Connectivity-Faithful Graph Drawing

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Contribution

- Introduce connectivity-faithful graph drawing

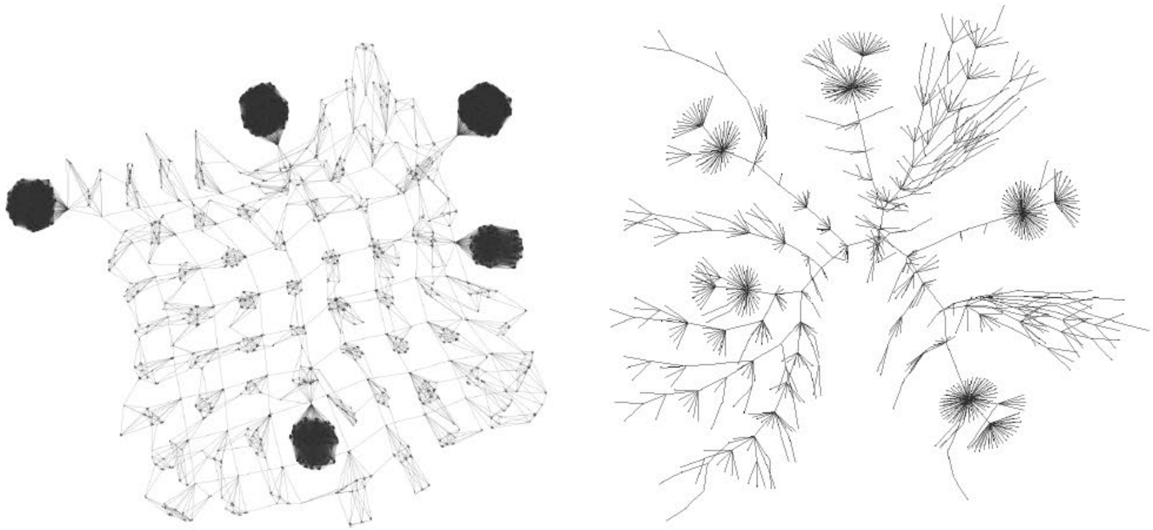
- Leverage NI (Nagamochi-Ibaraki) Algorithm

- Compute a k-connectivity preserving sparsification of a k-connected graph G
- Sampling: NI outperform SS (Spectral Sparsification)
- Drawing: NIGD computes similar quality drawing for biconnected graphs

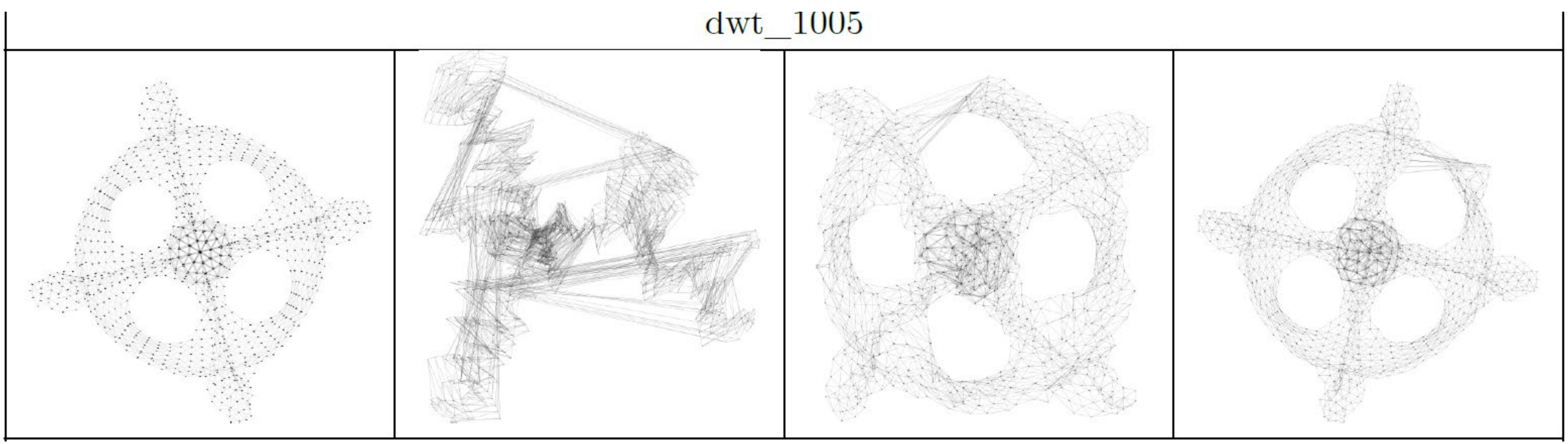
- Modification of NI Algorithm

- Preserve global k-connectivity of G and local h-connectivity of h-connected components of G
- CFNI outperforms NI: 66% improvement in sampling quality metrics and 73% improvement in proxy quality metrics.
- CFGD runs 51% faster than drawing the whole graph G with a similar quality, and outperforms NIGD, with 62% improvement in stress metrics

