Revisiting ILP Models for Exact Crossing Minimization in Storyline Drawings

Alexander Dobler[1], Michael Jünger[2], Paul J. Jünger[3], Julian Meffert[3], Petra Mutzel[3], Martin Nöllenburg[1]

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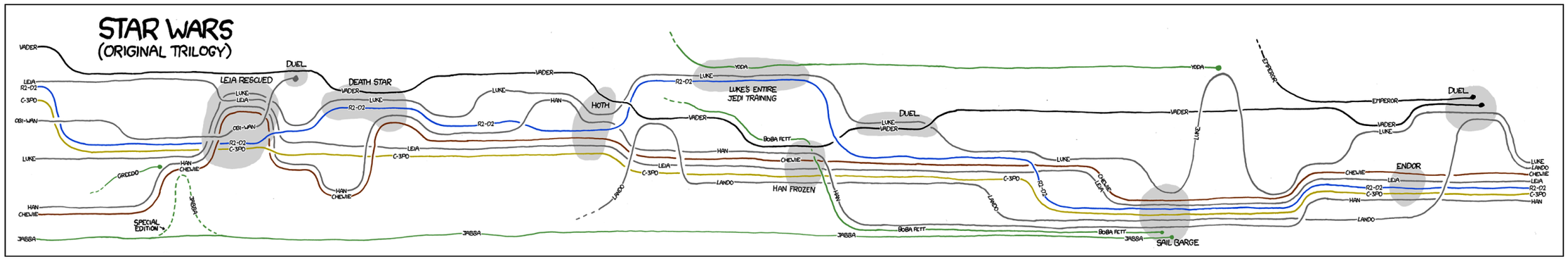
[1] TU WIEN ac ALGORITHMS AND COMPLEXITY GROUP

[3] UNIVERSITÄT BONN

[2] UNIVERSITÄT ZU KÖLN

Storylines



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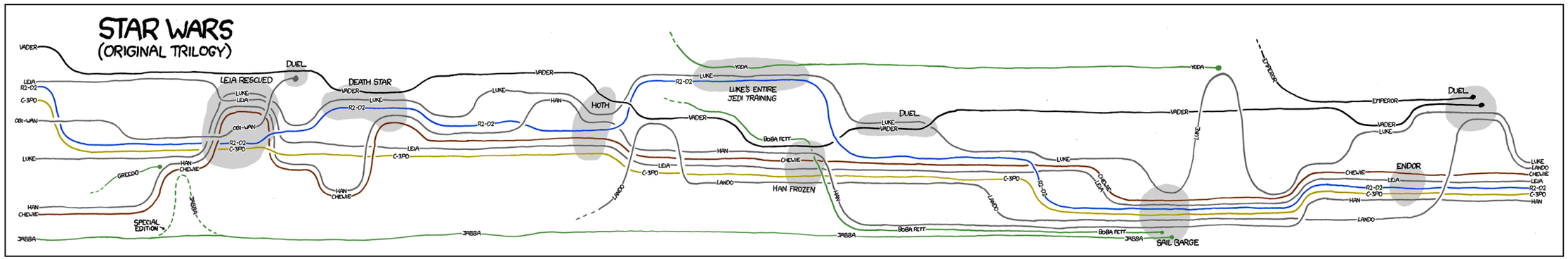

Time

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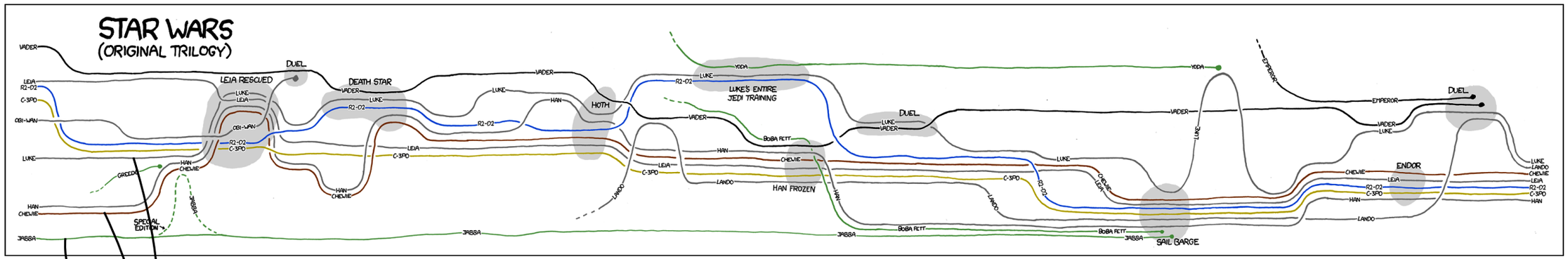

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Characters

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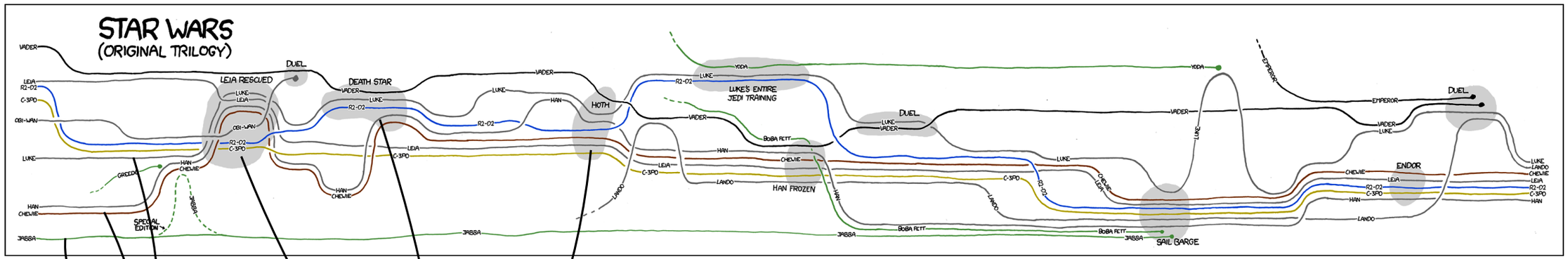

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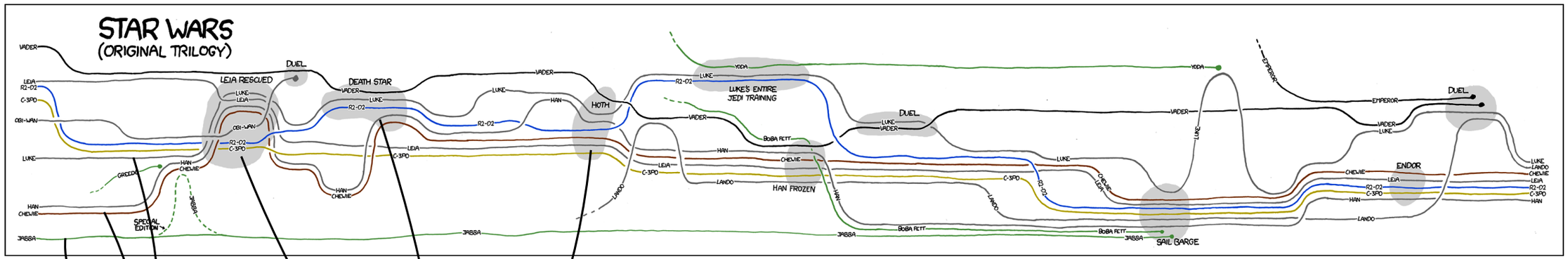

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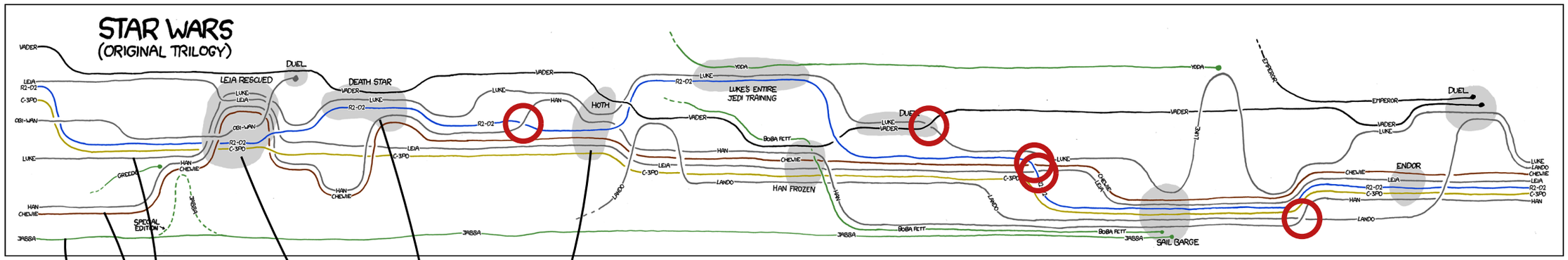

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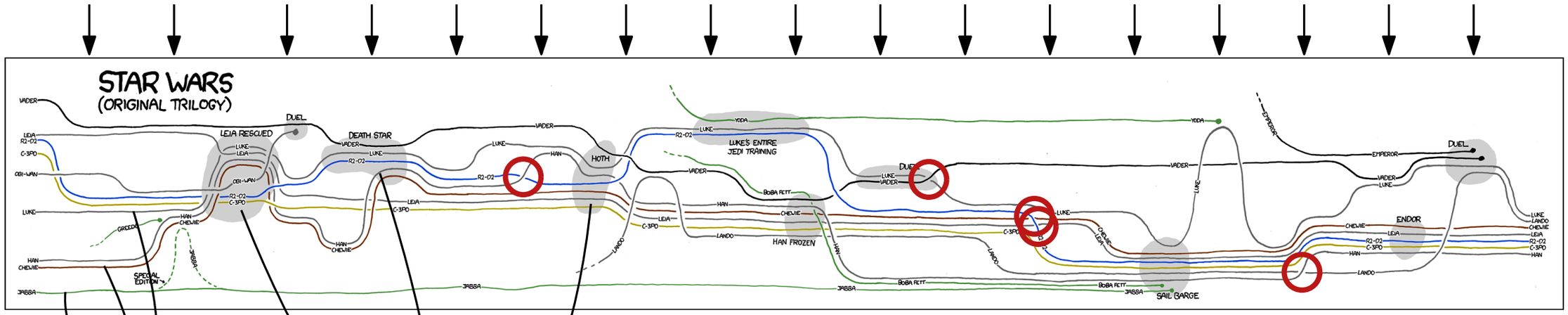

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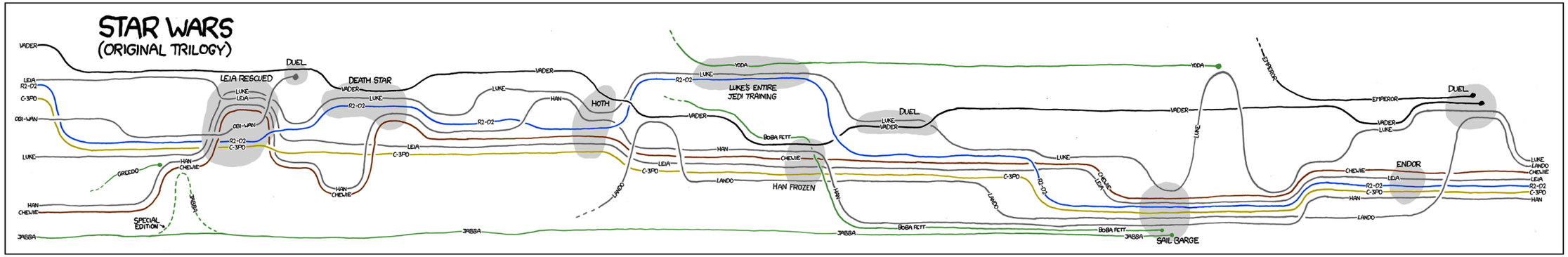
Degree of freedom: vertical **order** of character curves over time

Previous Work on Crossing Minimization



- [Ogawa and Ma, 2010]: greedy algorithm
- [Tanahashi and Ma, 2012]: genetic algorithm
- [Liu, Wu, Wei, Liu, and Liu, 2013]: pipeline approach
- [Kostitsyna, Nöllenburg, Polishchuk, Schulz, and Strash, 2015]: NP-hardness and fixed-parameter tractability by #characters
- [Gronemann, Jünger, Liers, Mambelli, 2016]: exact integer programming formulations and experimental evaluation of one formulation

Our Contribution

[GJLM, 2016]: exact integer programming formulations and experimental evaluation of algorithm based on one formulation

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Barbie, Oppenheimer, Harry Potter, Titanic, ...

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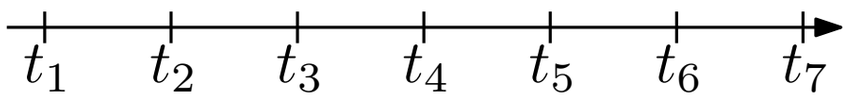

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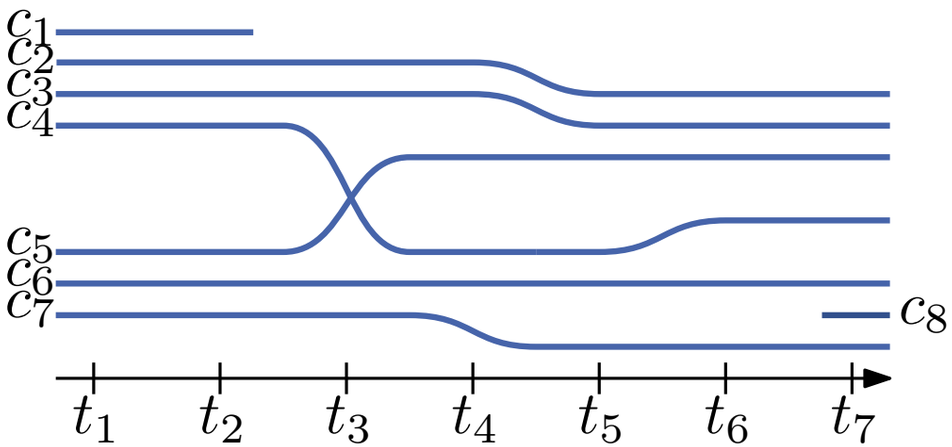

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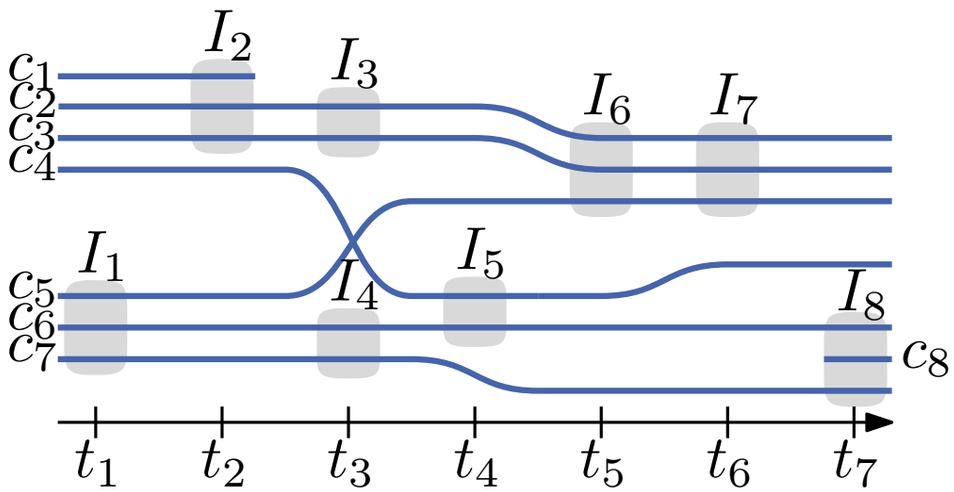

