MSO_2

Goal: $\varphi_{\mathcal{F}}(F) \models G \Leftrightarrow$ G has $(F, \mathcal{F}, c)$-good drawing

$\rightarrow$ express existence of suitable planarization



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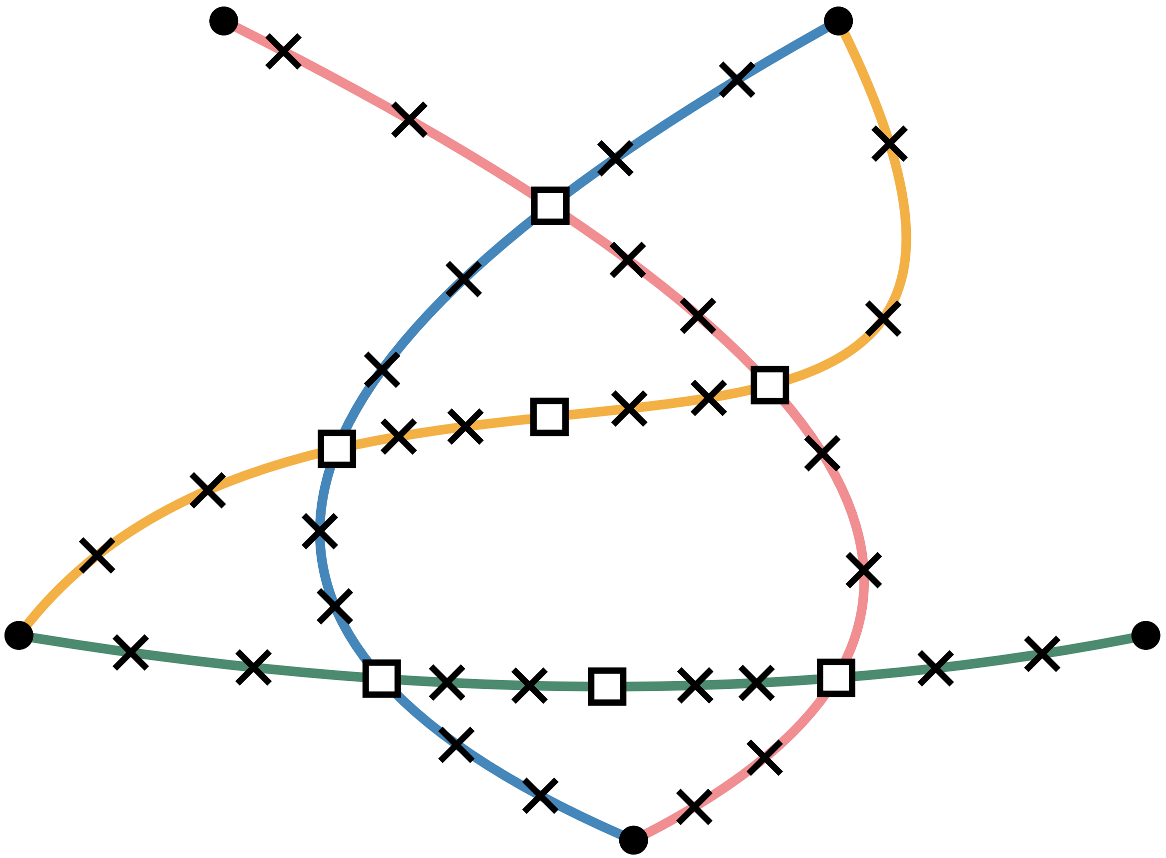
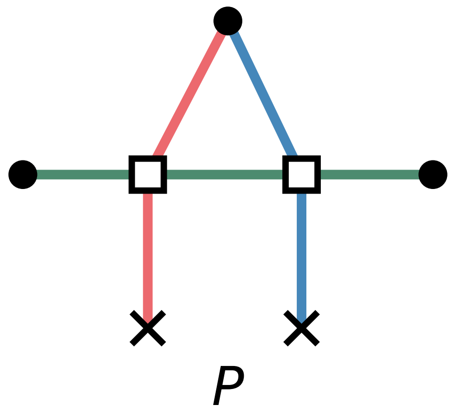
Input: free variables $x_1, \ldots x_c$, $y_1, \ldots, y_c$ vertices in C, x_i identified with y_i