Example

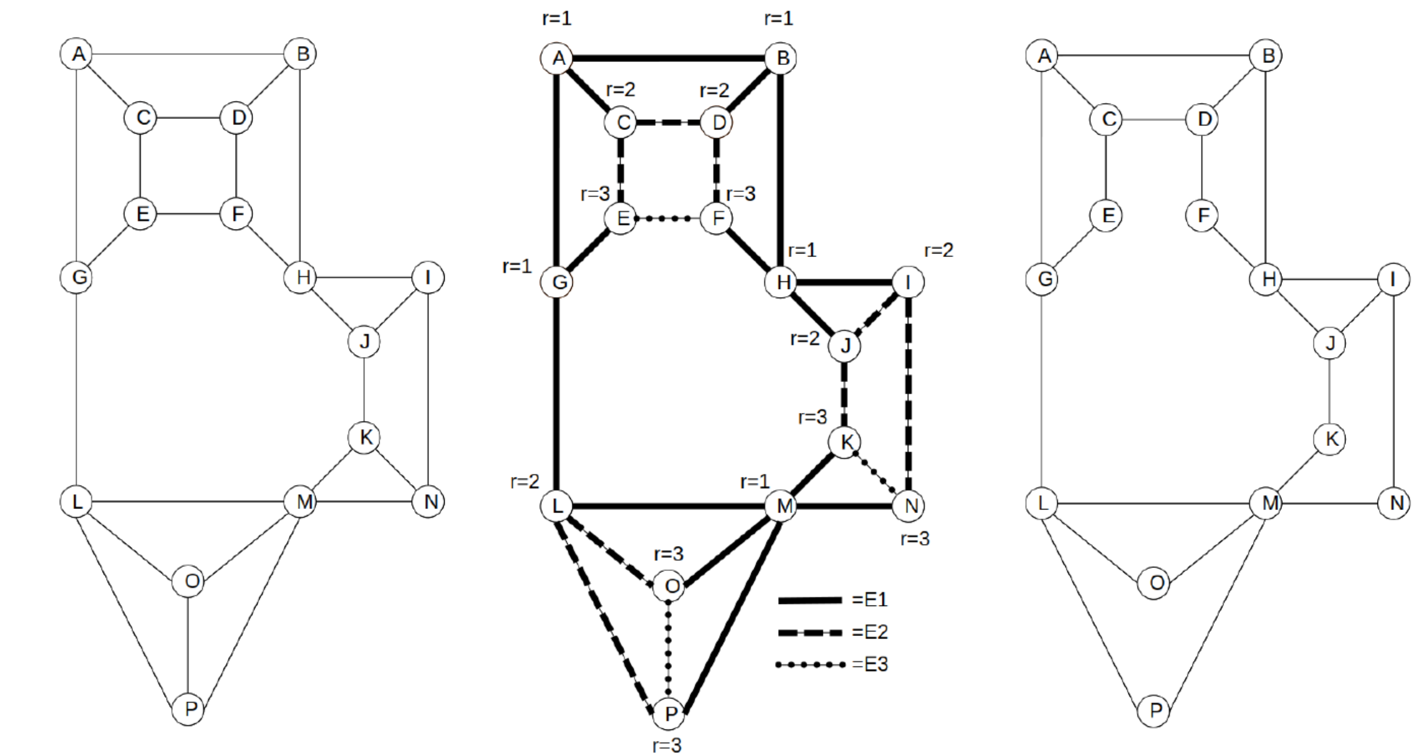


(a) G **(b)** G_1 (computed by NI) **(c)** G_3 (computed by CFNI)



NI Algorithm

Computes a k-connectivity preserving sparsification of a k-connected graph G



(a) Biconnected graph G **(b)** G after running NI **(c)** Biconnected subgraph G_k

NIGD Algorithm

NIGD

Step 1: Compute k-connected spanning subgraph $G_k = (V, E')$ of $G = (V, E)$ using NI.

Step 2: Apply a graph layout algorithm to G_k to obtain drawing D_{G_k} of G_k.

Step 3: Add all edges in $E_r = E \setminus E'$ to D_{G_k} to obtain drawing $D_{G_k+E_r}$ of G.