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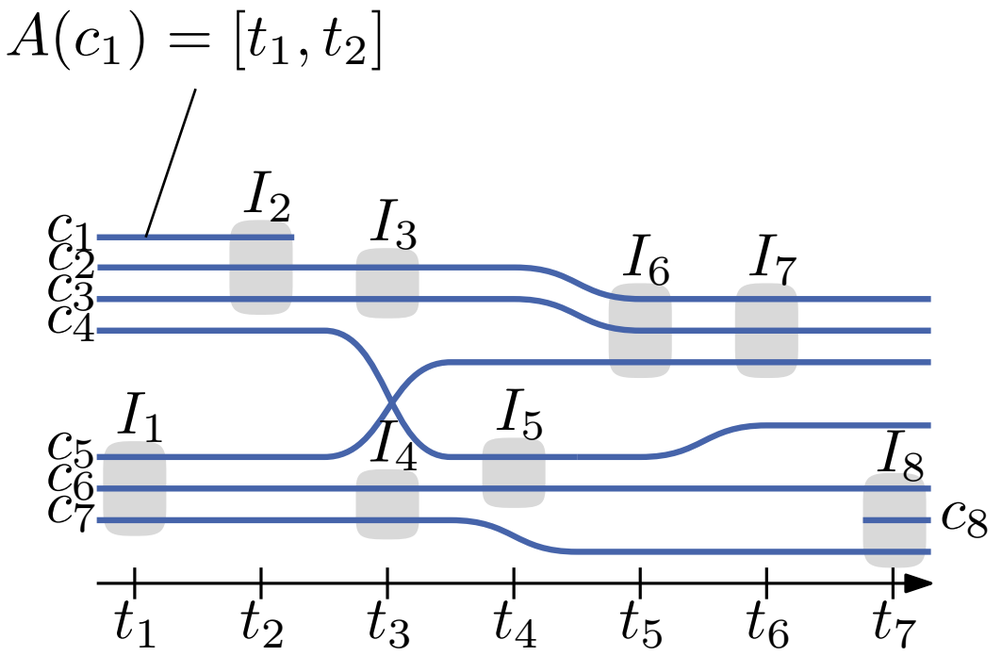

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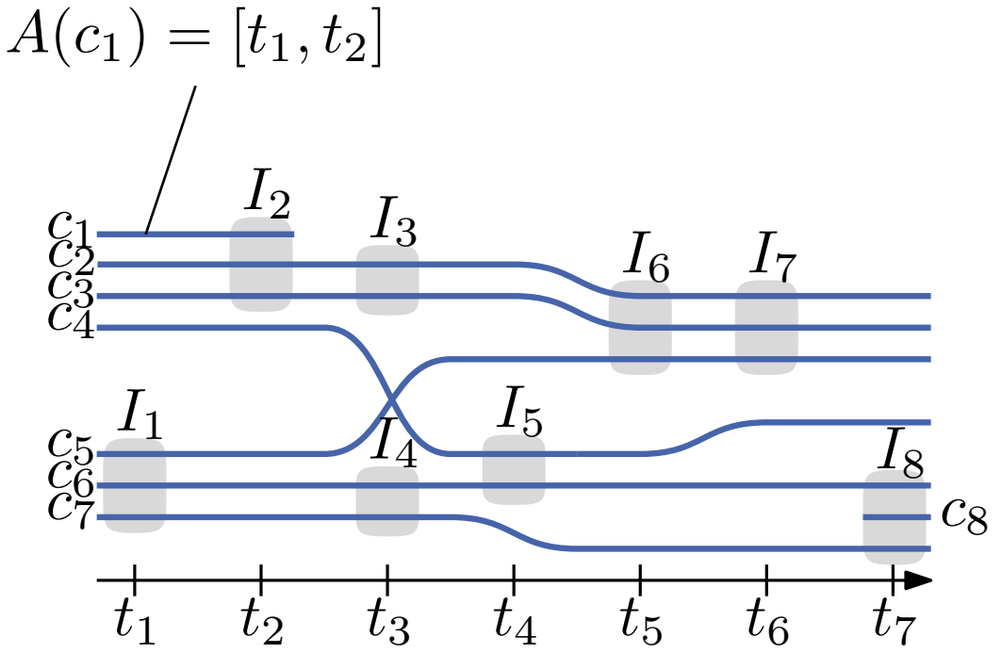

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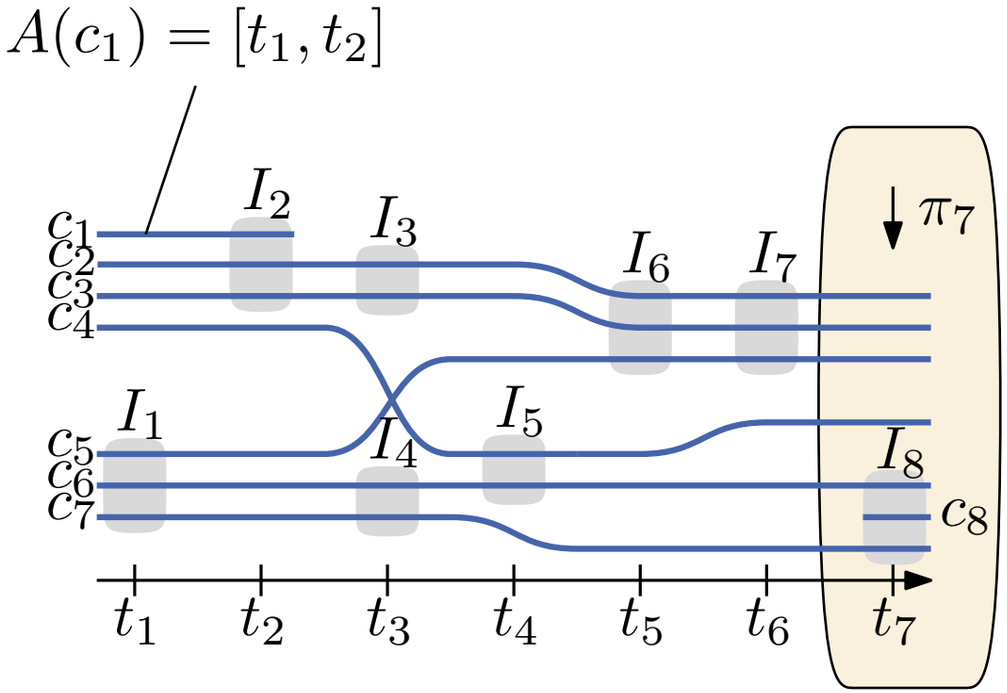

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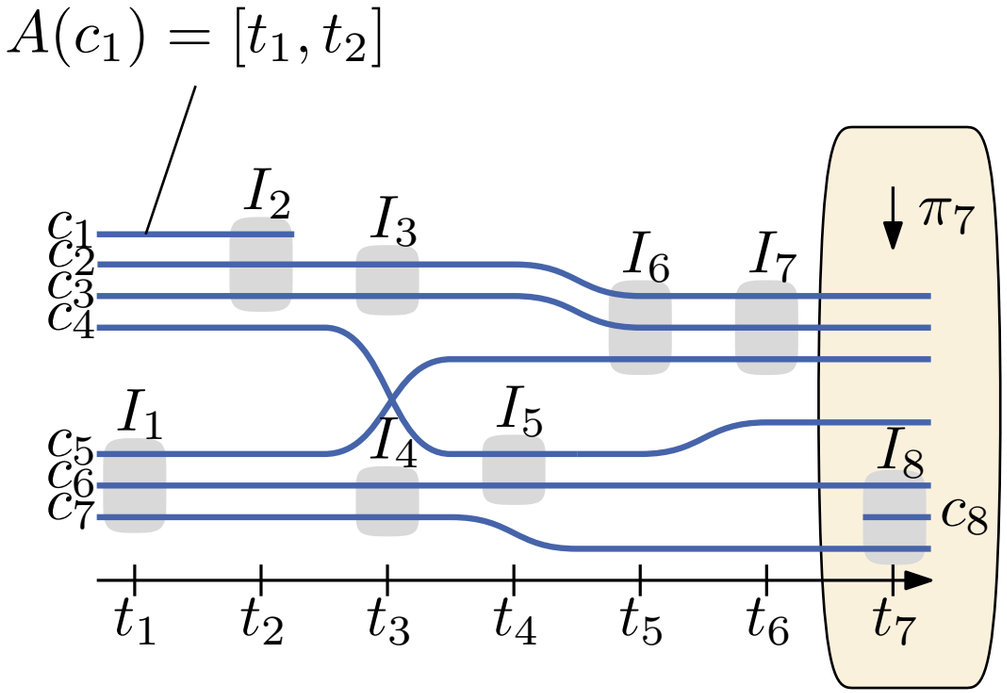

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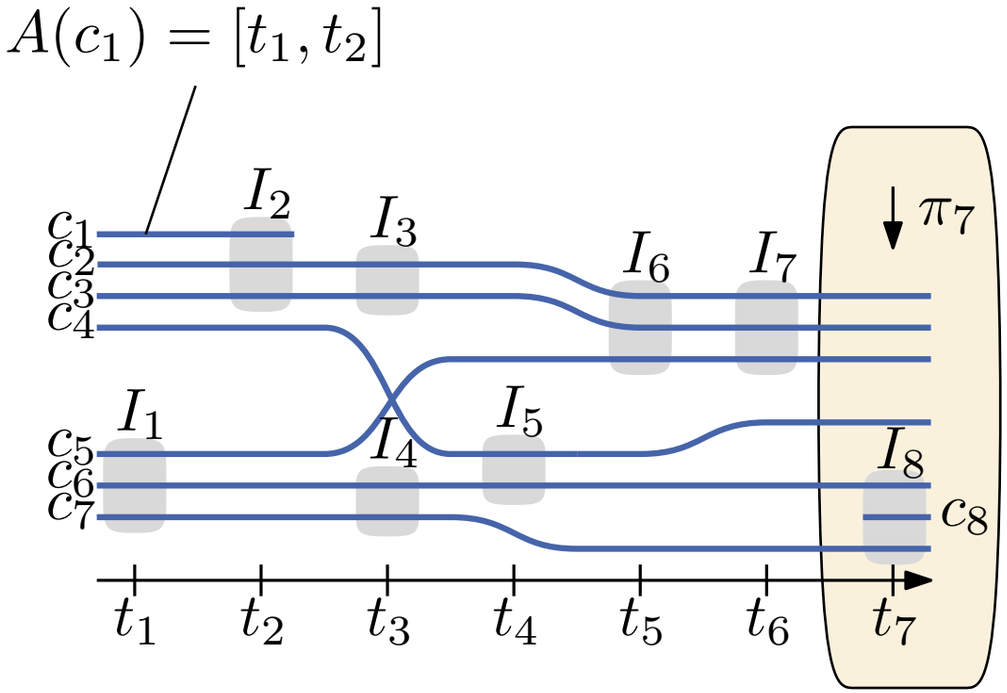

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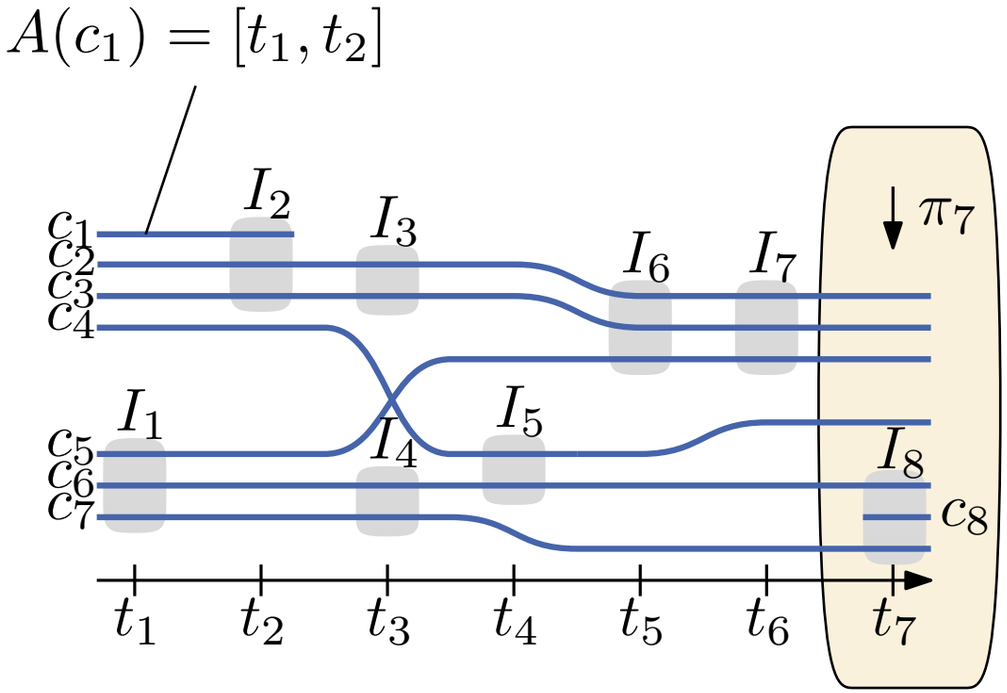

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combinatorial problem!

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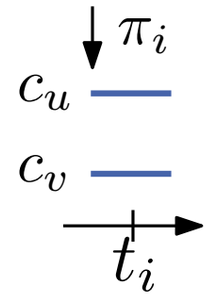

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[GJLM, 2016] implement this

$$\max \sum_{e \in E_M} w_e z_e \qquad \text{(CUT)}$$

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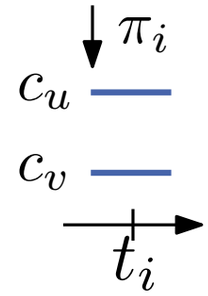

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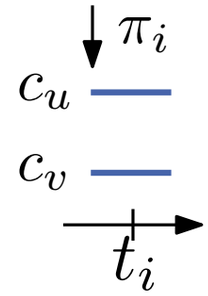

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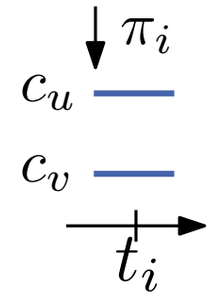

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We work with **simple** linear and quadratic model!