- pairwise distinct (except $x_i = y_i$)
- no self-crossings
- described graph is planar

Forbid Single Pattern





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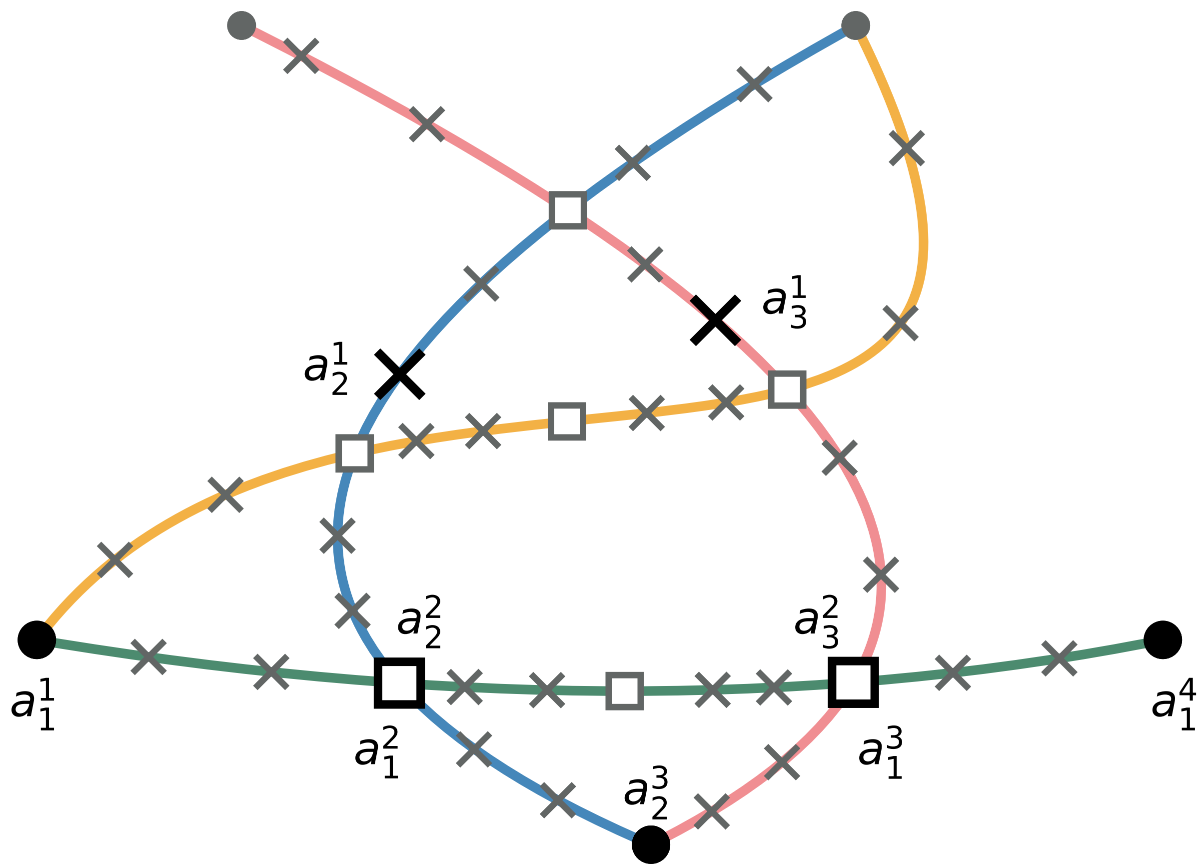

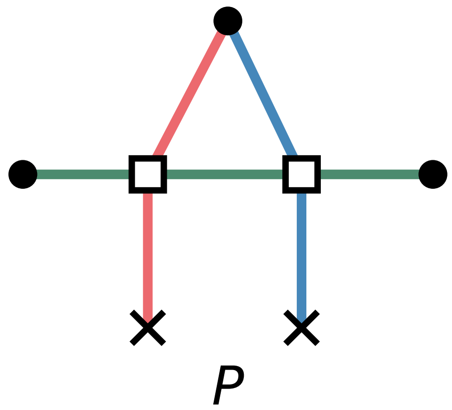

$$\varphi_P = \exists a_1^1 \in R \;\; \exists a_1^2, a_1^3 \in C \;\; \exists a_1^4 \in R \;\; \exists a_2^1, a_3^1 \in S \;\; \exists a_2^2, a_3^2 \in C \;\; \exists a_2^3 \in R$$

