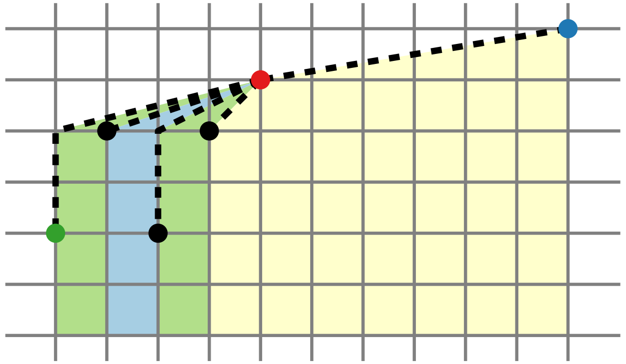
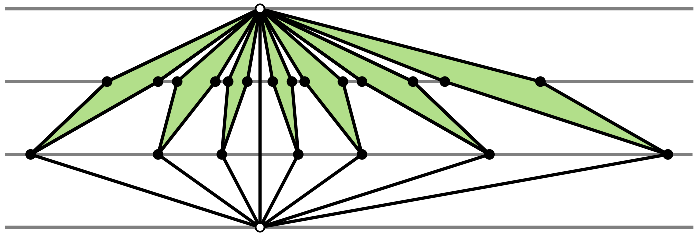
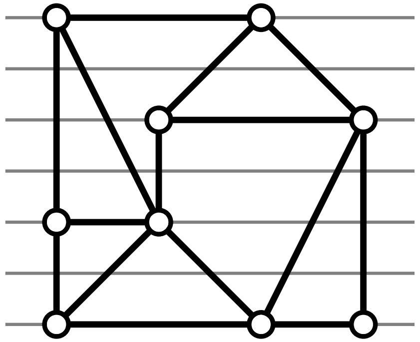
UNIVERSITÄT TRIER

ALGO

Weakly Leveled Planarity with Bounded Span







Michael Bekos

Giordano Da Lozzo

Fabrizio Frati

Siddharth Gupta

Philipp Kindermann

Giuseppe Liotta

Ignaz Rutter

Ioannis Tollis

GNV 2023
Graph & Network Vis. Workshop

GD 2024
Graph Drawing & Network Vis.

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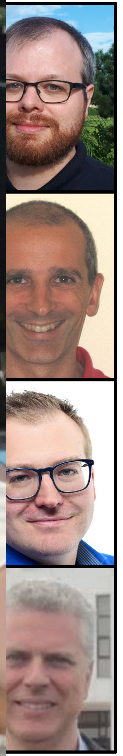

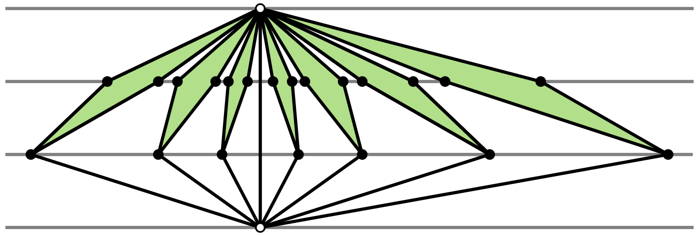

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Weakly Leveled Planarity with Bounded Span

(Weakly) leveled planar drawings

Leveled planar drawing

(Weakly) leveled planar drawings

Leveled planar drawing

- Set of horizontal lines (*levels*)

(Weakly) leveled planar drawings

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(Weakly) leveled planar drawings

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- Set of horizontal lines (*levels*)
- Every vertex is a point on a level

(Weakly) leveled planar drawings

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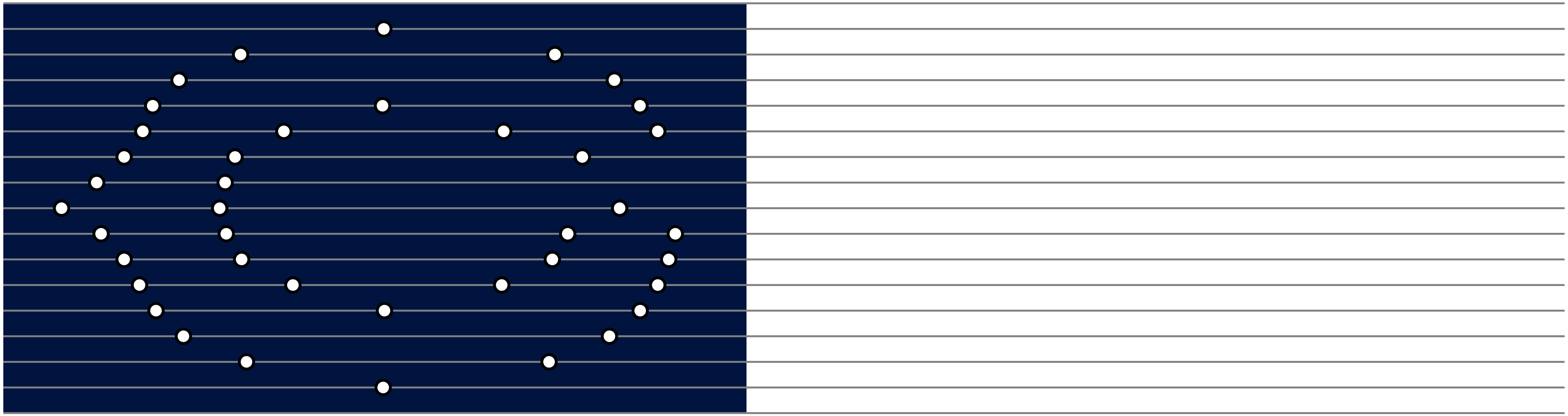
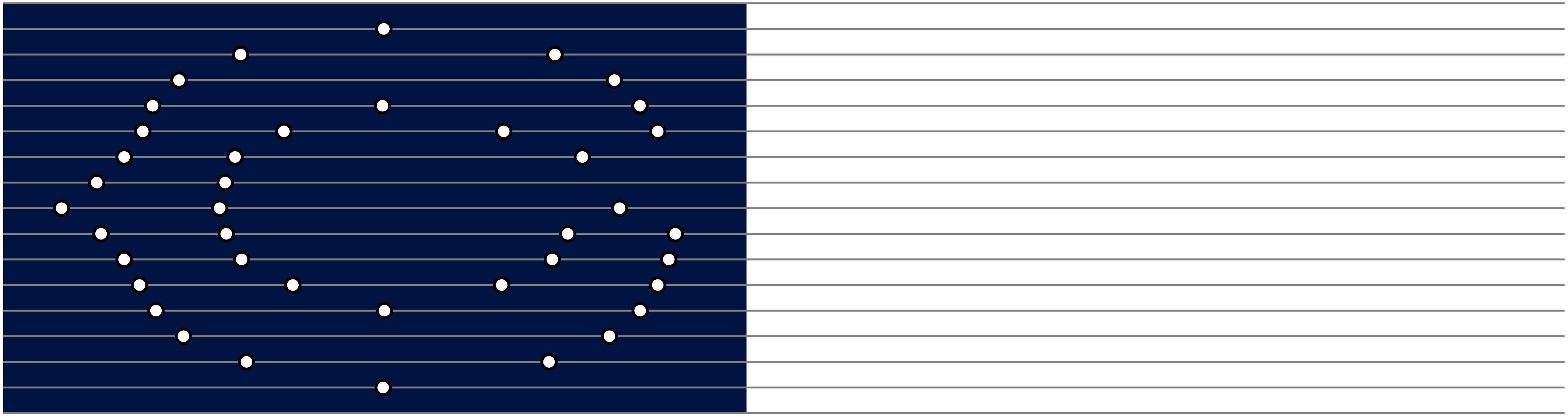


(Weakly) leveled planar drawings

Leveled planar drawing

- Set of horizontal lines (*levels*)
- Every vertex is a point on a level
- Edges are only between adjacent levels



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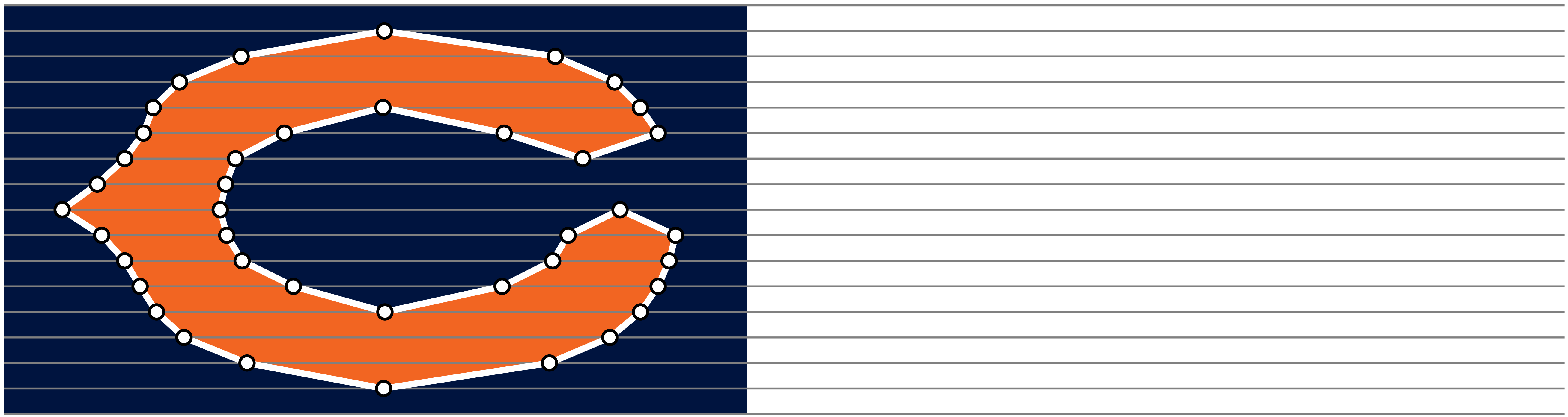
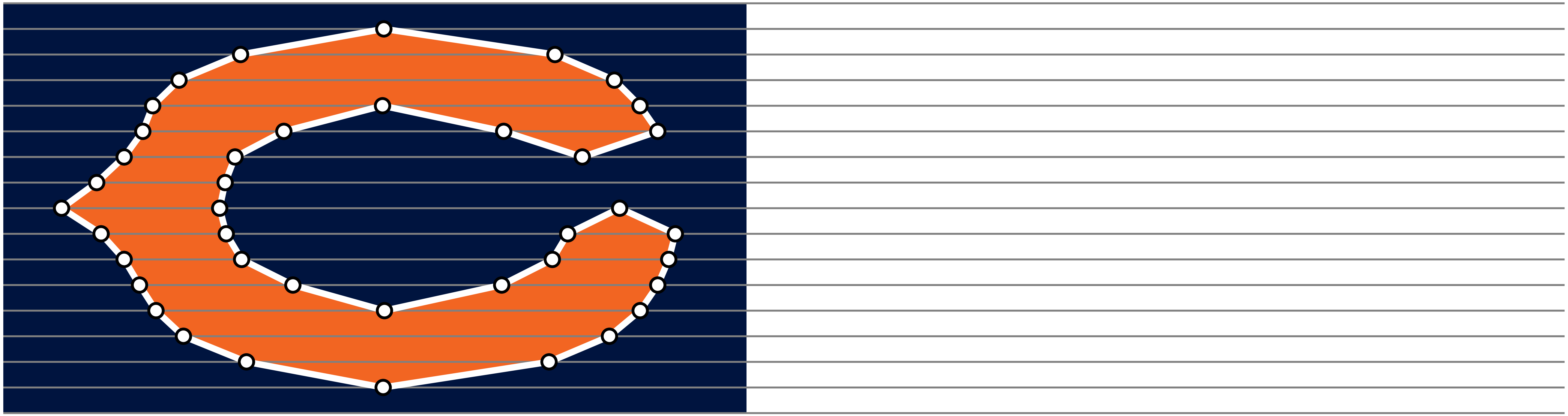


(Weakly) leveled planar drawings

Leveled planar drawing

- Set of horizontal lines (*levels*)
- Every vertex is a point on a level
- Edges are only between adjacent levels

- Drawing is planar



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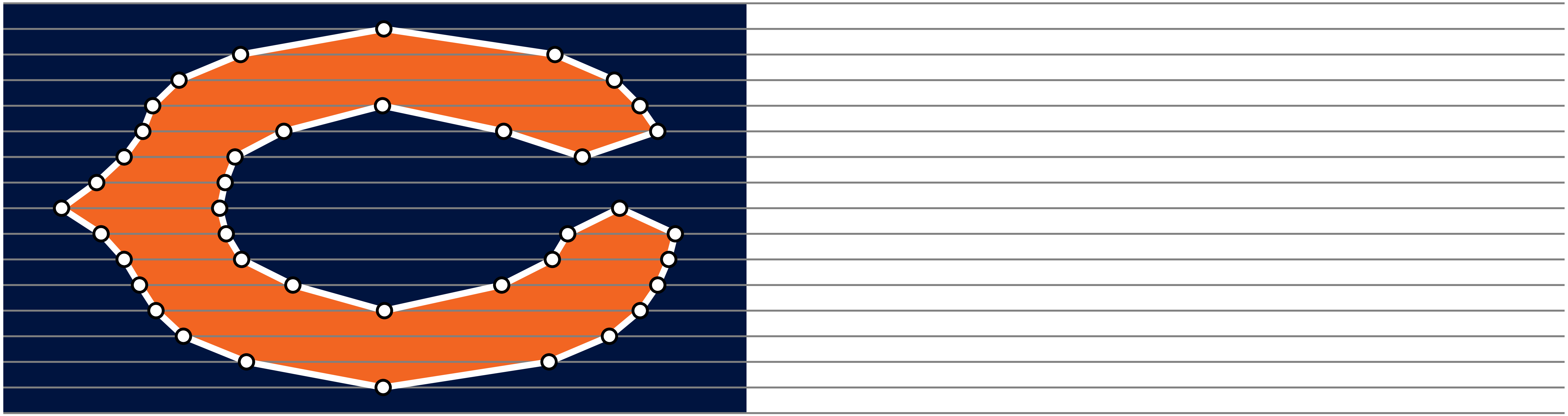

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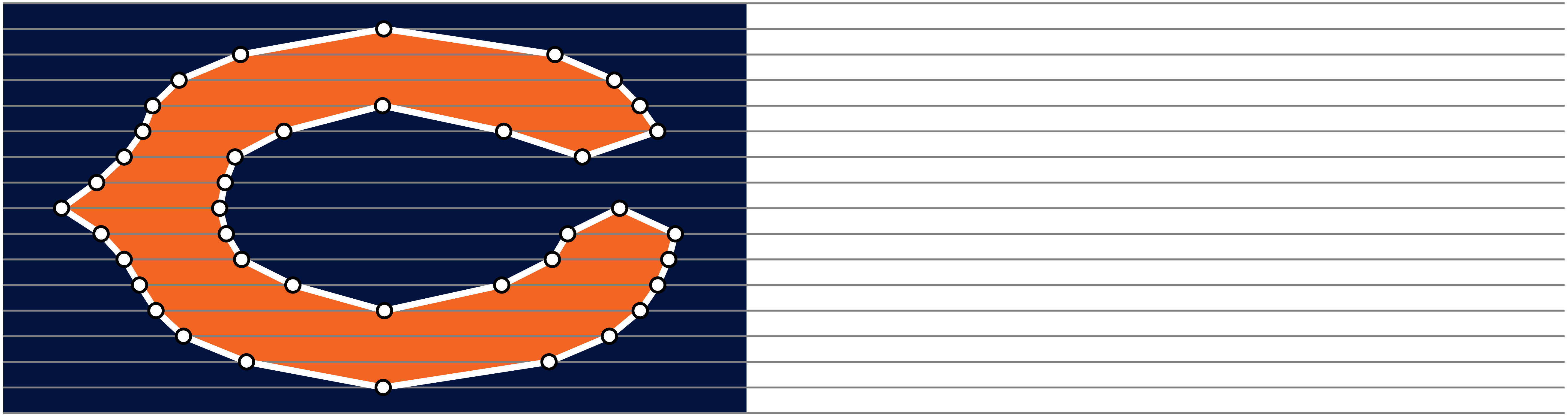
Weakly leveled planar drawing

- . . . or horizontal segments



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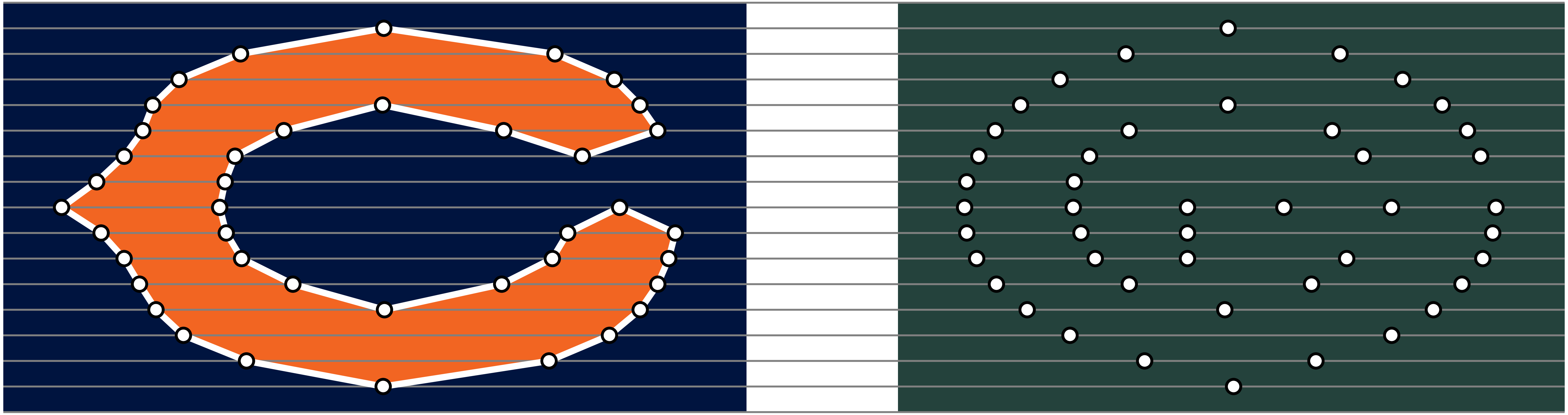

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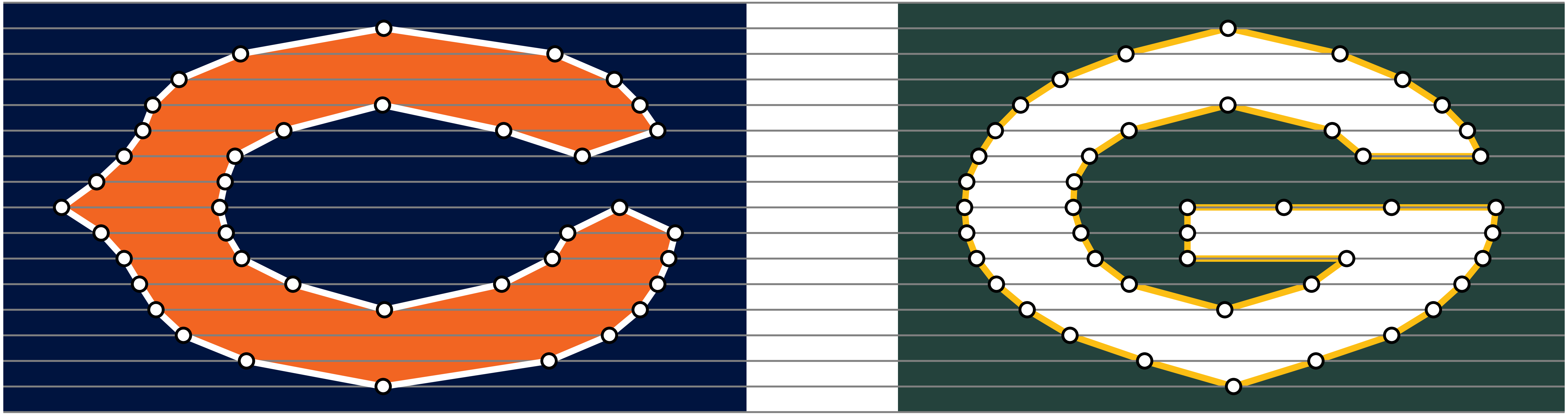

(Weakly) leveled planar drawings

s-span leveled planar drawing

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- Edges are only between adjacent levels
- Drawing is planar

s-span weakly leveled planar drawing

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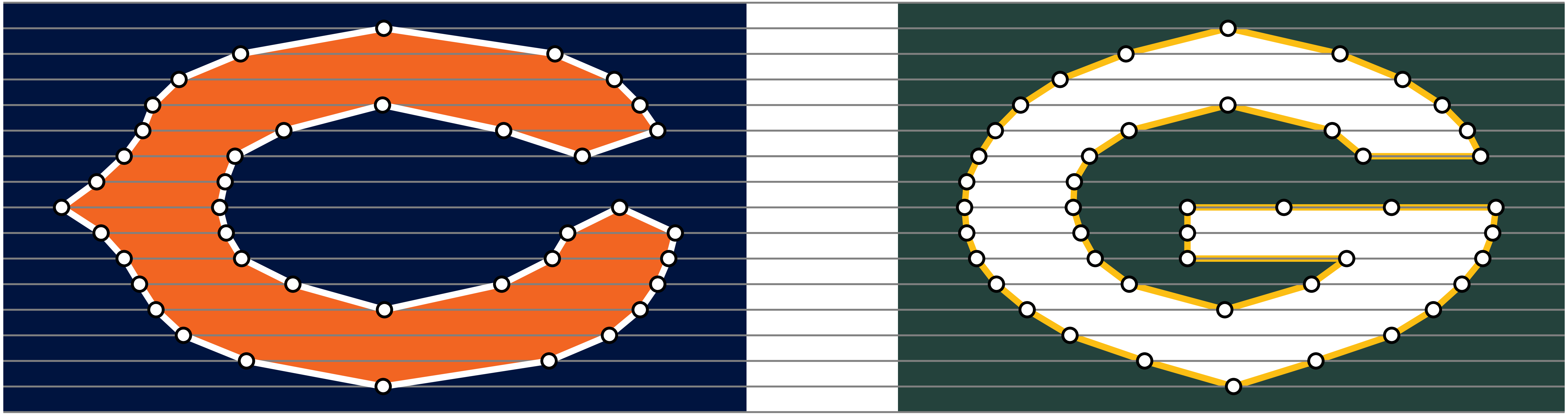

(Weakly) leveled planar drawings

s-span leveled planar drawing

- Set of horizontal lines (*levels*)
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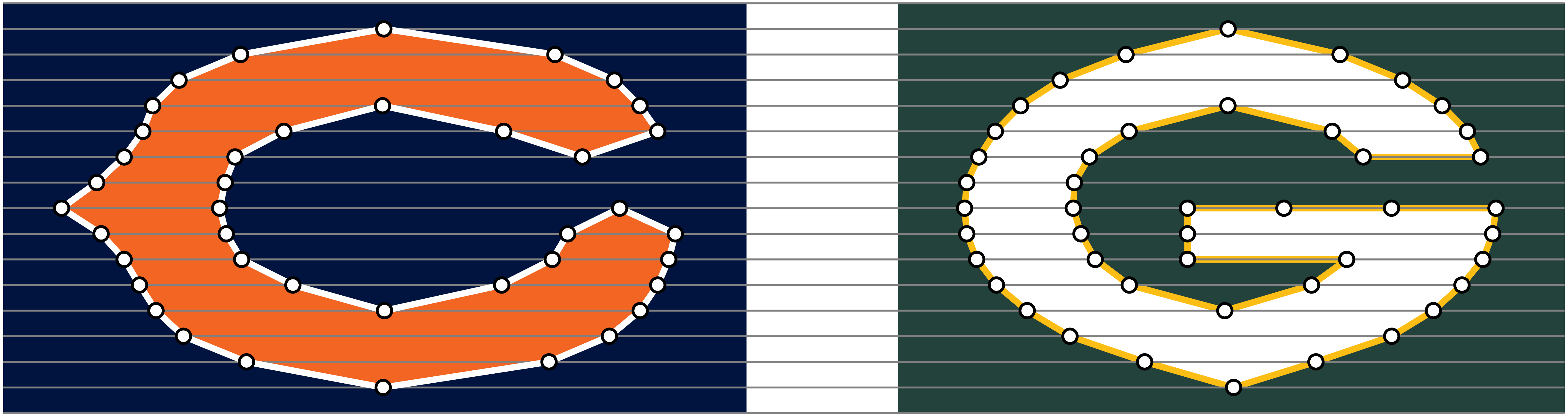
- Edges are y-monotone curves
- Drawing is planar

s-span weakly leveled planar drawing

- ... or horizontal segments



(Weakly) leveled planar drawings

s-span leveled planar drawing

- Set of horizontal lines (*levels*)
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- Edges are y-monotone curves and span at most s levels
- Drawing is planar

s-span weakly leveled planar drawing

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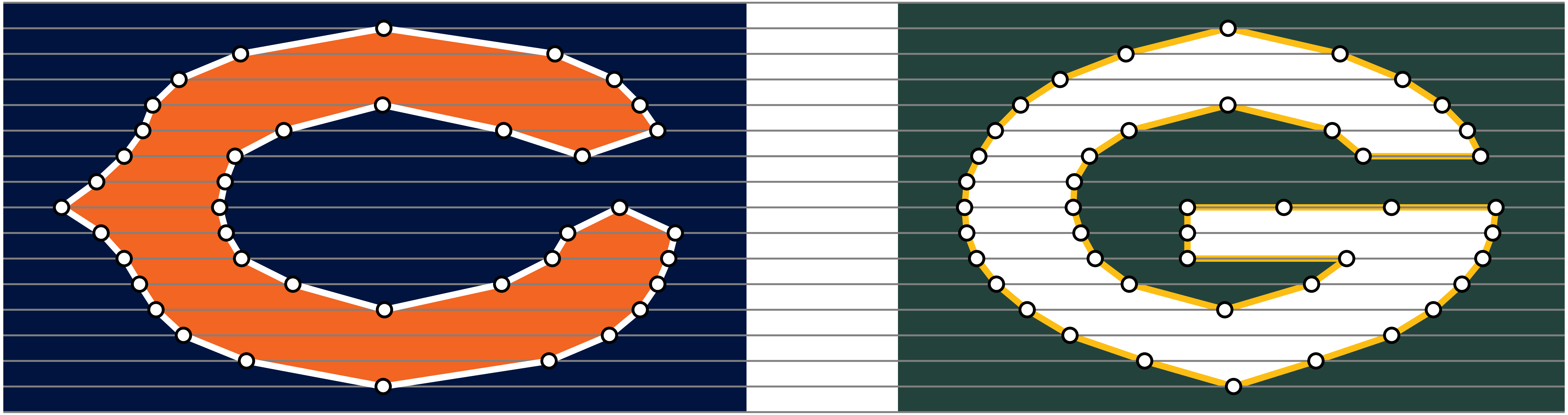

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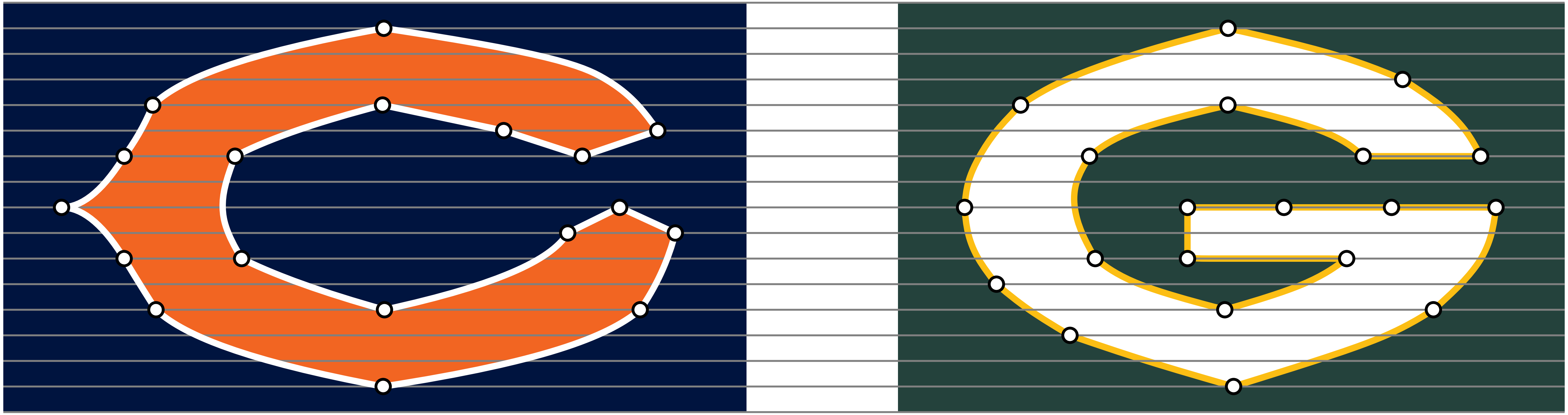


Overview

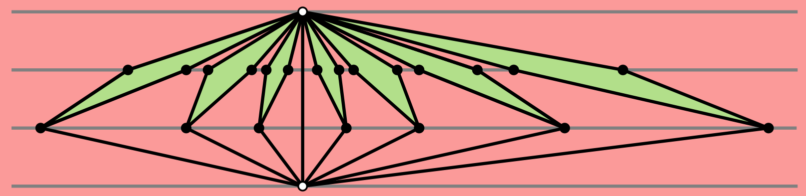
Overview

NP-hardness

Overview

NP-hardness

Reduction from leveled planar

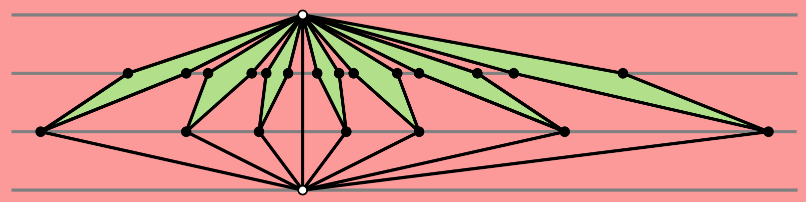


Overview

FPT algorithms

NP-hardness

Reduction from leveled planar



Overview

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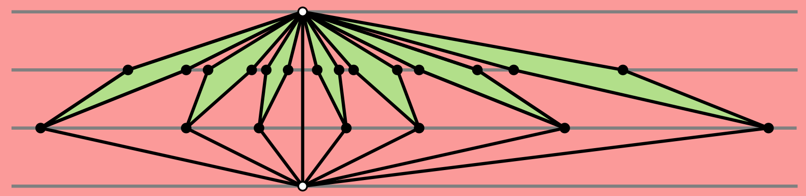
Reduction from leveled planar



Combinatorial results

Overview

FPT algorithms

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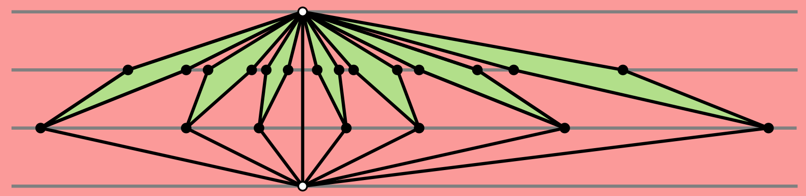


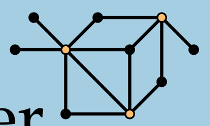
Combinatorial results

Implications

Overview

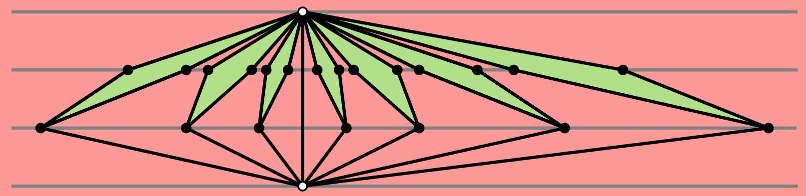
FPT algorithms

Linear kernel in size of vertex cover



NP-hardness

Reduction from leveled planar



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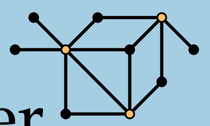
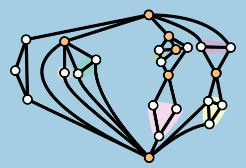
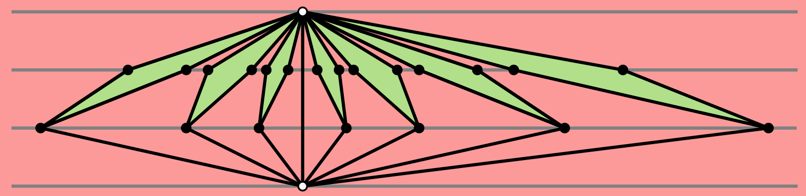


FPT in size of b-modulator $(+b)$



NP-hardness

Reduction from leveled planar



Combinatorial results

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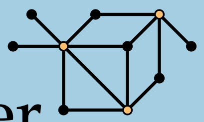
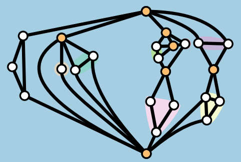
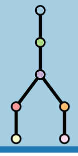
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

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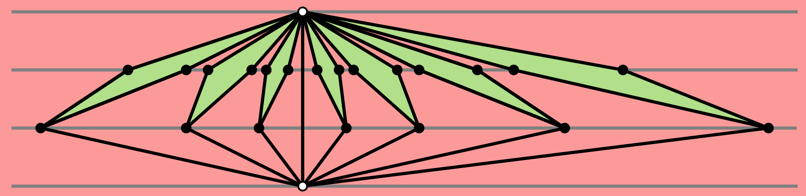

Combinatorial results

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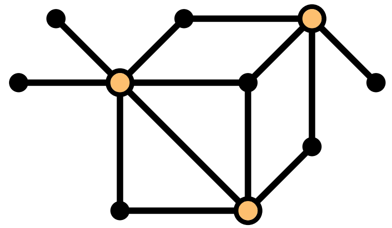
Vertex Cover

A **vertex cover** of a graph G is a set V' of vertices such that, for every edge $uv \in E$, $u \in V'$ or $v \in V'$.

Vertex Cover



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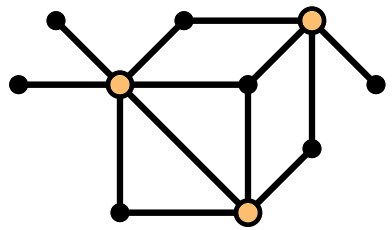

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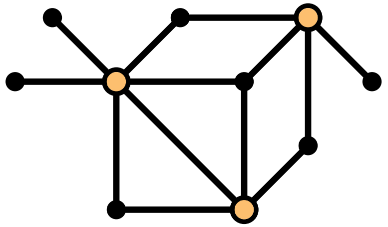

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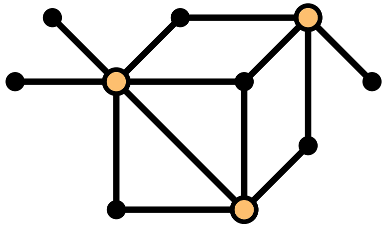
Lemma.
Every graph with a vertex cover of size k admits a $6k$-span (weakly) leveled planar drawing.