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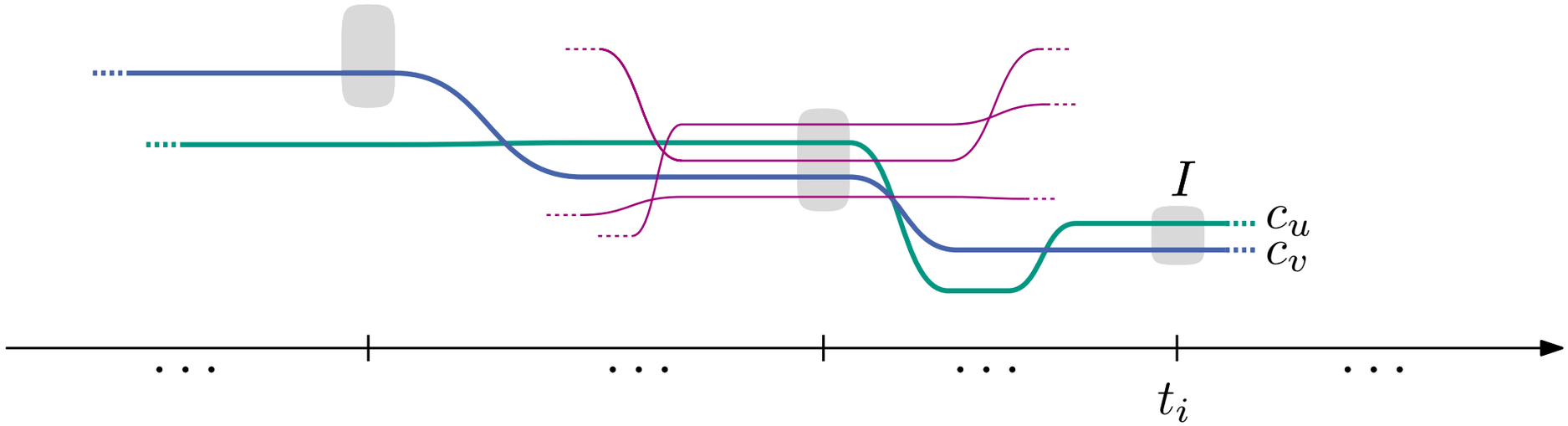

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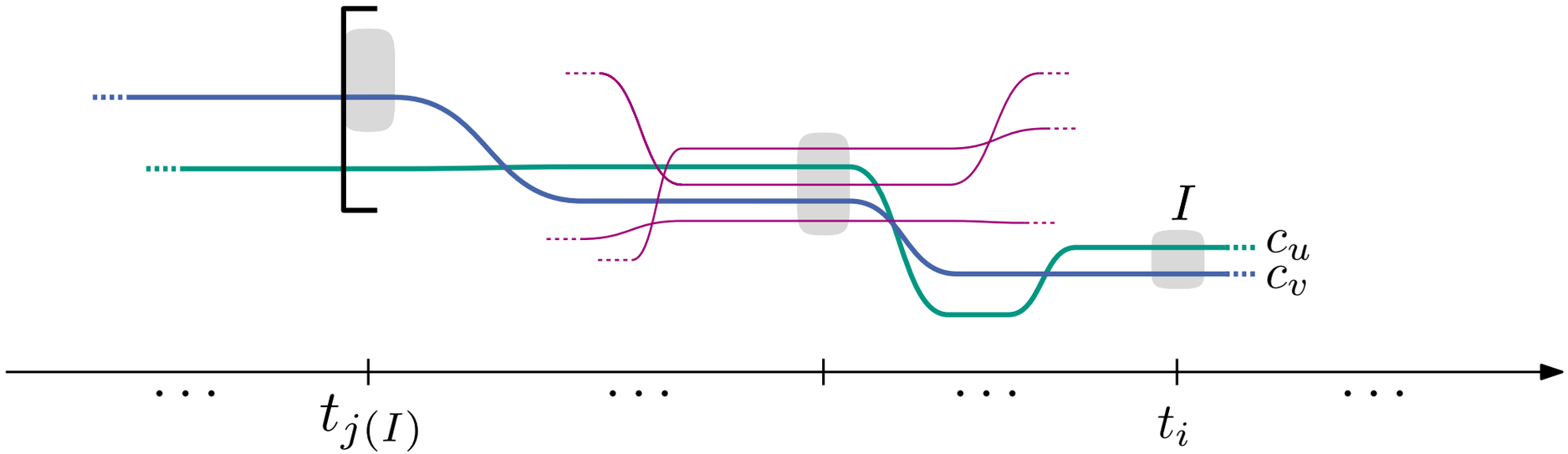

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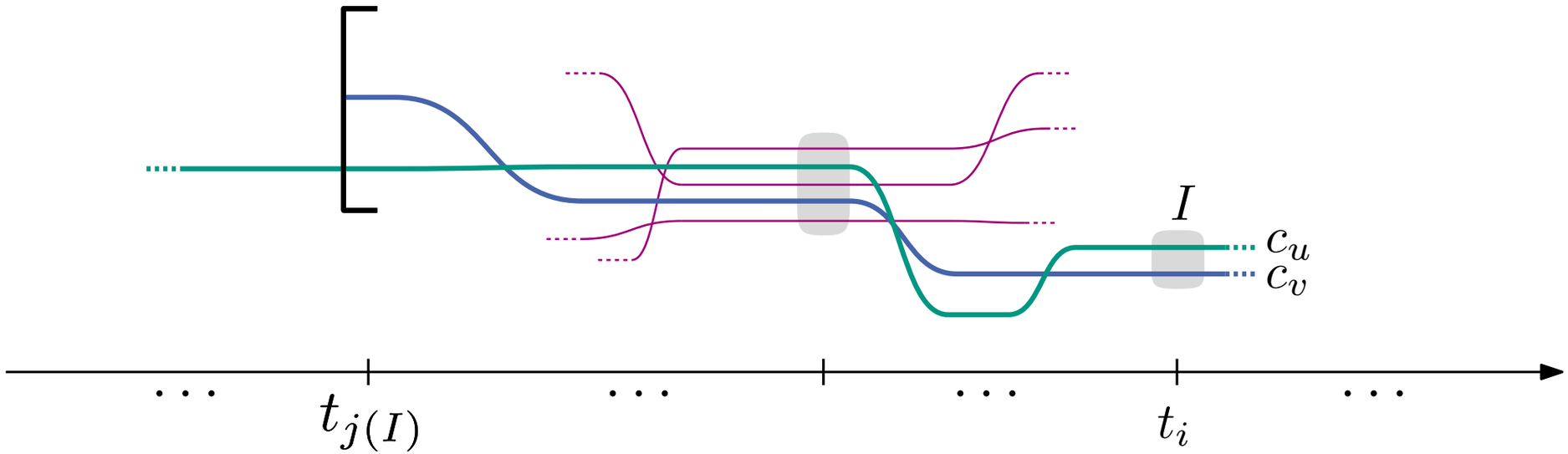

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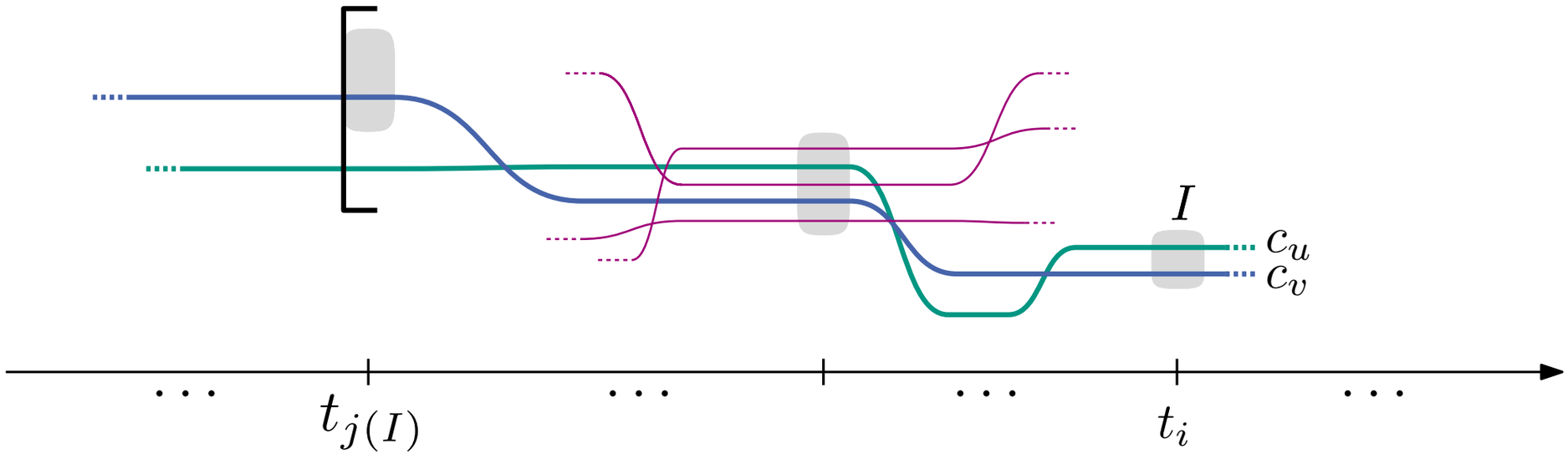

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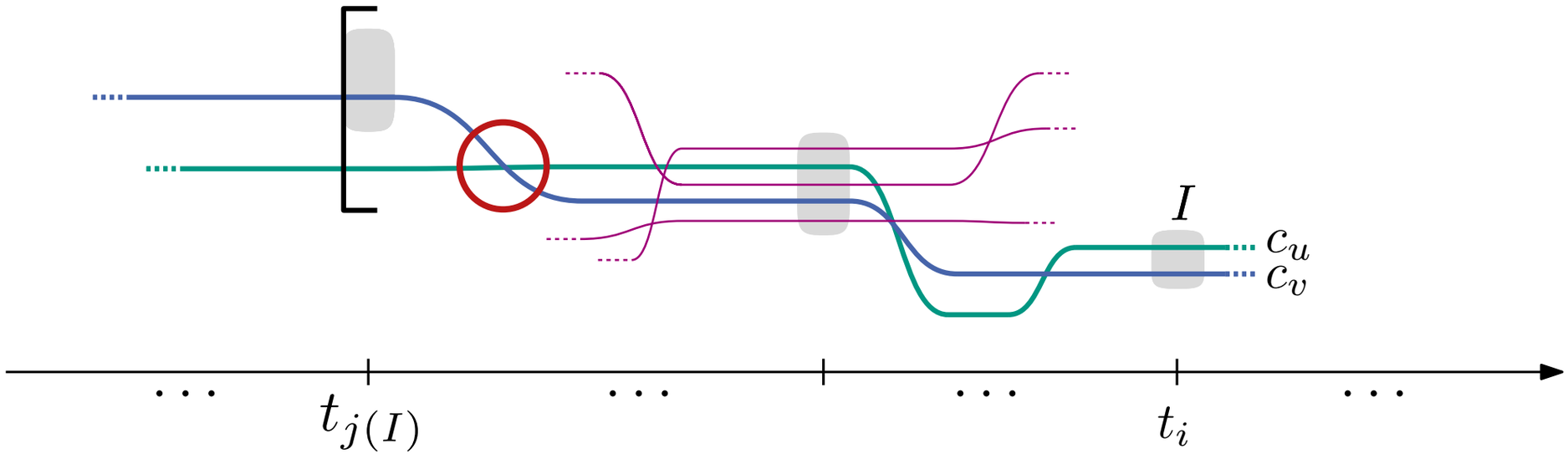

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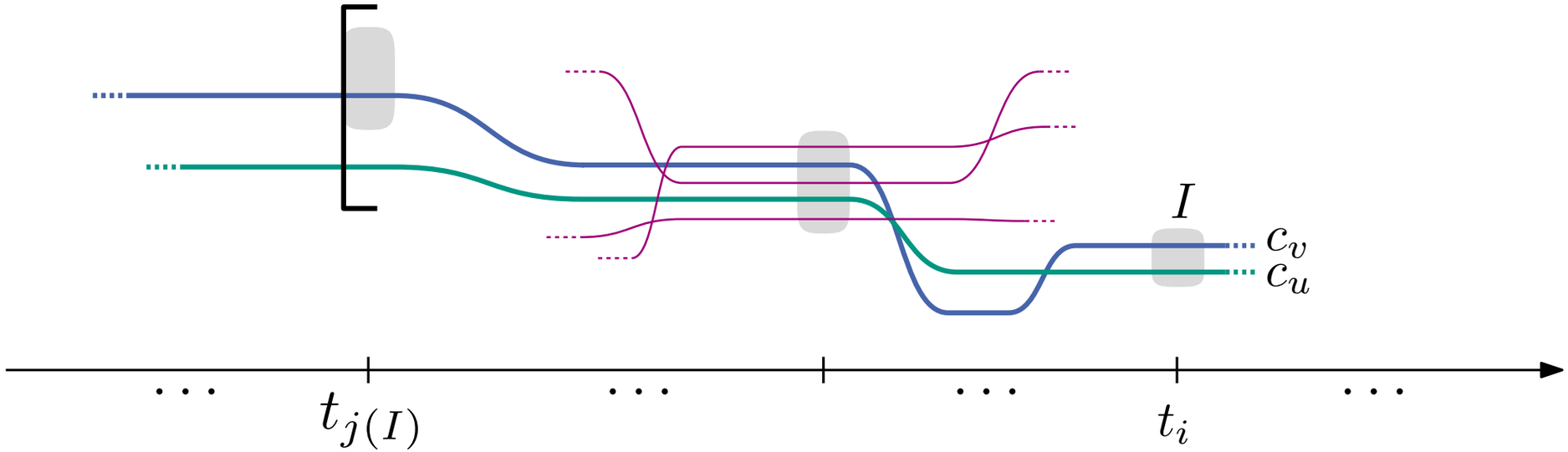

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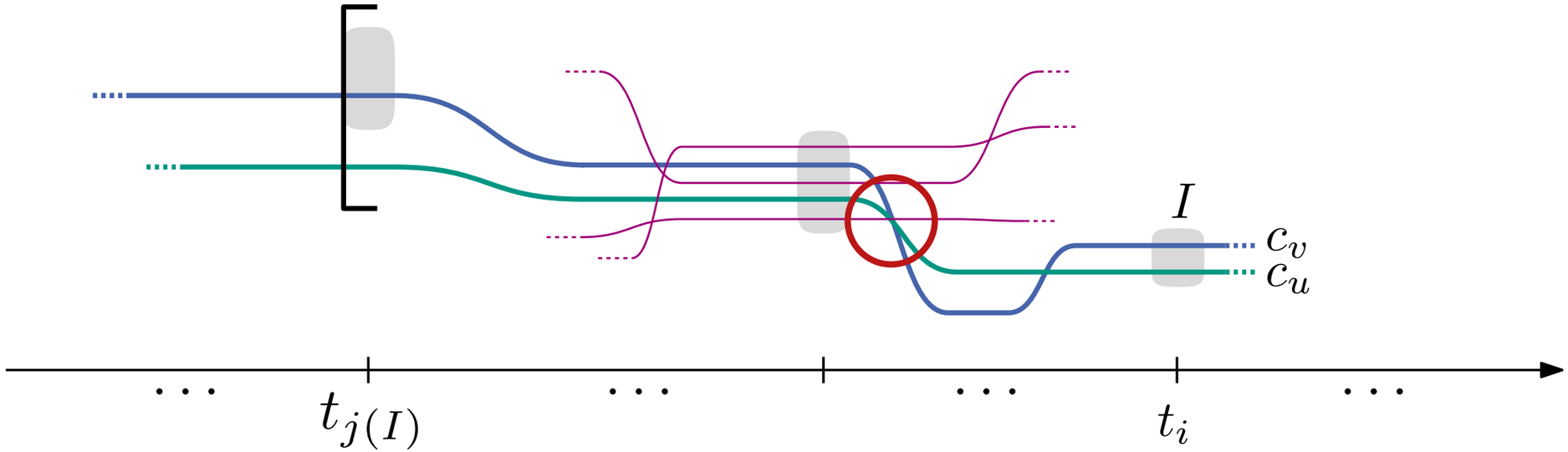

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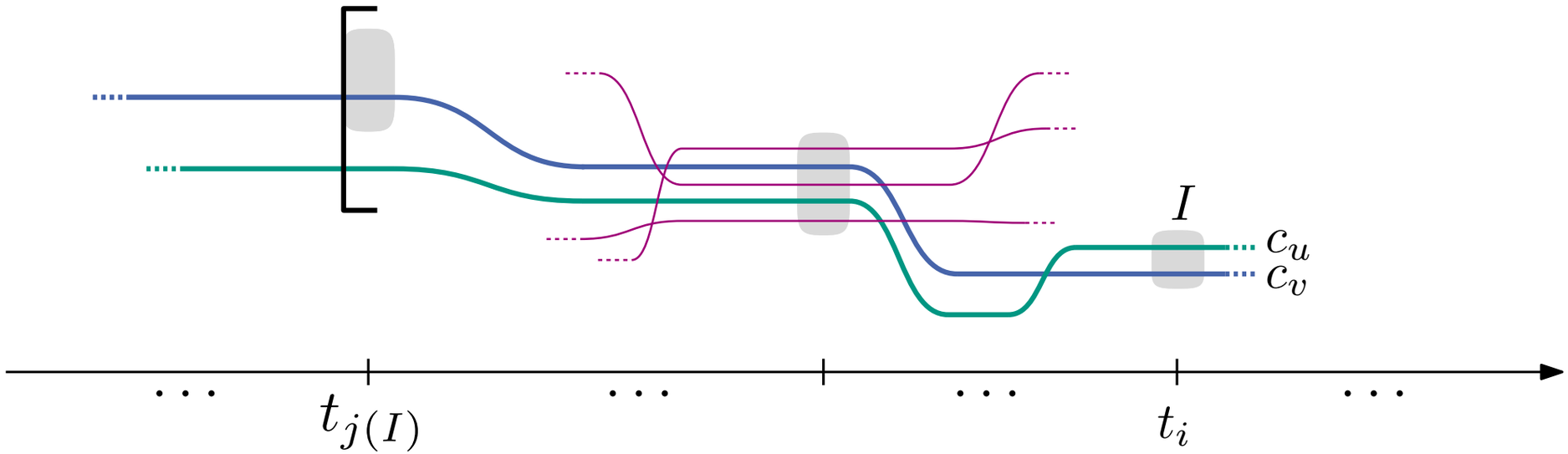