NI vs. SS Comparison Experiment

(a) Scale-free

G	$\|V\|$	$\|E\|$
jazz	193	2737
EVA	234	373
migrations	817	4152
oflights	1920	14458
tvcg	1925	7163
block_2000	2000	9912
CA-GrQc	2651	10480
us_powergrid	3040	4555
as19990606	3213	7937
facebook	3698	85963

(b) Mesh

G	$\|V\|$	$\|E\|$
mesh3e1	289	800
dwt_1005	1005	3808
cage8	1015	4994
bcsstk09	1083	8677
nasa1824	1824	18692
plat1919	1919	15240
sierpinski3d	2050	6144
data	2851	15093
3elt	4720	13722

(c) GION

G	$\|V\|$	$\|E\|$
2_gion	1159	6424
5_gion	1748	13957
6_gion	1785	20459
7_gion	3010	41757
8_gion	4924	52502
4_gion	5953	186279
1_gion	5452	118404
3_gion	7885	427406

(d) Black-hole

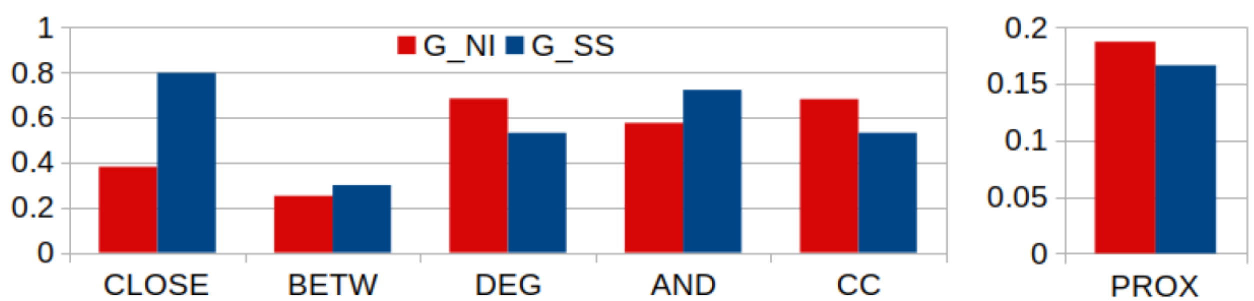
G	$\|V\|$	$\|E\|$
G443	285	2009
Cycle759	377	4790
G462	733	62509
Cycle907	823	14995
Cycle896	1031	22638
G500	1080	17636
G887	4784	38135

Quality Metrics Comparison

Sampling quality metrics: quality of samples

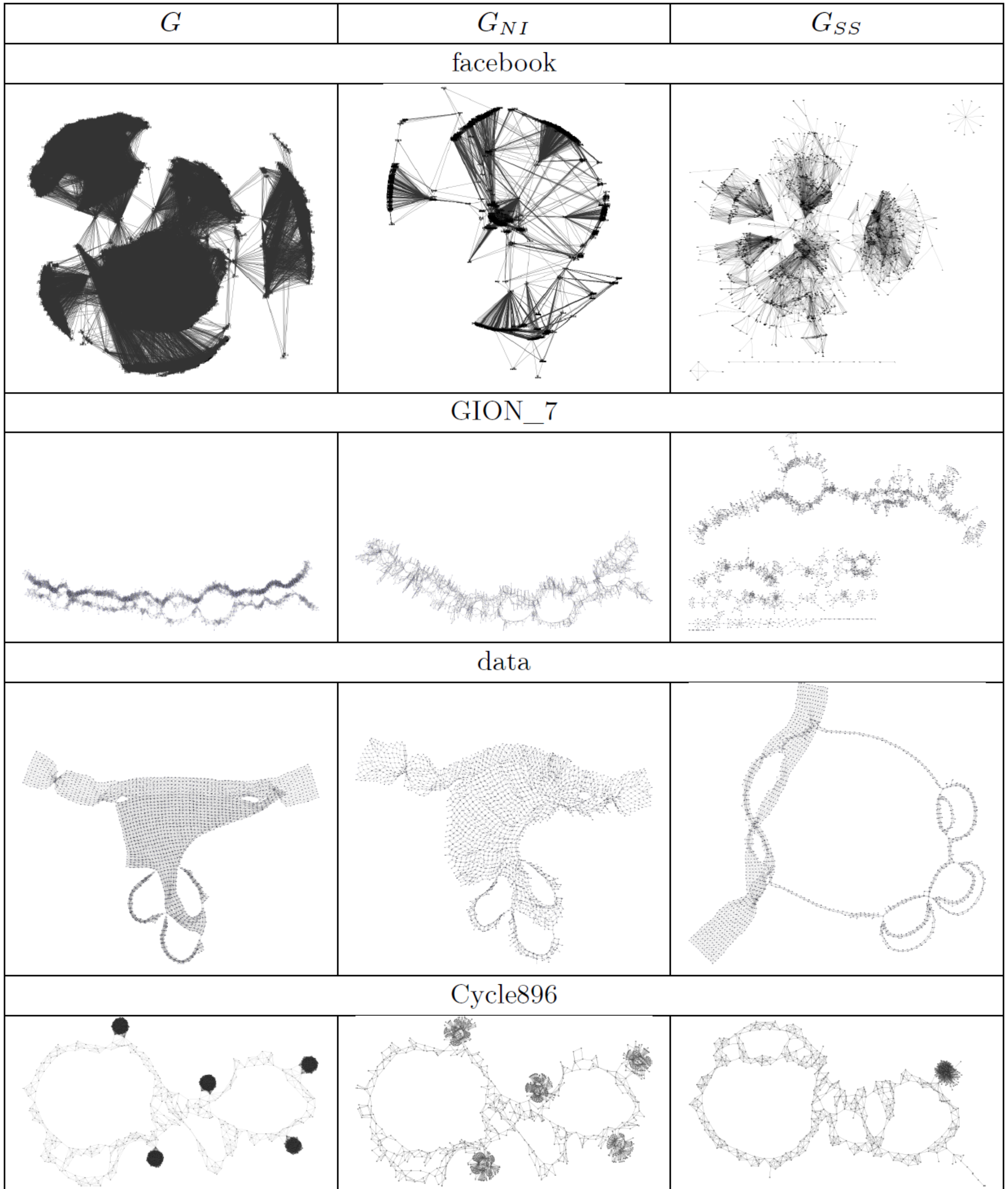
- Closeness centrality
- Betweenness centrality
- Degree Correlation
- Average Neighbor Degree
- Clustering Coefficient