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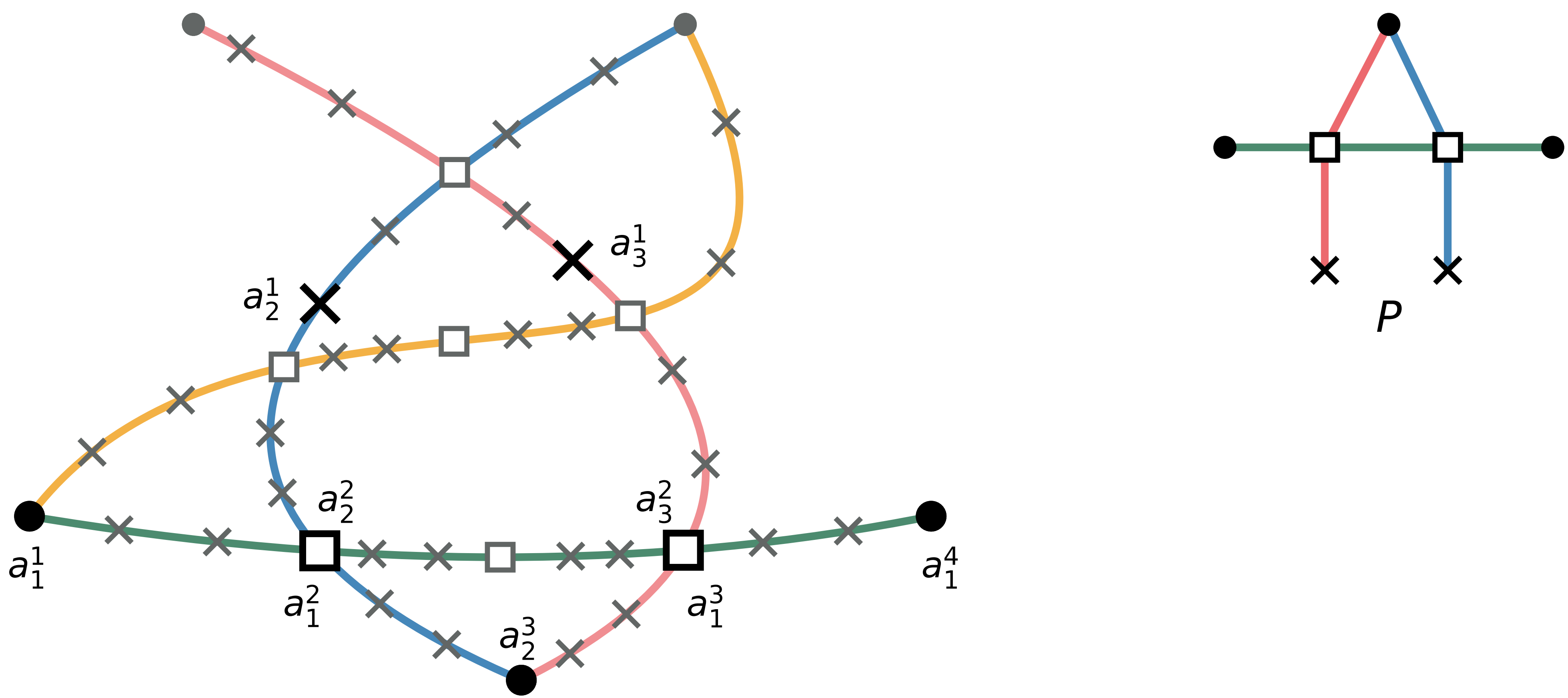

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$$\text{isCrossing}(a_1^2, a_2^2) \wedge \text{isCrossing}(a_1^3, a_3^2)$$