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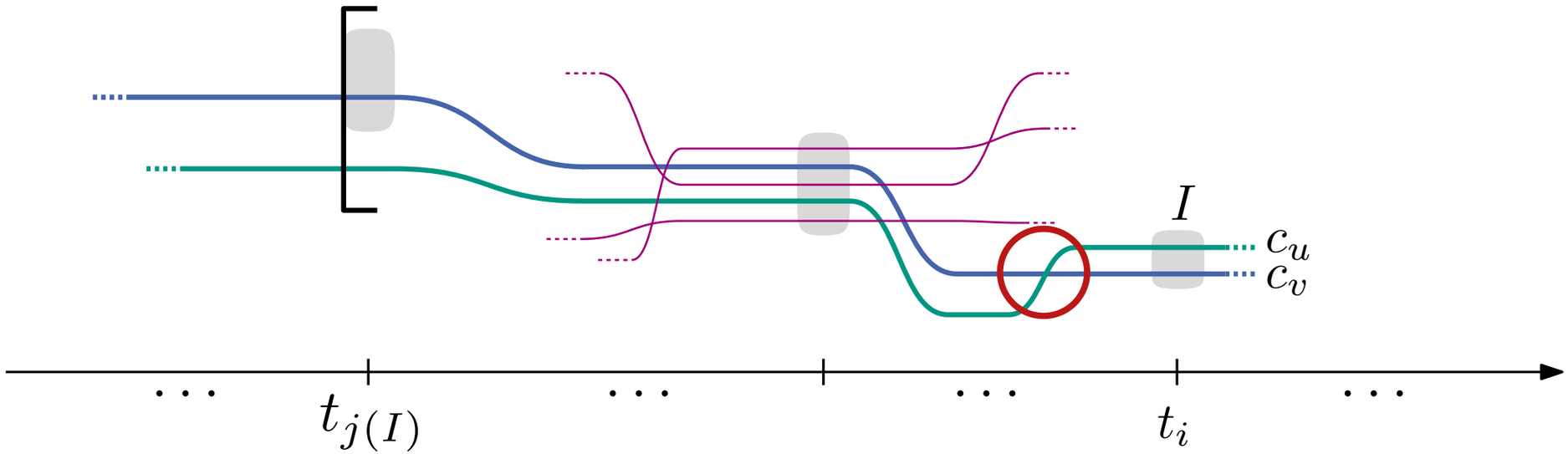

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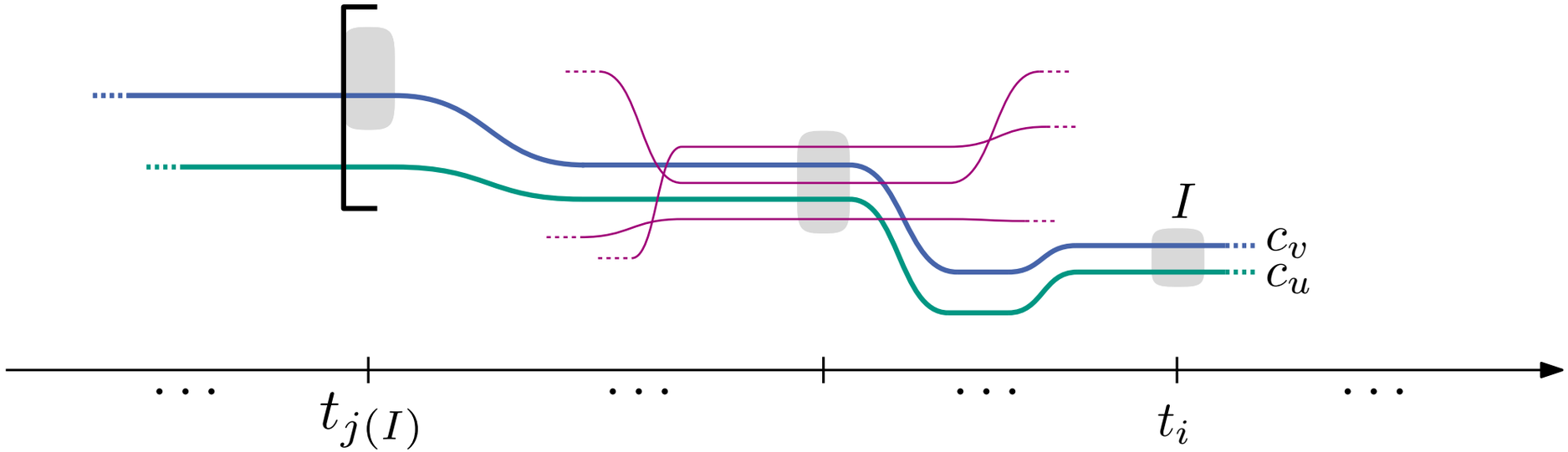

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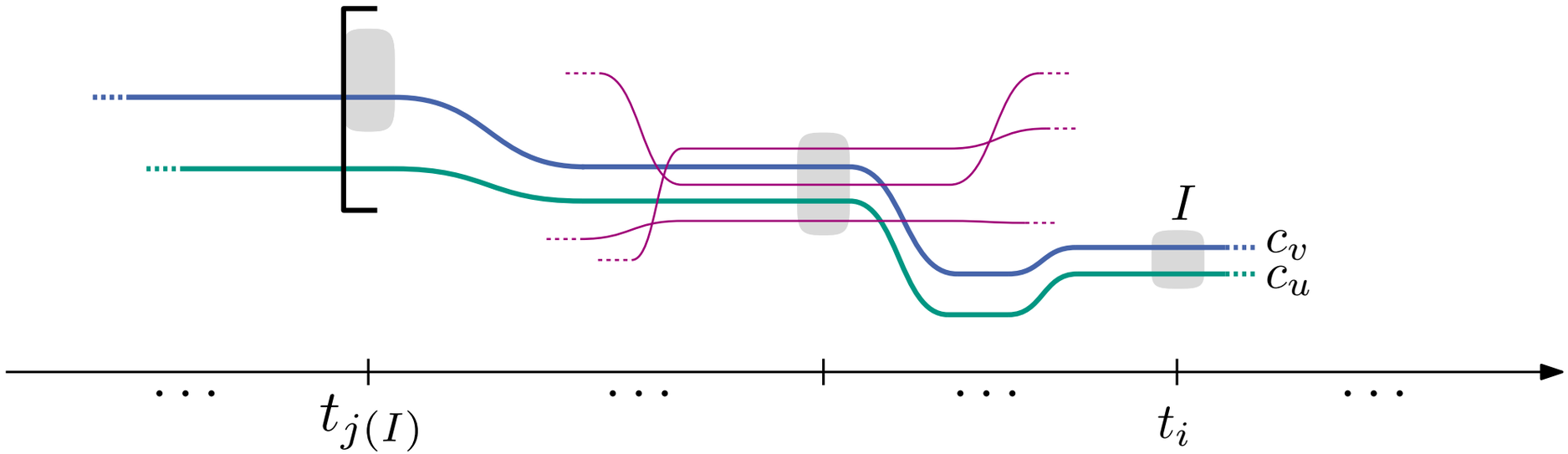

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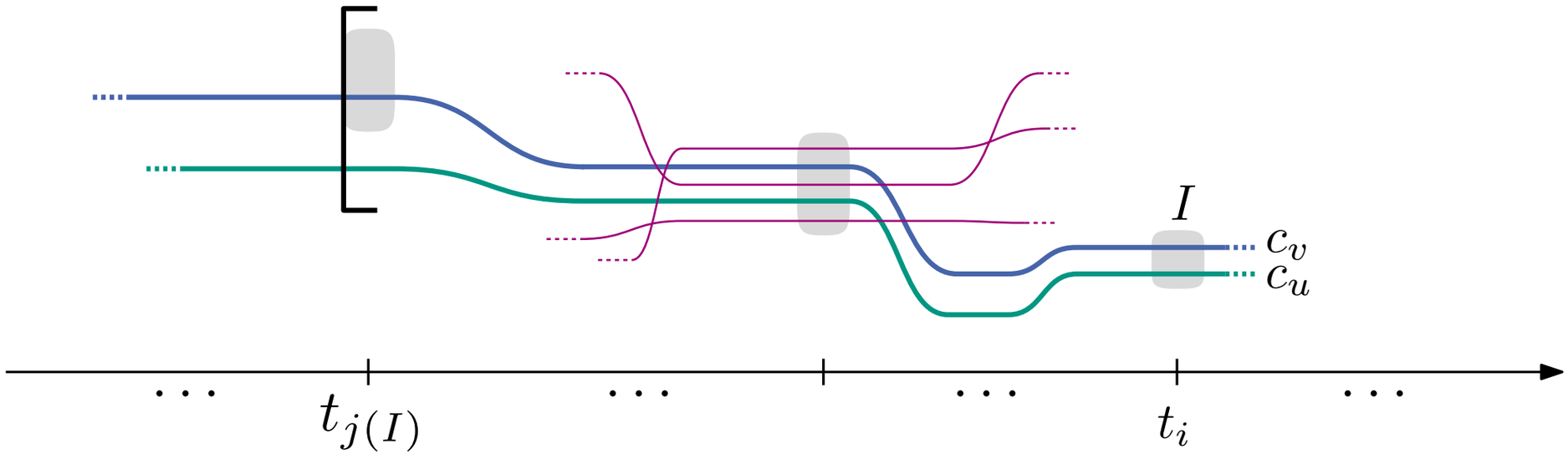

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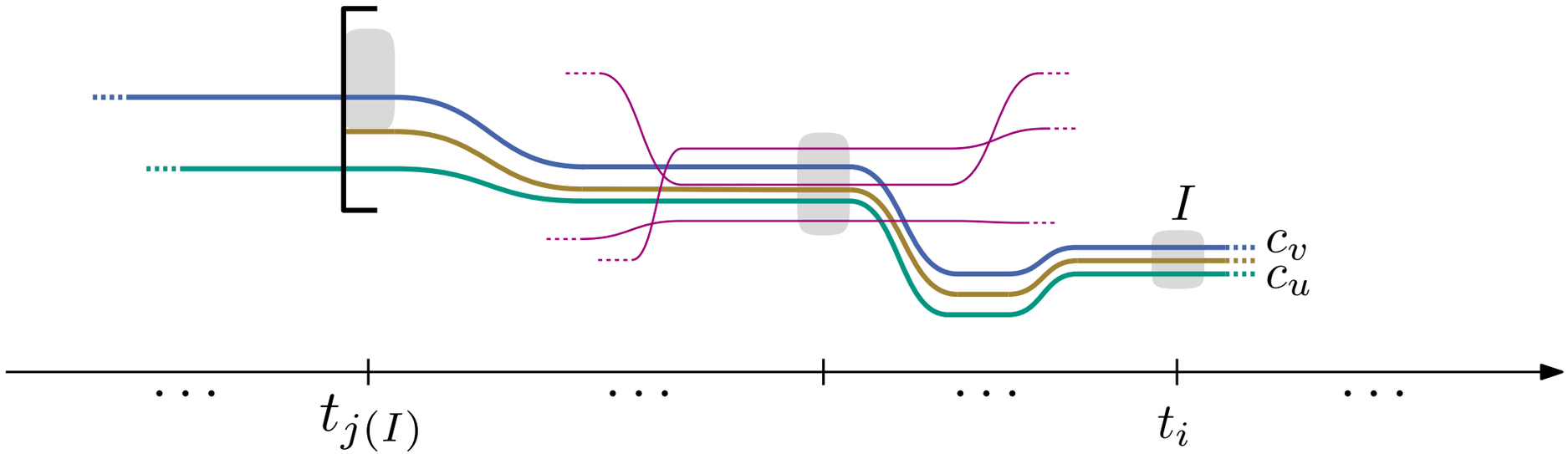

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This also works for **more than two characters**!

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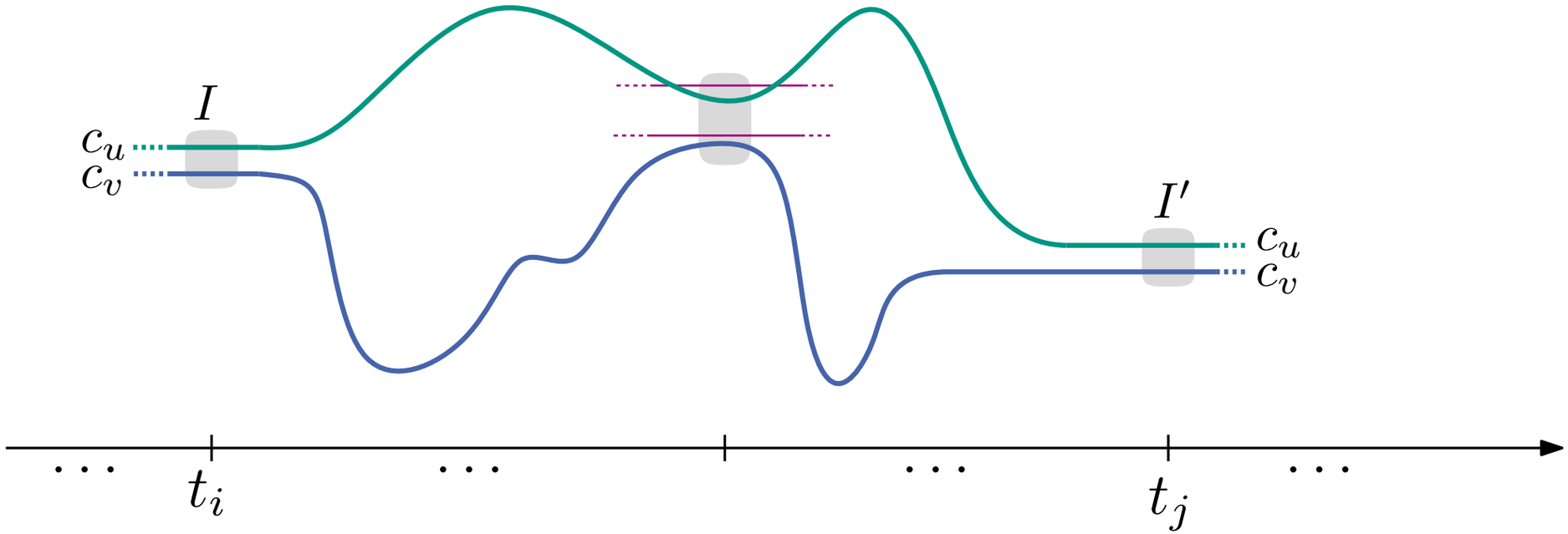

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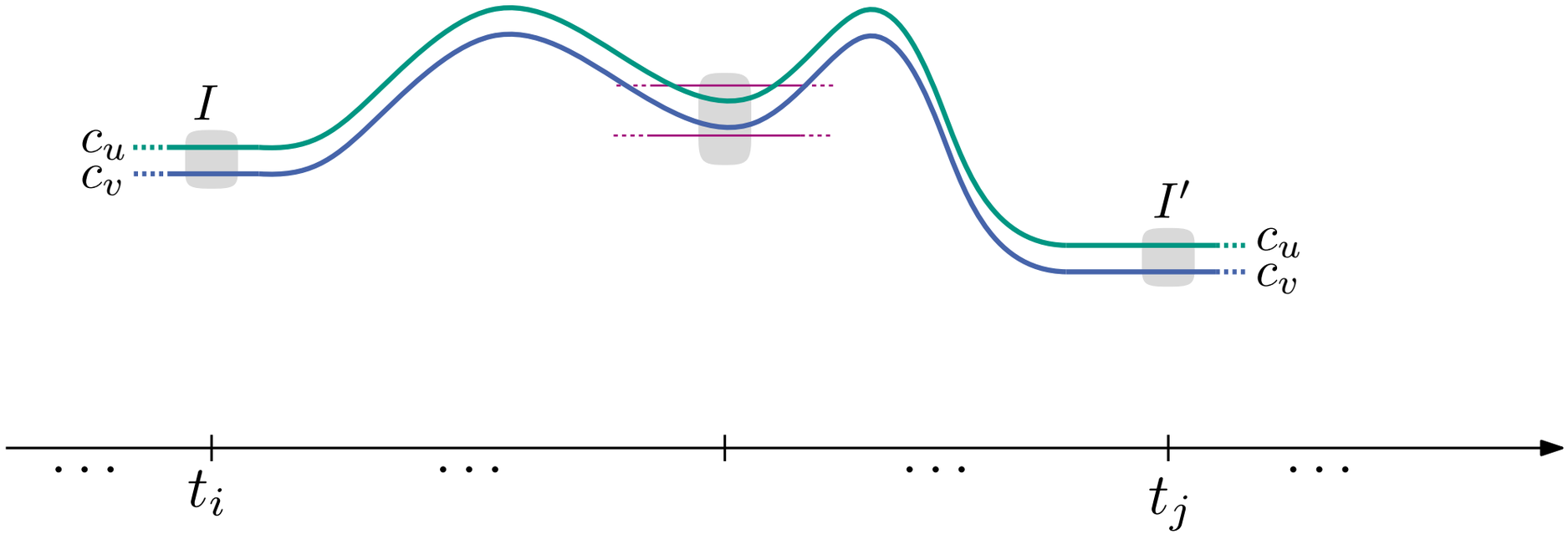