Proxy quality metrics: faithfulnesss of shapes in drawings of samples



(a) Sampling quality metrics **(b)** Proxy quality

Figure 8 Average sampling quality metrics (lower is better) and proxy quality metrics (higher is better) for NI and SS. NI obtains better metrics especially on connectivity-related quality metrics CLOSE and BETW, as well as on AND and proxy quality metrics.


G
G_{NI}
G_{SS}
facebook
GION_7
data
Cycle896

NIGD Experiment: Quality Metrics

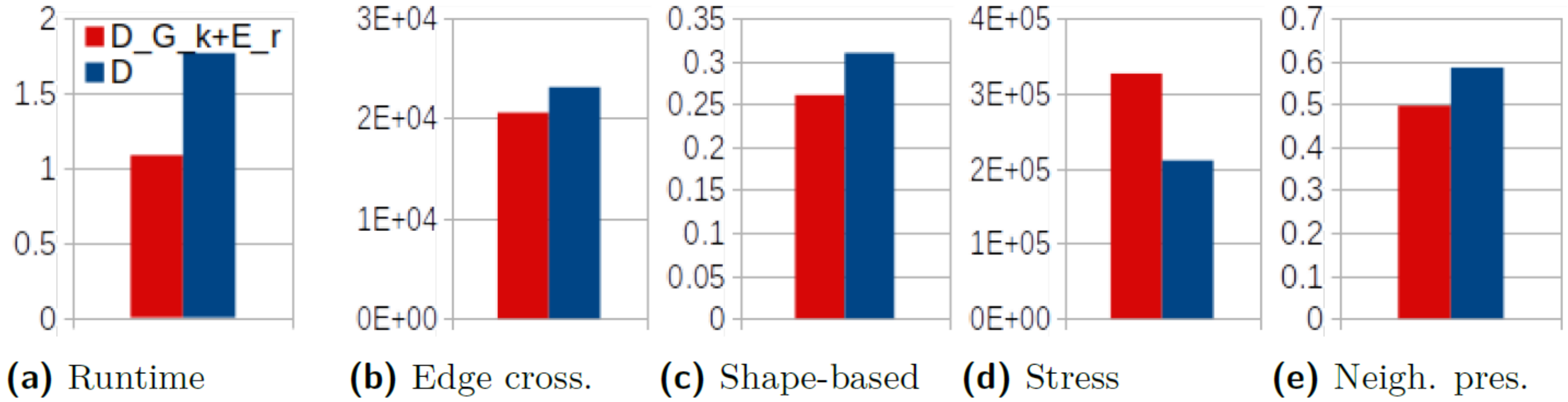


Figure 9 Average runtime and quality metrics obtained by drawing $D_{G_k+E_r}$ by NIGD and drawing D directly from G. NIGD runs significantly faster than drawing G directly, at 30% faster, with 11% better edge crossing and only 15% lower shape-based metrics and neighborhood preservation, however with on average 55% higher stress.