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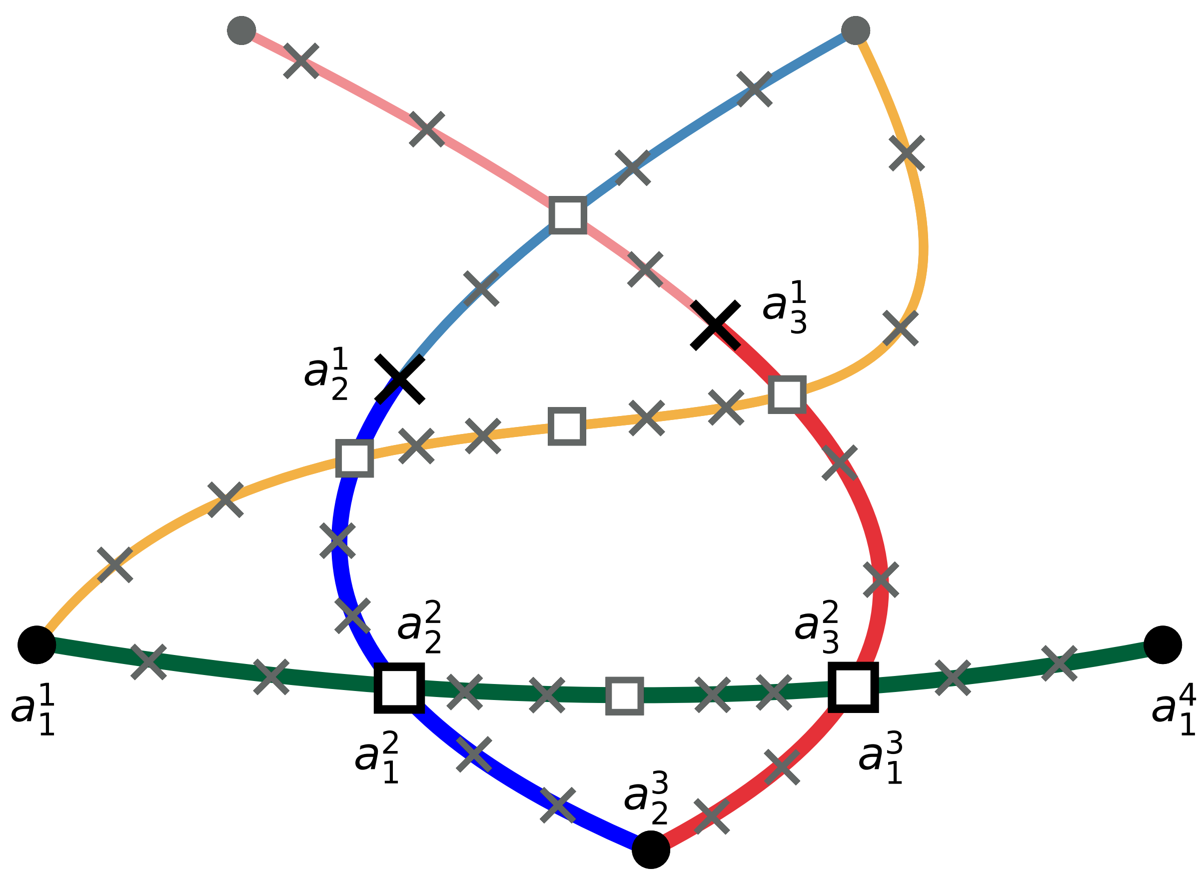