Vertex Cover



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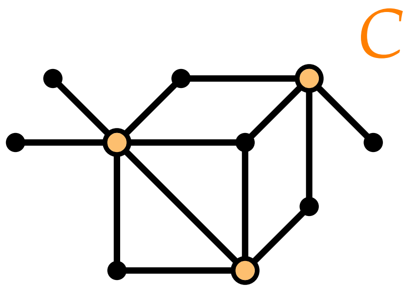

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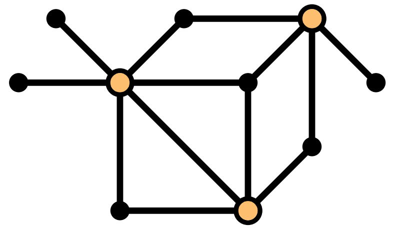

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Let C be a vertex cover of G of size k.

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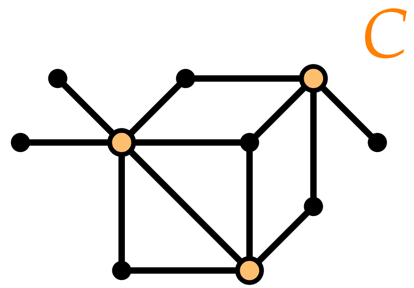

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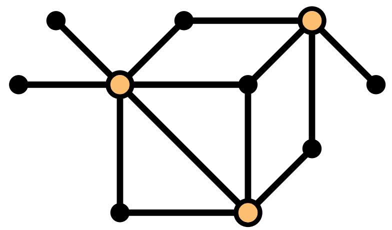

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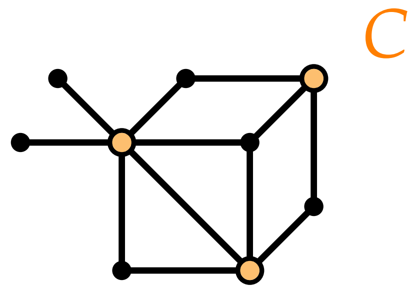

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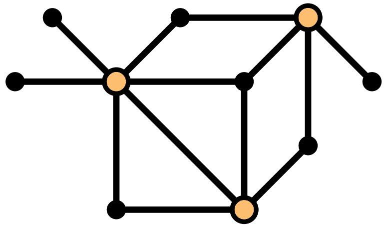

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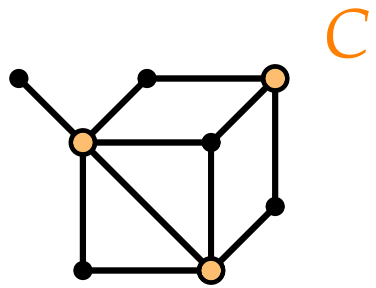

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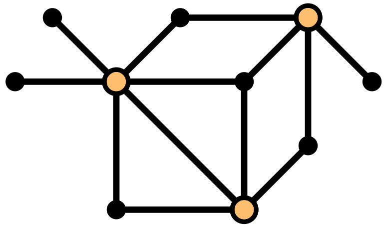

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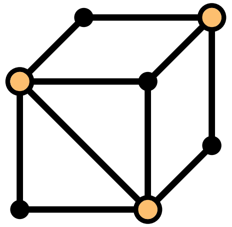
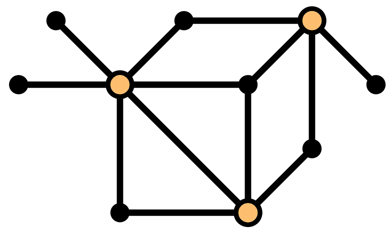


Vertex Cover



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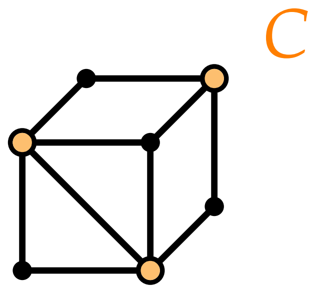

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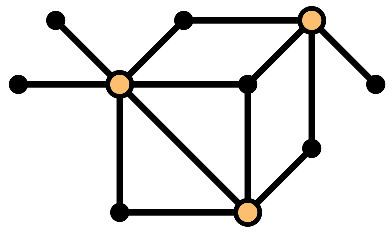

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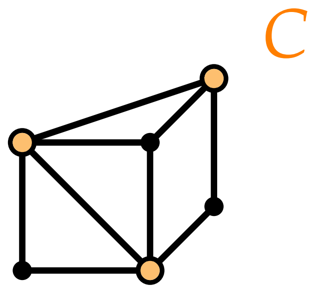

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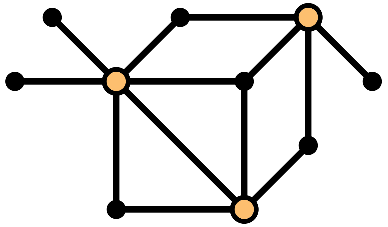

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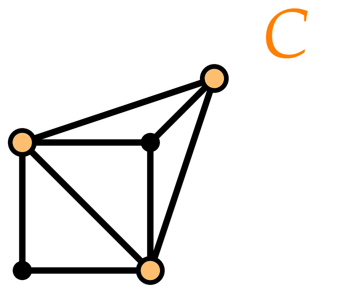

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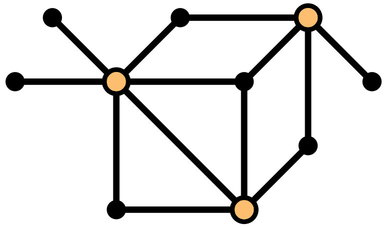

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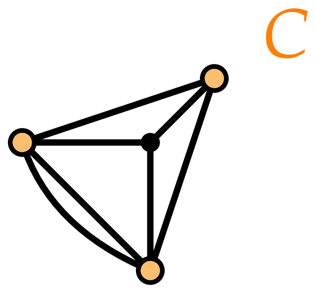

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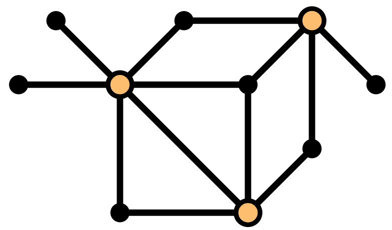

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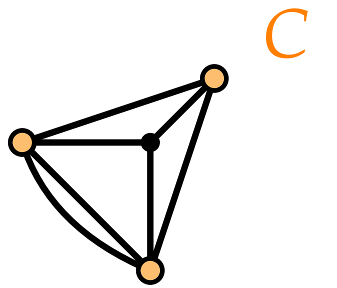

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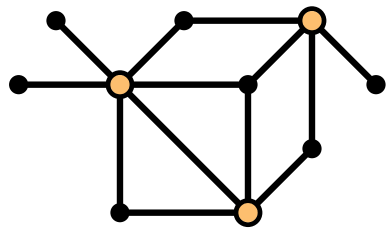

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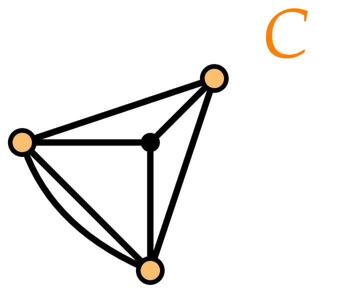

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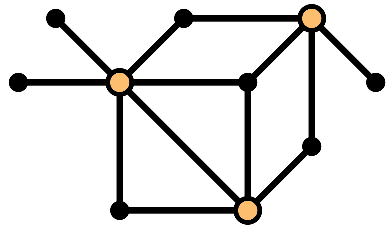

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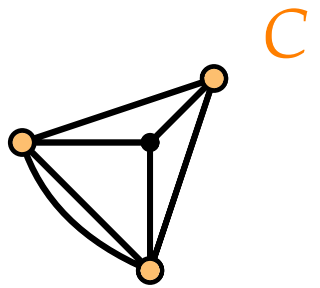

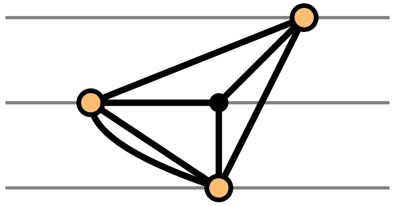

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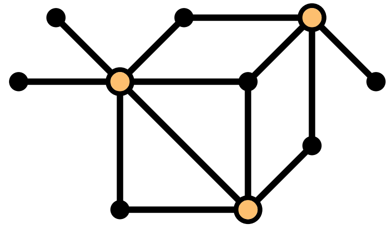

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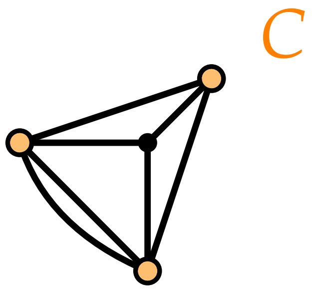

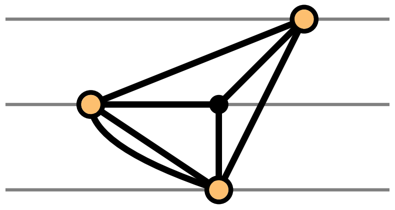

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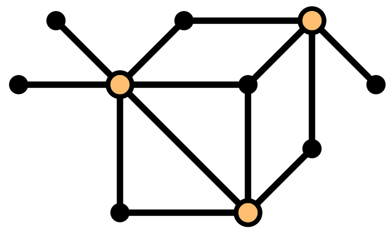

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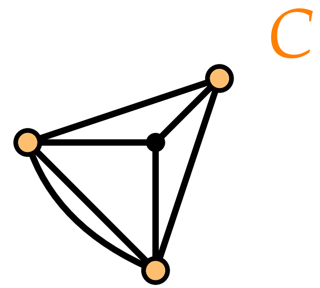

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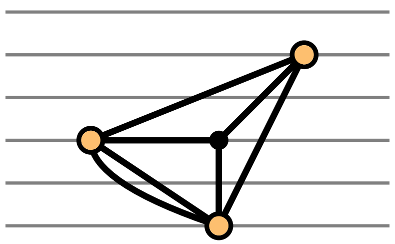

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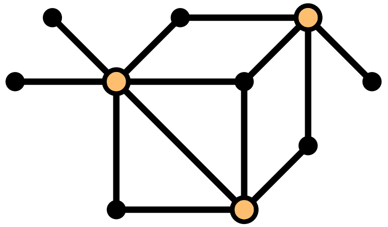

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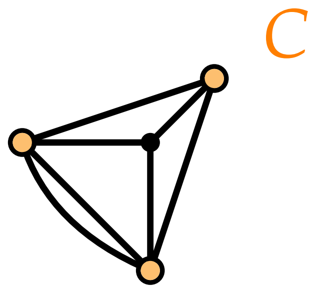

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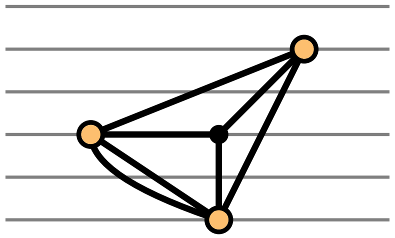

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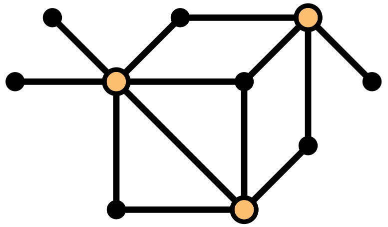

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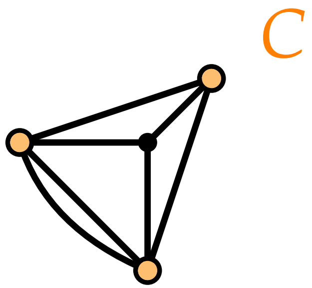

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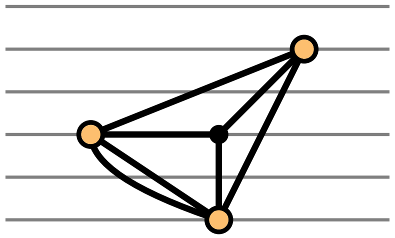

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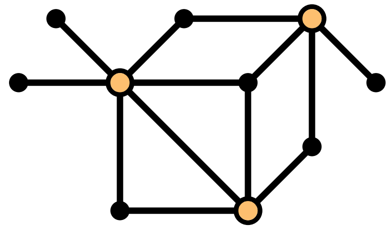

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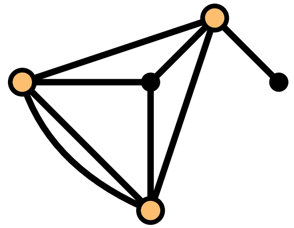

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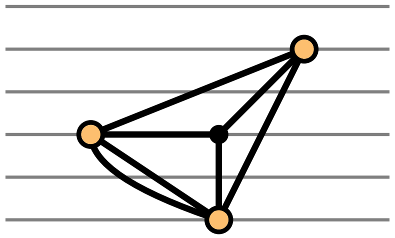

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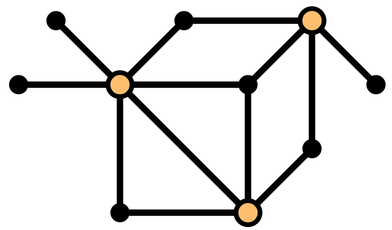

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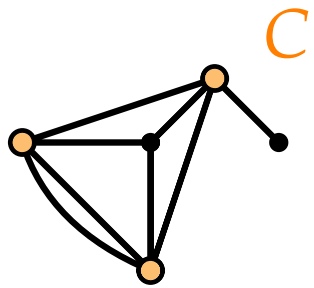

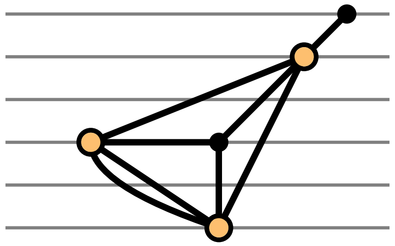

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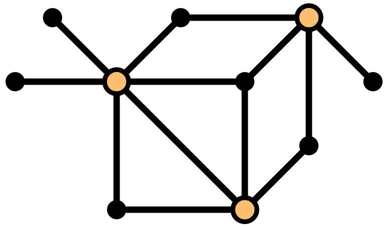

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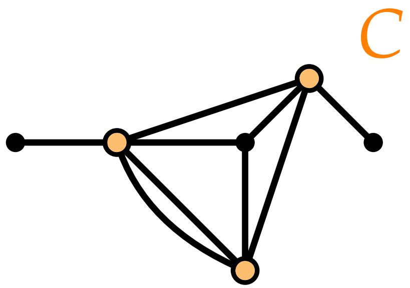

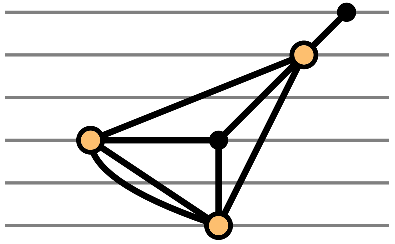

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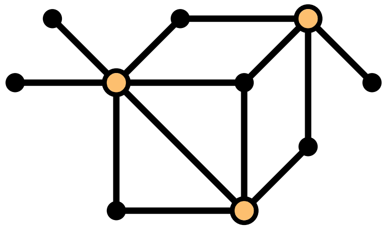

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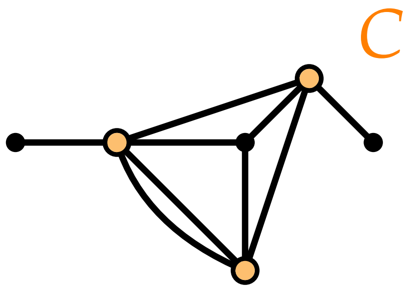

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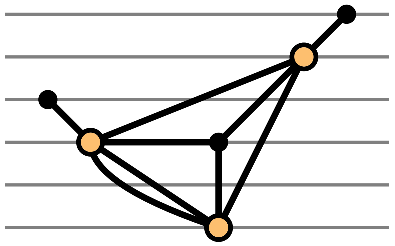

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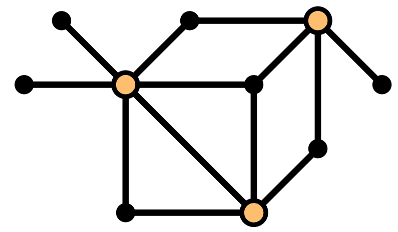

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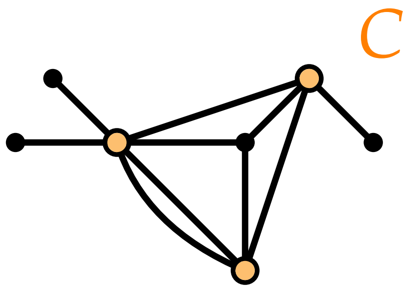

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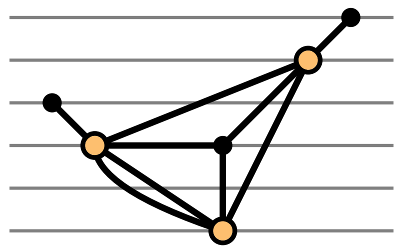

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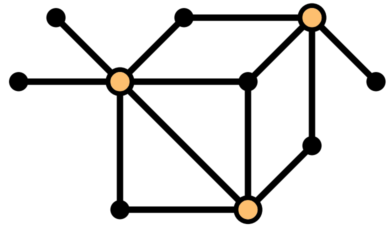

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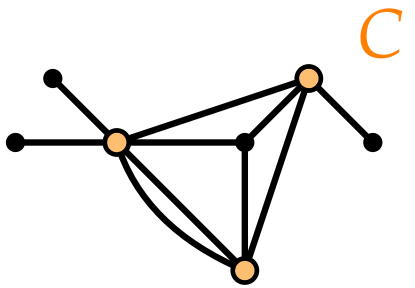

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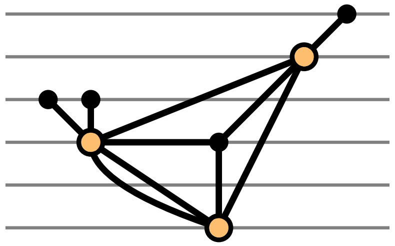

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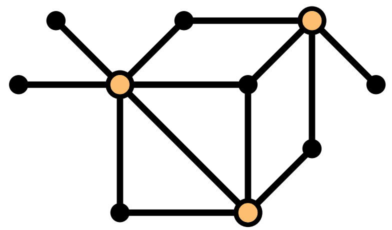

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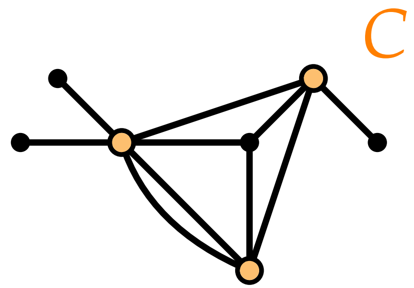

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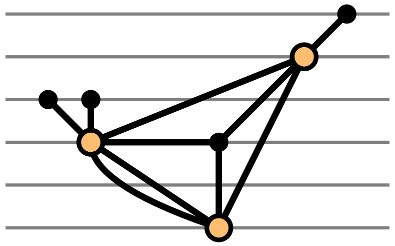

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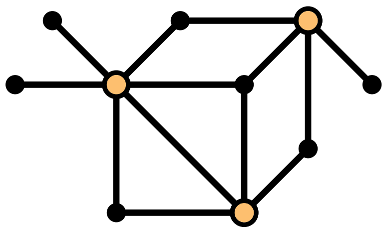

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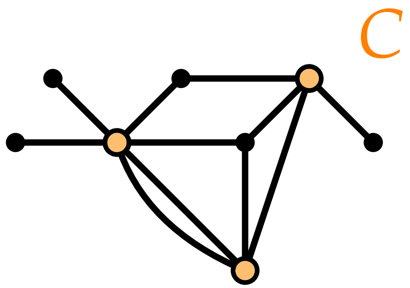

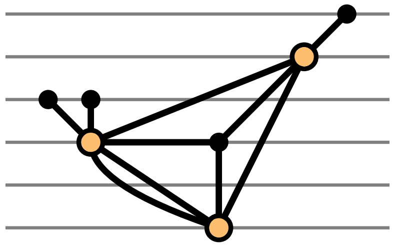

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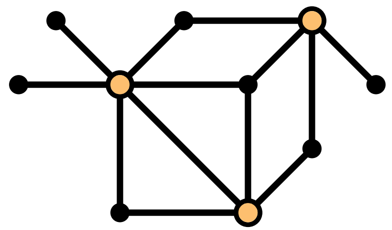

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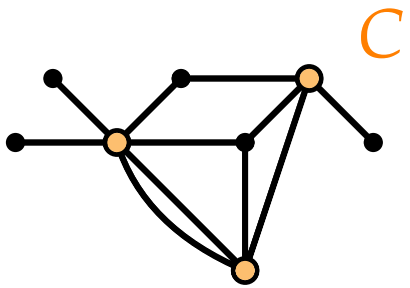

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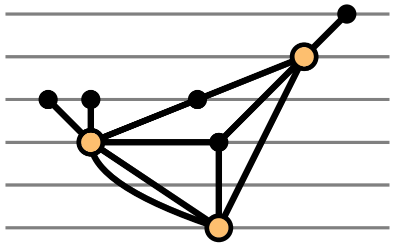

5. Re-insert degree-1 vertices above their neighbor
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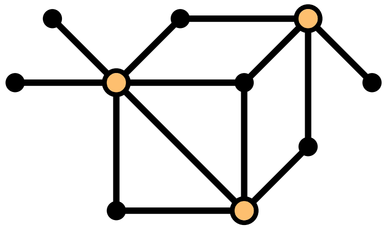
Vertex Cover



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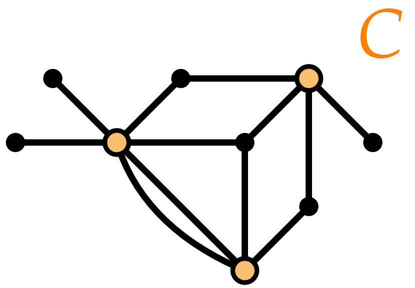
Lemma.
Every graph with a vertex cover of size k admits a $6k$-span (weakly) leveled planar drawing.

Let C be a vertex cover of G of size k.



1. Remove all degree-1 vertices from $G \setminus C$
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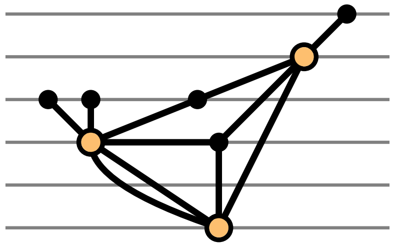

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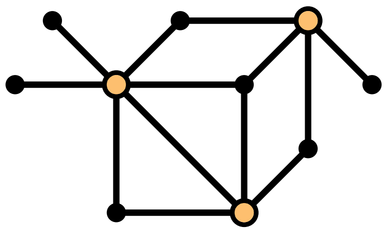

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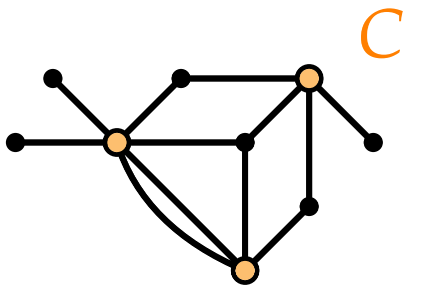
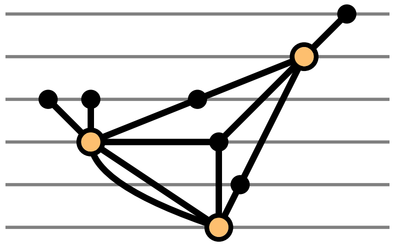
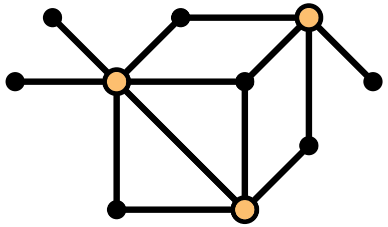




Vertex Cover



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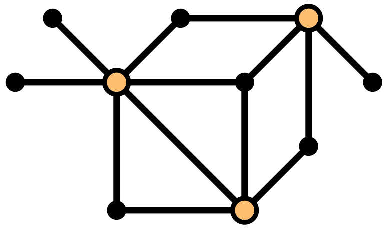

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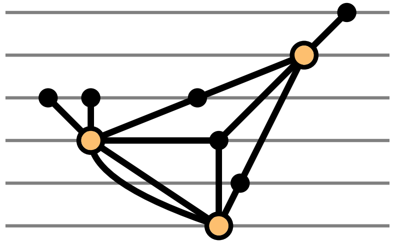
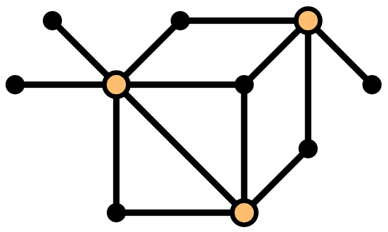


Vertex Cover



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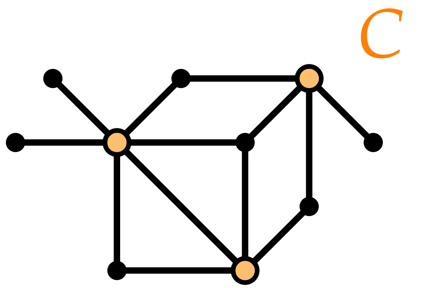

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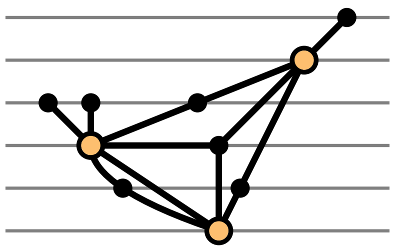

Vertex Cover: Reduction Rules

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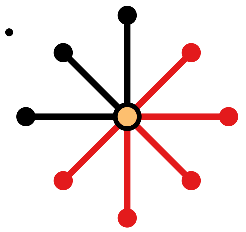

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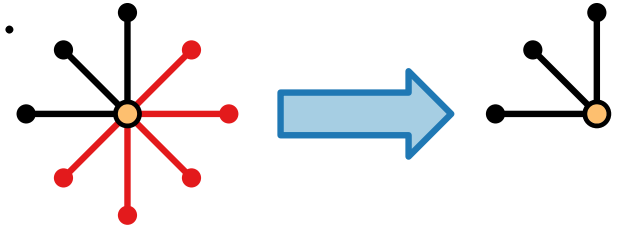

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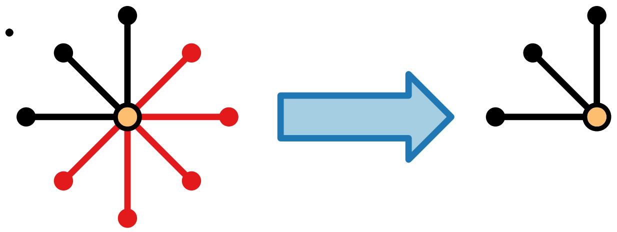

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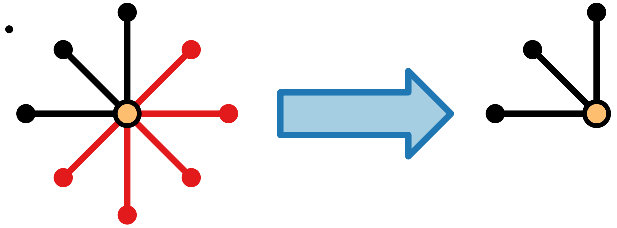

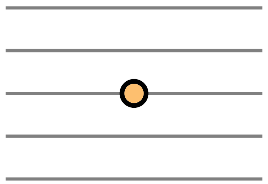

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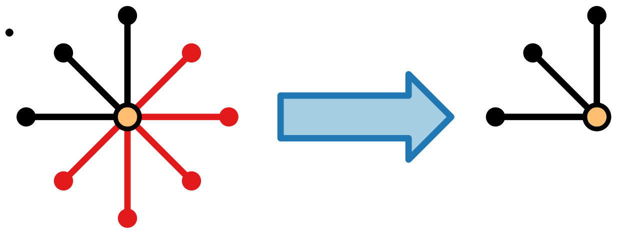

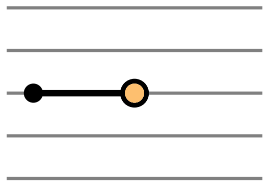

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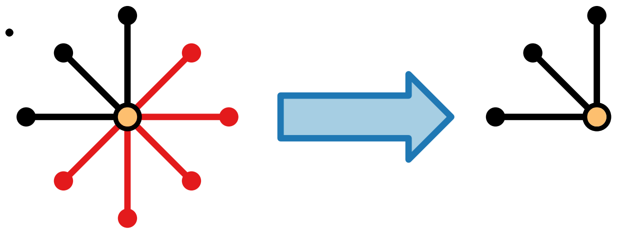

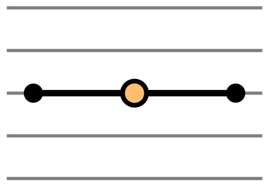

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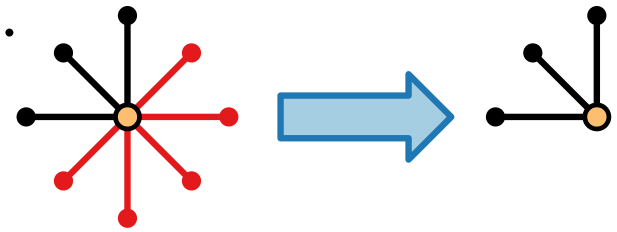

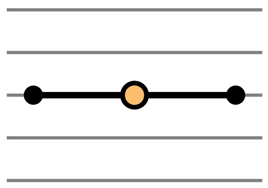

At most 2 neighbors on same level

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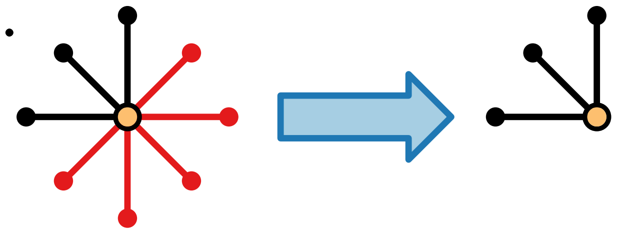

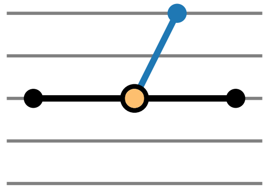

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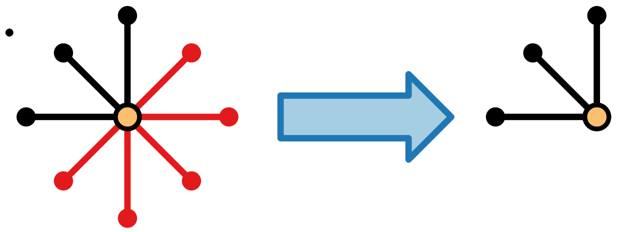

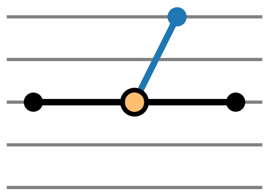

At most 2 neighbors on same level ➡ insert all other next to third neighbor

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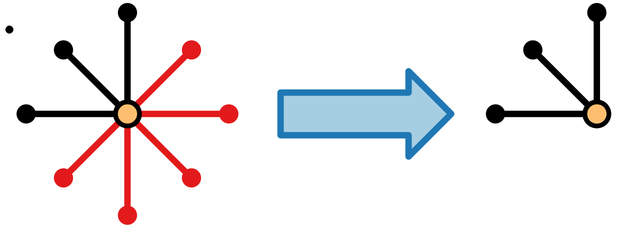

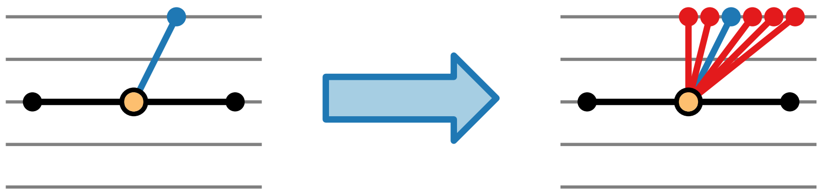

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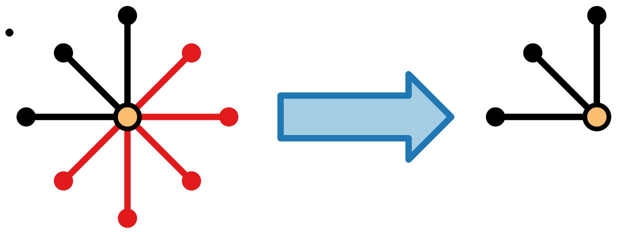

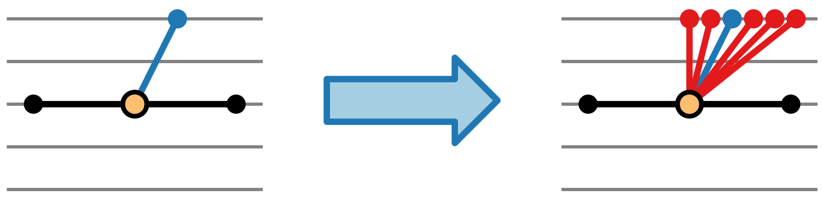

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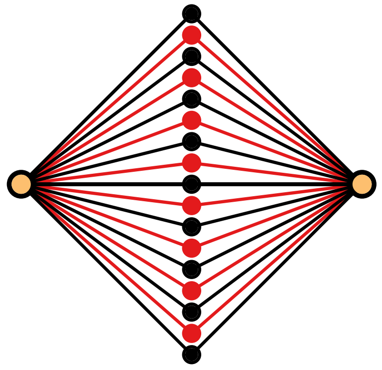

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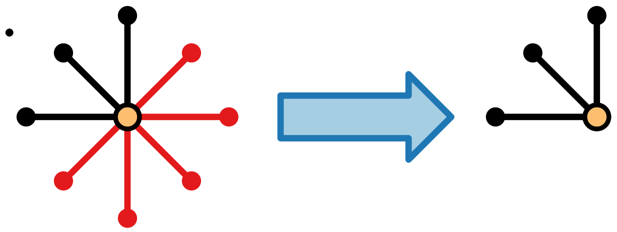