NIGD Experiment

<table>
<tr><th>D</th><th>$D_{G_k+E_r}$</th><th>D</th><th>$D_{G_k+E_r}$</th></tr>
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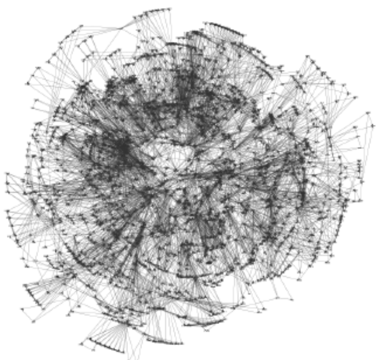
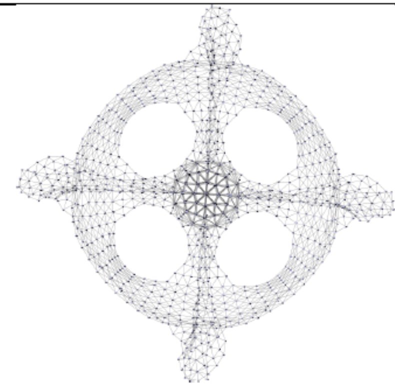
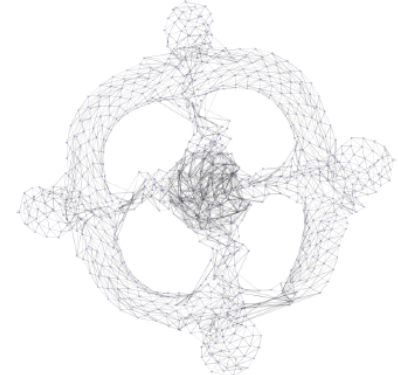
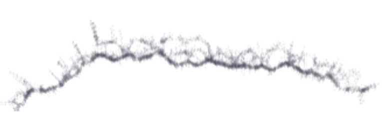
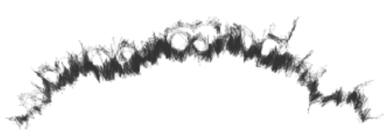
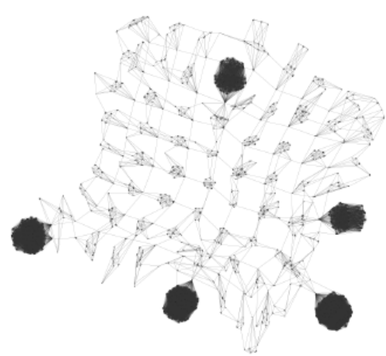
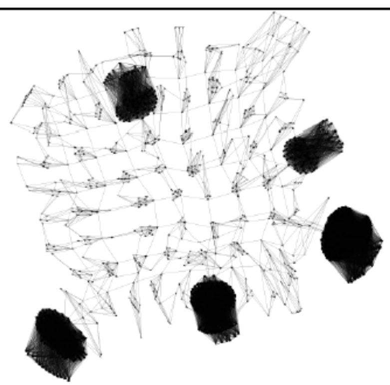
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">GION_8</td><td colspan="2">G500</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
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CFNI Algorithm

- divide-and-conquer algorithm
- computes a sparsification, preserving the global k-connectivity of a graph G and the local h-connectivity of h-connected components of G
- Example: 1-connected graph G

- G1: computed by NI
- G2: run NI for each biconnected component
- G3: run NI for each triconnected components

CFGD Algorithm

Algorithm CFGD

Step 1: Compute subgraph $G_{CFNI} = (V, E'_h)$ preserving global k-connectivity and local h-connectivity of k-connected graph G using CFNI.

Step 2: Compute a drawing $D_{G_{CFNI}}$ of G_{CFNI} using a graph drawing algorithm.

Step 3: Add all edges in $E_{r_h} = E \setminus E'_h$ to $D_{G_{CFNI}}$ to obtain a drawing D of G.

CFNI vs. NI Comparison Experiment

(a) Scale-free

G	$\|V\|$	$\|E\|$
soc_h	2000	16097
block_2000	2000	3992
oflights	2905	15645
tvcg	3213	10140
facebook	4039	88234
CA-GrQc	4158	13422
EVA	4475	4652
us_powergrid	4941	6594
as19990606	5188	9930
migrations	6025	9378
lastfm_asia	7624	27806

(b) Mesh

G	$\|V\|$	$\|E\|$
dwt_1005	1005	4813
cage8	1015	4994
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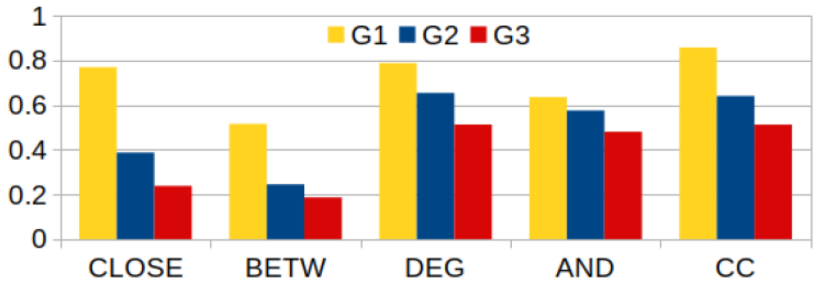
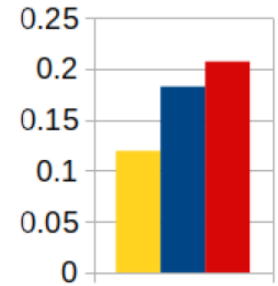
Quality Metrics Comparison

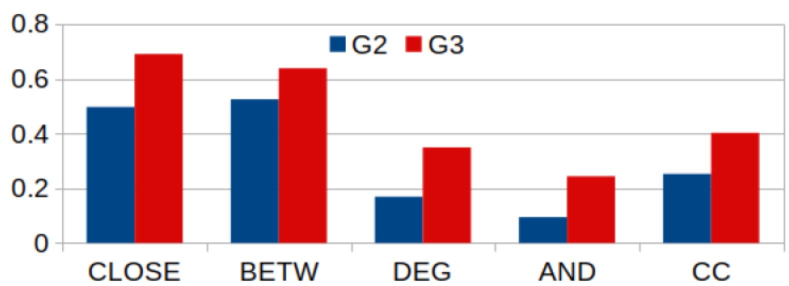


(a) Sampling quality metrics

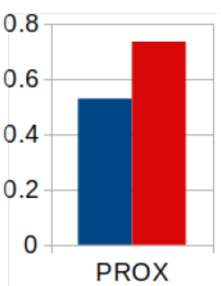


(b) Prox. qual.