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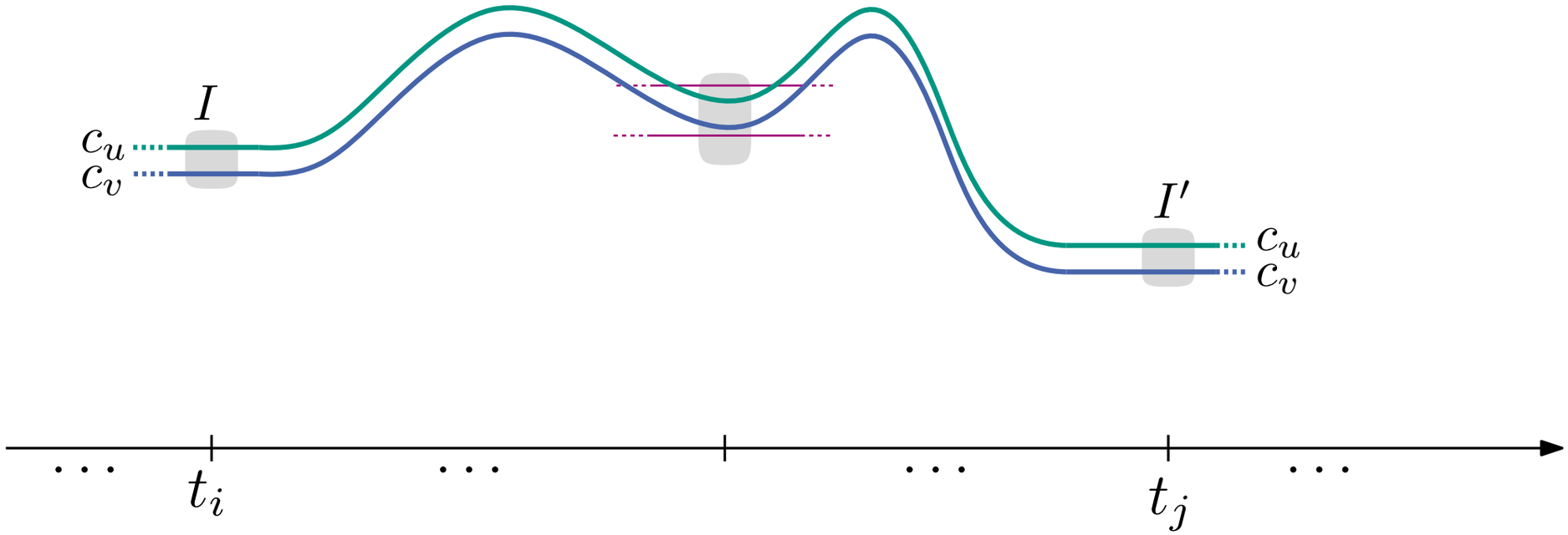

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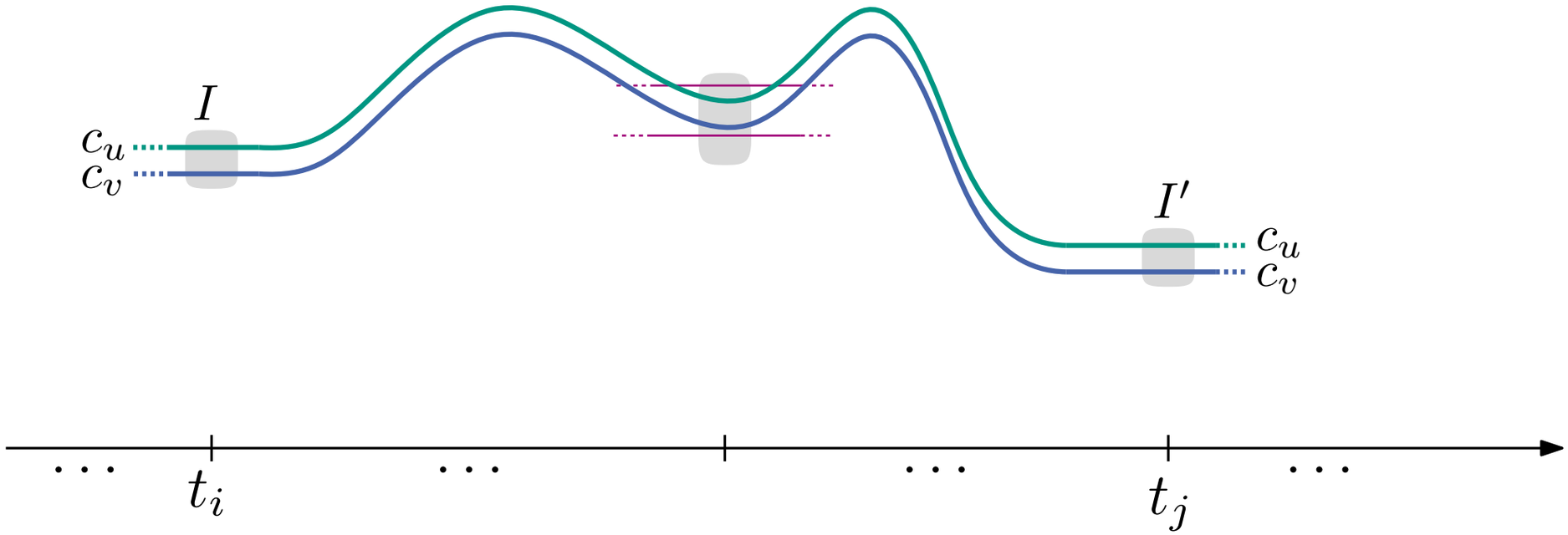

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Also **extends** to interaction with **more characters**!

Structural Property 3 and New ILP Model

Models have **cubic** number of
transitivity constraints

$$x_{i,u,v} + x_{i,v,w} + x_{i,w,u} \leq 2$$
$$\forall i \ldots, \ell; c_u, c_v, c_w \in \mathrm{AC}(t_i)$$

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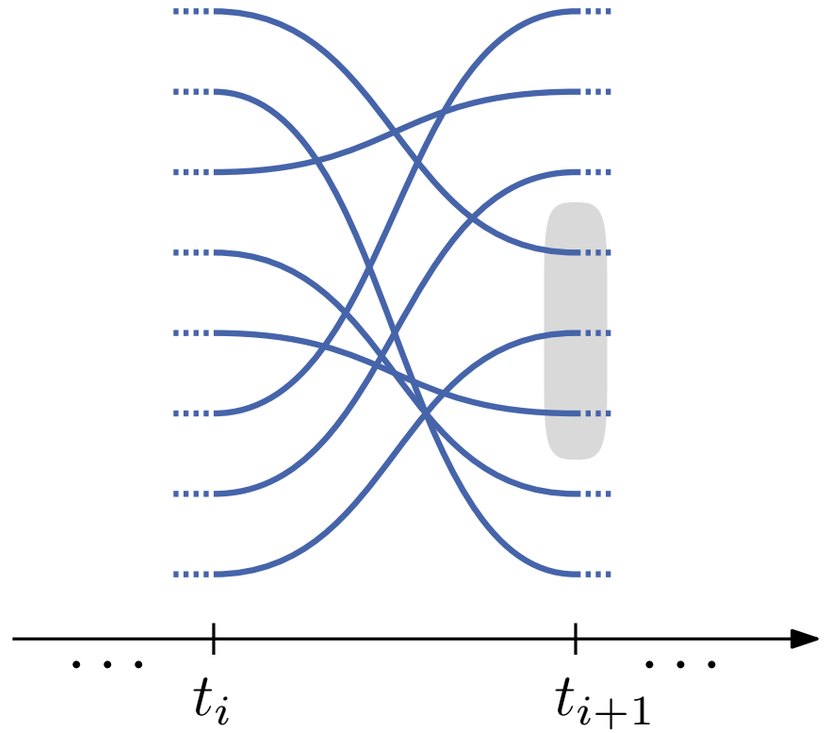

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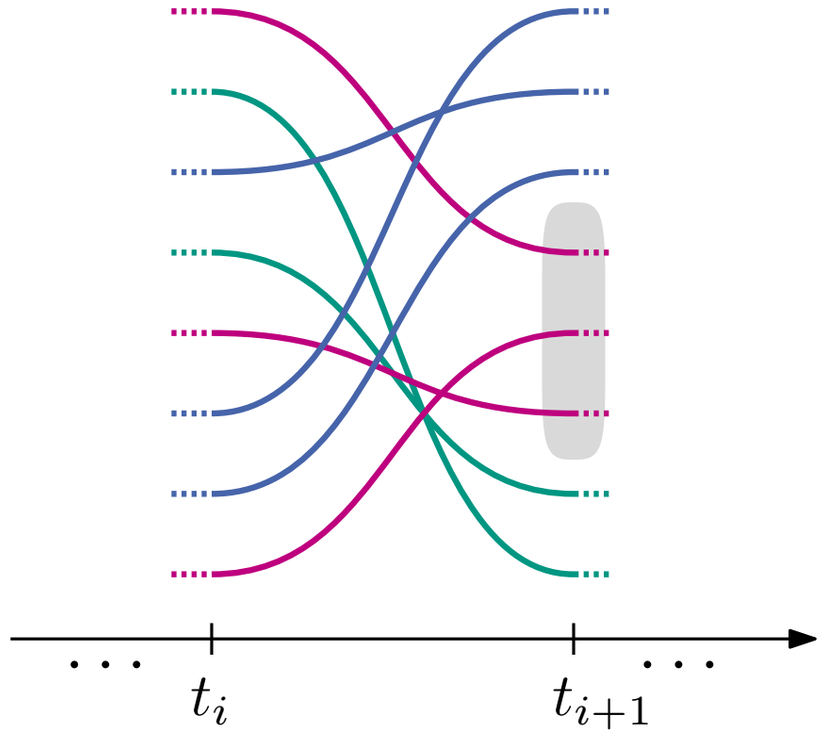

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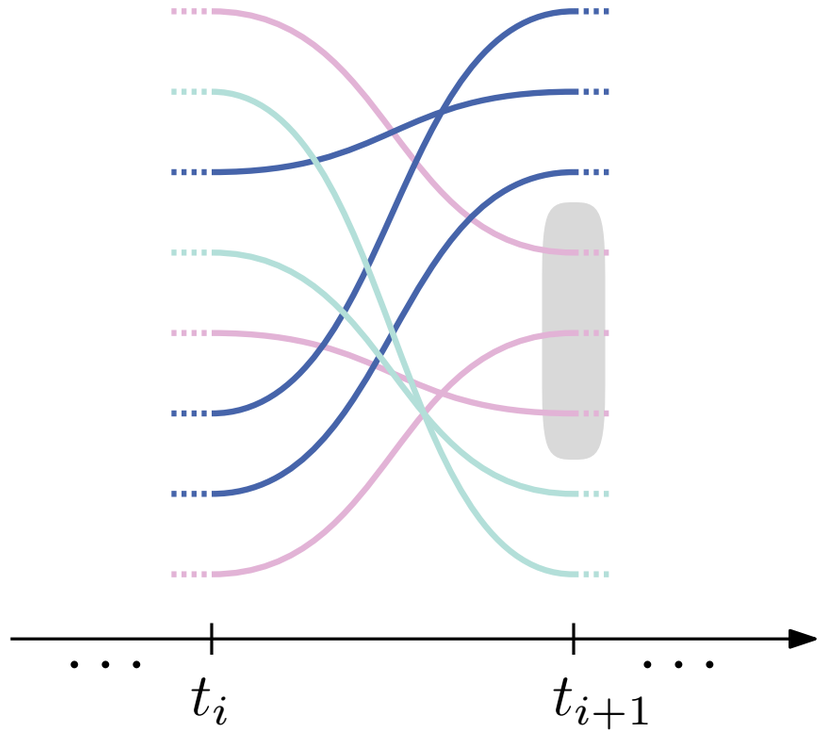

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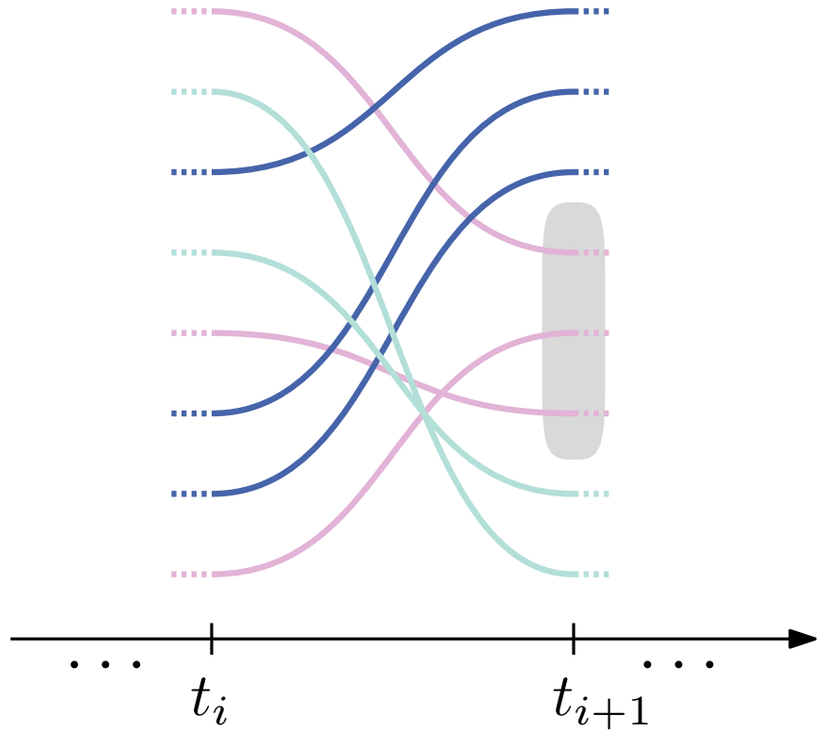

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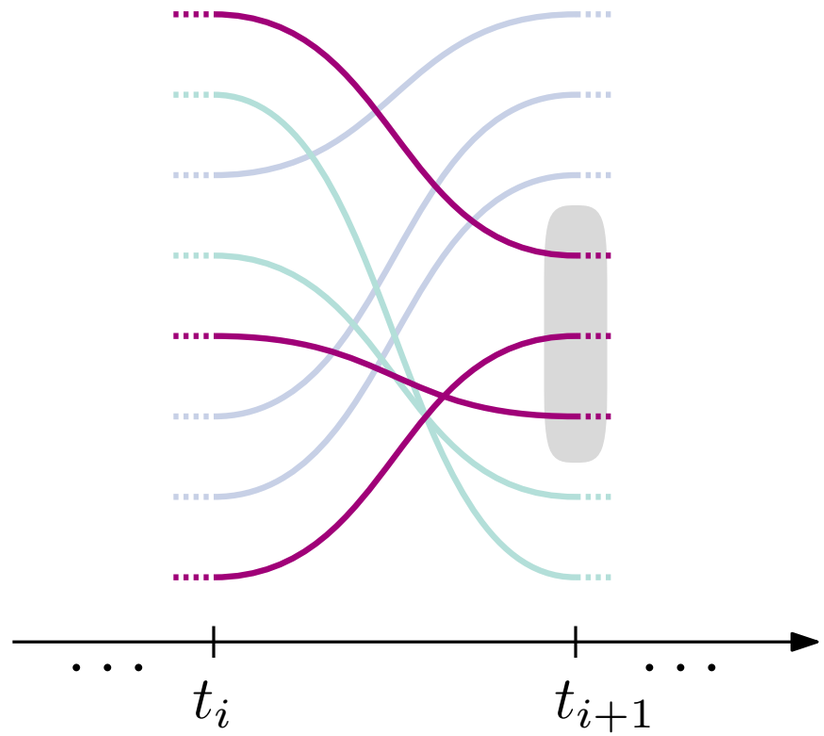