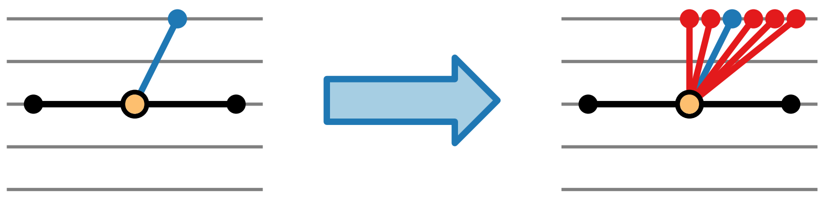

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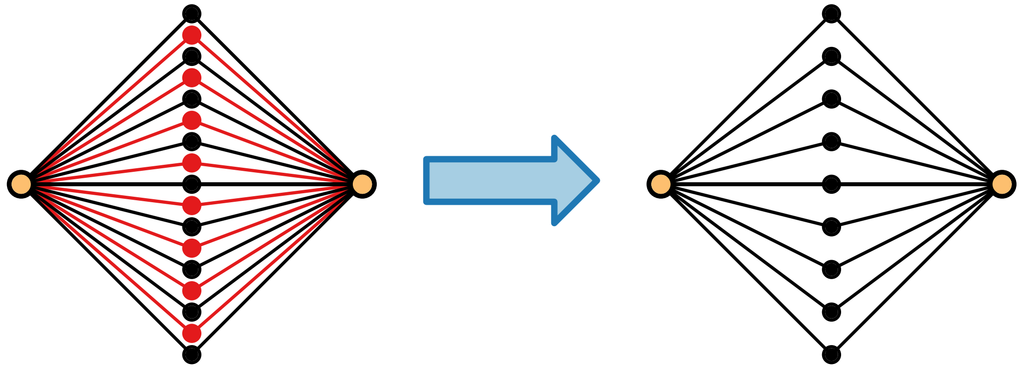

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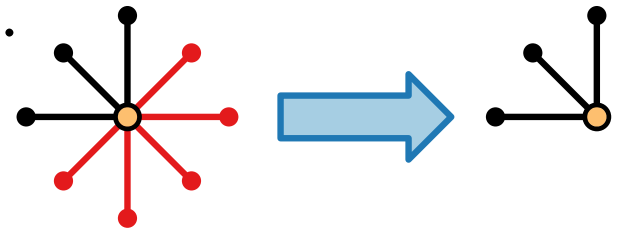

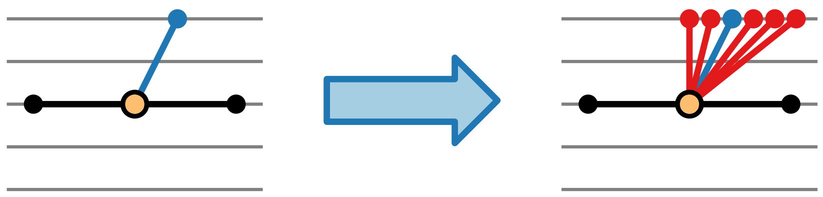

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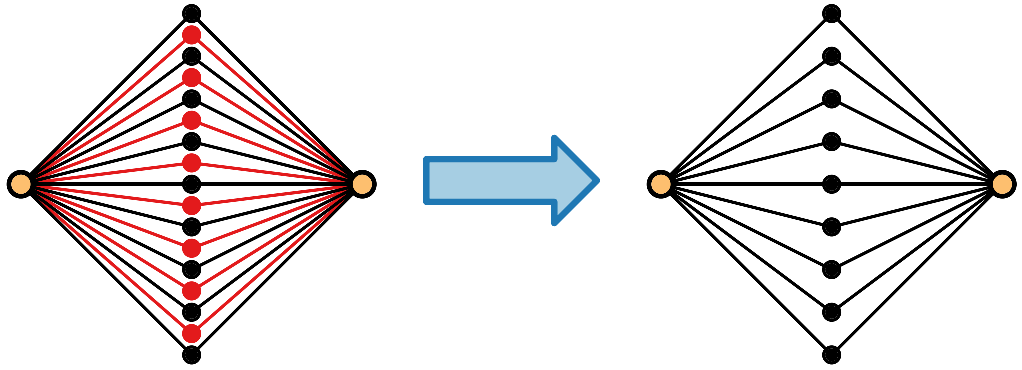

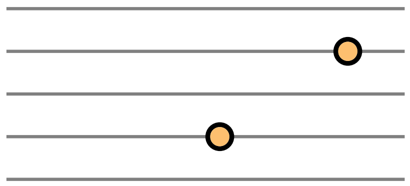

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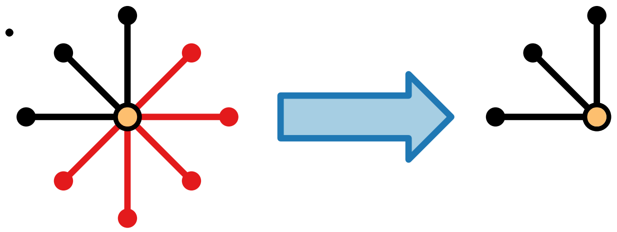

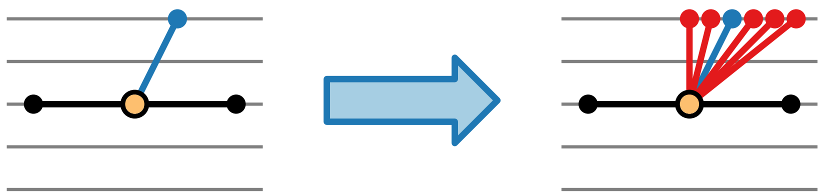

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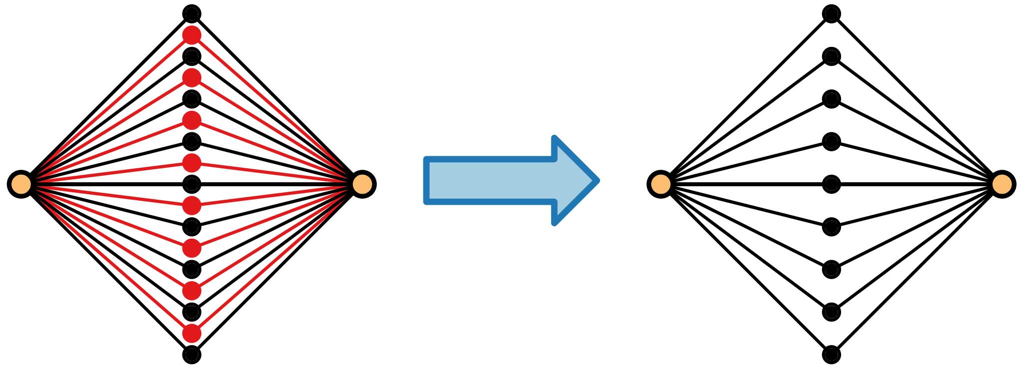

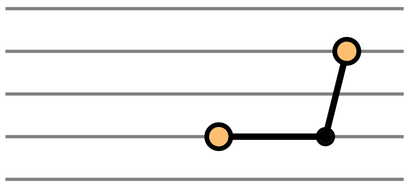

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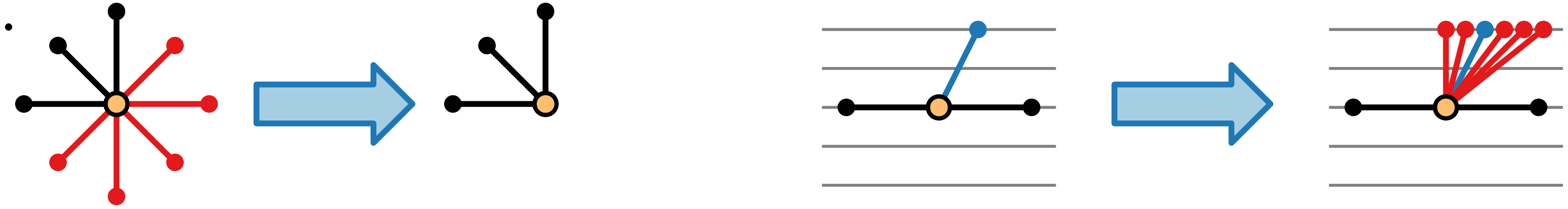

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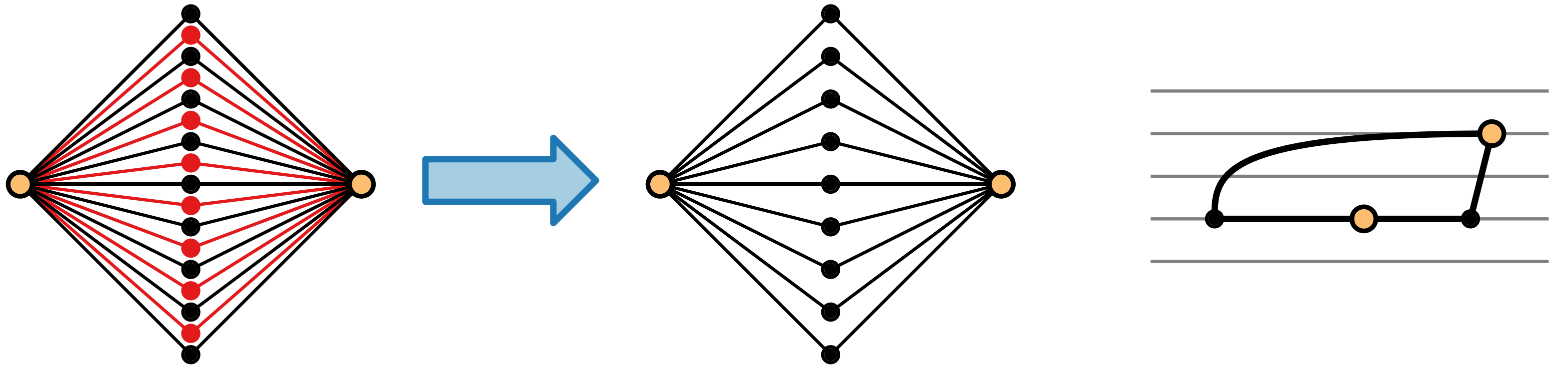

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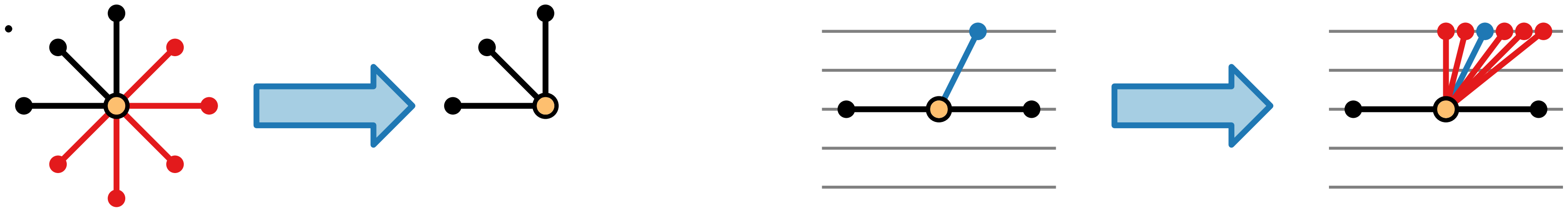

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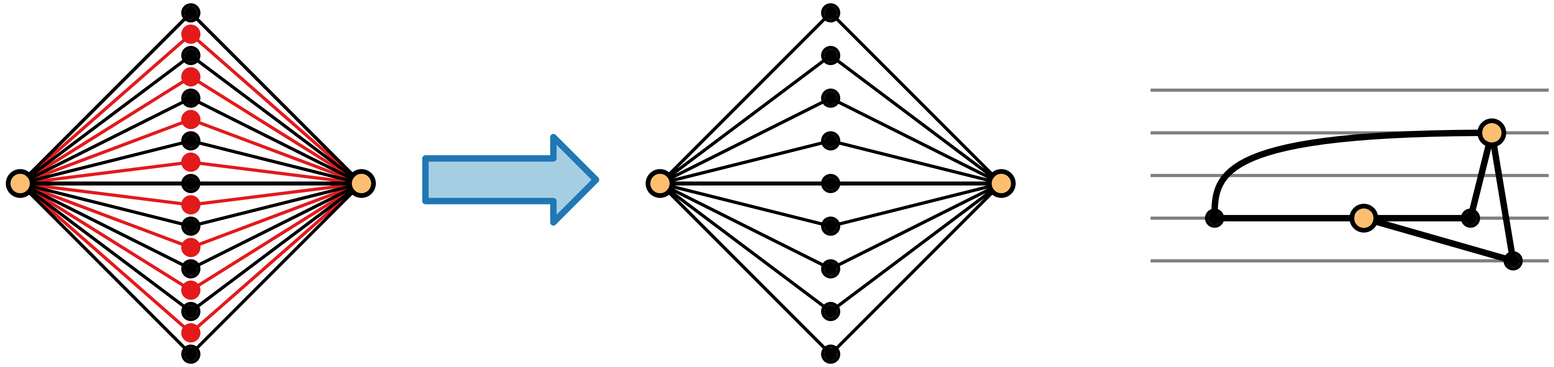

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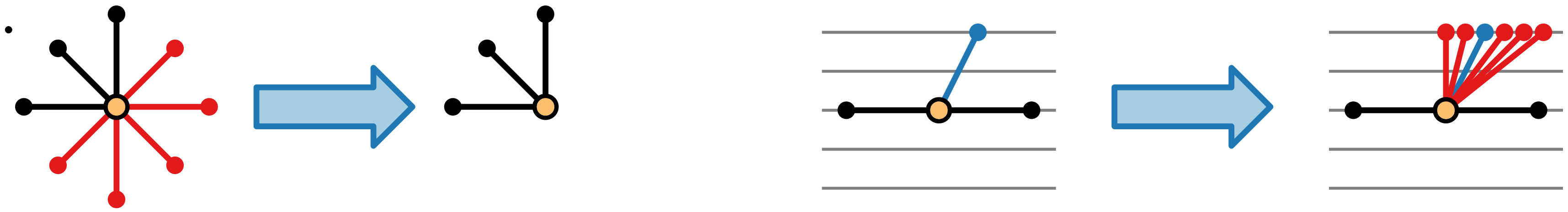

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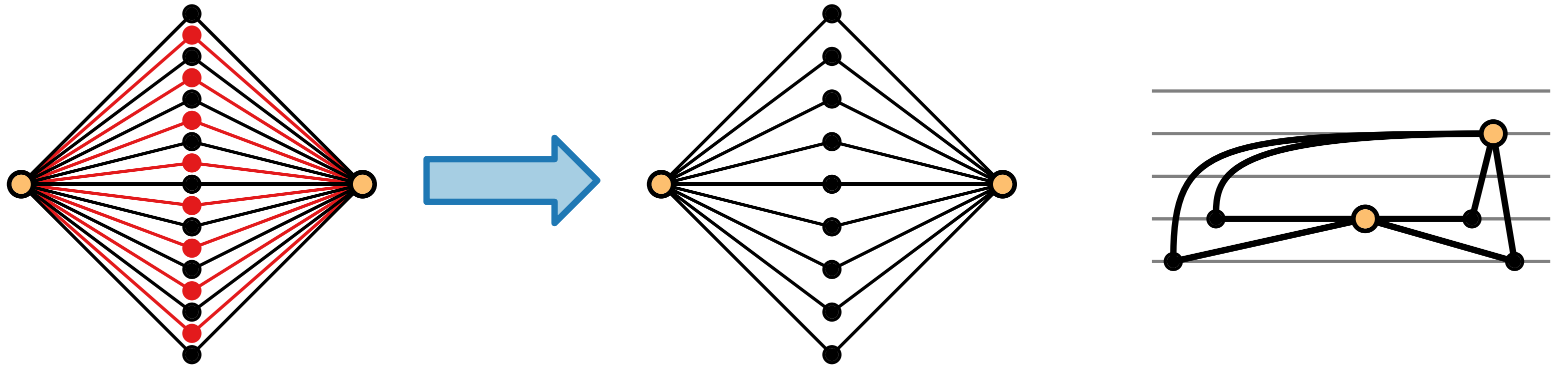

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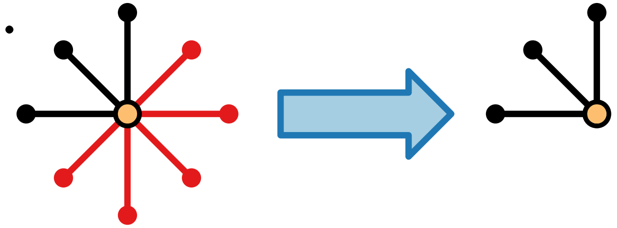

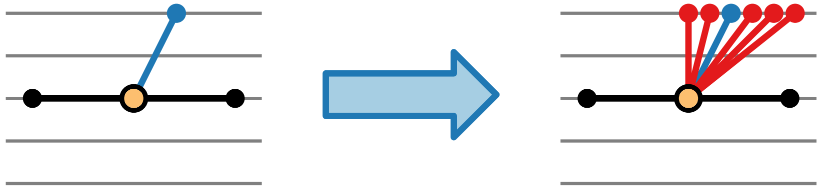

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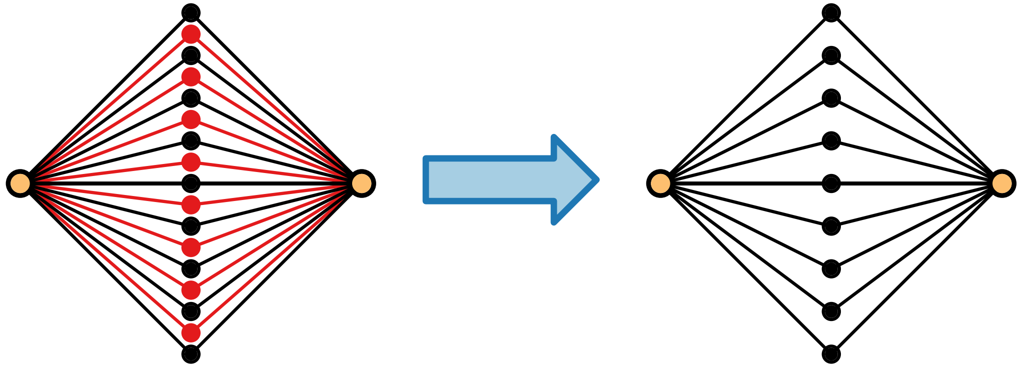

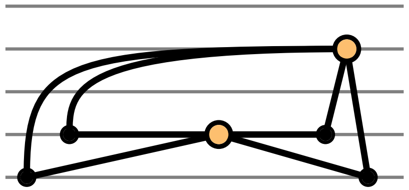

At most 2 neighbors on each level not between c and d

Vertex Cover: Reduction Rules

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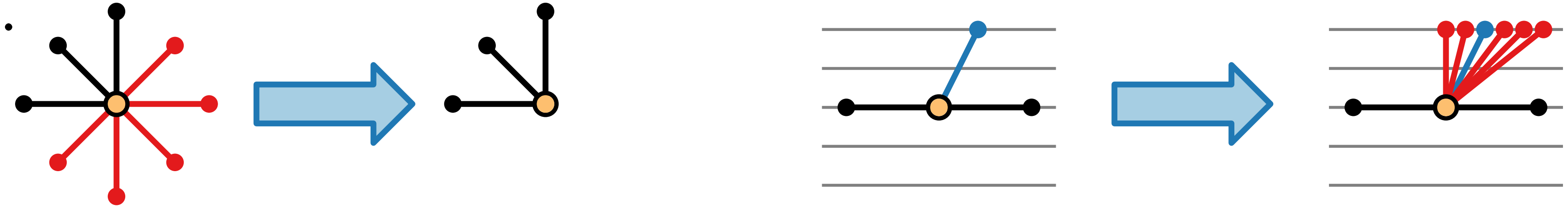
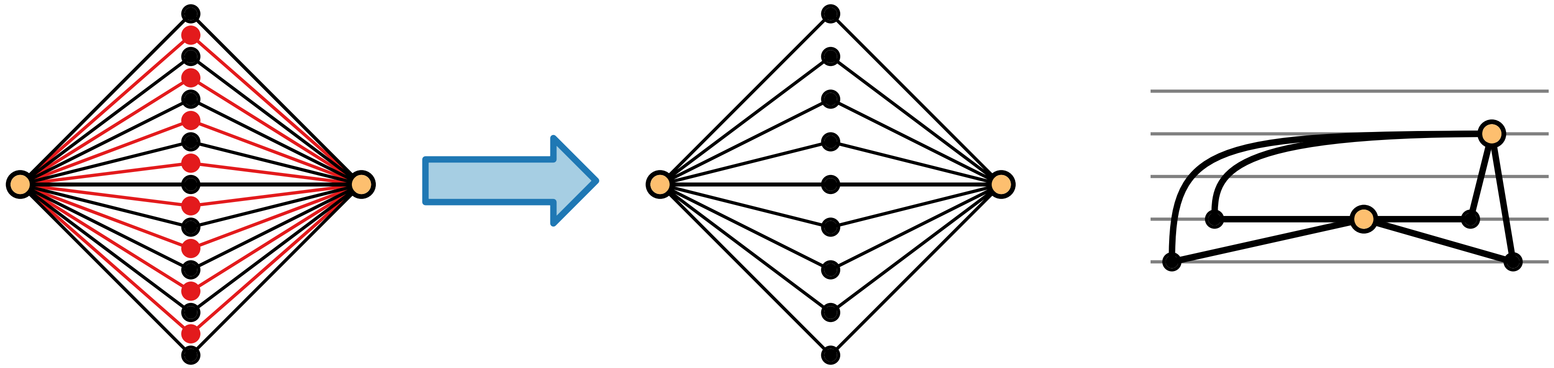


At most 2 neighbors on same level ➡ insert all other next to third neighbor

Rule 2: If any two vertices $c, d \in C$ have more than $4s + 5$ common degree-2 neighbors, remove all but $4s + 5$ of them.

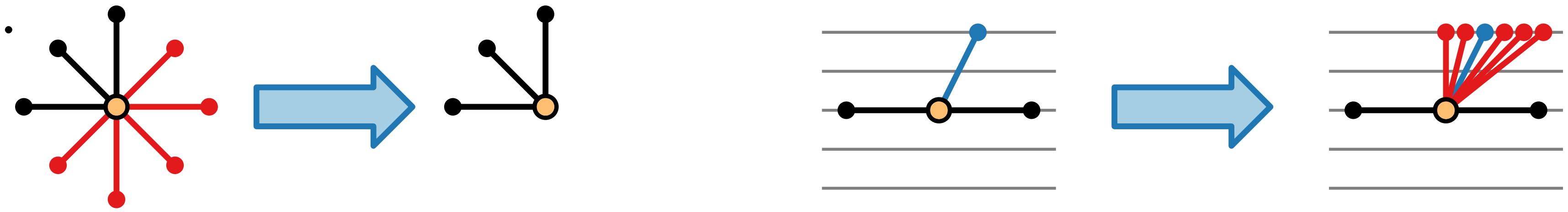


At most 2 neighbors on each level not between c and d

At most $2s + 2$ levels within span s of c and d

Vertex Cover: Reduction Rules

Rule 1: If any vertex $c \in C$ has more than 3 degree-1 neighbors in $V \setminus C$, remove all but 3 of them.



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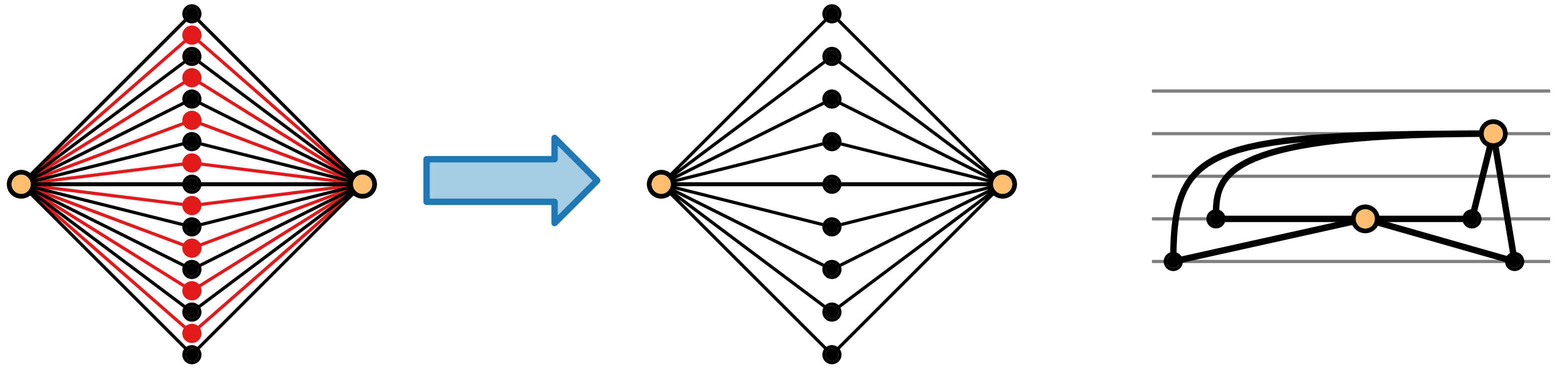

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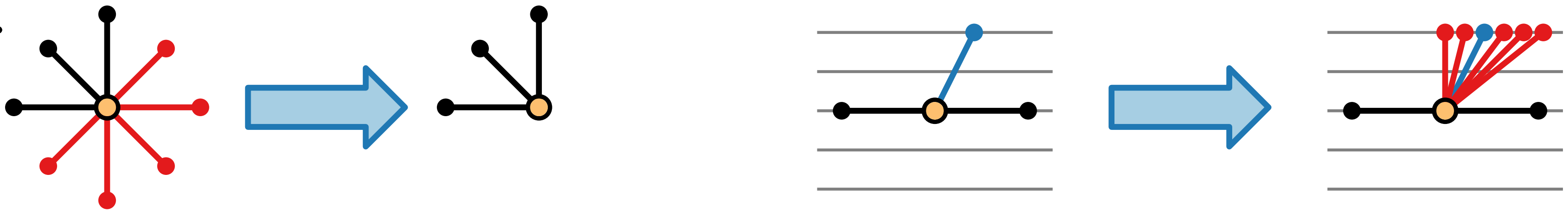

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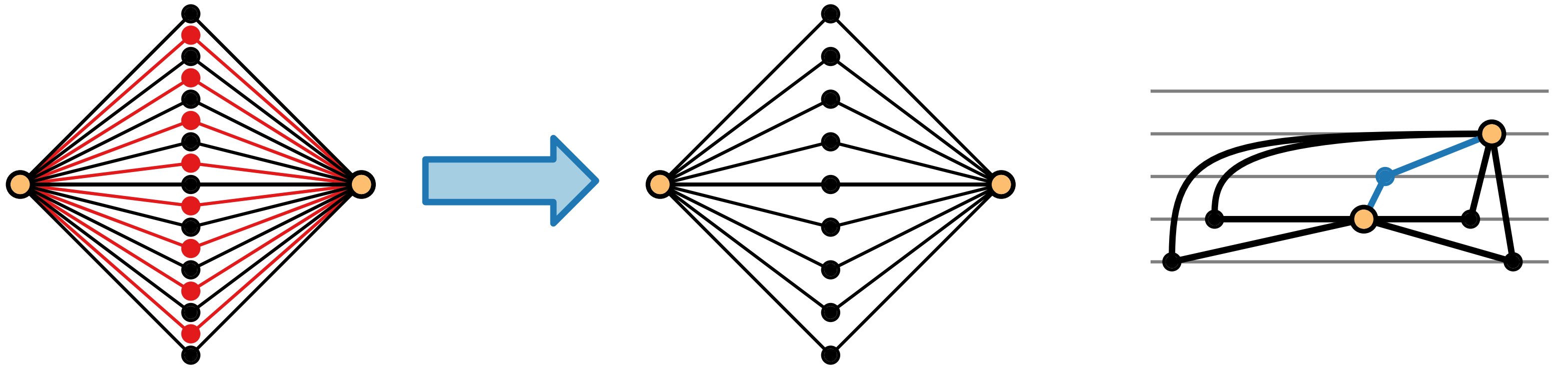

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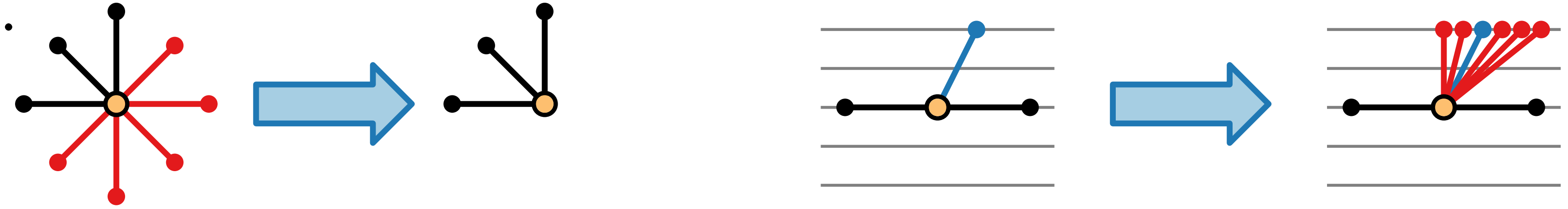

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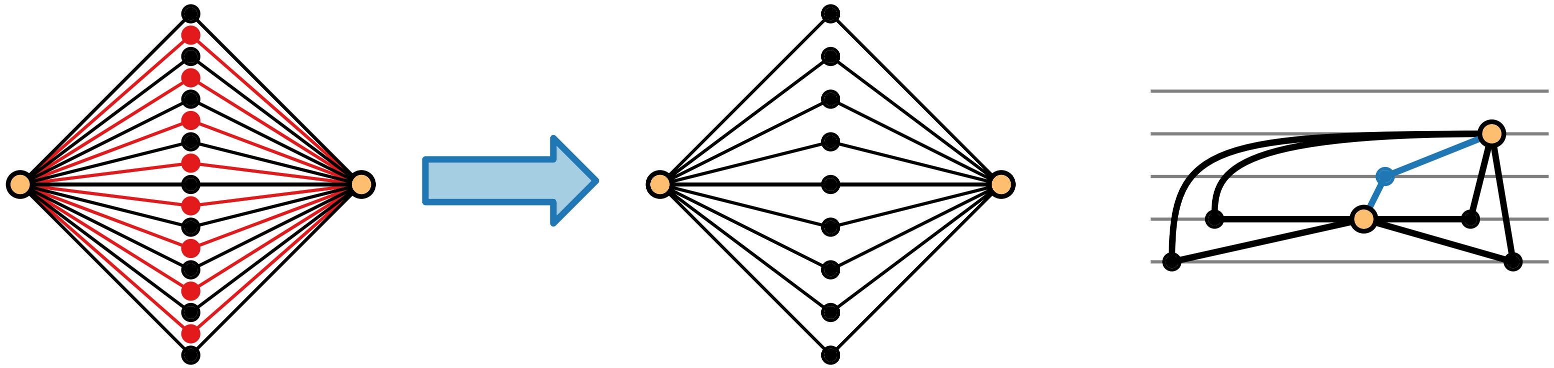

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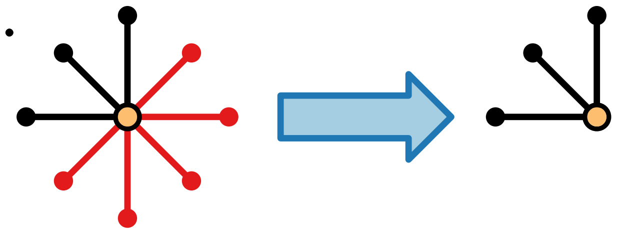

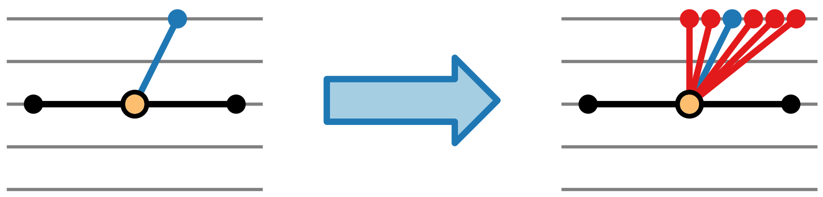

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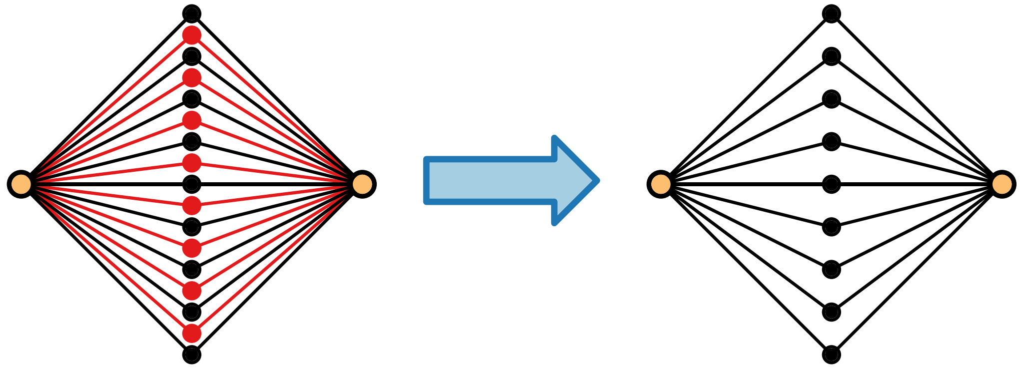

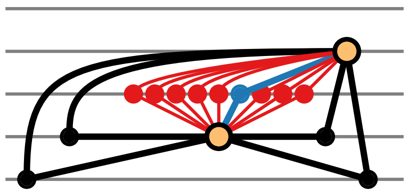

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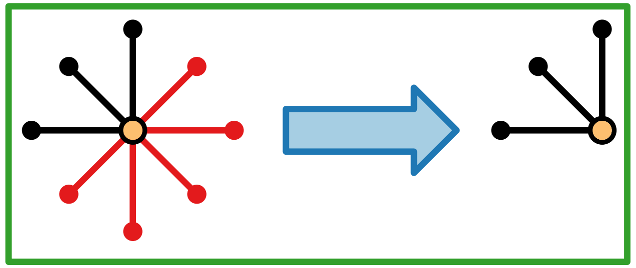

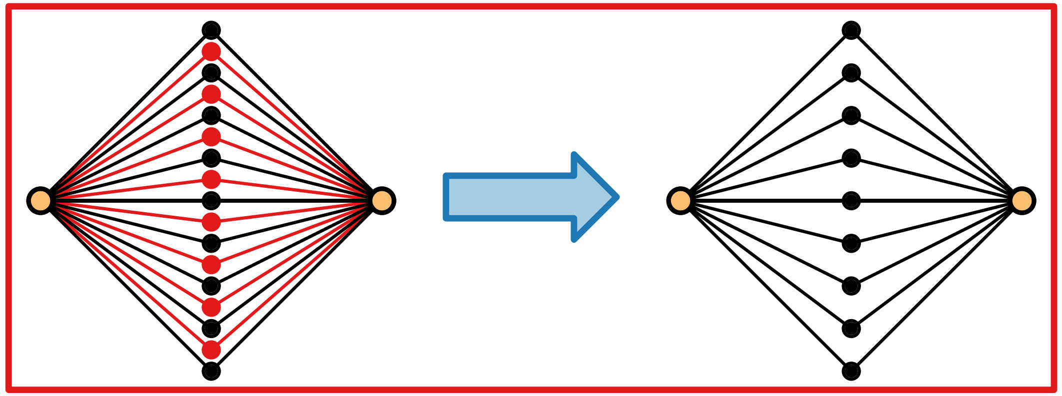

The reduced graph has $\mathcal{O}(k \cdot s)$ vertices.