Figure 3 Average sampling (lower = better) and proxy quality metrics (higher = better) for G_1, G_2, and G_3. On average, G_3 obtains significantly better metrics than G_1 (i.e., NI), especially on connectivity-related metrics CLOSE and BETW at 66% better on average.

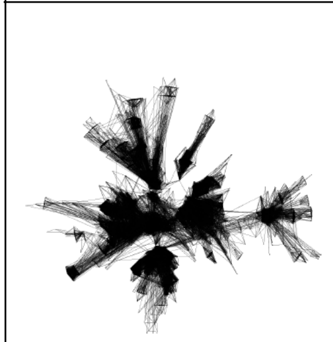
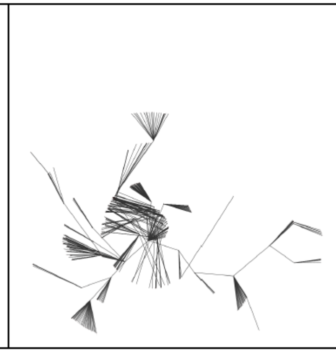
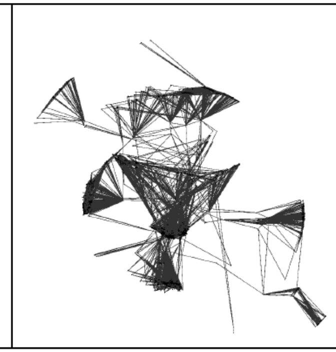
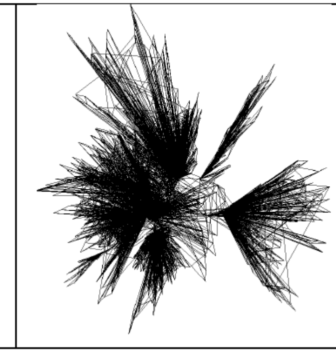
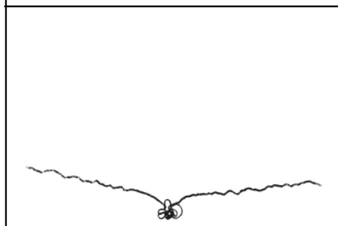
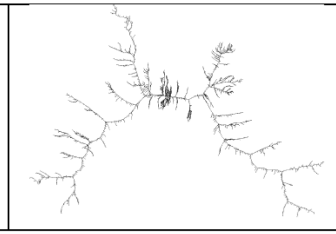
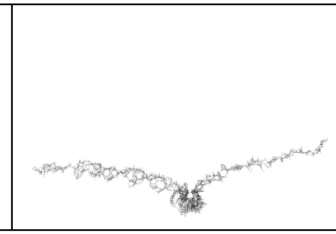
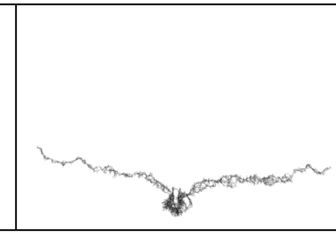
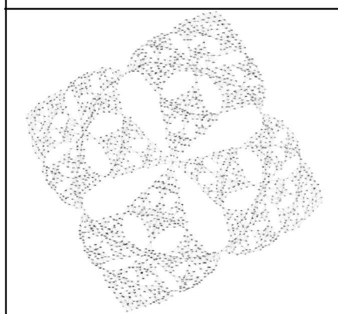
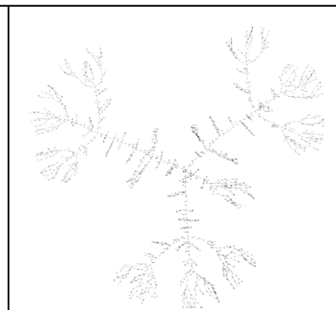
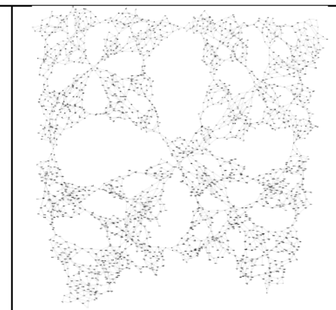
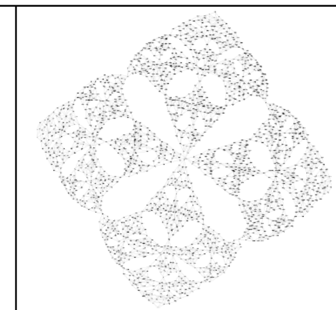
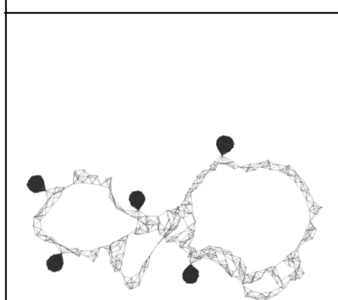
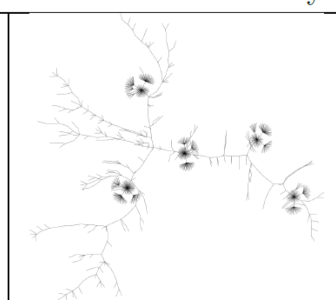
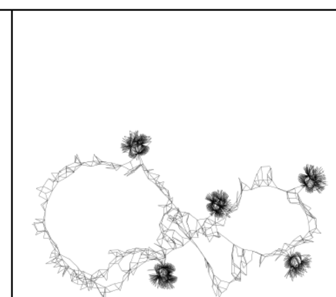
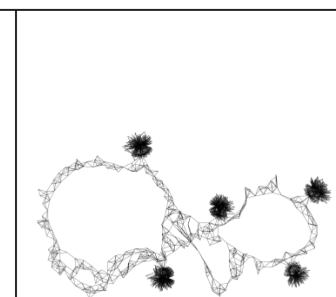