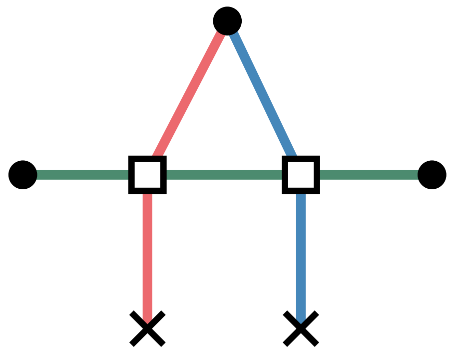

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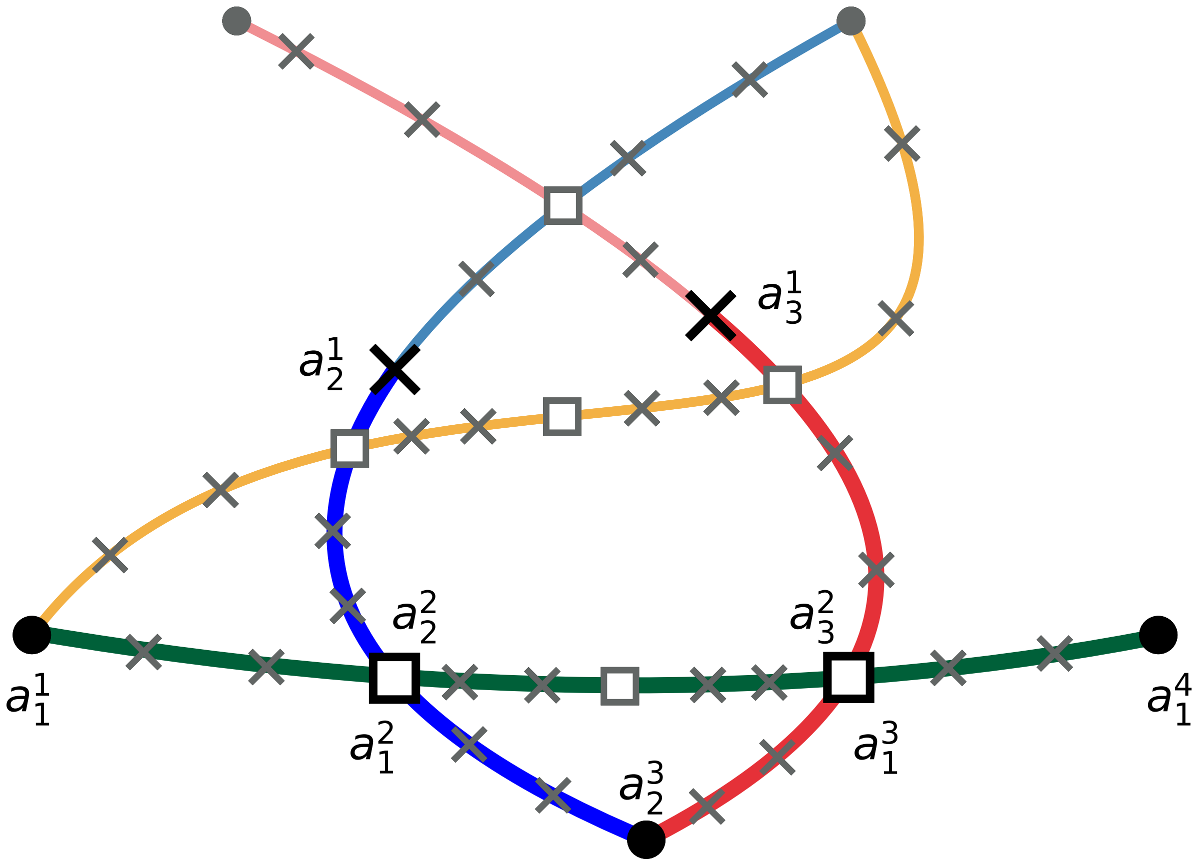
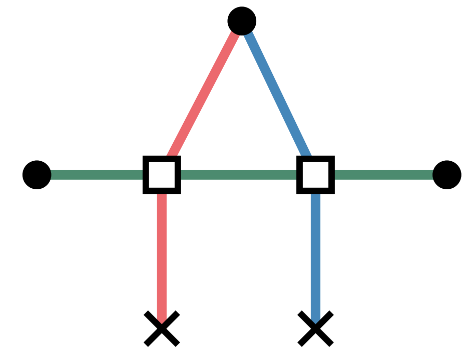
$$\text{isCrossing}(a_1^2, a_2^2) \wedge \text{isCrossing}(a_1^3, a_3^2)$$

$$\exists E_1, E_2, E_3 \subseteq E$$

Forbid Single Pattern





P

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$$\text{isCrossing}(a_1^2, a_2^2) \wedge \text{isCrossing}(a_1^3, a_3^2)$$