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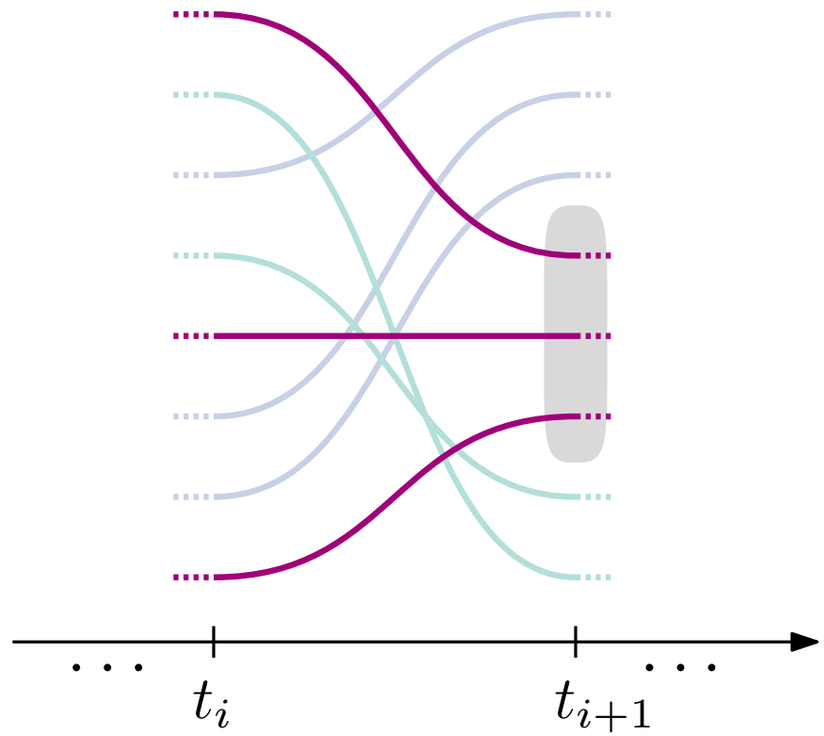

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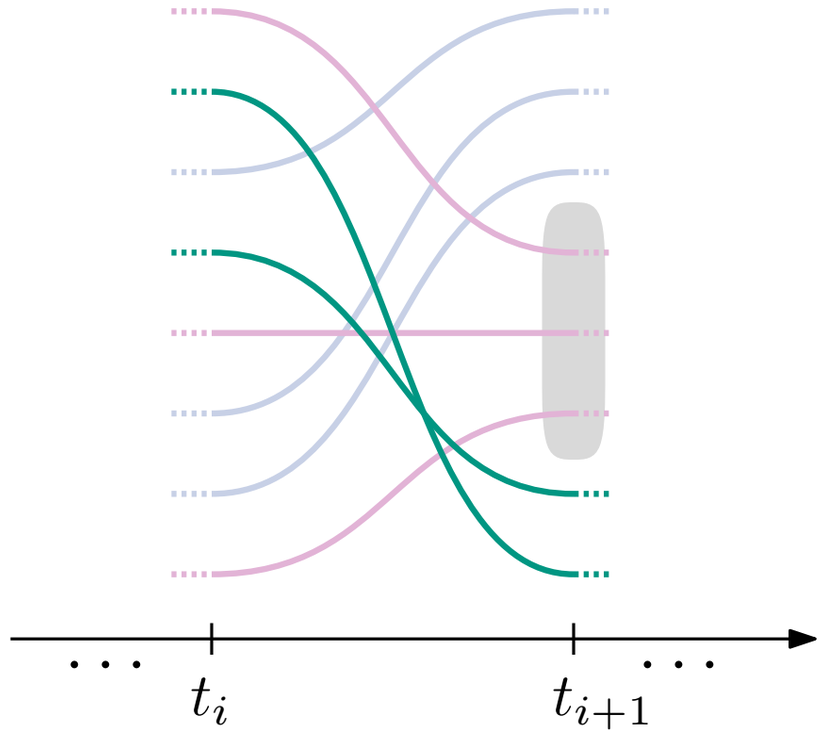

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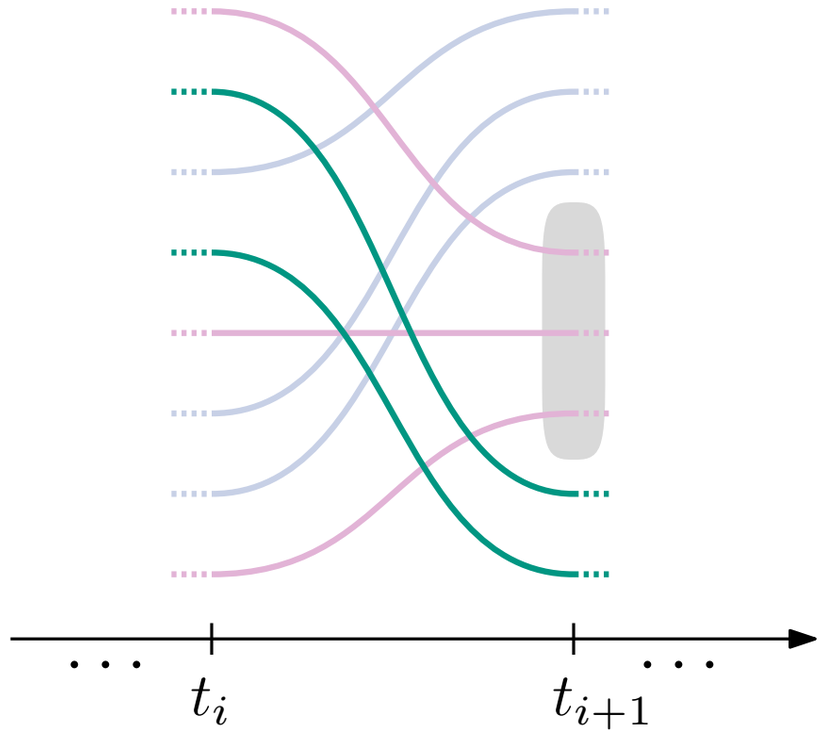

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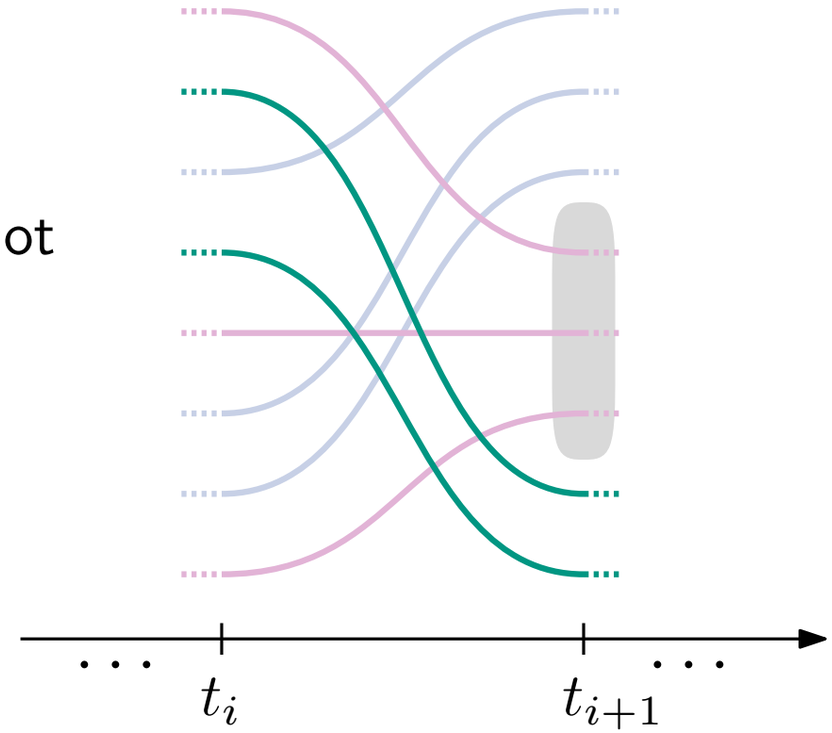

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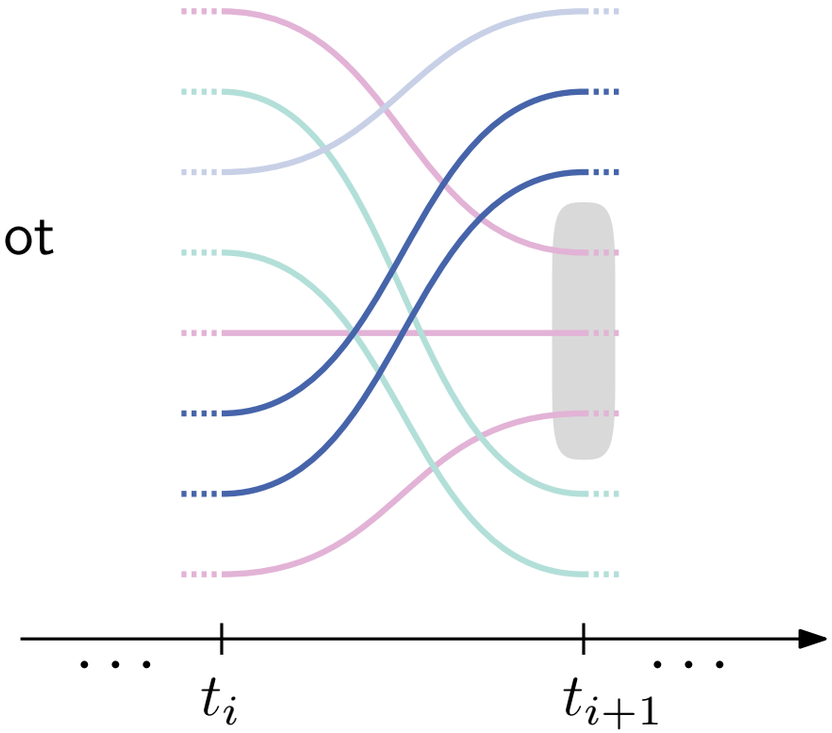

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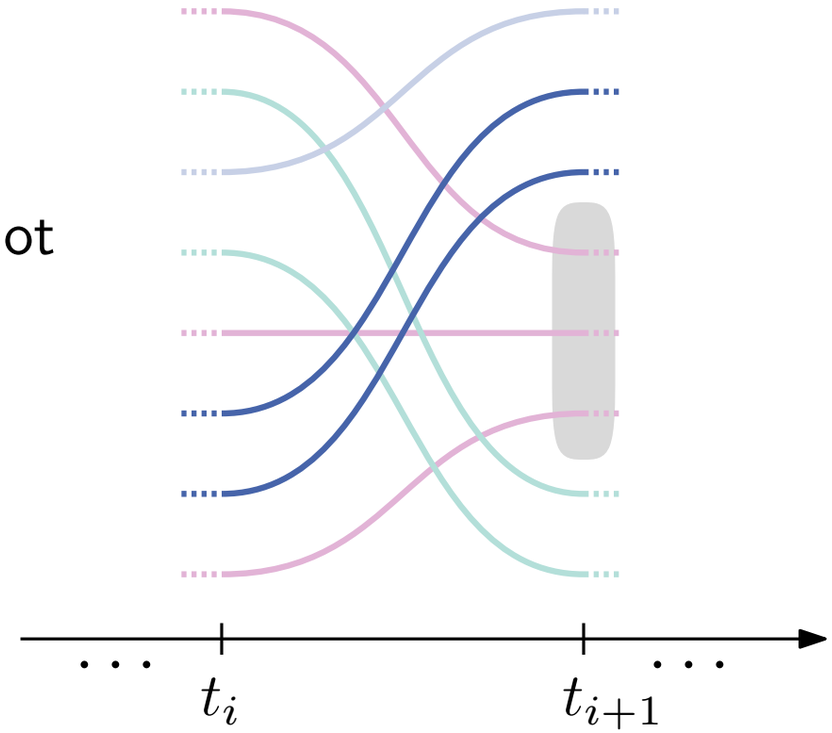

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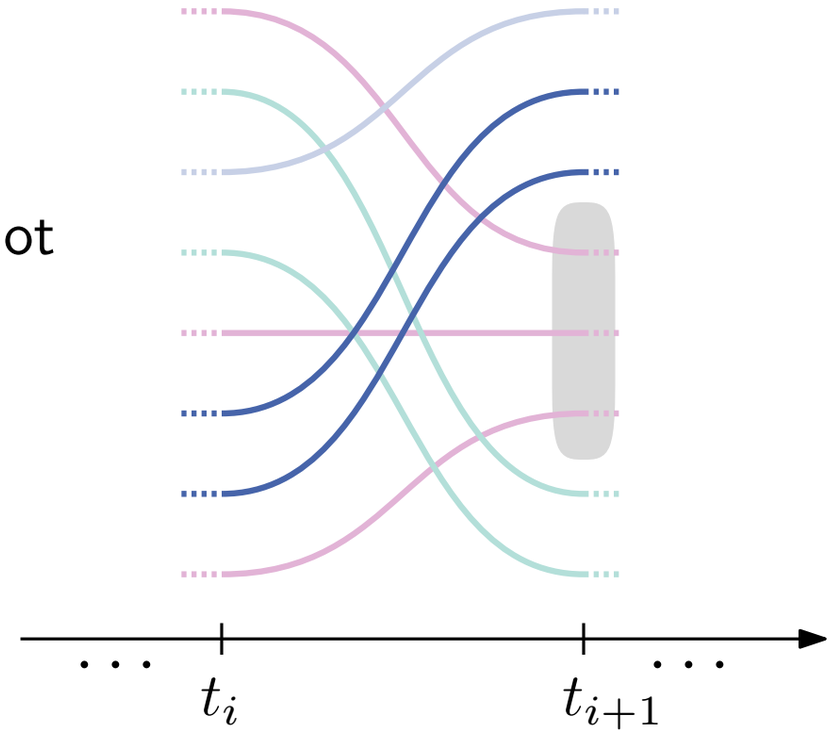

New Model: **PLO**

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- **structural** problem-specific **insights**,
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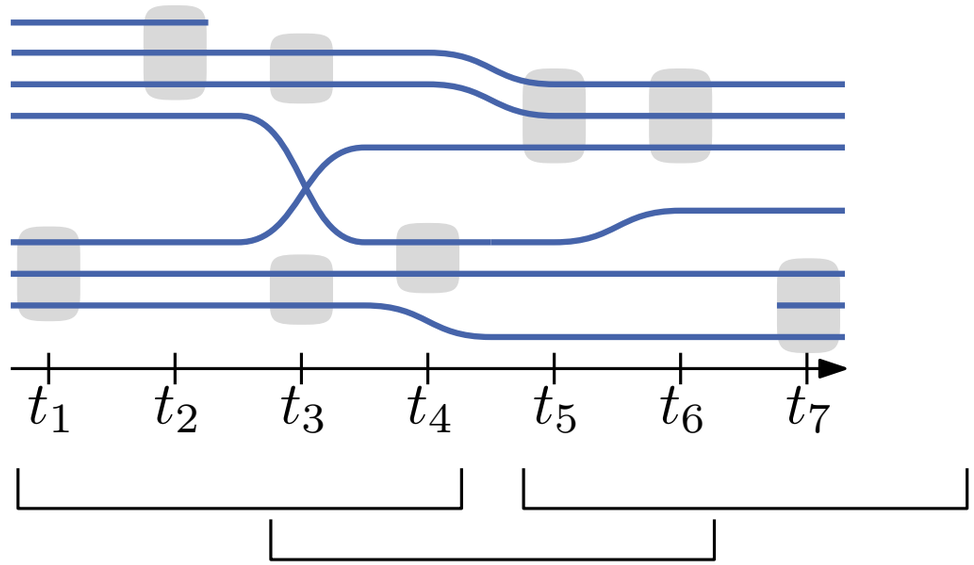