(a) Sampling metrics

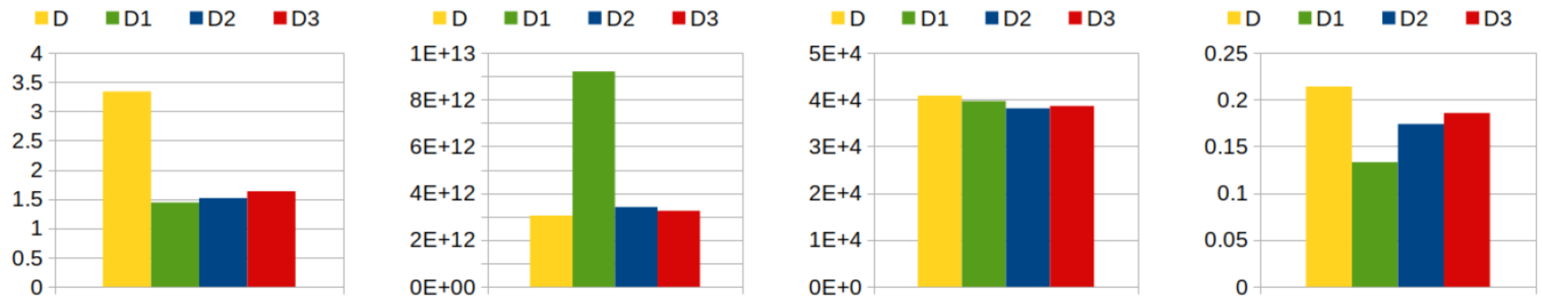


(b) Proxy metrics

Figure 4 Average improvements by G_2 and G_3 (computed by CFNI) over G_1 (computed by NI). CFNI obtains improvement over NI on all metrics, most significantly on connectivity-related sampling quality metrics CLOSE and BETW.