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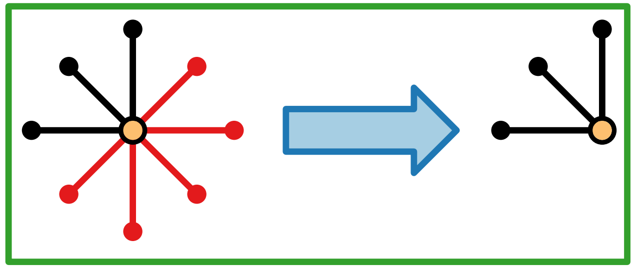

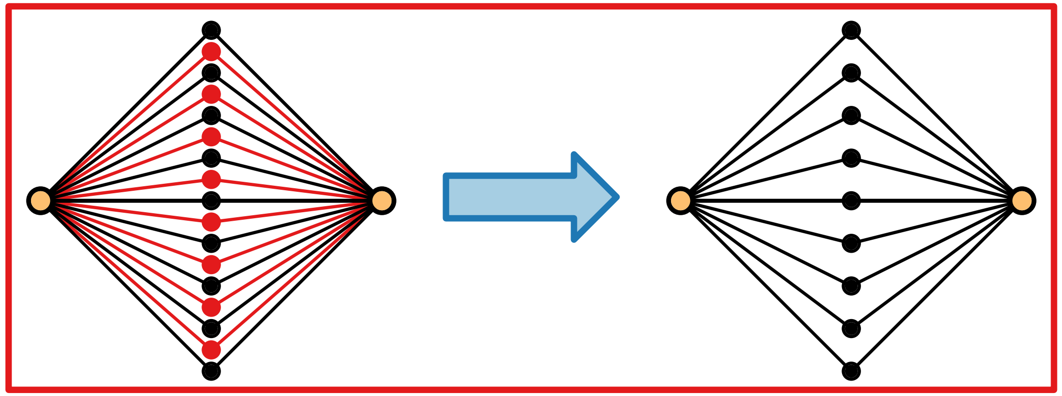

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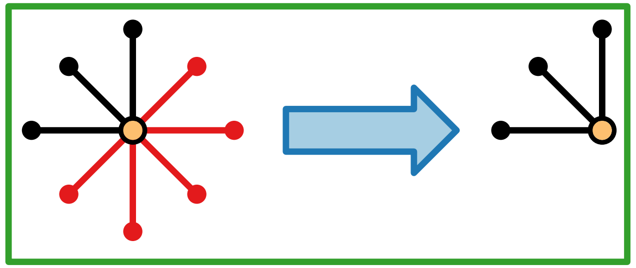

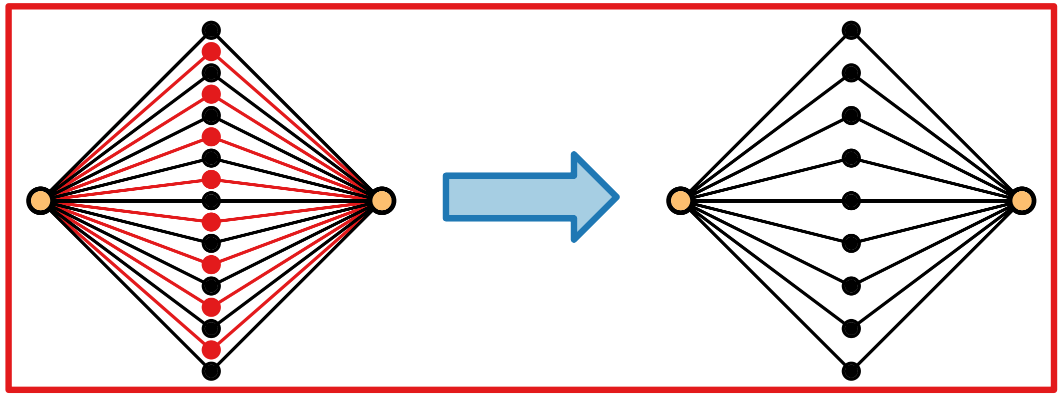

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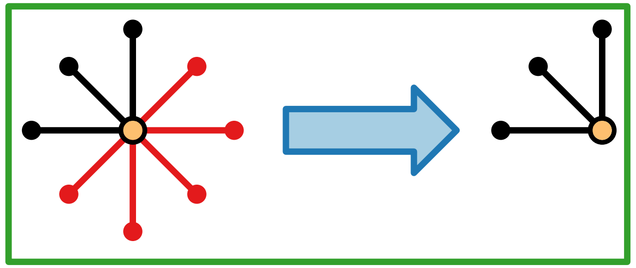

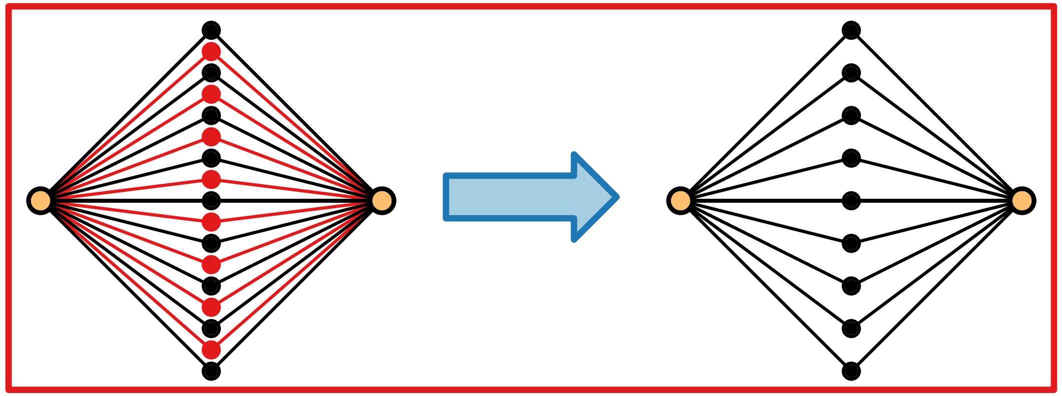

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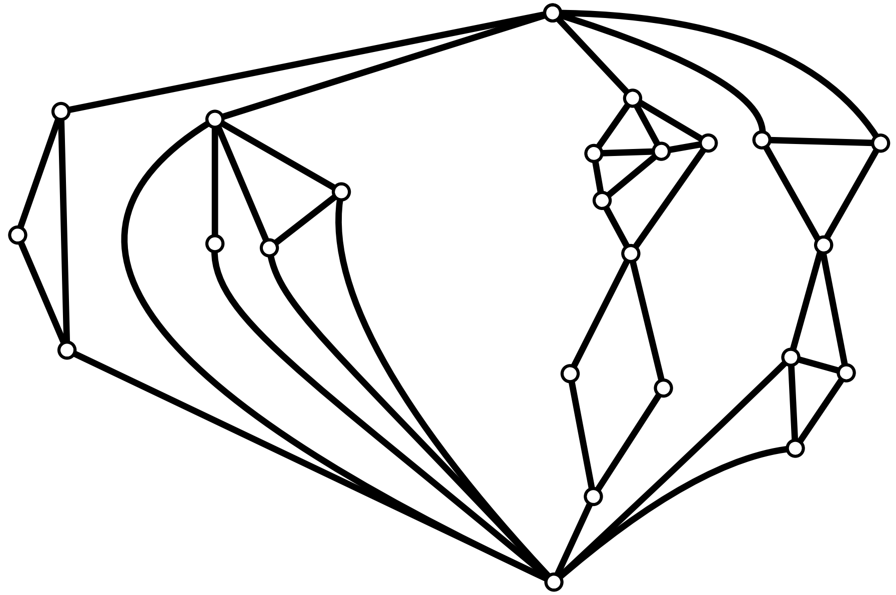
A ***b*-modulator** of a graph G is a set V' of vertices such that every connected component of $G - V'$ has size at most b.

b-modulator



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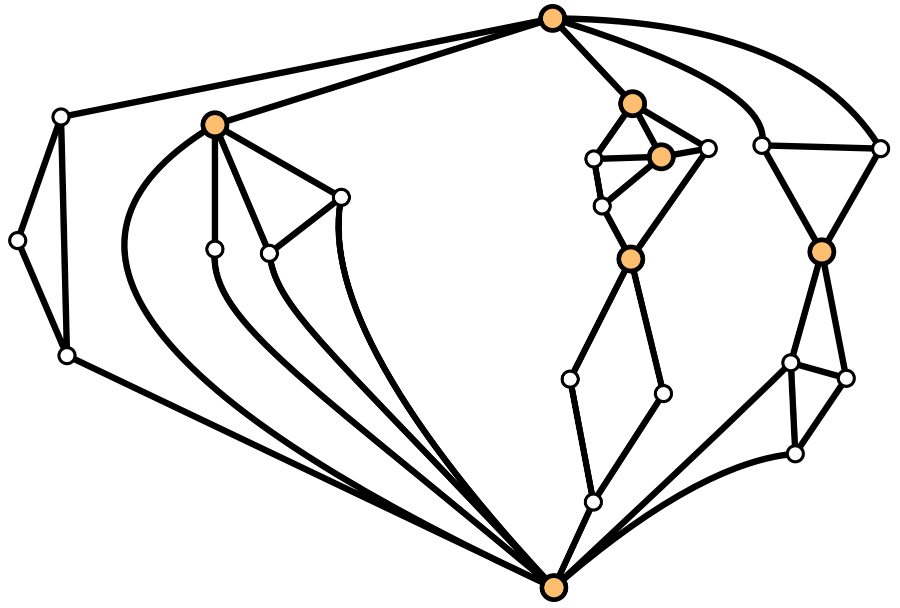

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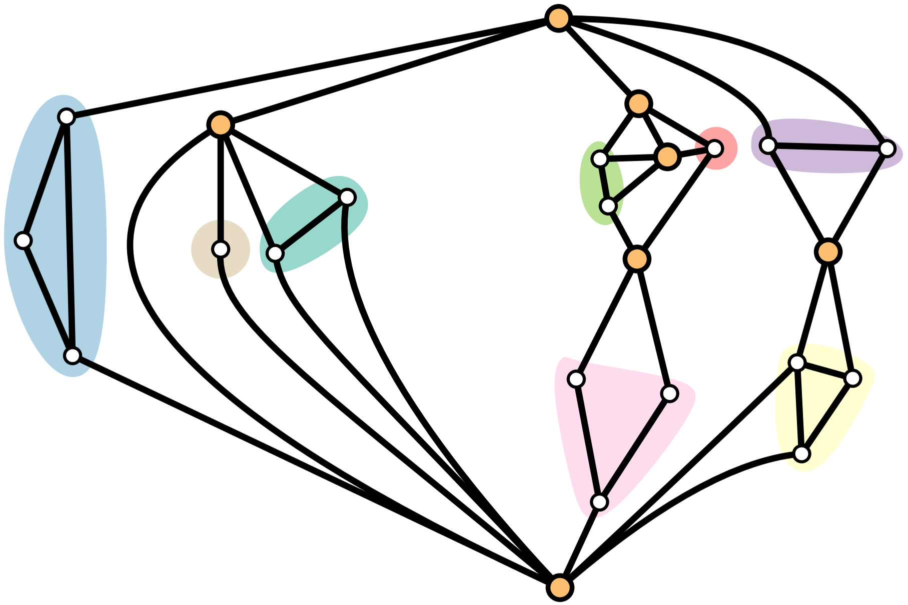

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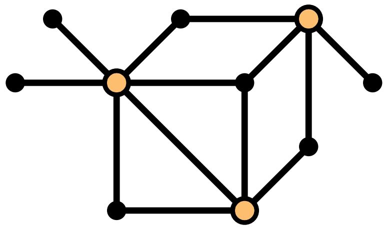

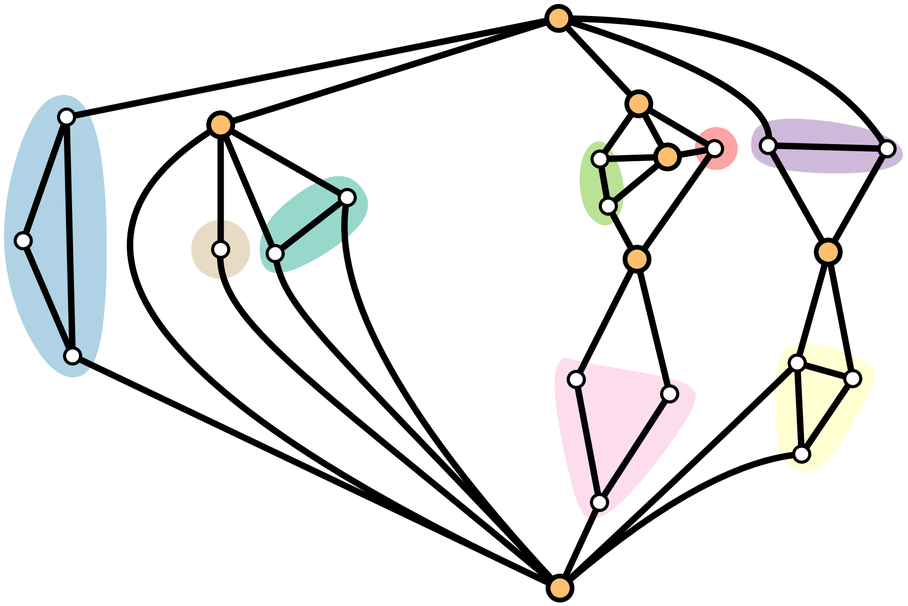

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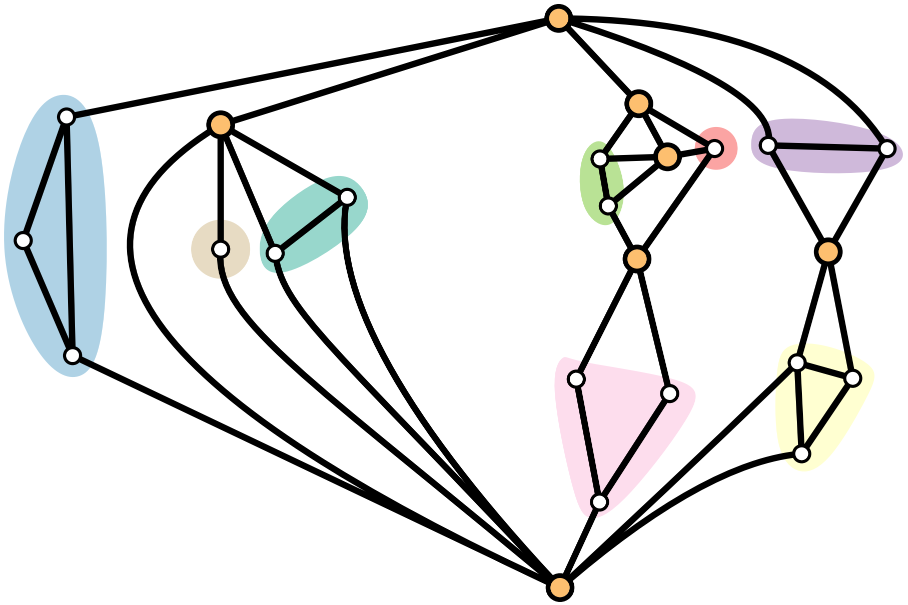

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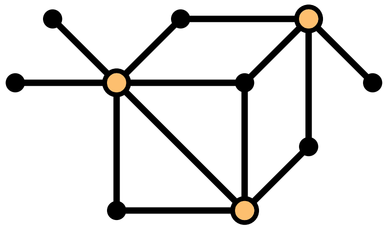

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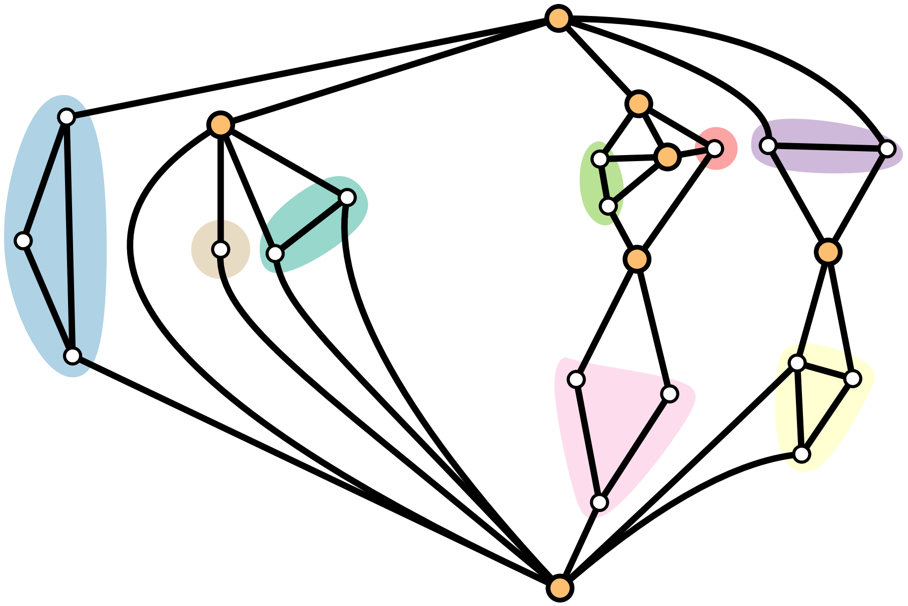

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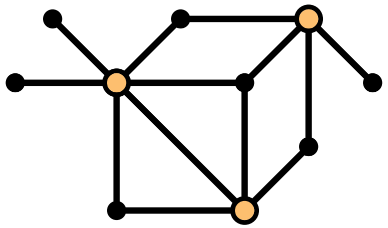

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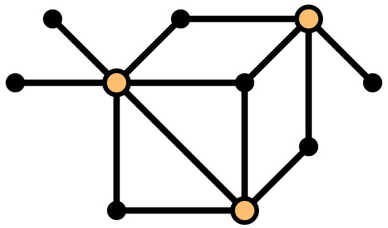

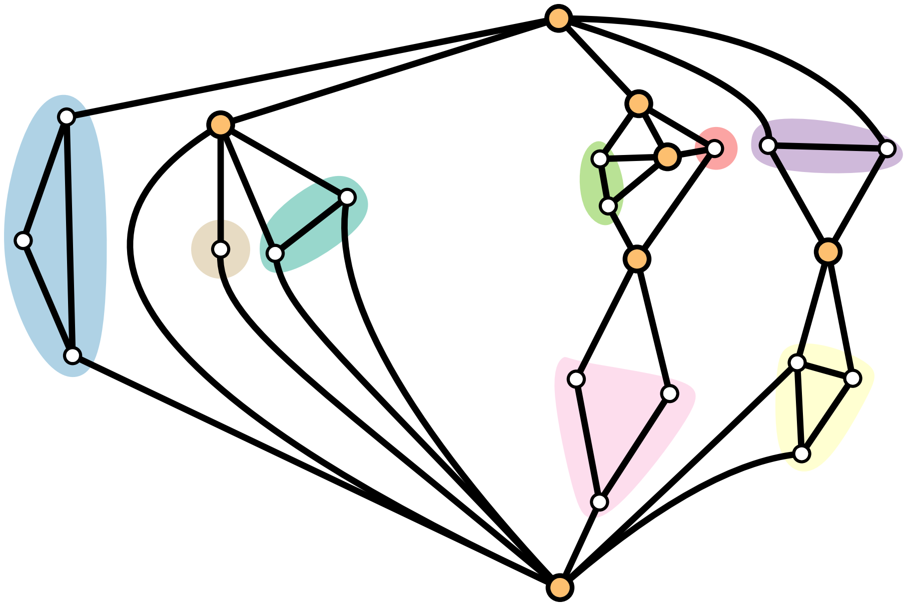

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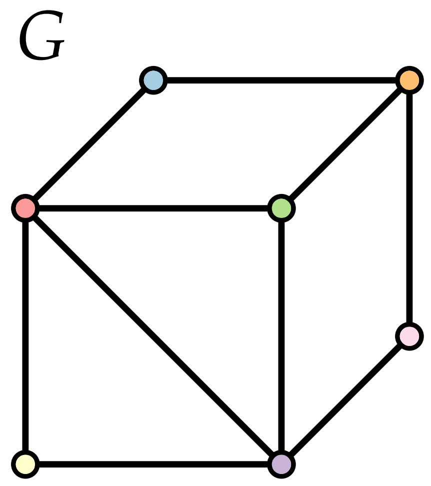

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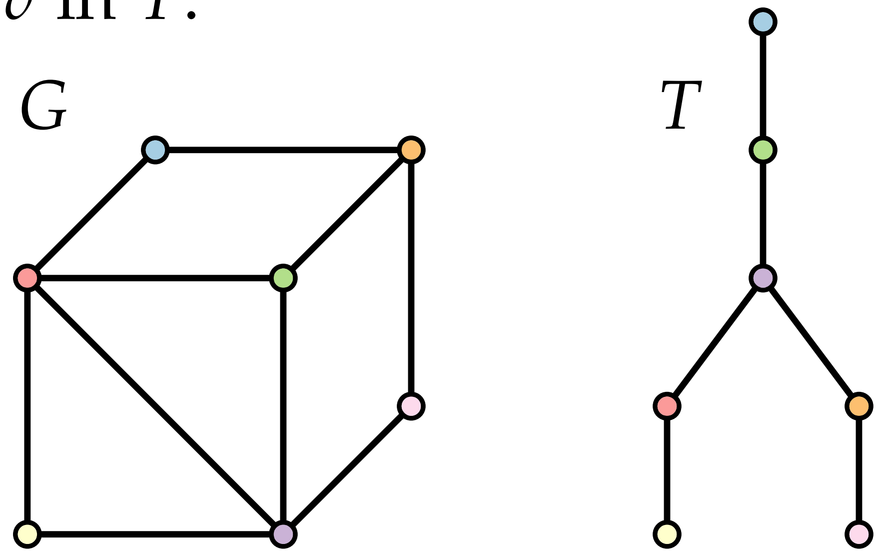

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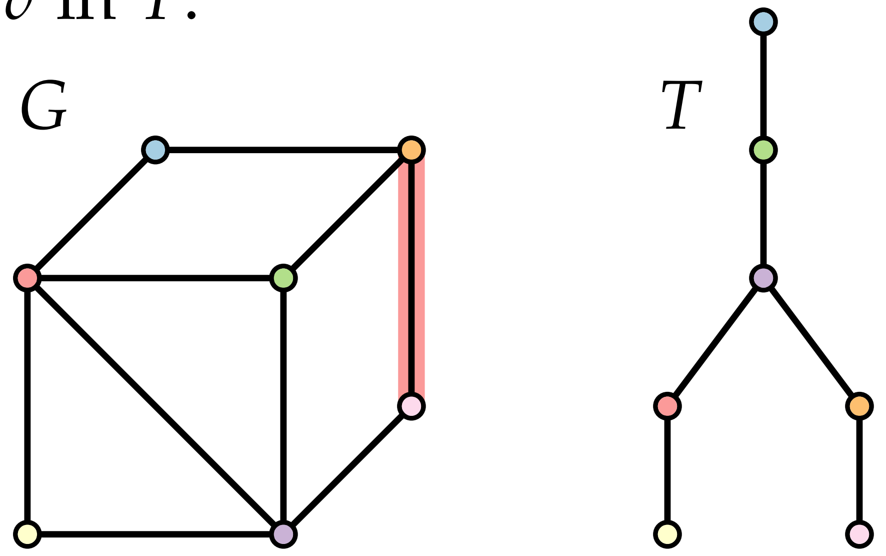

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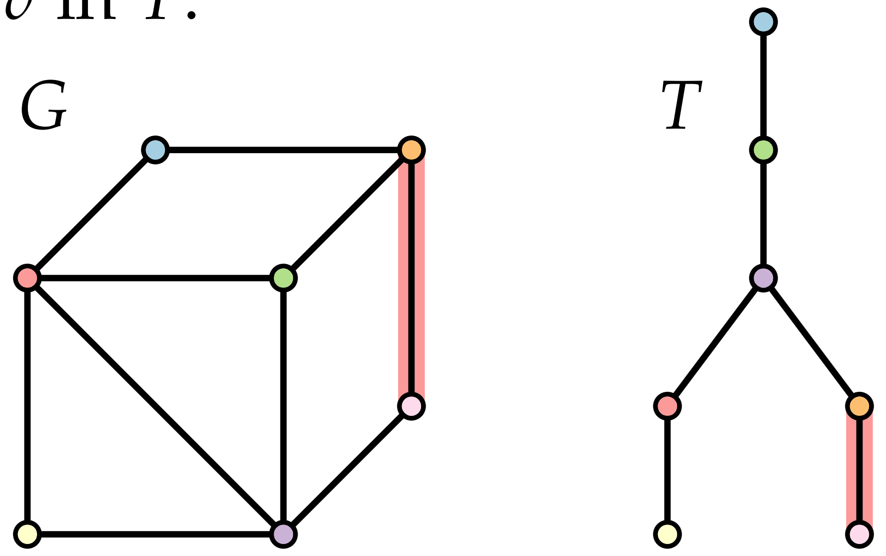

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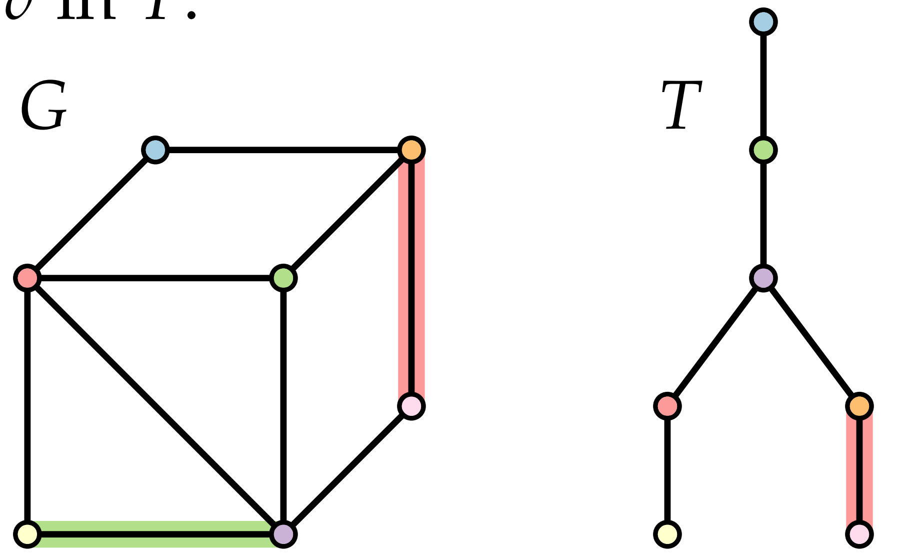

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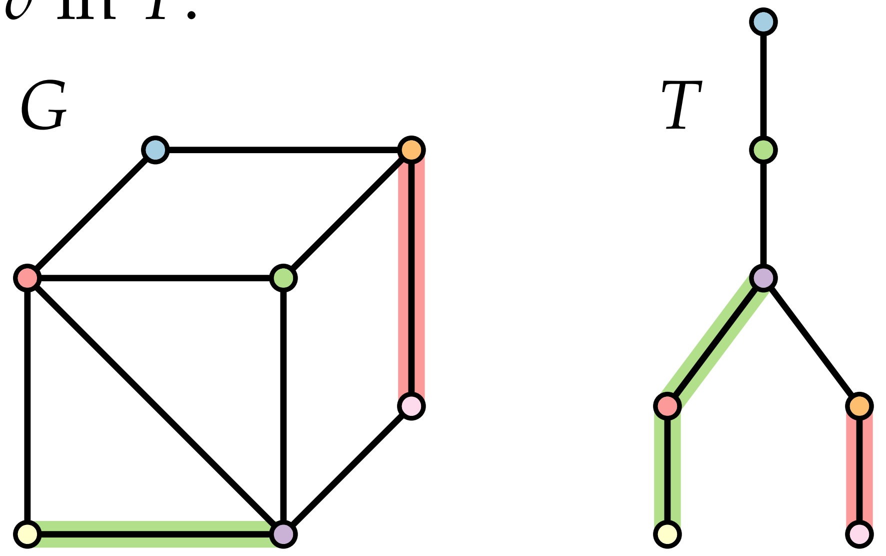

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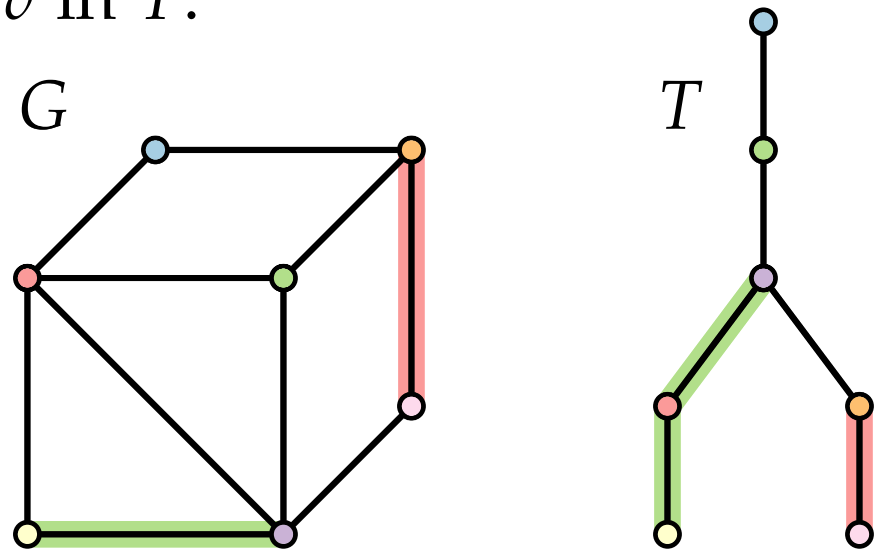

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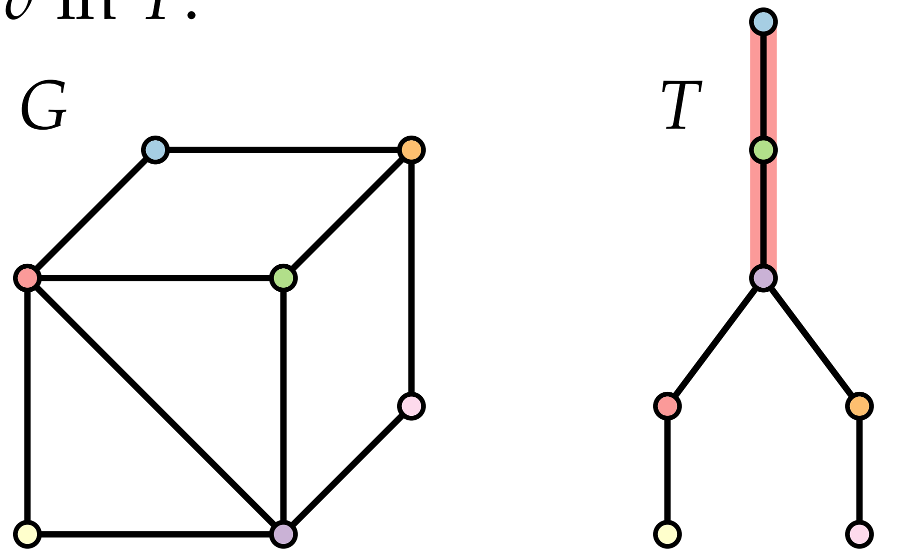

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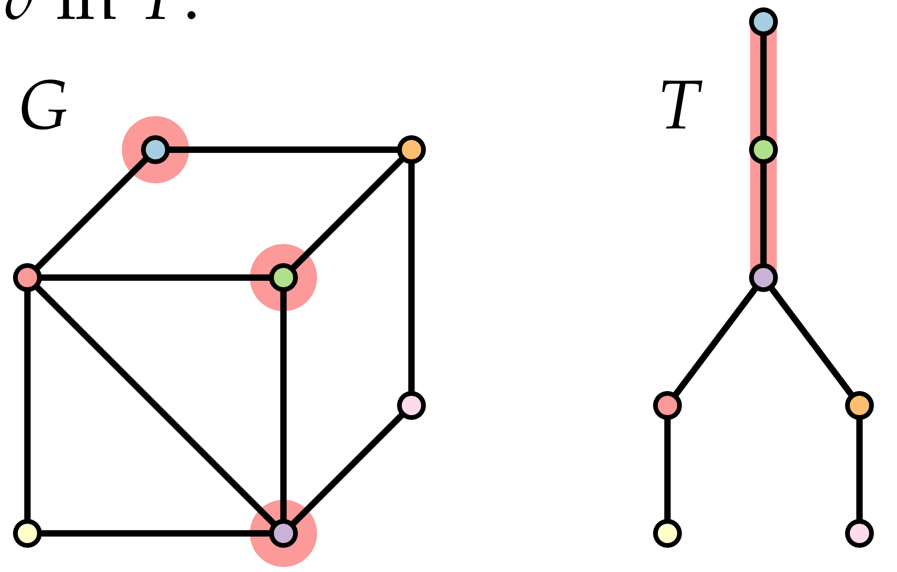

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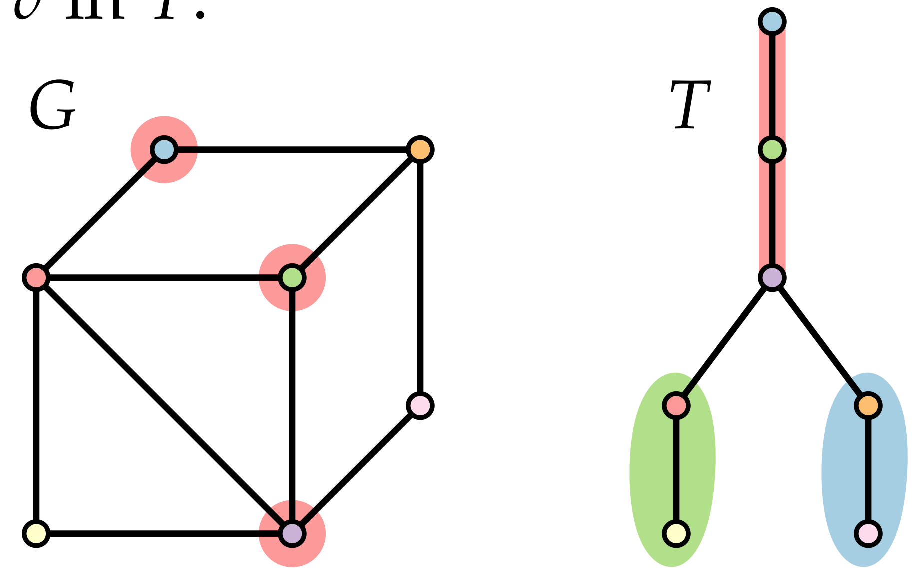

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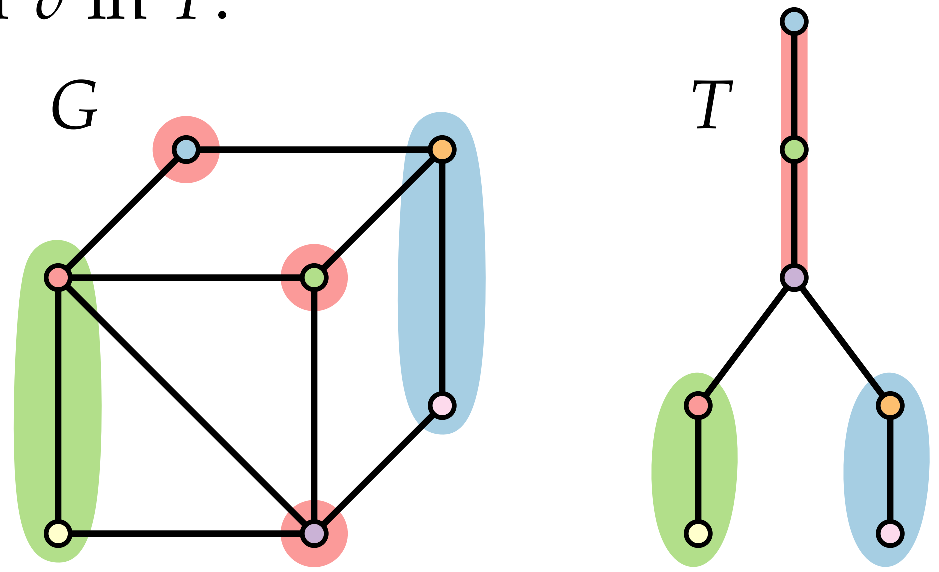

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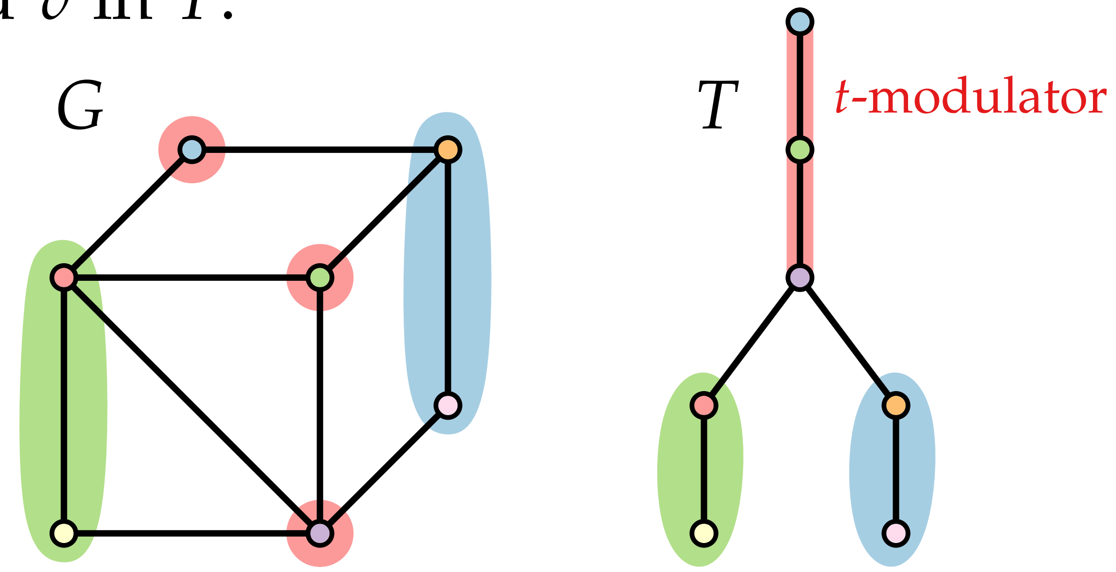

Trim until every vertex has outdegree $\leq 5t$.