$$\exists E_1, E_2, E_3 \subseteq E$$

$$\bigwedge_{i \neq j} \text{disjoint}(E_i, E_j)$$

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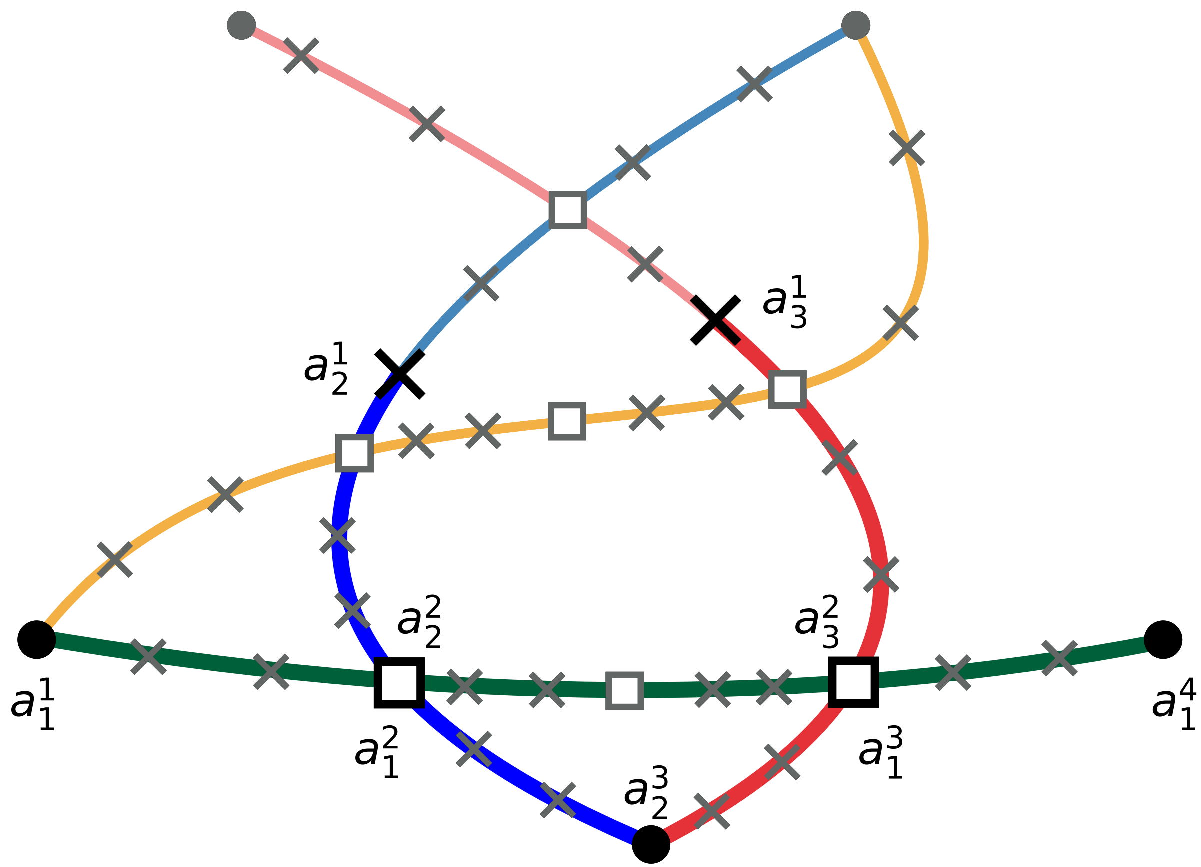