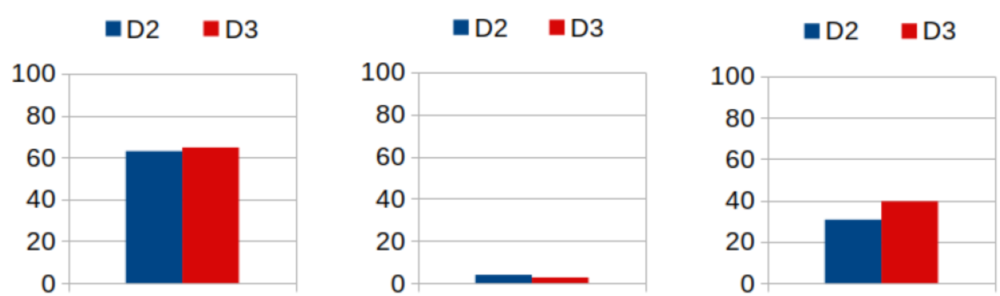
G	G_1	G_2	G_3
facebook			
			
GION_1			
			
sierpinski3d			
			
Cycle896			
			

CFGD vs. NIGD Experiment: Quality Metrics



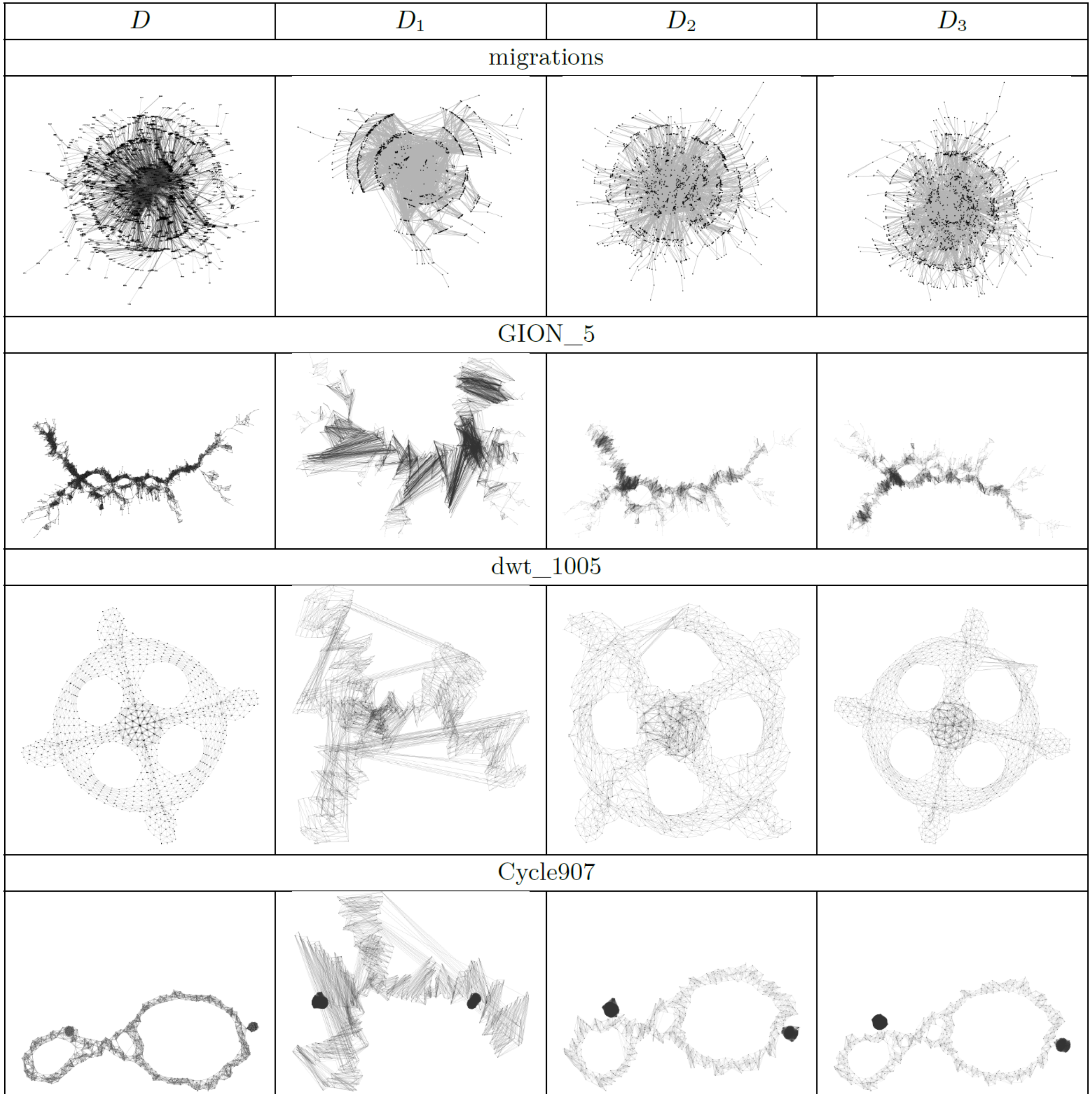
(a) Runtime **(b)** Stress **(c)** Edge crossing **(d)** Shape-based

Figure 5 Average runtime and quality metrics (lower is better for stress and edge crossing, and higher is better for shape-based) of computing D_1, D_2, and D_3 compared to computing D directly on G. CFGD (D_2 and D_3) obtains significant runtime improvements over computing D directly on G, while obtaining significantly lower stress than D_1 and similar metrics to D.



(a) Stress **(b)** Edge crossing **(c)** Shape-based

Figure 6 Average improvements (in %) in quality metrics computed by D_2 and D_3 over D_1, i.e., improvement of CFGD over a naive application of NI to graph drawing. CFGD obtains improvements on all quality metrics, with the largest improvement on stress at over 63%.


D
D_1
D_2
D_3
migrations
GION_5
dwt_1005
Cycle907

Conclusion

- Introduce connectivity-faithful graph drawing

- Leverage NI (Nagamochi-Ibaraki) Algorithm

- NI outperform SS (Spectral Sparsification), esp. Closeness metrics
- NIGD runs fast with similar quality drawing for biconnected graphs

- Improvement of NI Algorithm

- CFNI outperforms NI
- CFGD runs fast with similar quality drawing and outperforms NIGD in stress metrics