Compute optimal solution for **overlapping slices** and **glue them together**

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Afterwards: apply **local search** based on **structural observations** and a **barycenter-type heuristic**

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- **MC**: Reimplemented Max-Cut Formulation of [GJLM, 2016]
- **LIN**: Linearized model
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All algorithms use **Gurobi** (Gurobi allows quadratic models)

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35–88 characters, 54–329 interactions

Results

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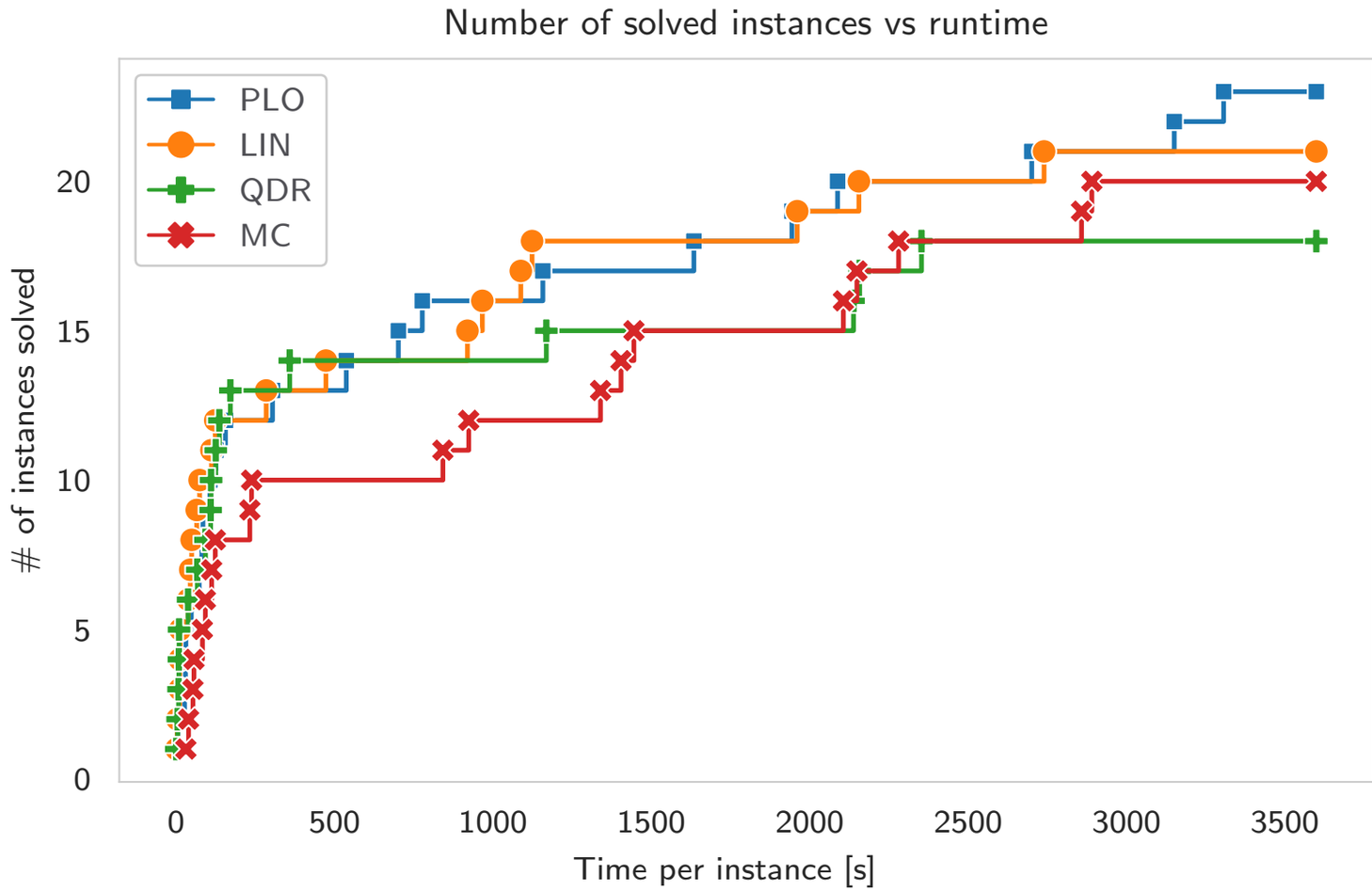

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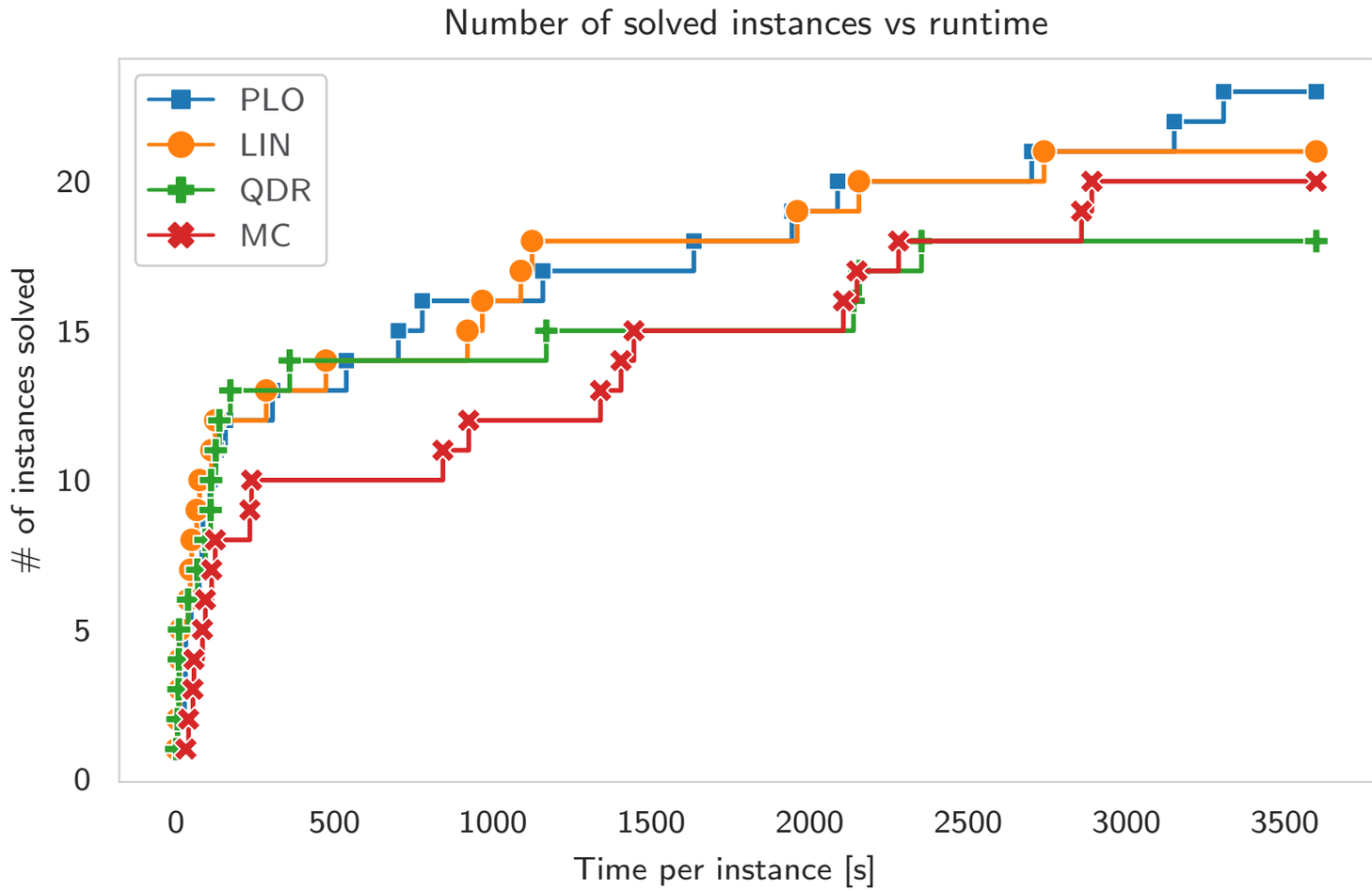

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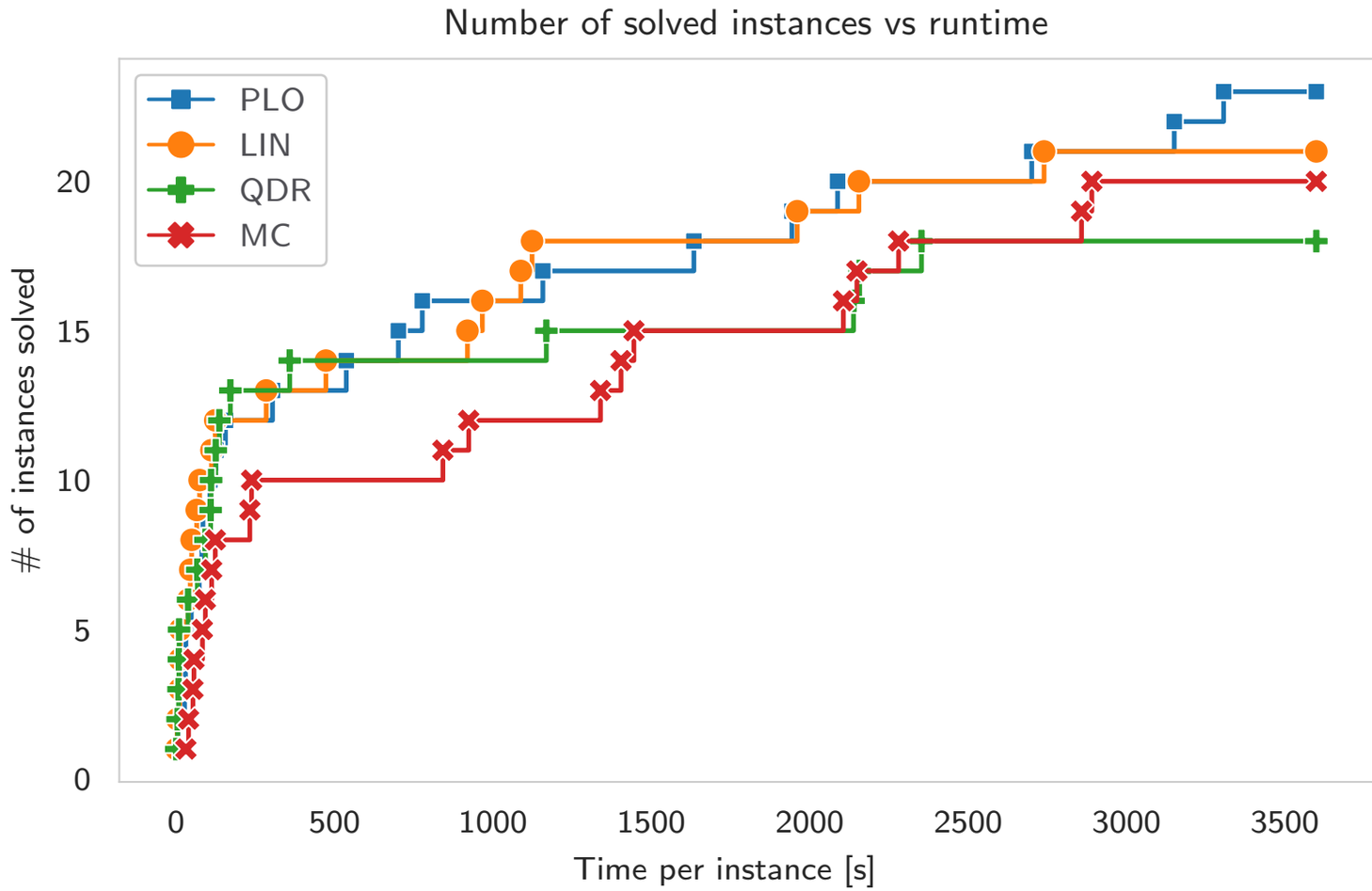

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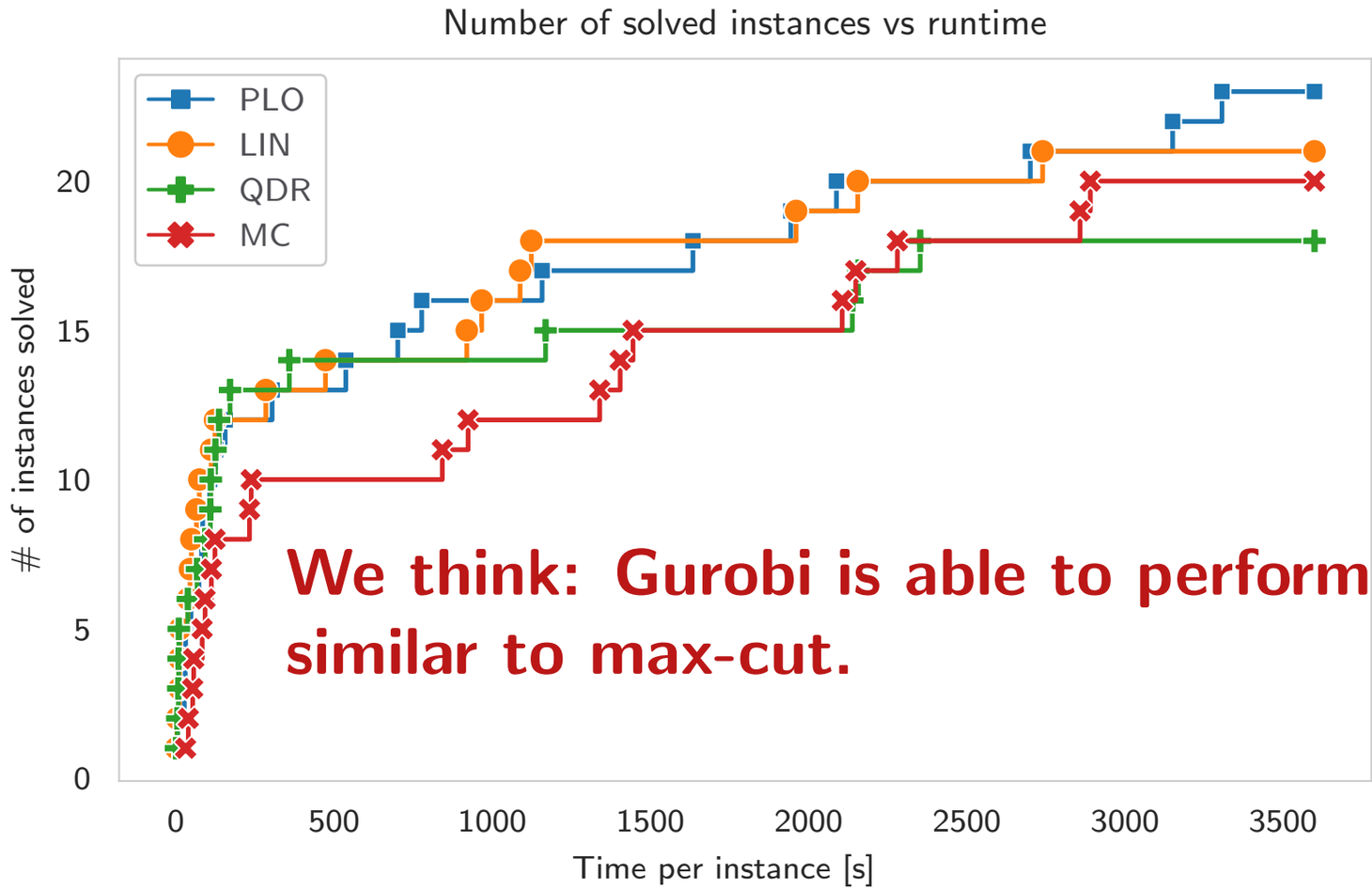

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We think: Gurobi is able to perform transformation similar to max-cut.