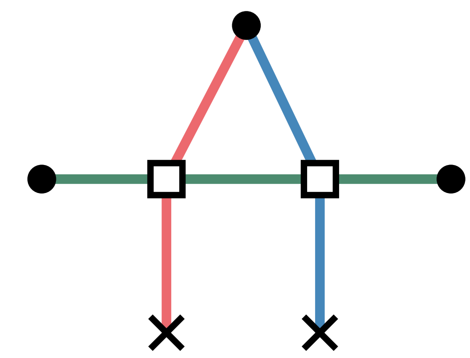

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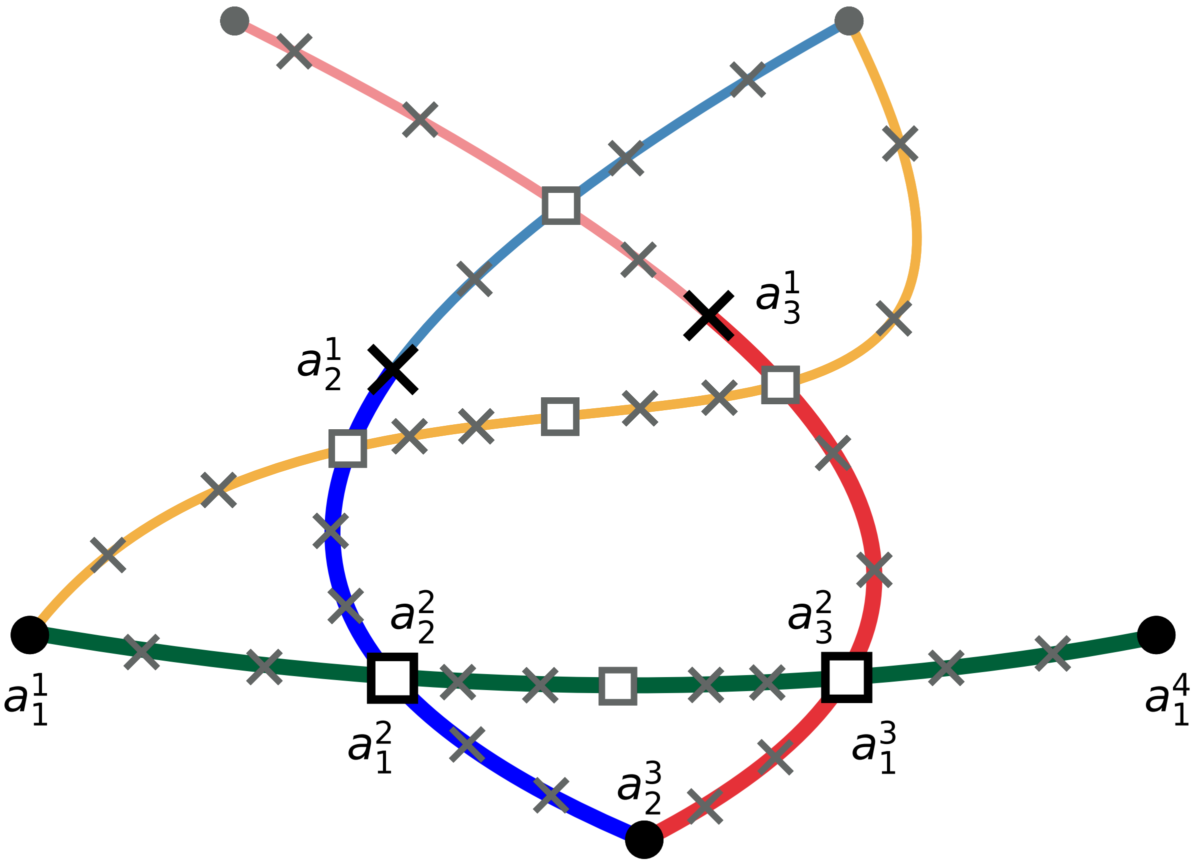
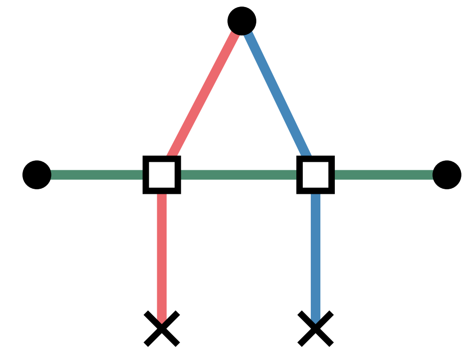
$$\text{chain}(E_1, a_1^1, a_1^2, a_1^3, a_1^4) \wedge \text{chain}(E_2, \ldots) \wedge \text{chain}(E_3, \ldots)$$

Forbid Single Pattern





P

chain = path s.t.