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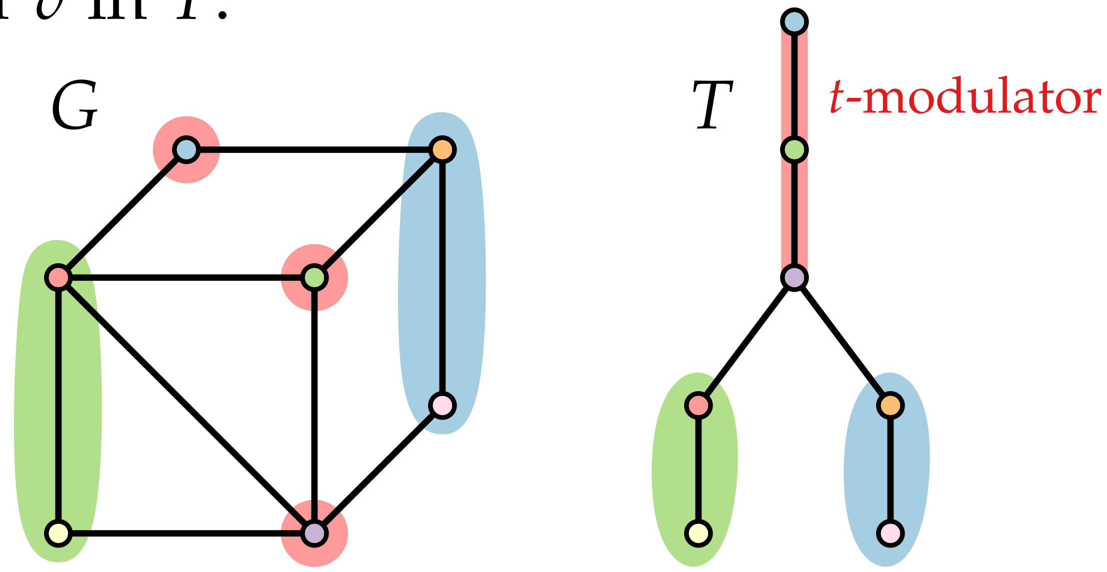

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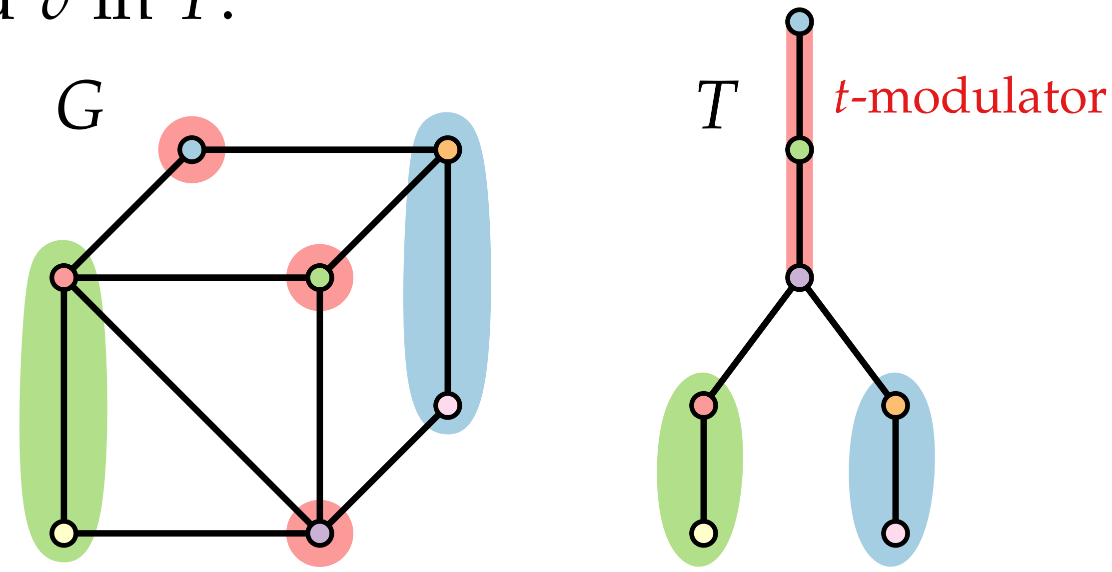

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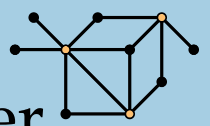
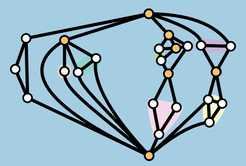
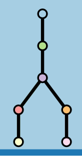
Overview

FPT algorithms

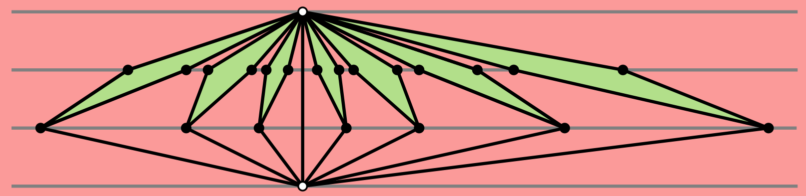
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

Reduction from leveled planar



Combinatorial results

Implications

Overview

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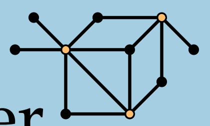
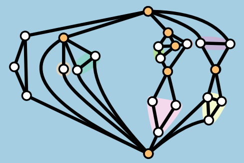
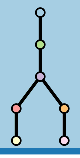
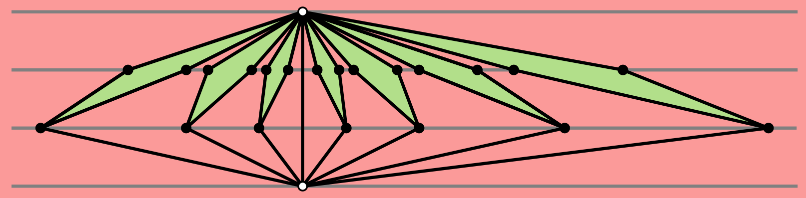
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

Reduction from leveled planar



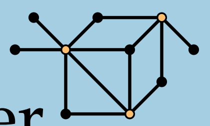
Combinatorial results

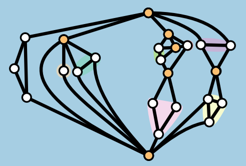
Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n

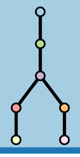
Implications

Overview

FPT algorithms

Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

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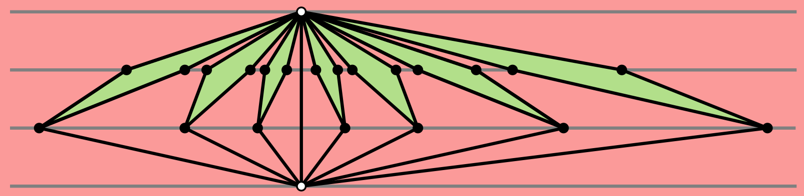
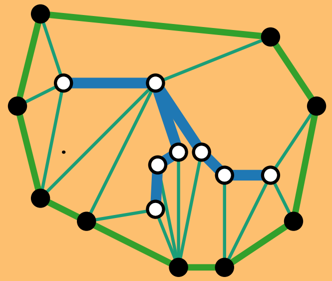


Combinatorial results

Graph class	LB	UB
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4

Implications



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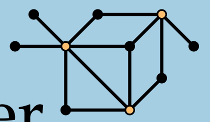
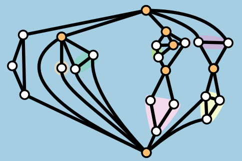
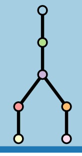
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

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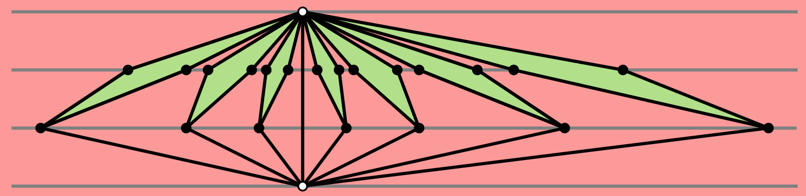

Combinatorial results

Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
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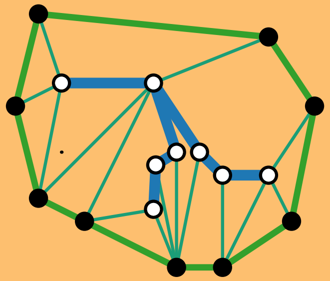
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$

Implications



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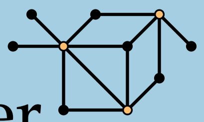
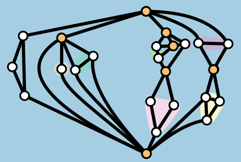
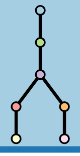
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

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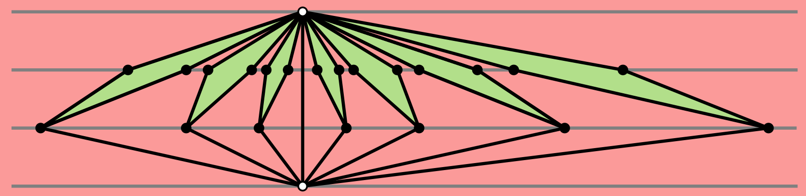

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Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4
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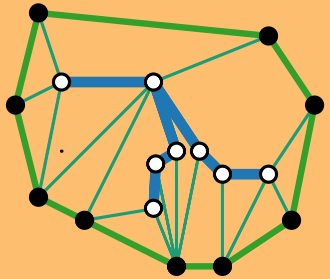
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

Implications



Combinatoric Bounds

Theorem. [Felsner, Liotta & Wismath '01]
Every outerplanar graph admits a 1-span weakly leveled planar drawing.

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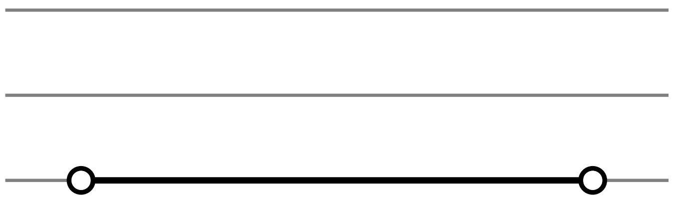

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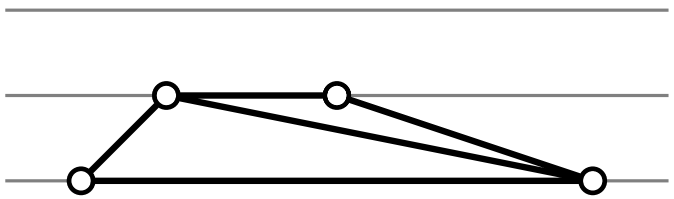

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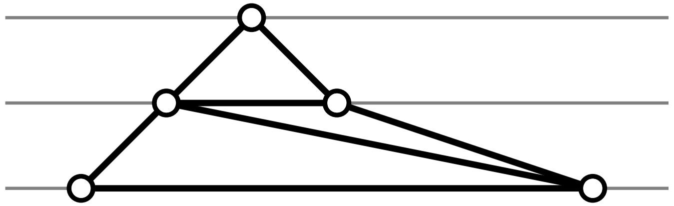

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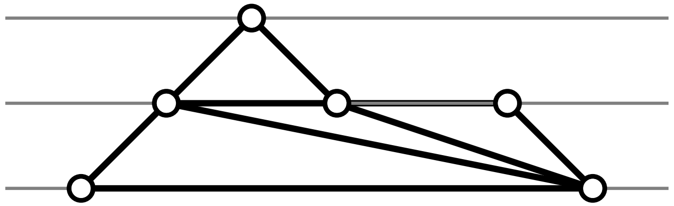

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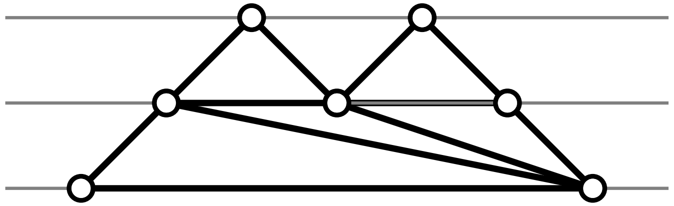

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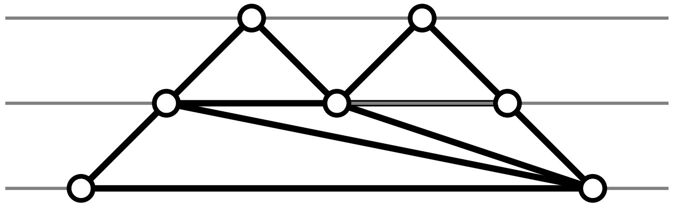

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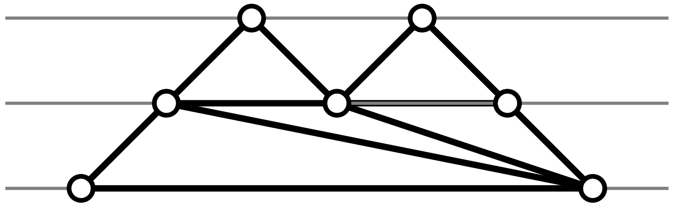

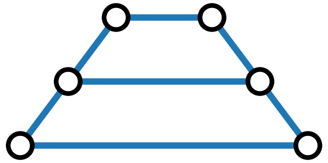

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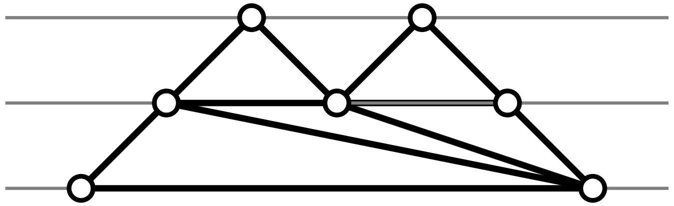

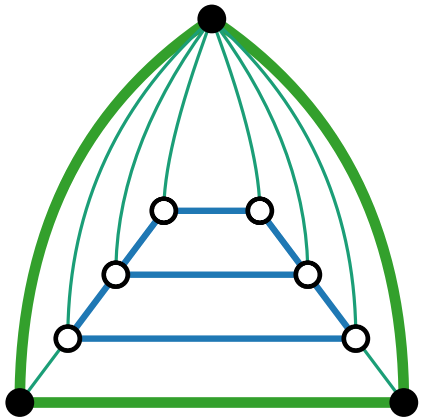

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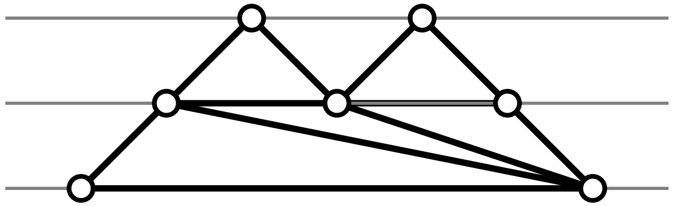

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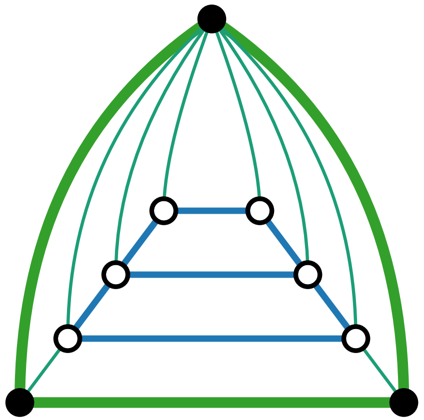

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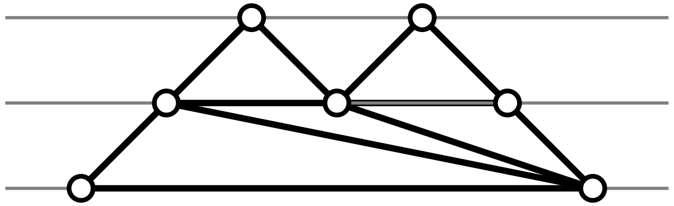

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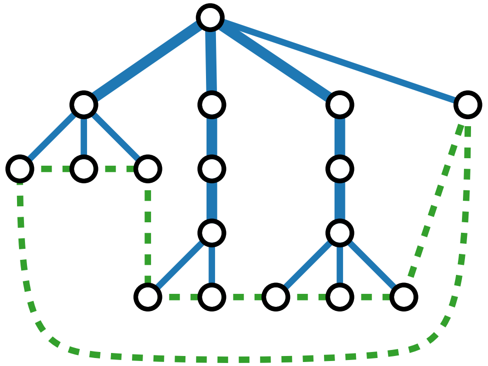

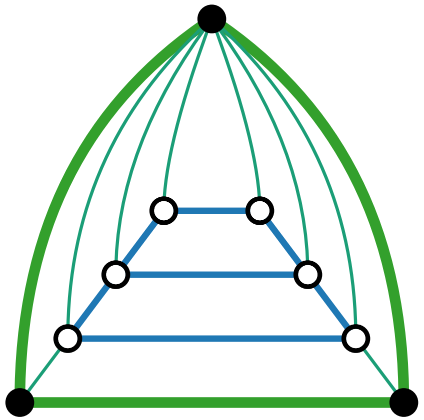

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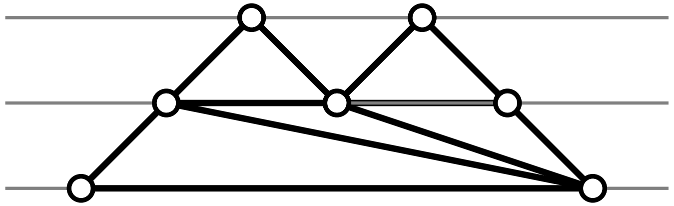

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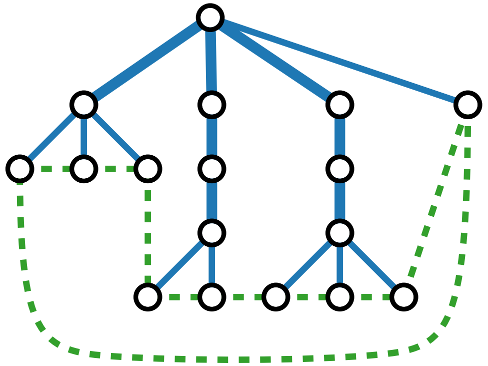
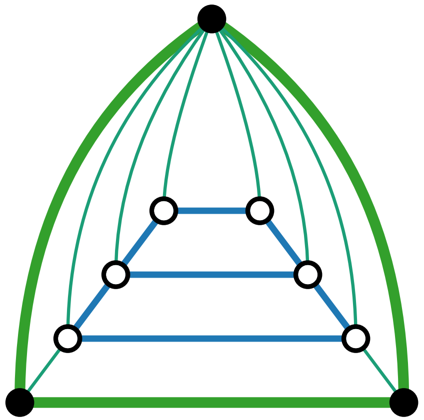


?



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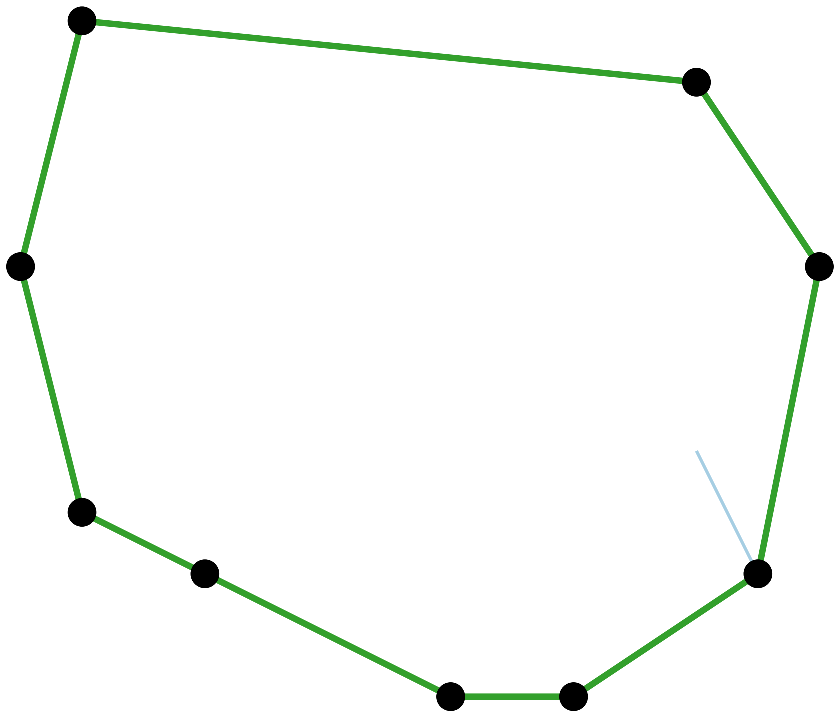
Cycle- and Path-trees

A **cycle-tree** consists of

Cycle- and Path-trees

A **cycle-tree** consists of

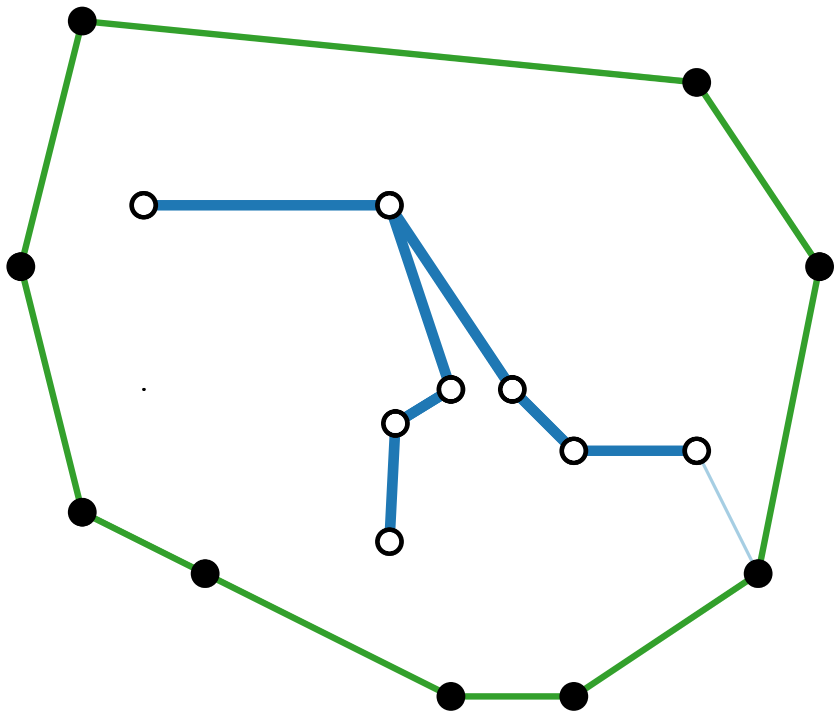
- A **cycle**



Cycle- and Path-trees

A **cycle-tree** consists of

- A **cycle**
- A **tree** inside the cycle



Cycle- and Path-trees

A **cycle-tree** consists of

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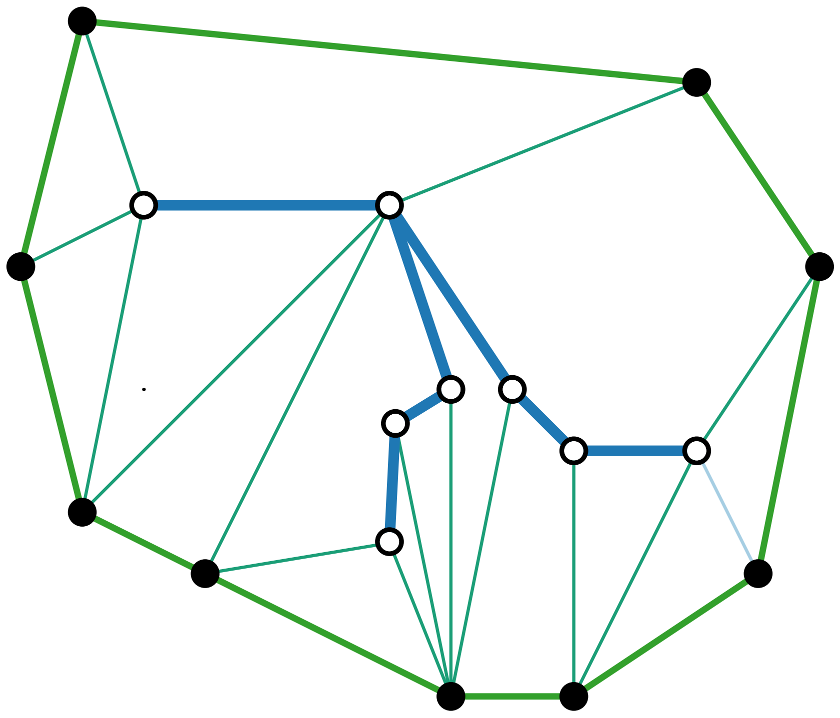
- A **tree** inside the cycle
- A **set of edges** between the cycle and the tree



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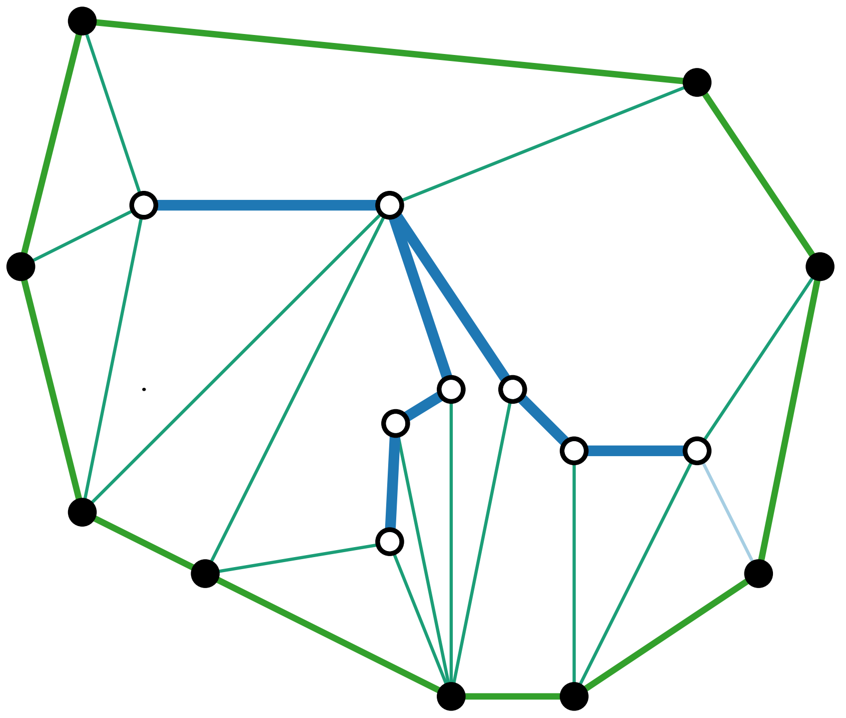

Every **Halin graph** is the subdivision of a 3-connected cycle-tree

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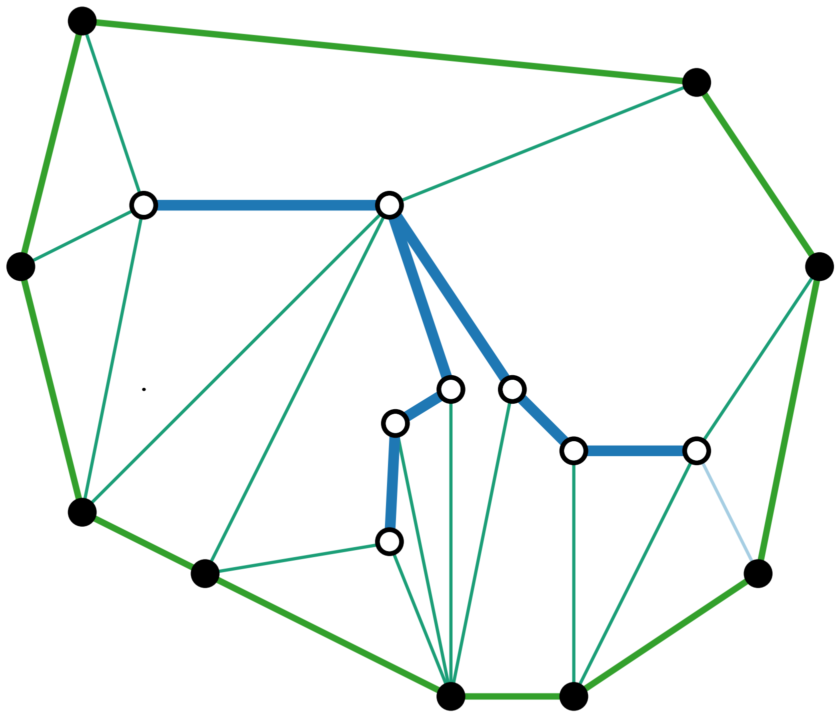

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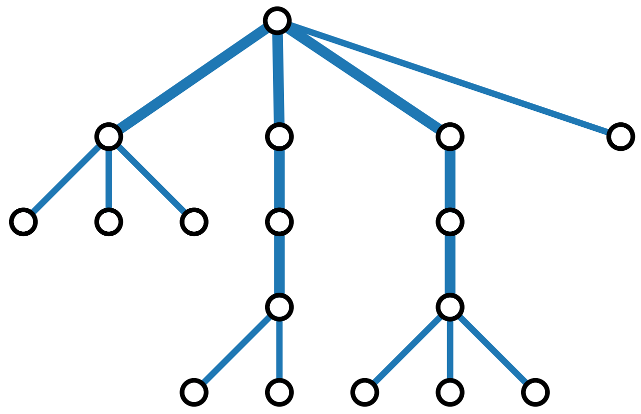

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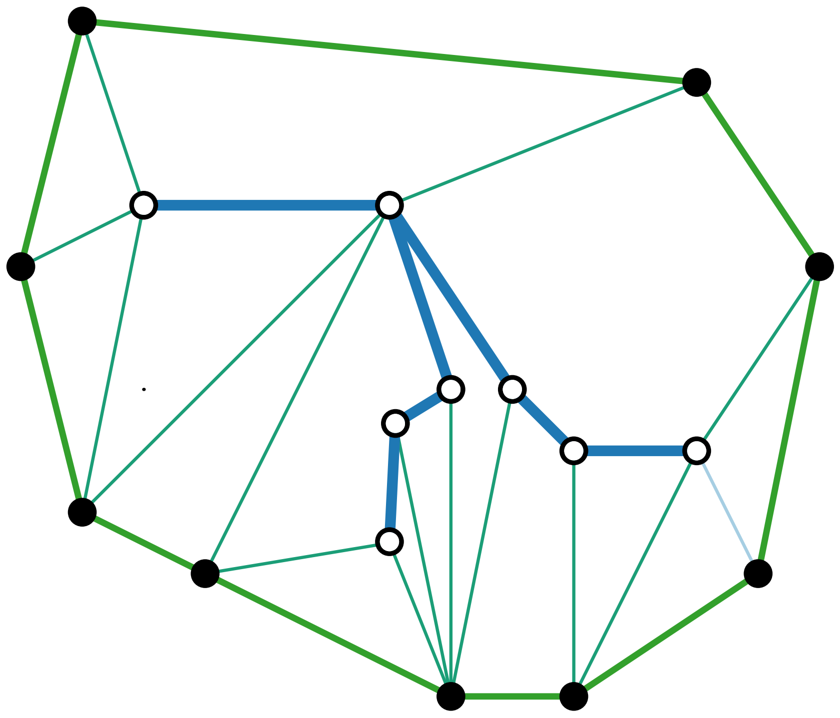