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speedup factor	**PLO**	**LIN**	**QDR**
SBC	1.12	1.35	1.42
INIT	1.05	1.09	0.97
RND	1.50	1.37	1.52

Conclusion

Enriching **simpler models** by problem-specific insights can **outperform more complex models** (at least when implemented in Gurobi).

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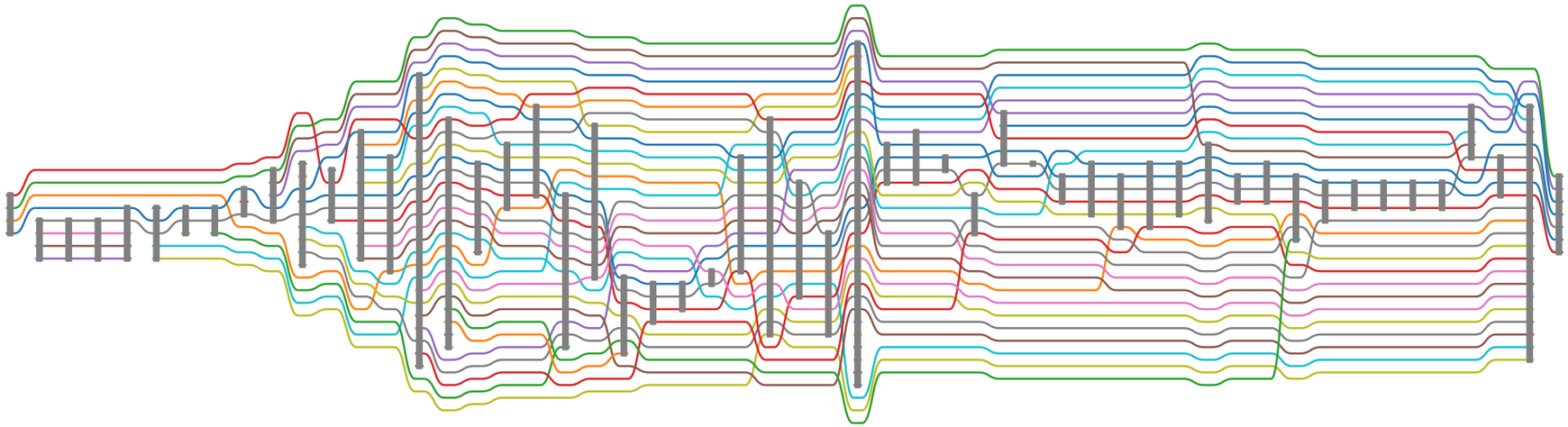

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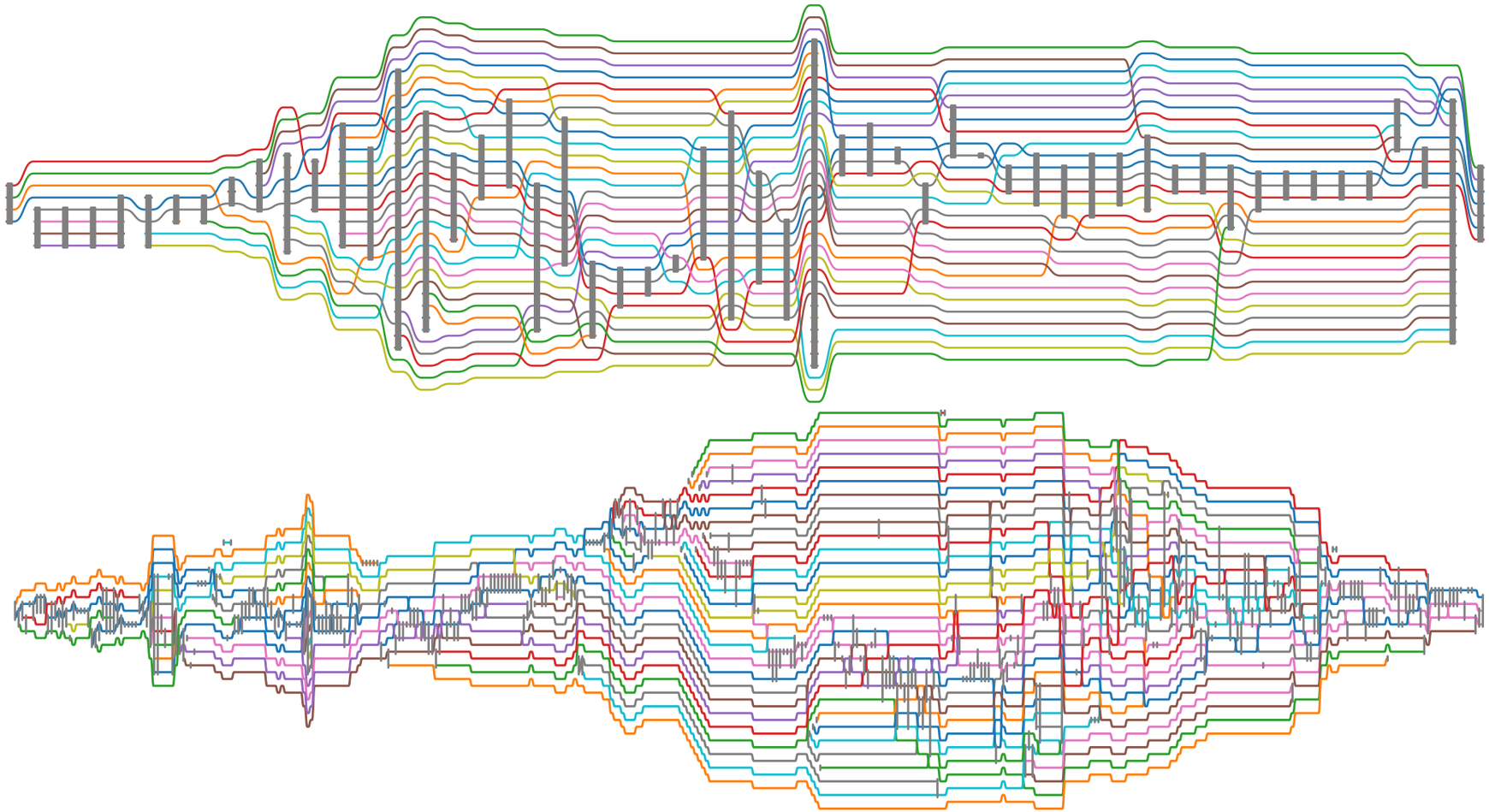

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Les Misérables
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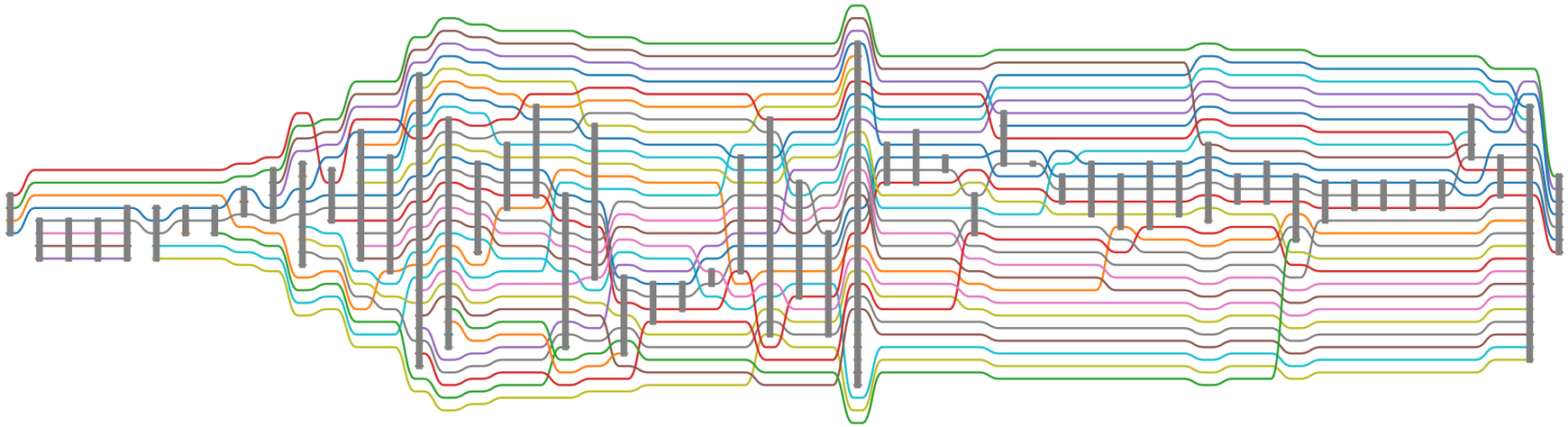

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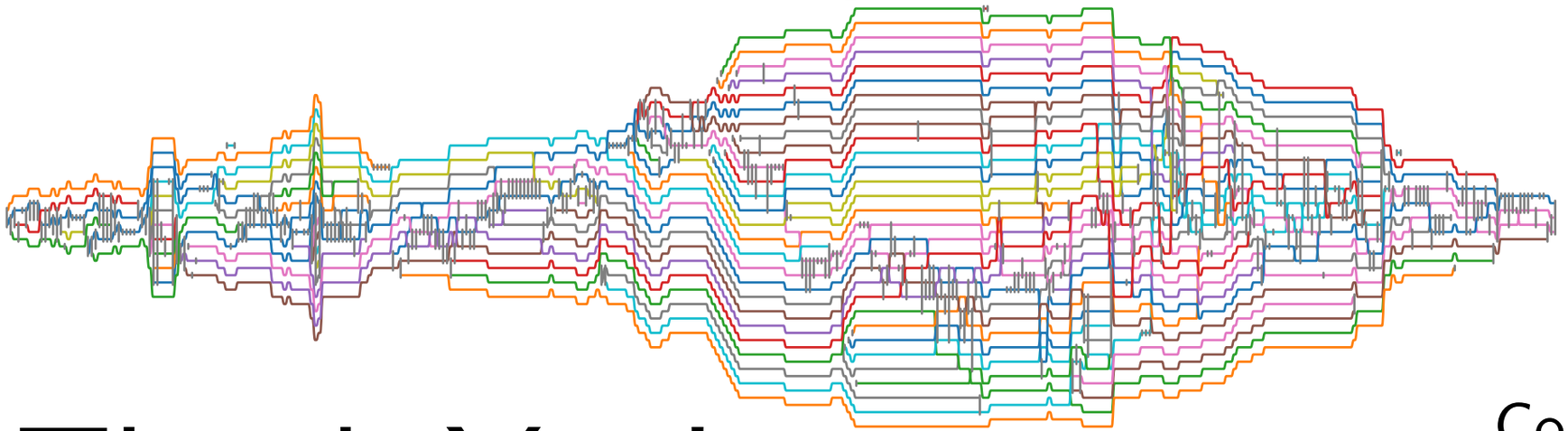

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Thank You!

Code and instances

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$$x_{i,u,v} \in \{0, 1\} \qquad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i), \qquad \text{(BIN)}$$

$\mathrm{AC}(t_i, t_{i+1})$ . . . characters active at t_i and t_{i+1}

A Simple Quadratic Integer Program

Binary variable $x_{i,u,v}$ which is 1 iff character c_u is before c_v in π_i

$$\min \sum_{i=1}^{\ell-1} \sum_{c_u, c_v \in \mathrm{AC}(t_i, t_{i+1})} x_{i,u,v} x_{i+1,v,u} \qquad \text{(QDR)}$$

Transitivity

$$x_{i,u,v} = 1 - x_{i,v,u} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i) \text{ with } u < v \qquad \text{(EQ)}$$

$$x_{i,u,v} + x_{i,v,w} + x_{i,w,u} \leq 2 \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v, c_w \in \mathrm{AC}(t_i) \qquad \text{(LOP)}$$

$$x_{i,u,w} = x_{i,v,w} \quad \text{for all } i = 1, \ldots, \ell; I \in \mathcal{I}(t_i); \qquad \text{(TREE)}$$
$$c_u, c_v \in \mathrm{char}(I), u < v; c_w \in \mathrm{AC}(t_i) \setminus \mathrm{char}(I)$$

$$x_{i,u,v} \in \{0, 1\} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i), \qquad \text{(BIN)}$$

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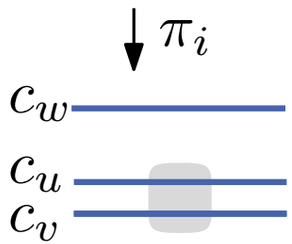
$$x_{i,u,v} \in \{0, 1\} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i), \qquad \text{(BIN)}$$

Tree contraints



$\mathrm{AC}(t_i, t_{i+1})$ . . . characters active at t_i and t_{i+1}

Linearized Model

linearized objective

$$\min \sum_{i=1}^{\ell-1} \sum_{c_u, c_v \in \mathrm{AC}(t_i, t_{i+1})} y_{i,u,v} \qquad \text{(LIN)}$$

$$y_{i,u,v} \geq x_{i,u,v} - x_{i+1,u,v} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i, t_{i+1}) \qquad \text{(CR)}$$

$$x_{i,u,v} = 1 - x_{i,v,u} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i) \text{ with } u < v \qquad \text{(EQ)}$$

$$x_{i,u,v} + x_{i,v,w} + x_{i,w,u} \leq 2 \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v, c_w \in \mathrm{AC}(t_i) \qquad \text{(LOP)}$$

$$x_{i,u,w} = x_{i,v,w} \quad \text{for all } i = 1, \ldots, \ell; I \in \mathcal{I}(t_i); \qquad \text{(TREE)}$$
$$c_u, c_v \in \mathrm{char}(I), u < v; c_w \in \mathrm{AC}(t_i) \setminus \mathrm{char}(I)$$

$$x_{i,u,v} \in \{0, 1\} \quad \text{for all } i = 1, \ldots, \ell; c_u, c_v \in \mathrm{AC}(t_i), \qquad \text{(BIN)}$$

Linear Max-Cut Model by [Gronemann et al., 2016]

Gronemann et al. reformulated the problem as a **Max-Cut** problem on specific weighted graph G and computed optimal solutions on real-world storylines

exponential number of constraints
$\rightarrow$ branch and cut, complicated to implement

$$\max \sum_{e \in E_M} w_e z_e \qquad \text{(CUT)}$$

$$\sum_{e \in F} z_e - \sum_{e \in C \setminus F} z_e \leq |F| - 1 \quad \text{for all cycles } C \subseteq E_M, F \subseteq C, |F| \text{ odd} \qquad \text{(CYC)}$$

$$0 \leq z_{(v^*, c^i_{uv})} + z_{(v^*, c^i_{vw})} - z_{(v^*, c^i_{uw})} \leq 1 \quad \text{for all } i = 1, \ldots, \ell; c^i_{uv}, c^i_{vw}, c^i_{uw} \in V_i \text{ with } c_u \prec_{\hat{\pi}_i} c_v \prec_{\hat{\pi}_i} c_w \qquad \text{(LOPC)}$$

$$\left.\begin{aligned} z_{(v^*, c^i_{uw})} = z_{(v^*, c^i_{vw})} \text{ if } c_u, c_v \prec_{\hat{\pi}_i} c_w \\ z_{(v^*, c^i_{wu})} = z_{(v^*, c^i_{wv})} \text{ if } c_u, c_v \succ_{\hat{\pi}_i} c_w \end{aligned}\right\} \quad \begin{aligned} &\text{for all } i = 1, \ldots, \ell; I \in \mathcal{I}(t_i); \\ &c_u, c_v \in \mathrm{char}(I); c_w \in \mathrm{AC}(t_i) \setminus \mathrm{char}(I) \end{aligned} \qquad \text{(TRC)}$$

$$z_e \in \{0, 1\} \quad \text{for all } e \in E_M \qquad \text{(BIC)}$$

More Results