- internal vertices in $C \cup S$
- given vertices in correct order

$$\varphi_P = \exists a_1^1 \in R \;\; \exists a_1^2, a_1^3 \in C \;\; \exists a_1^4 \in R \;\; \exists a_2^1, a_3^1 \in S \;\; \exists a_2^2, a_3^2 \in C \;\; \exists a_2^3 \in R$$

$$\text{isCrossing}(a_1^2, a_2^2) \wedge \text{isCrossing}(a_1^3, a_3^2)$$

$$\exists E_1, E_2, E_3 \subseteq E$$

$$\bigwedge_{i \neq j} \text{disjoint}(E_i, E_j)$$

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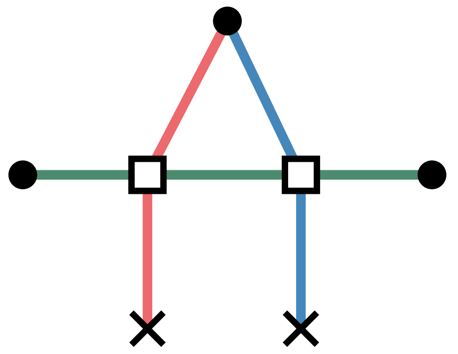
$\varphi_{\mathcal{F}} = \text{planar}^{\times} \wedge \bigwedge_{P \in \mathcal{F}} \neg \varphi_P$



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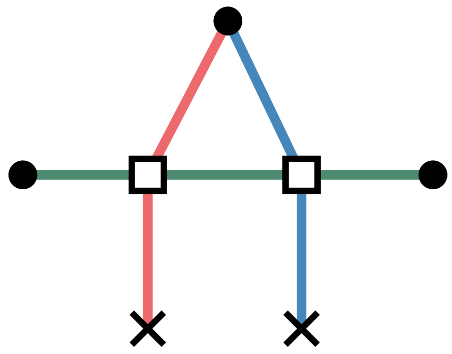

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- size depends only on c and pattern-size
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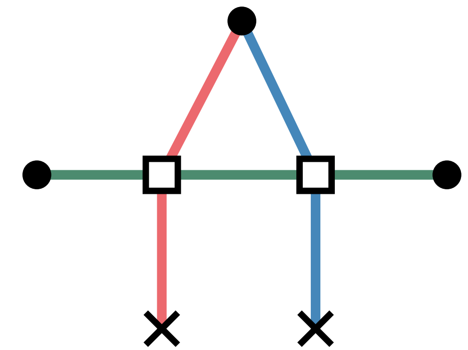

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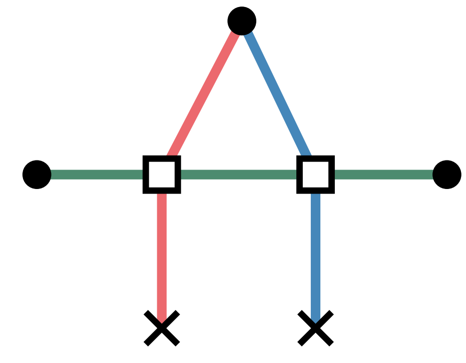
↓ Courcelle



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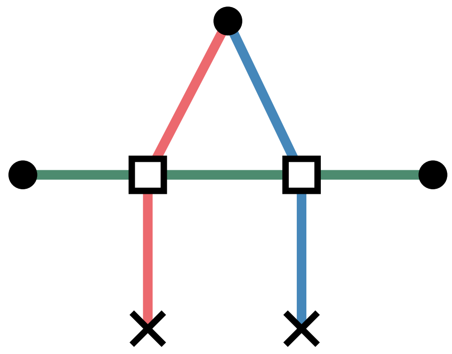

Theorem
For any fixed set of crossing patterns $\mathcal{F}$, testing whether a graph G admits drawing with $\leq c$ crossings that avoids all patterns in $\mathcal{F}$ is FPT w.r.t. c.

chain = path s.t.

- internal vertices in $C \cup S$
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Summary and Outlook

- combinatorial formalization crossing patterns

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Meta-Theorem

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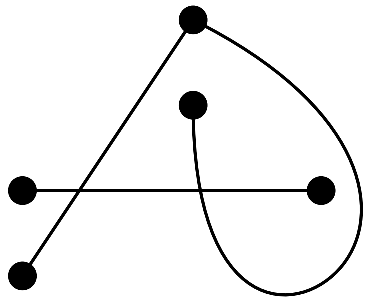

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