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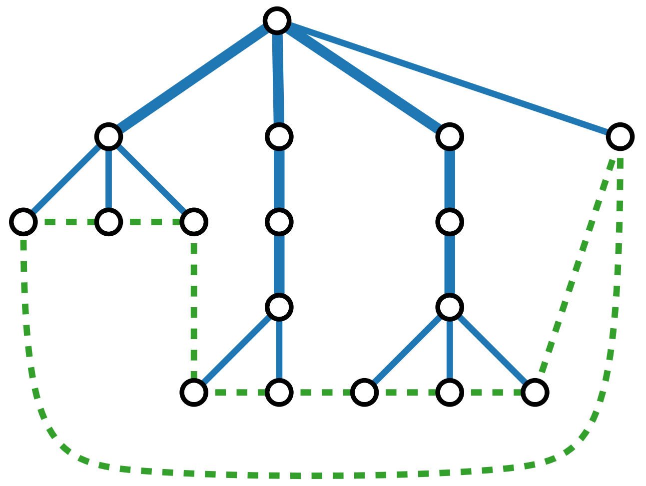

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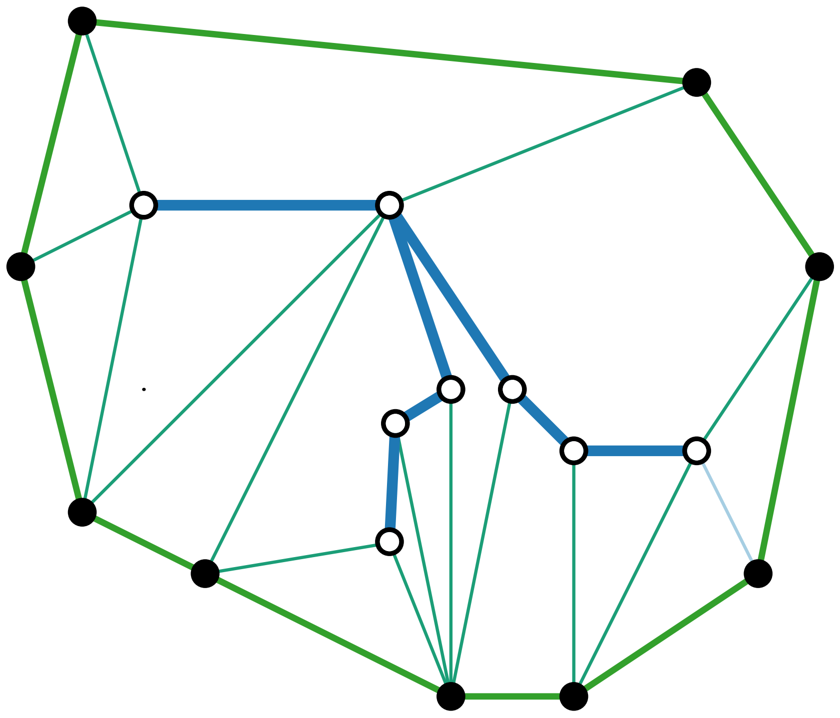

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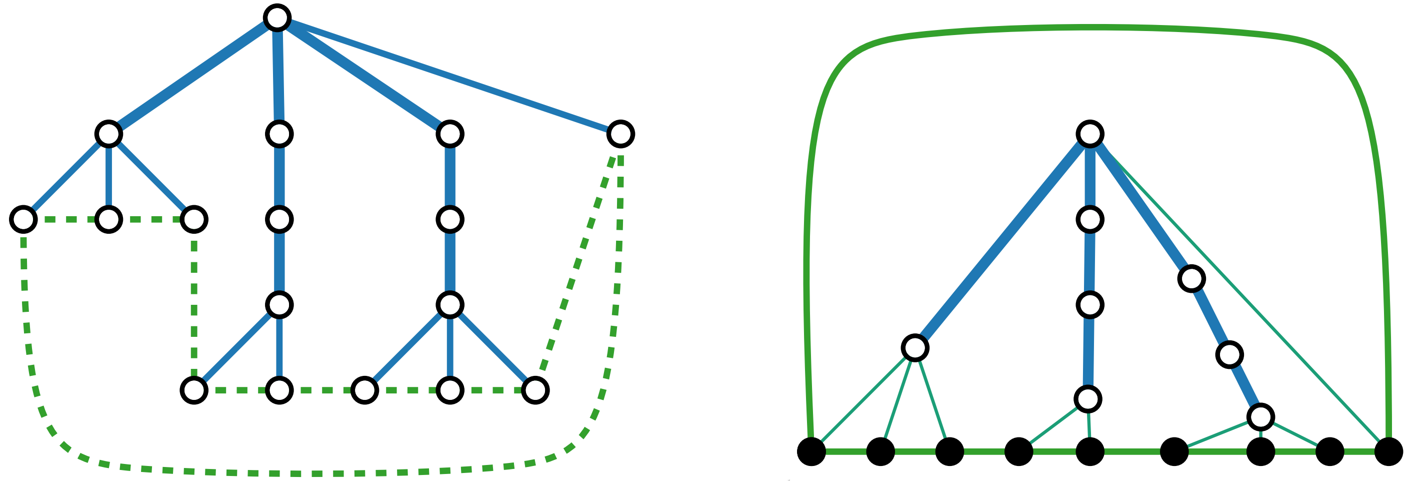

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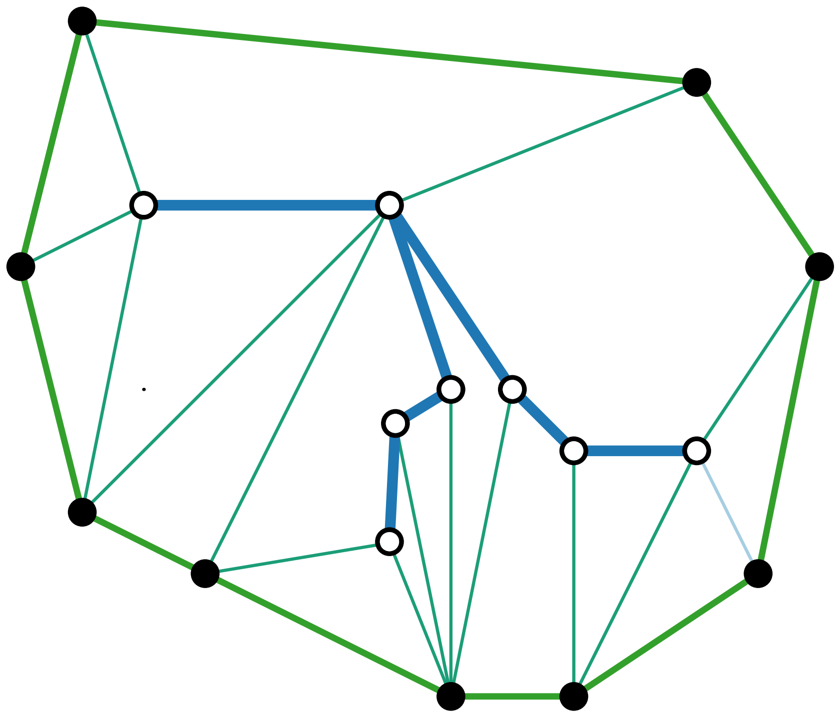

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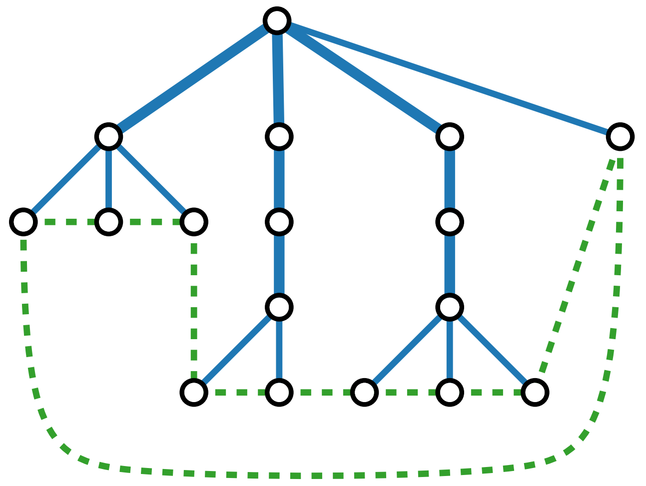

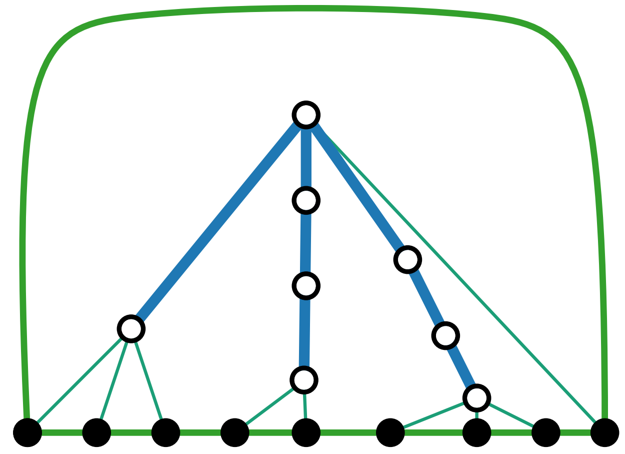

Cycle- and Path-trees

A **cycle-tree** consists of

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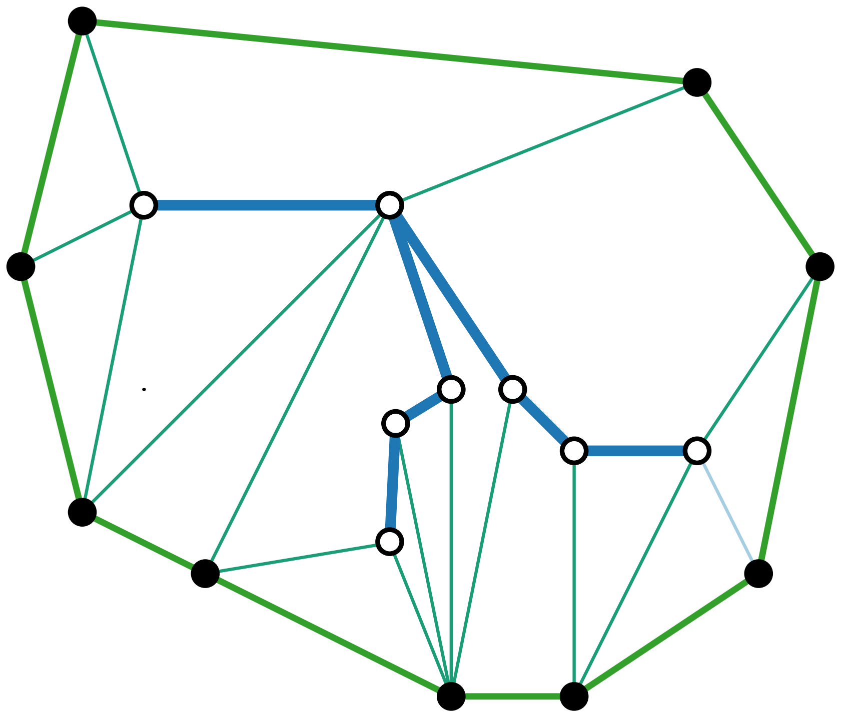
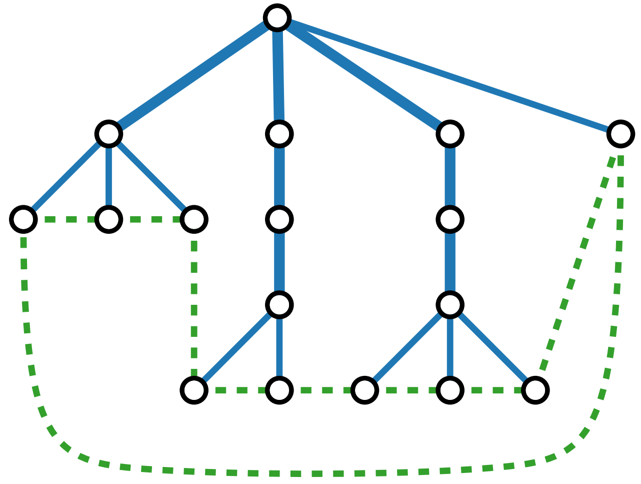
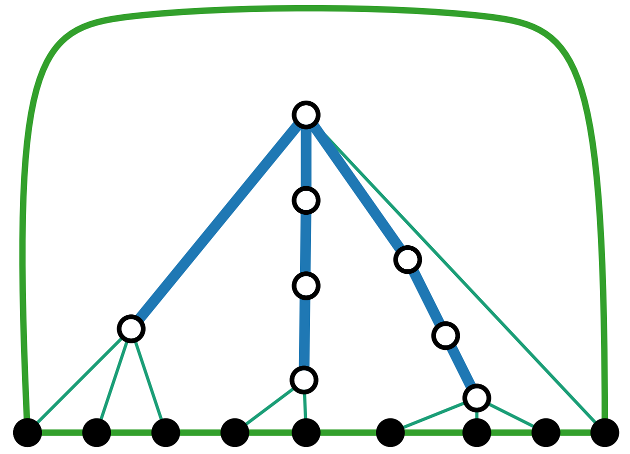


Outer face shares an edge with every other face

Every **Halin graph** is the subdivision of a 3-connected cycle-tree

Outer face shares a vertex with every other face





Cycle- and Path-trees

A **cycle-tree** consists of

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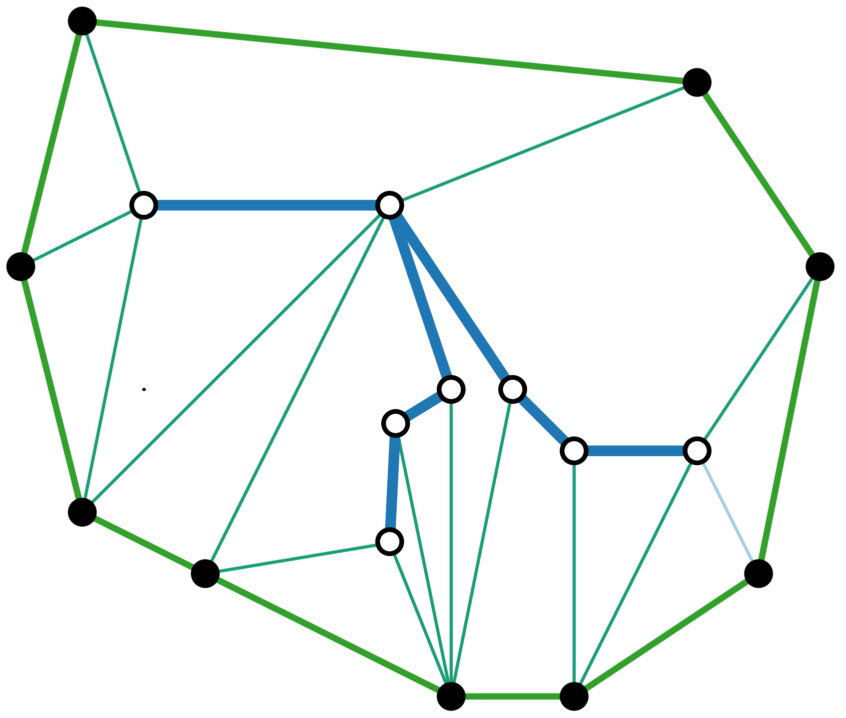
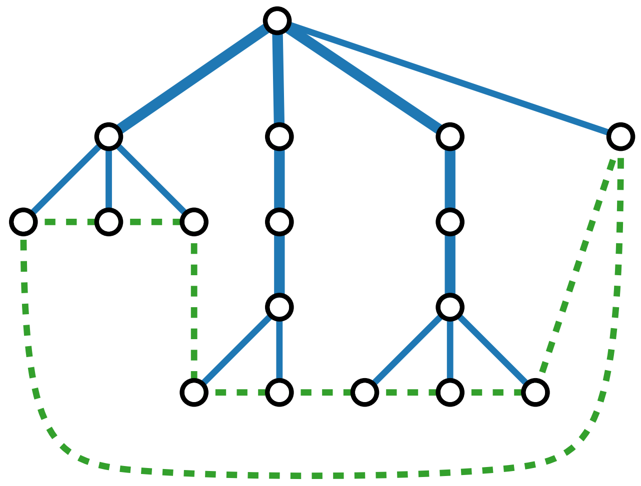
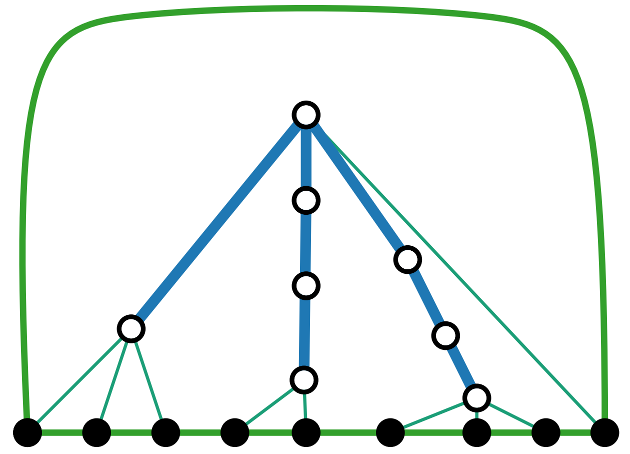


Outer face shares an edge with every other face

Every **Halin graph** is the subdivision of a 3-connected cycle-tree

Outer face shares a vertex with every other face





A **path-tree** is a cycle-tree minus one cycle edge

Cycle- and Path-trees

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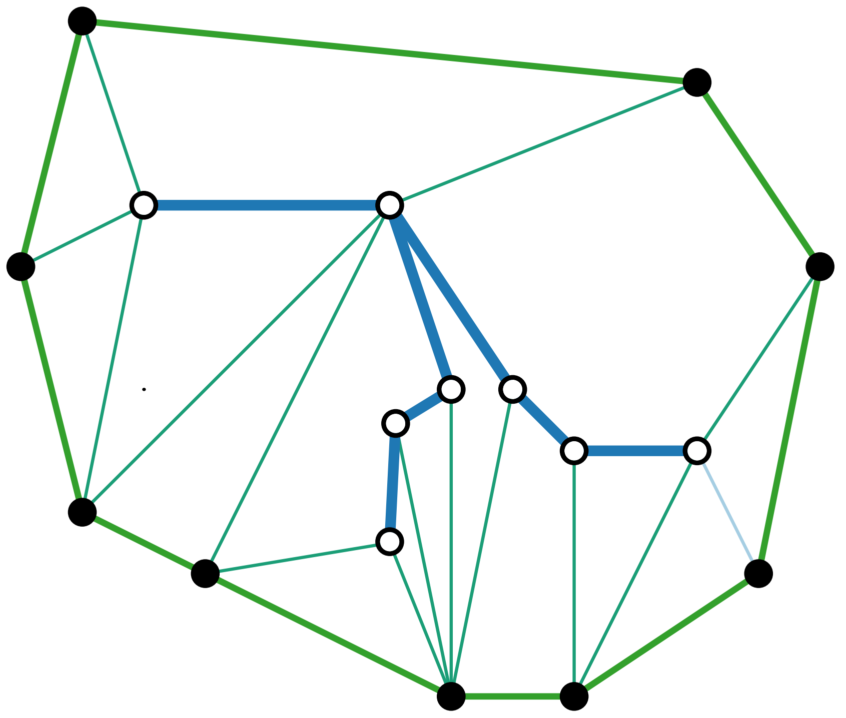
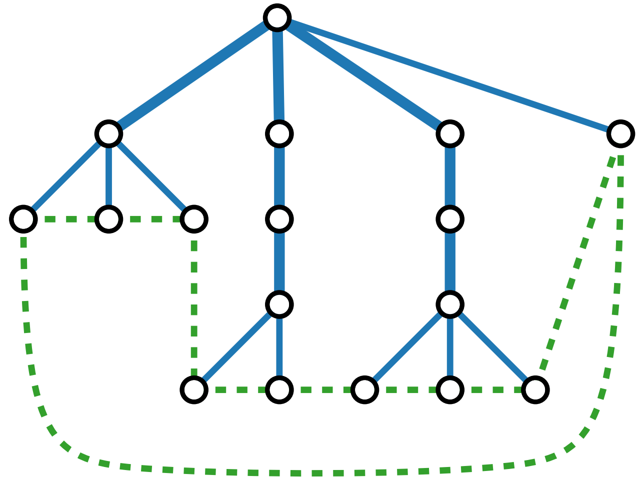
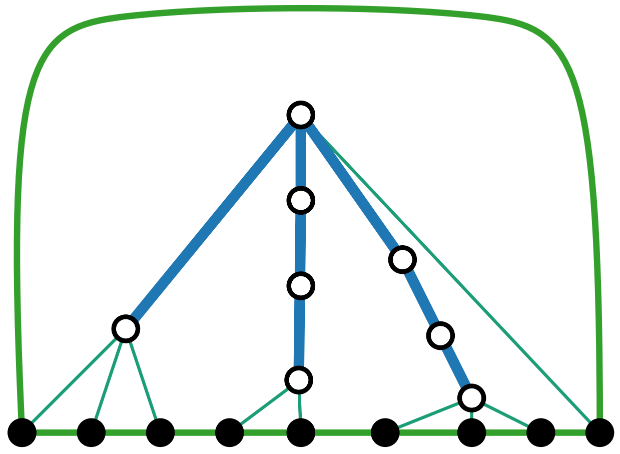


Outer face shares an edge with every other face

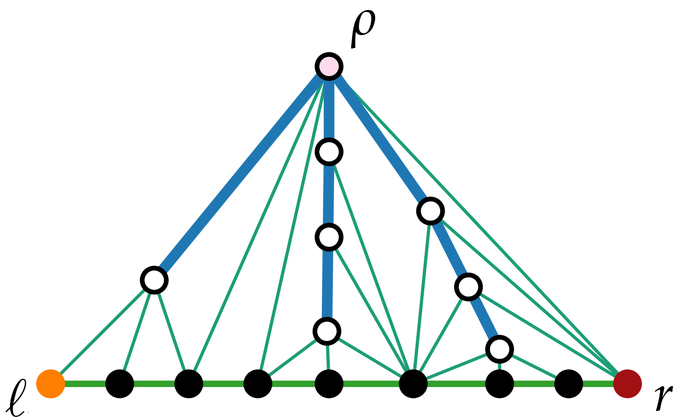
Every **Halin graph** is the subdivision of a 3-connected cycle-tree

Outer face shares a vertex with every other face





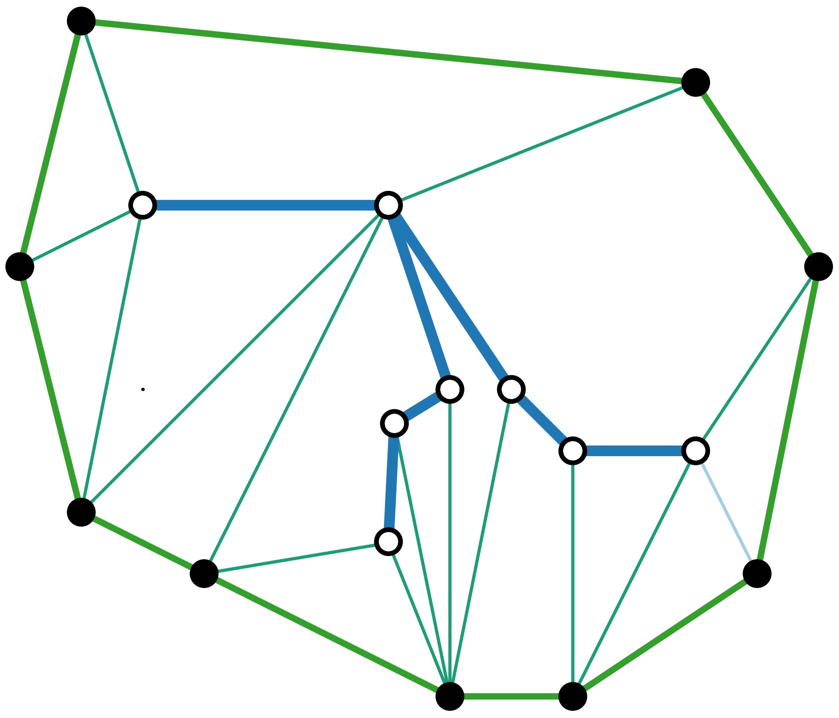
A **path-tree** is a cycle-tree minus one cycle edge



Cycle- and Path-trees

A **cycle-tree** consists of

- A **cycle**
- A **tree** inside the cycle
- A **set of edges** between the cycle and the tree



Outer face shares an edge with every other face

Every **Halin graph** is the subdivision of a 3-connected cycle-tree

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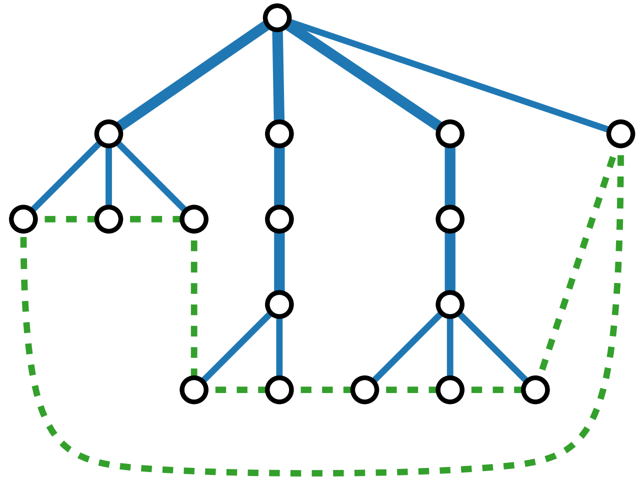
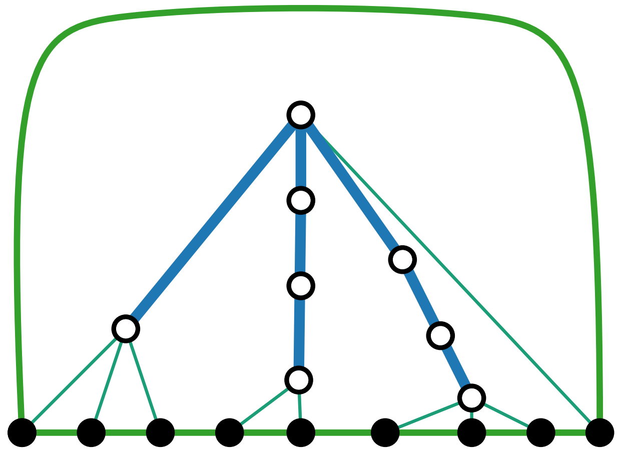
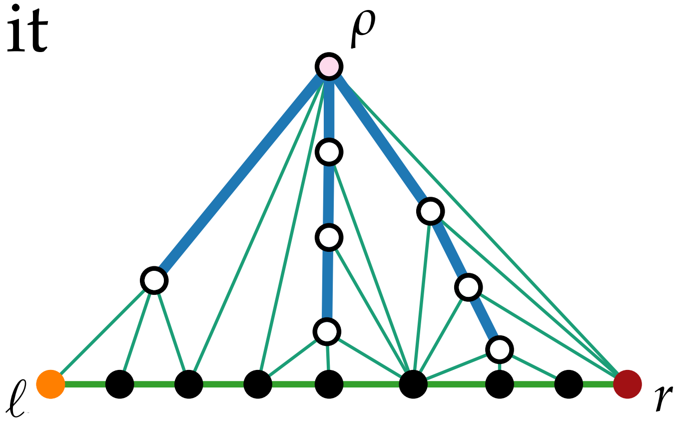




A **path-tree** is a cycle-tree minus one cycle edge
It is **almost-3-connected** if it becomes 3-connected by connecting ρ, ℓ, r



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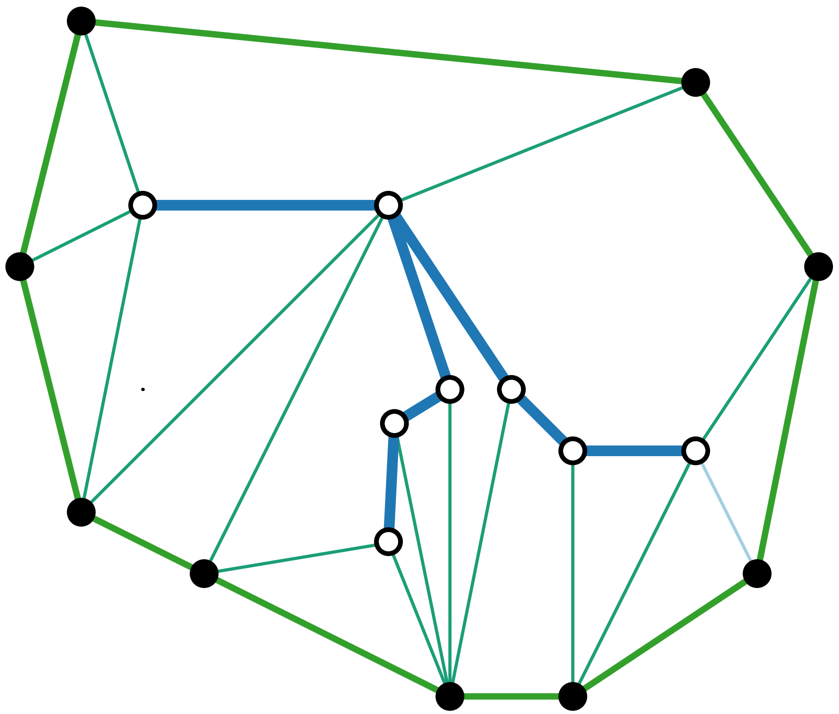

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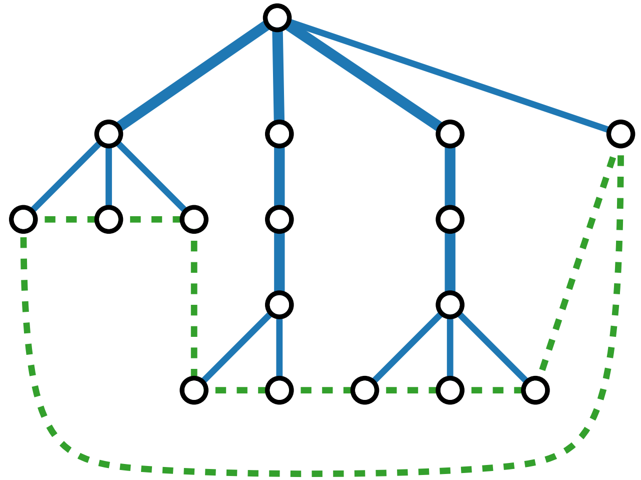

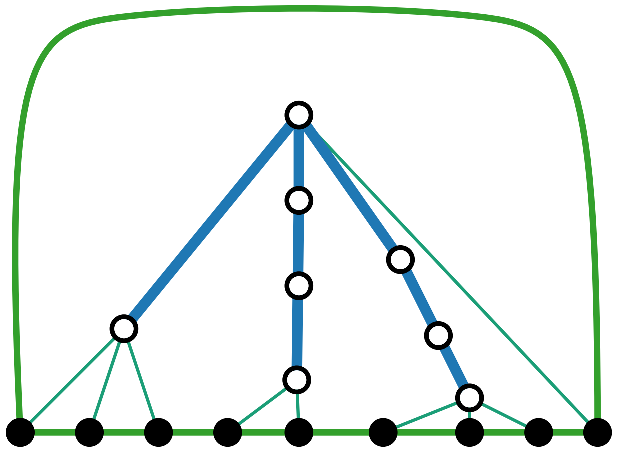

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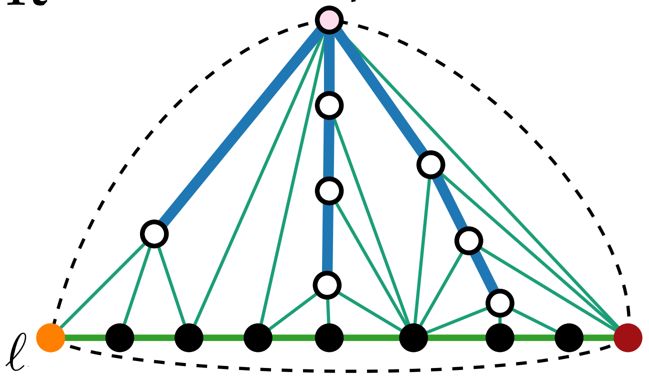
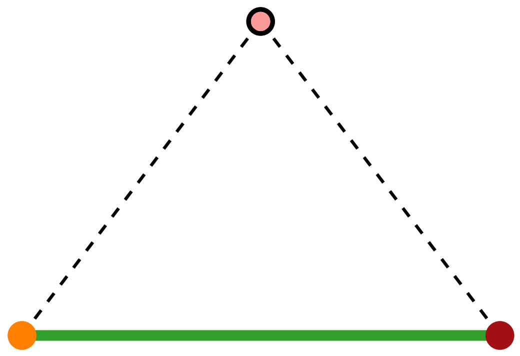


SPQ-(De)composition of Path-Trees

Every almost-3-connected path-tree has an SPQ-decomposition.



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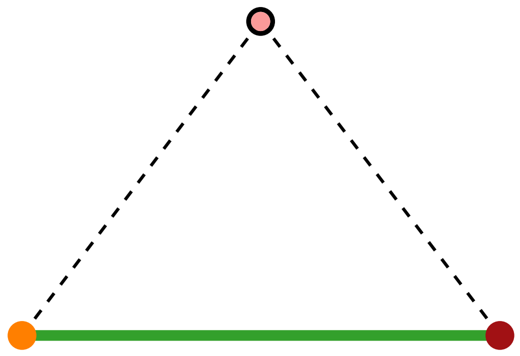
Solid edges must exist, dashed edges might exist



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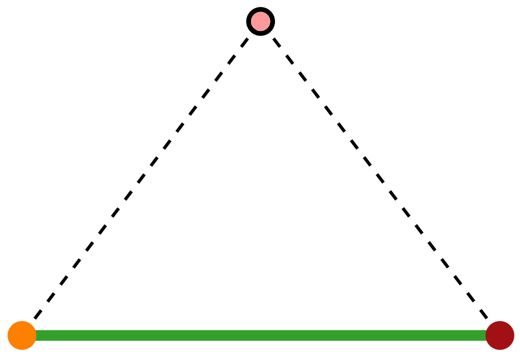

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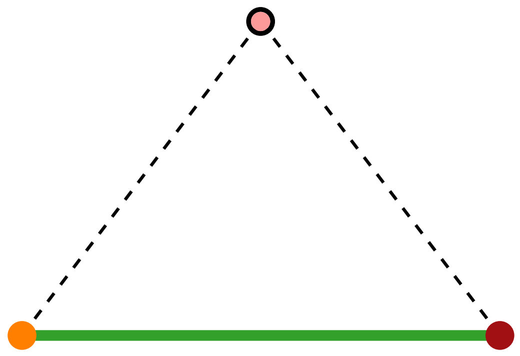

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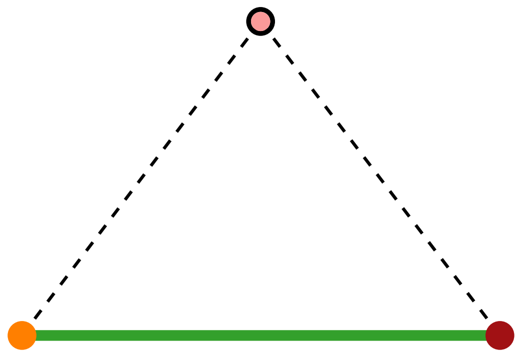

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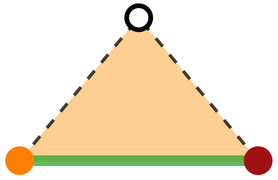

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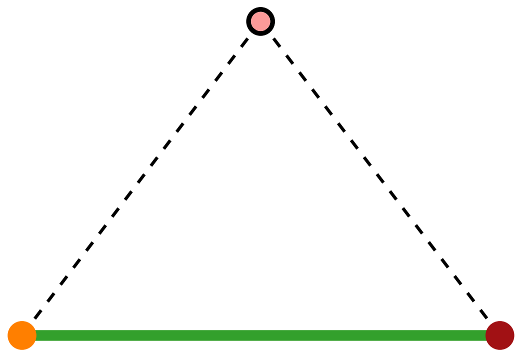

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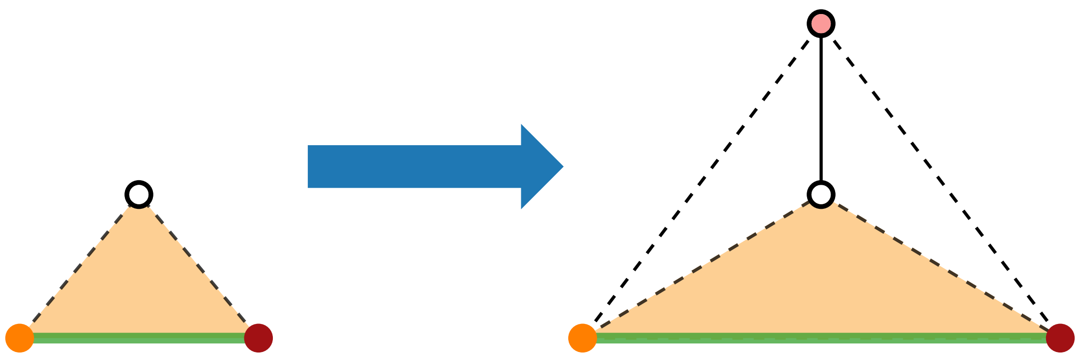

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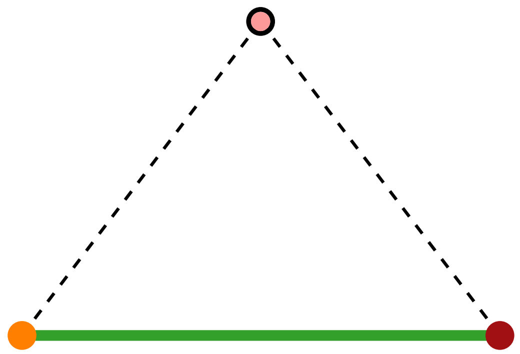

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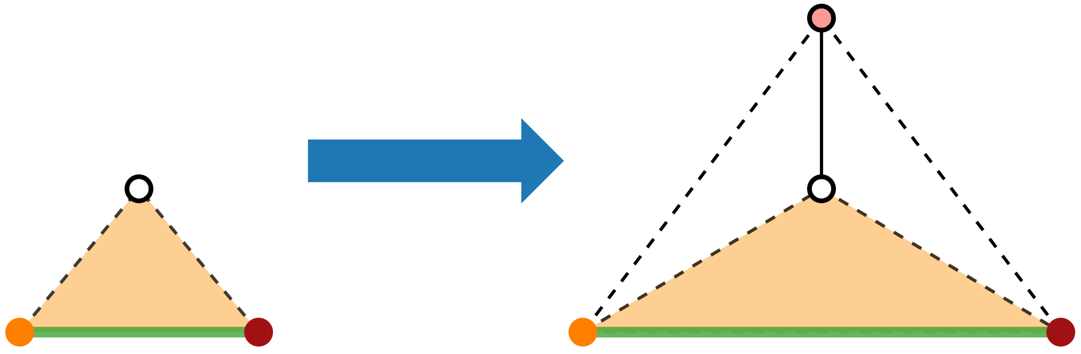

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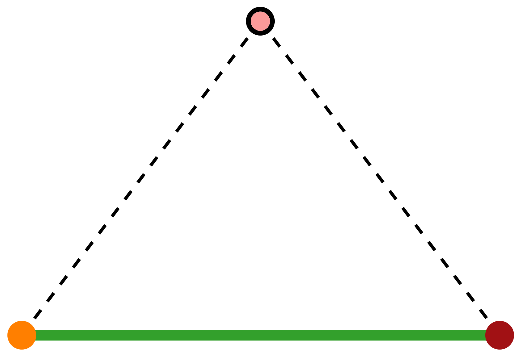

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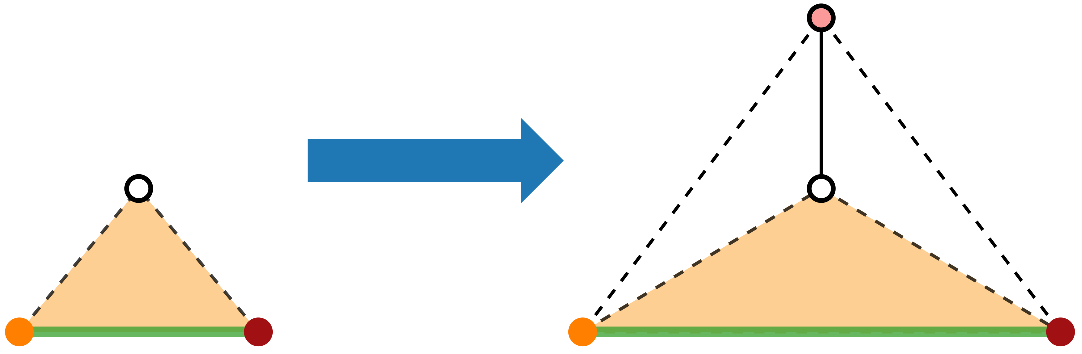

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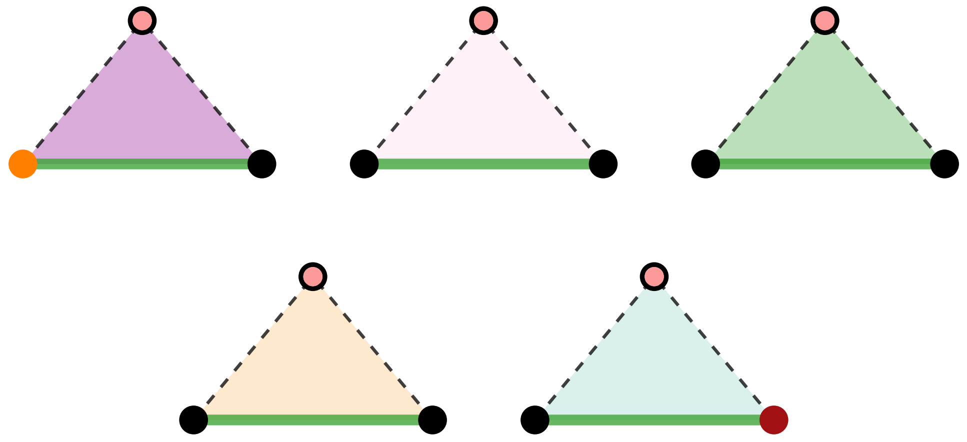

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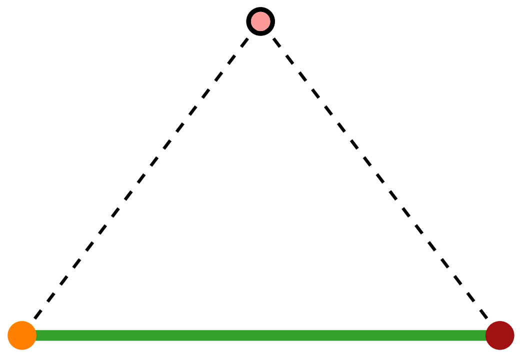

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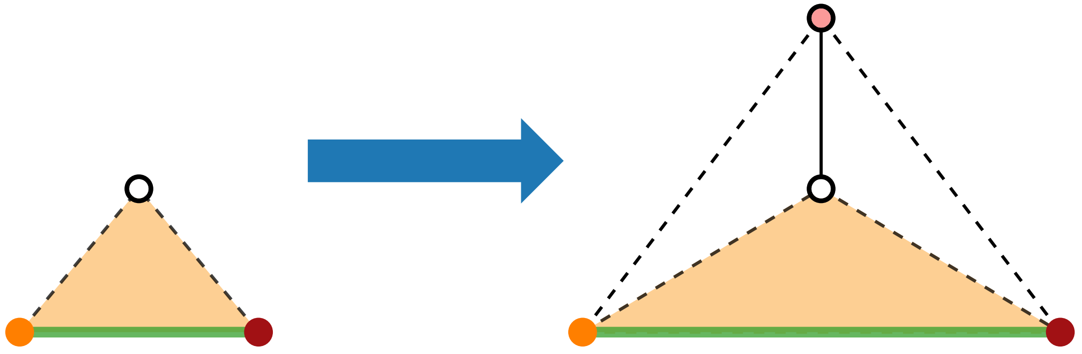

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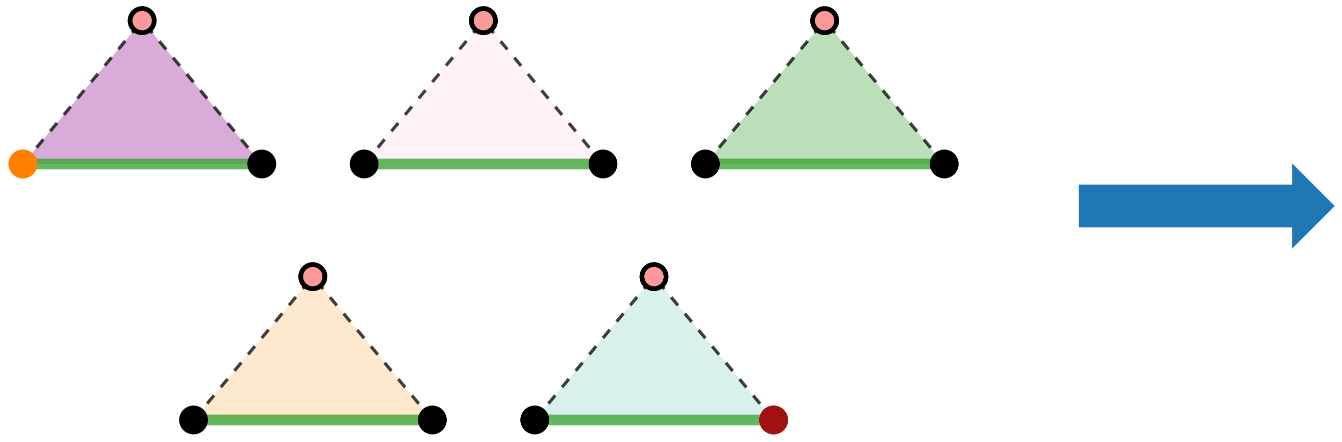

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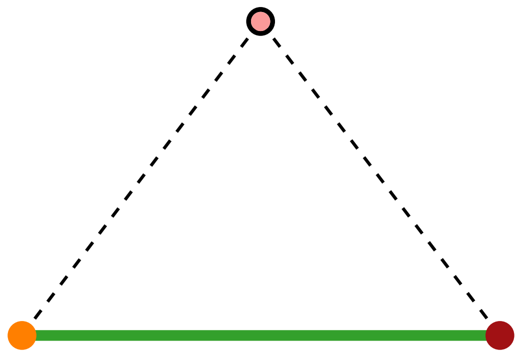

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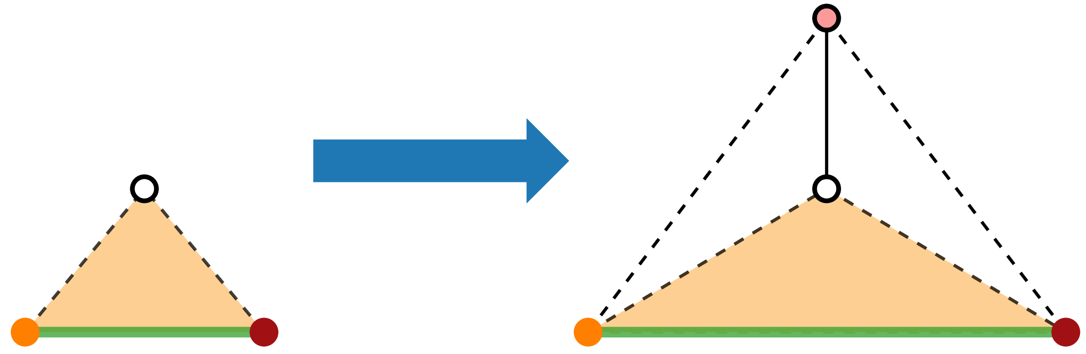

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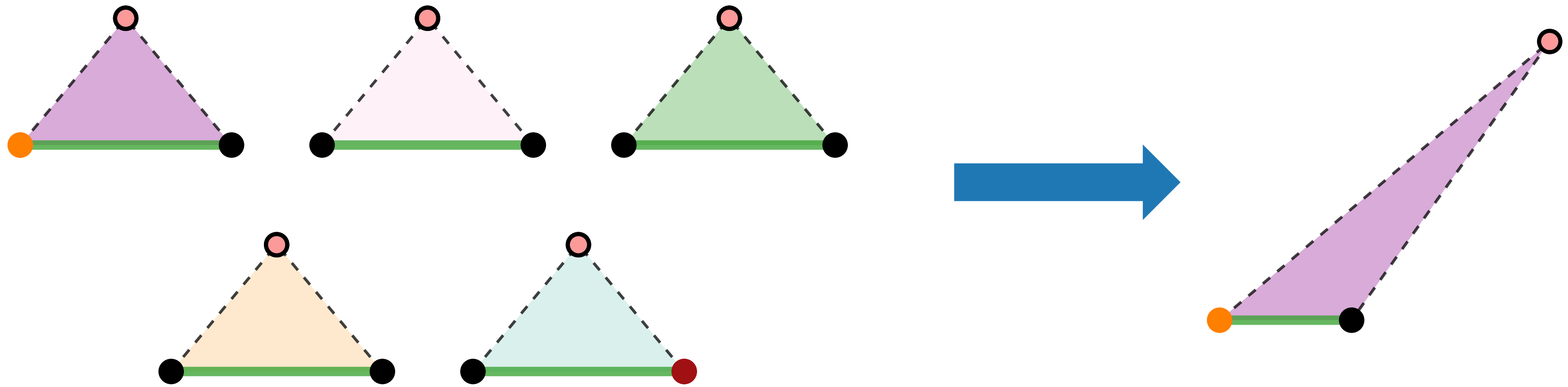

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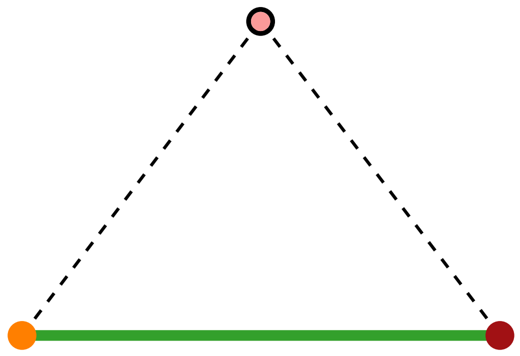

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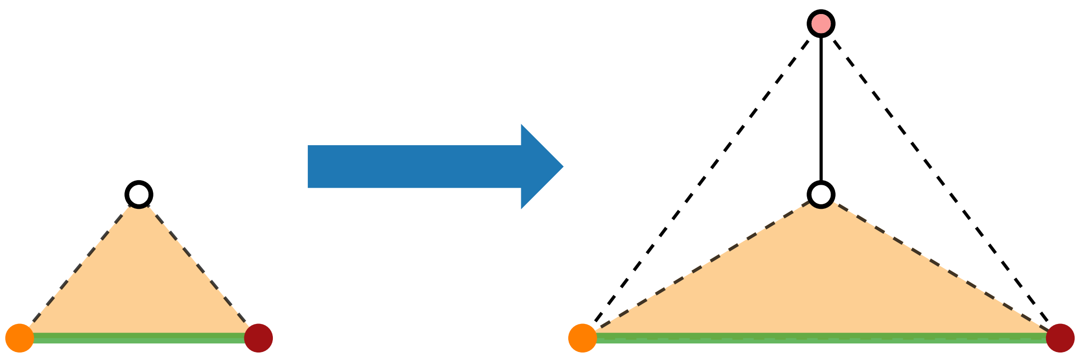

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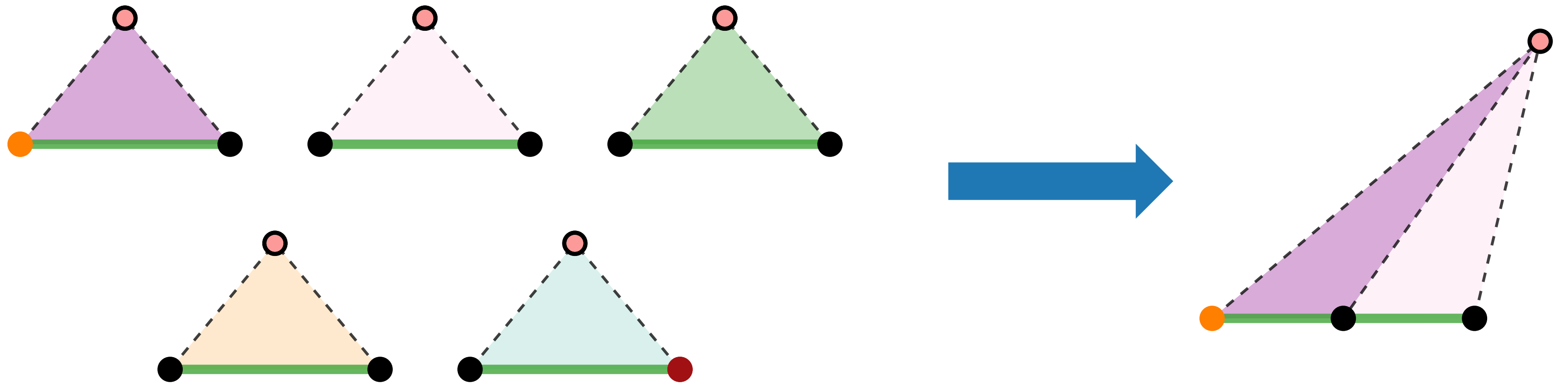

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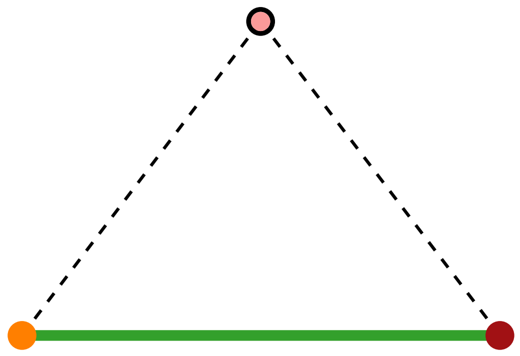

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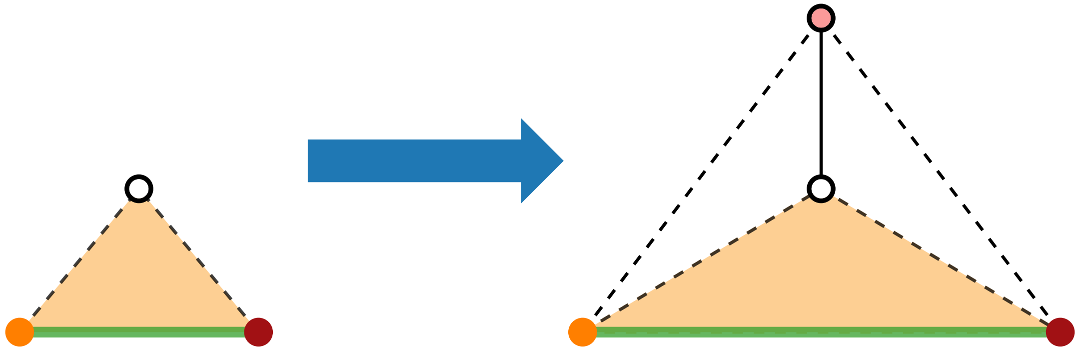

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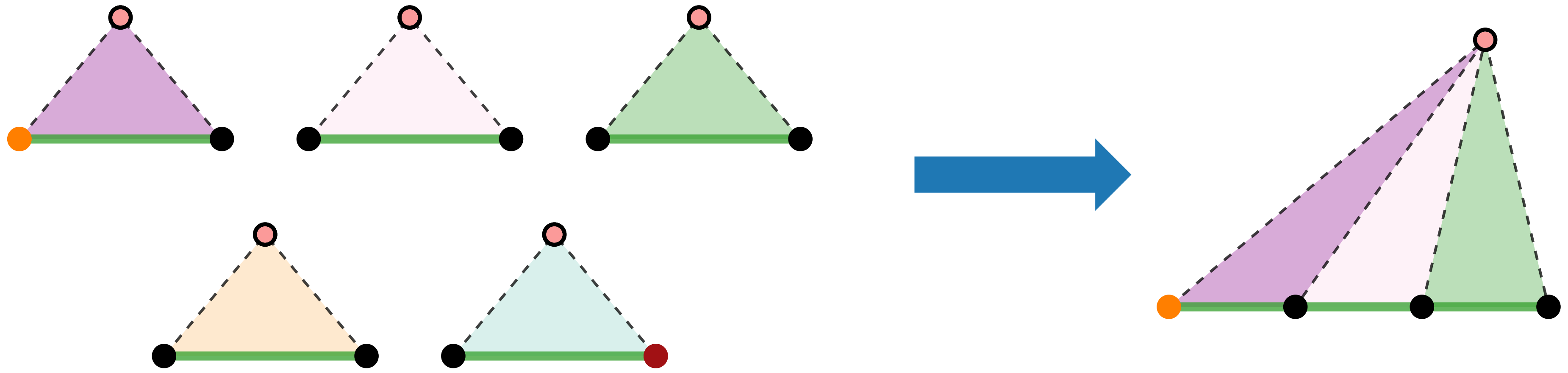

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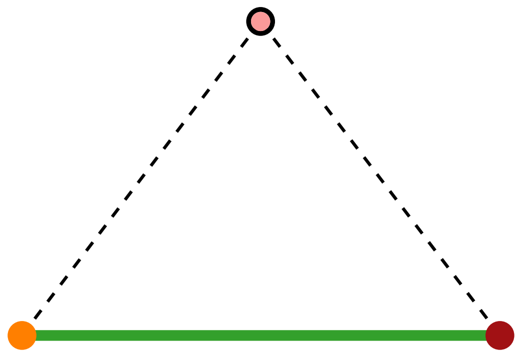

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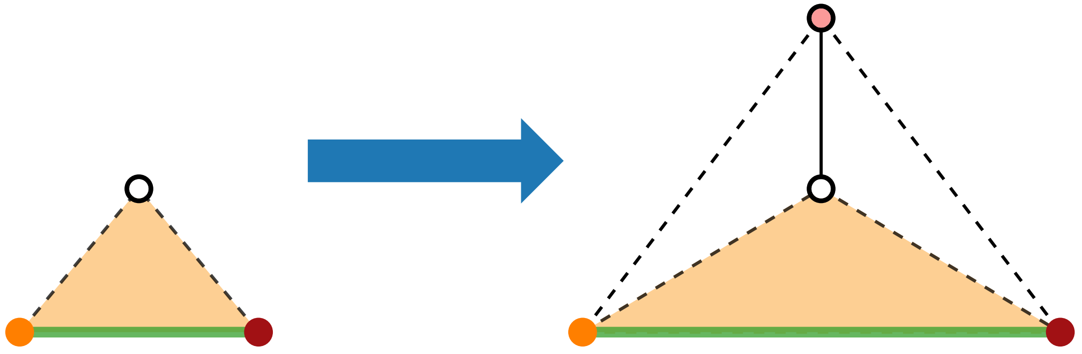

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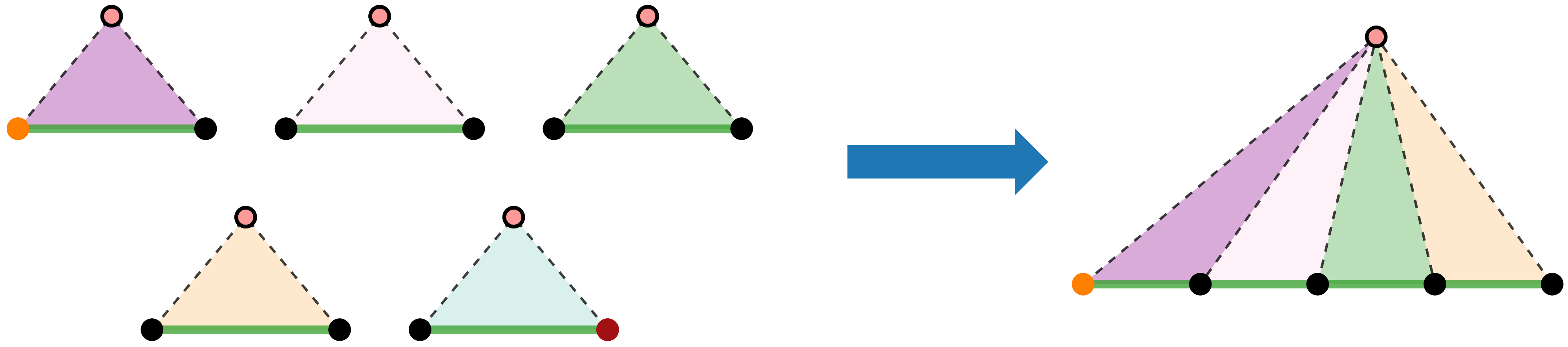

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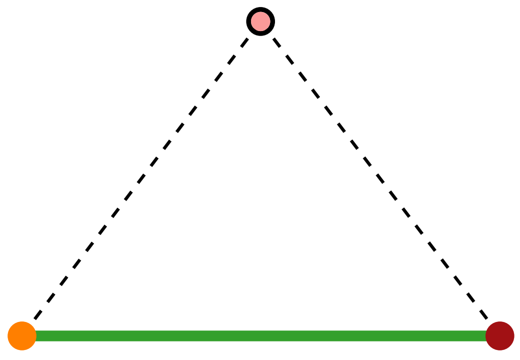

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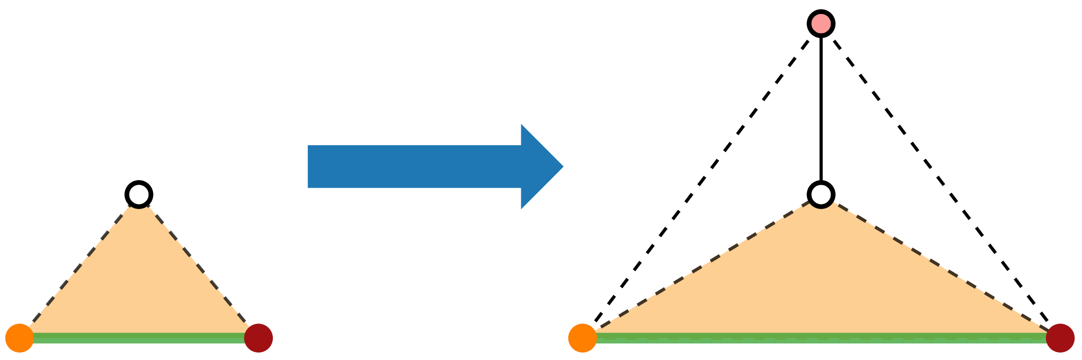

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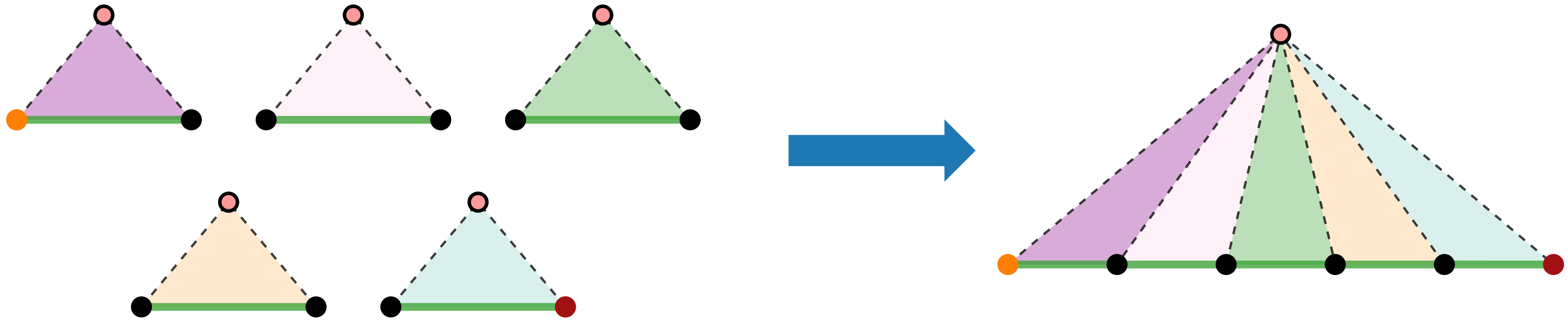

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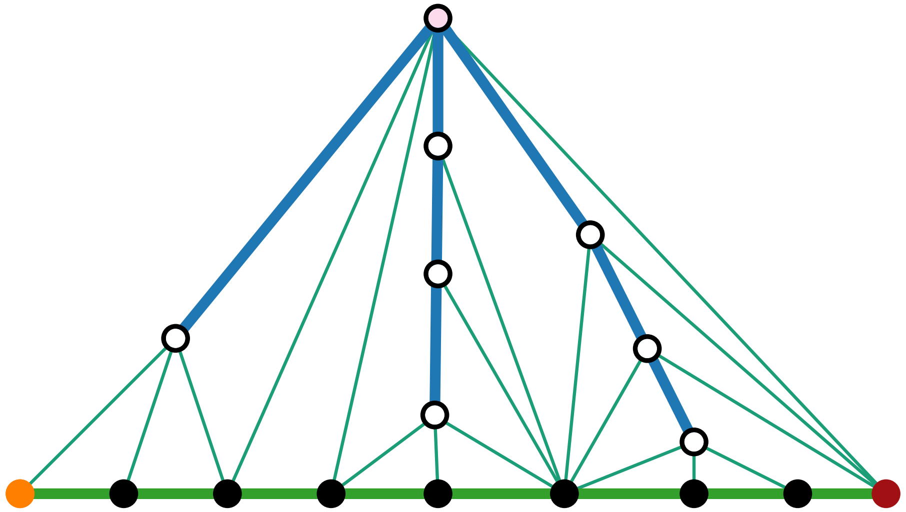

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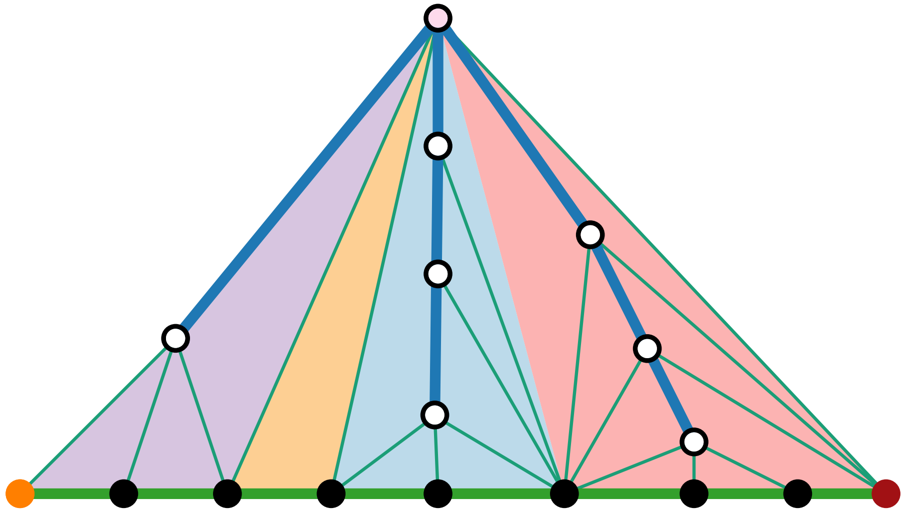

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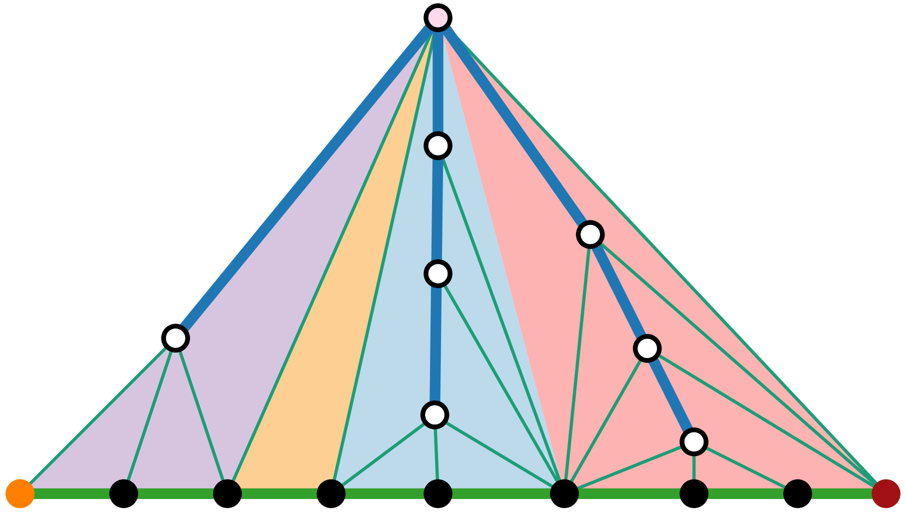

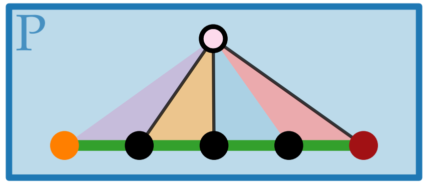

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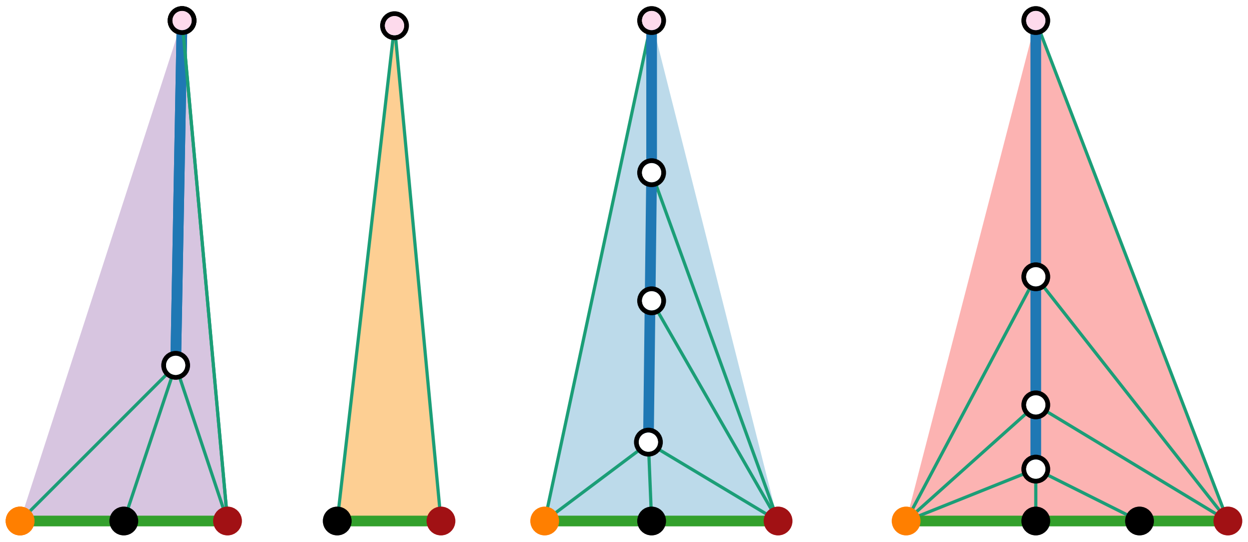

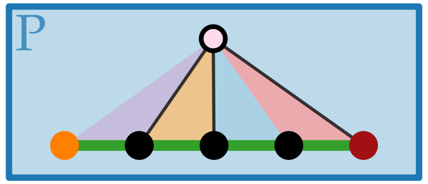

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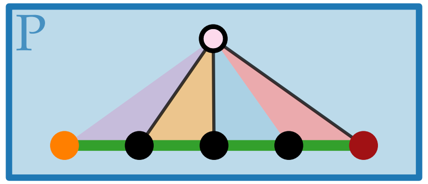

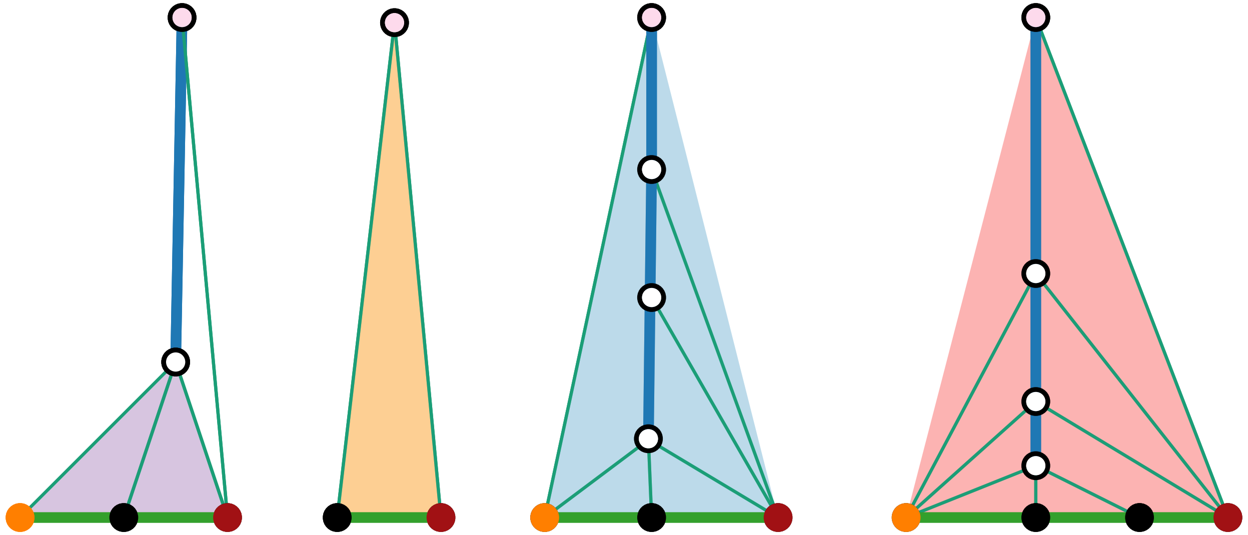

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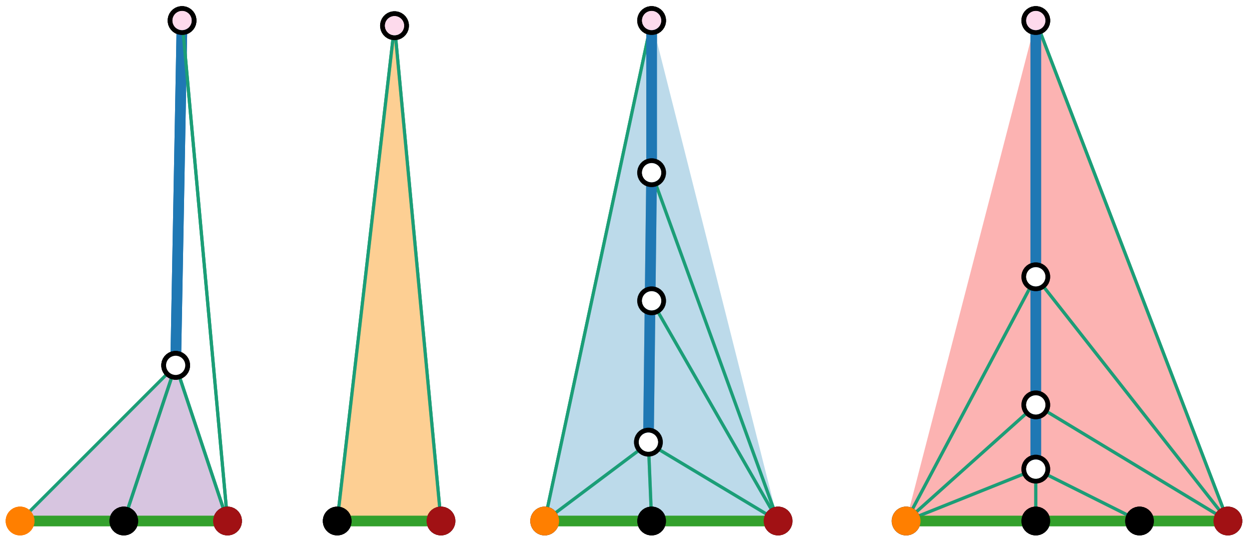

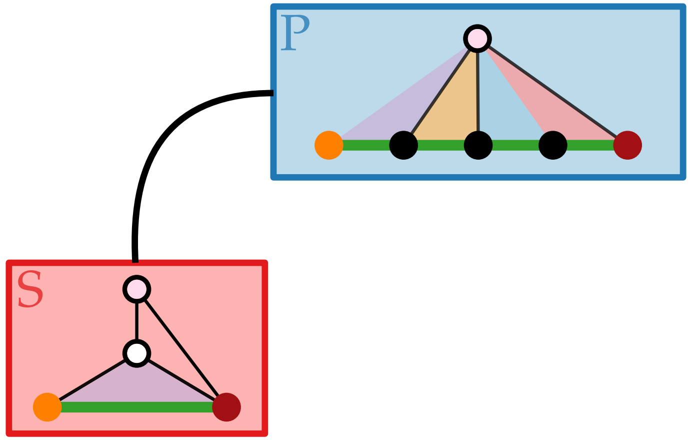

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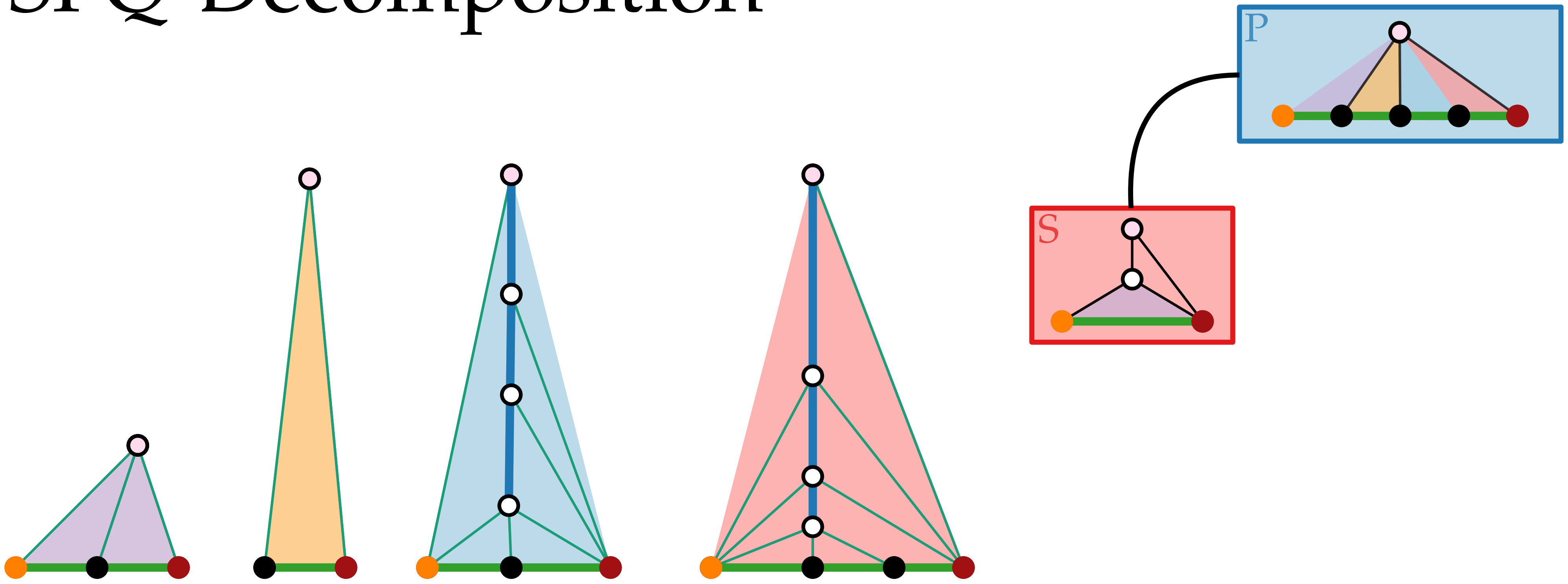

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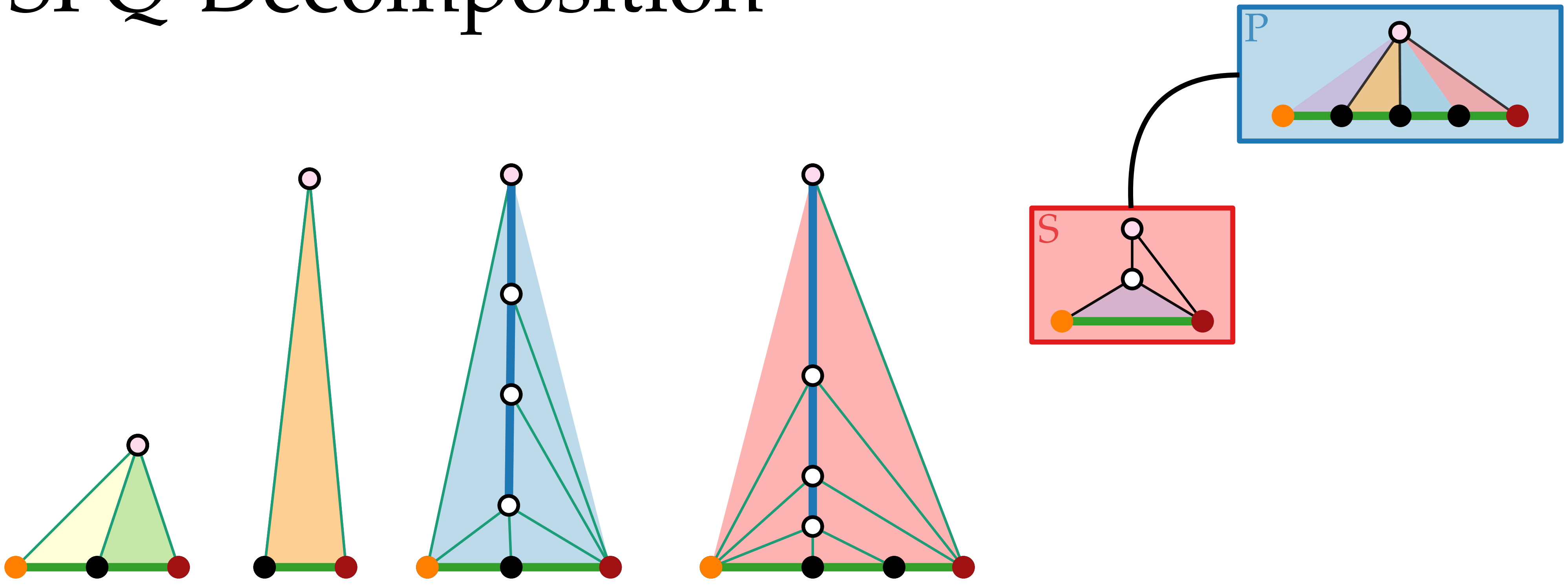

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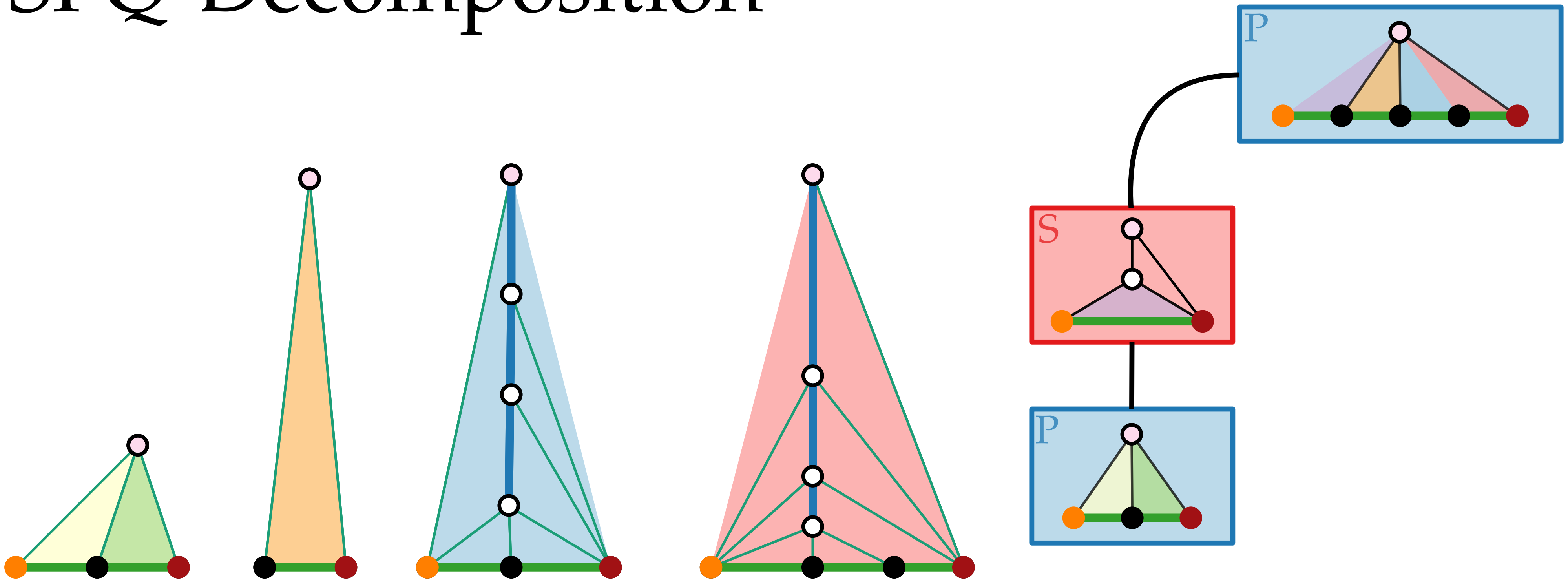

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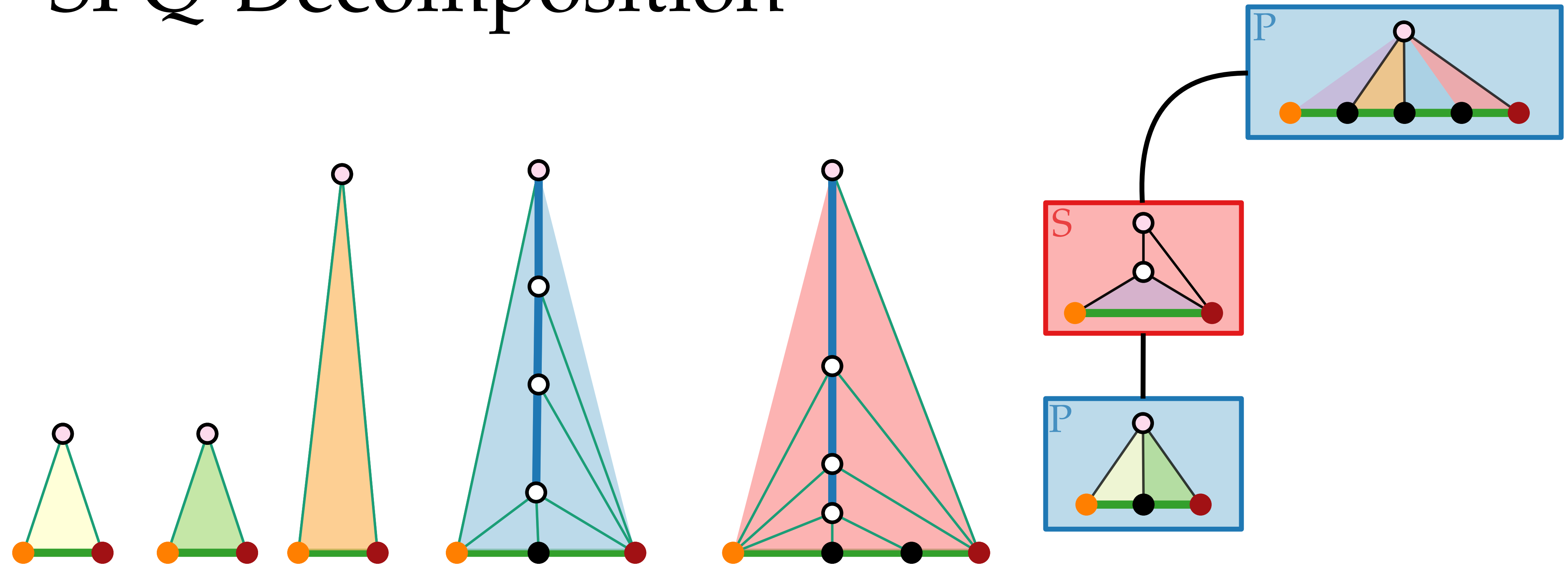

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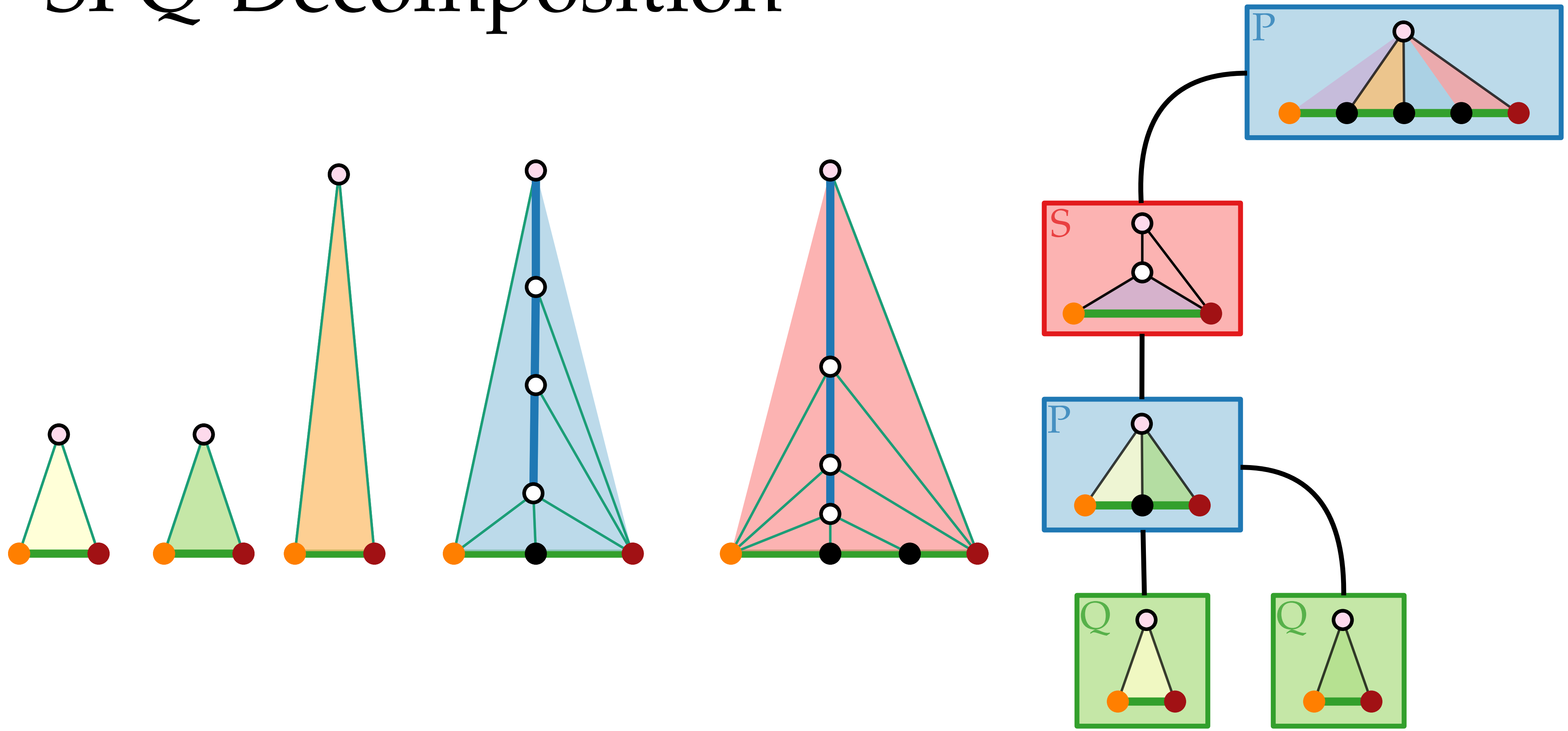

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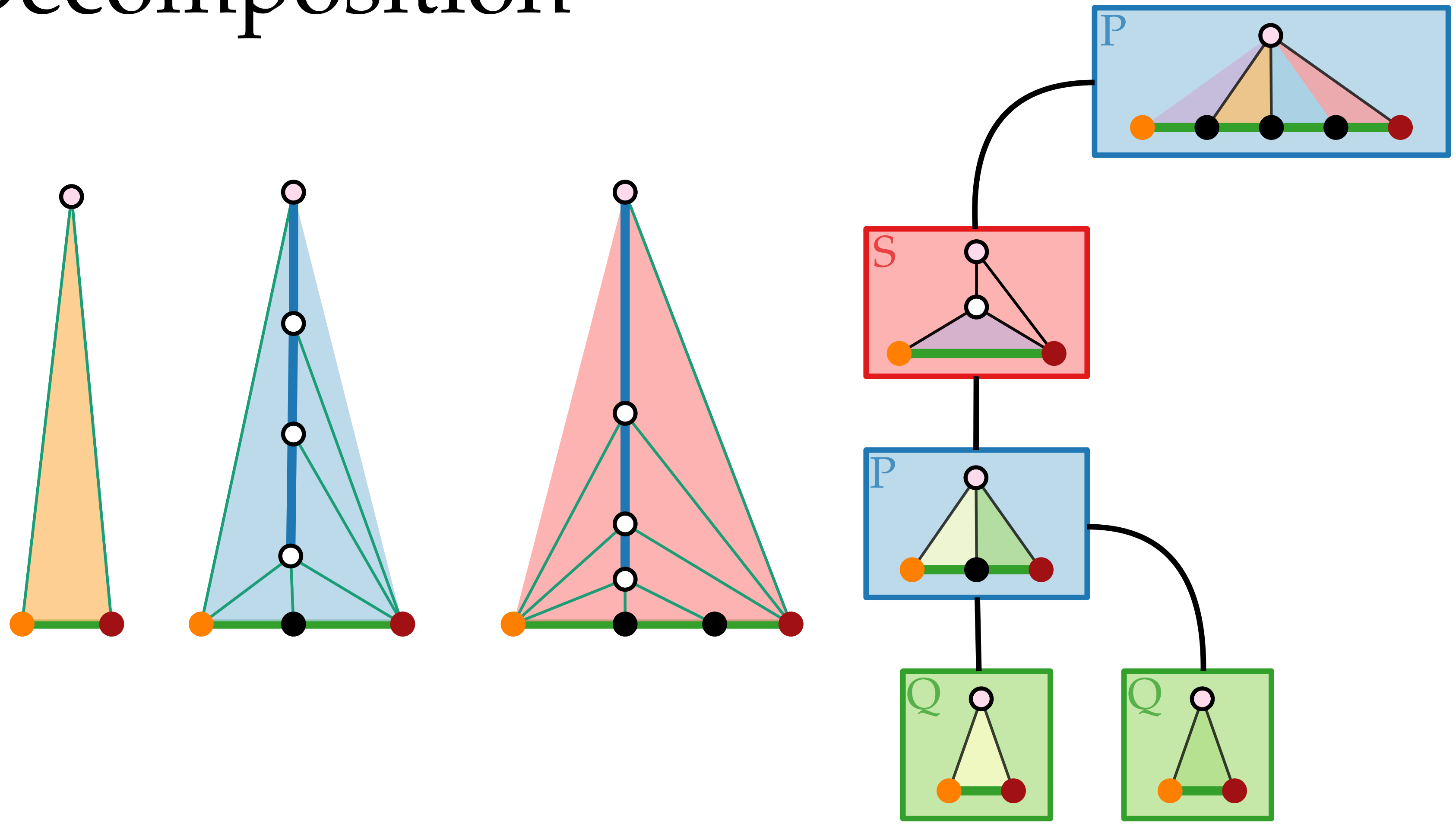

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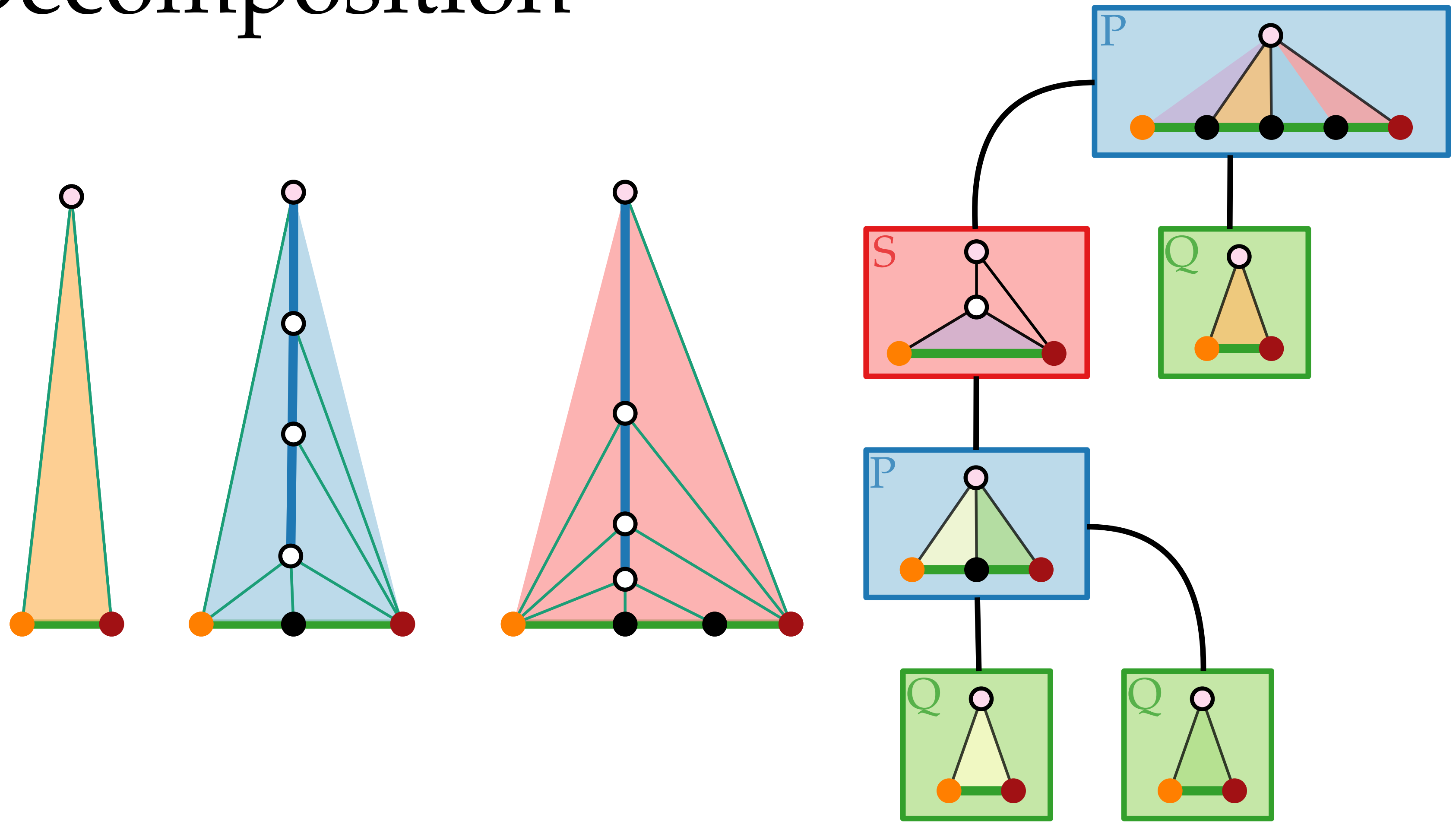

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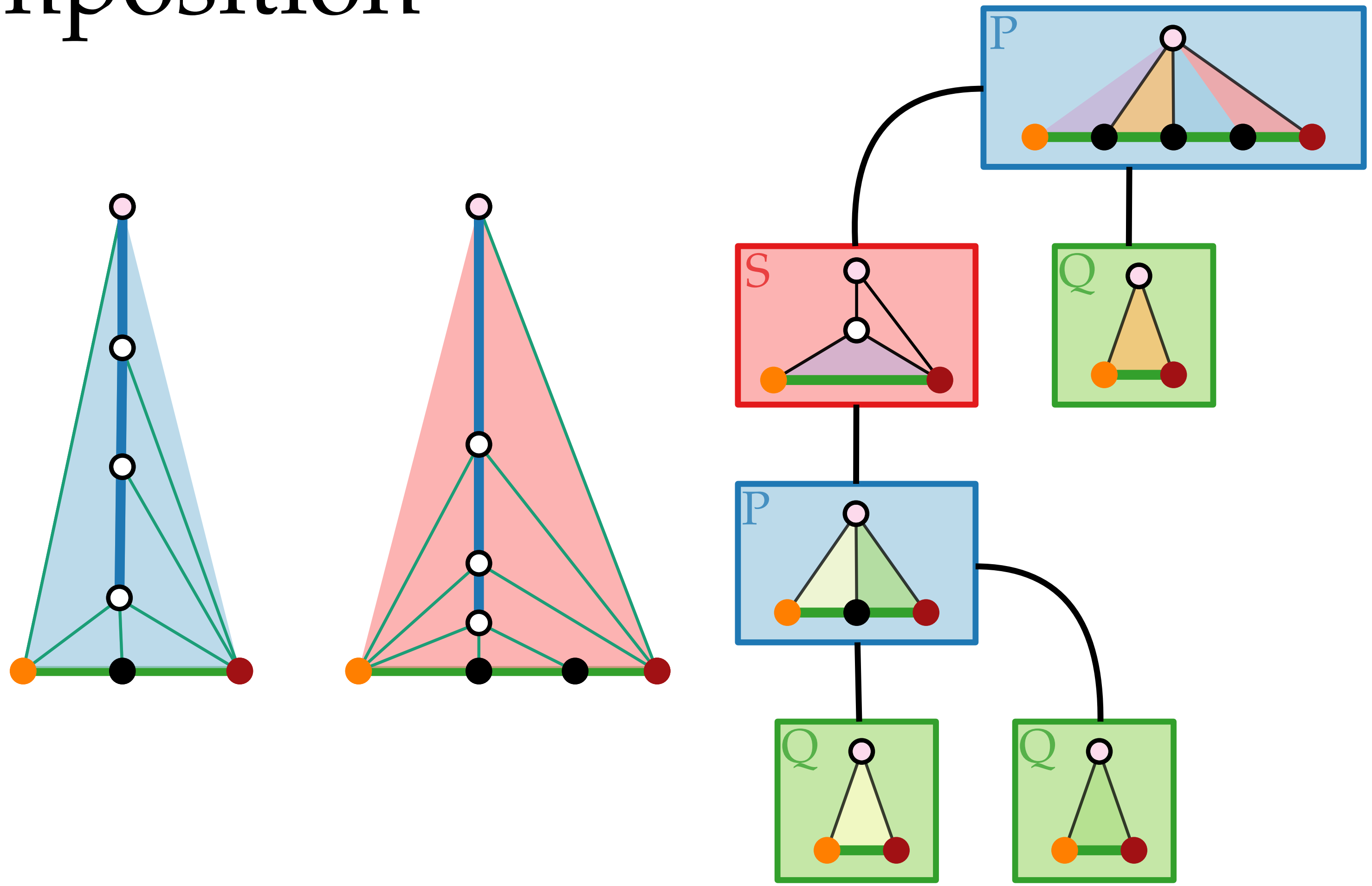

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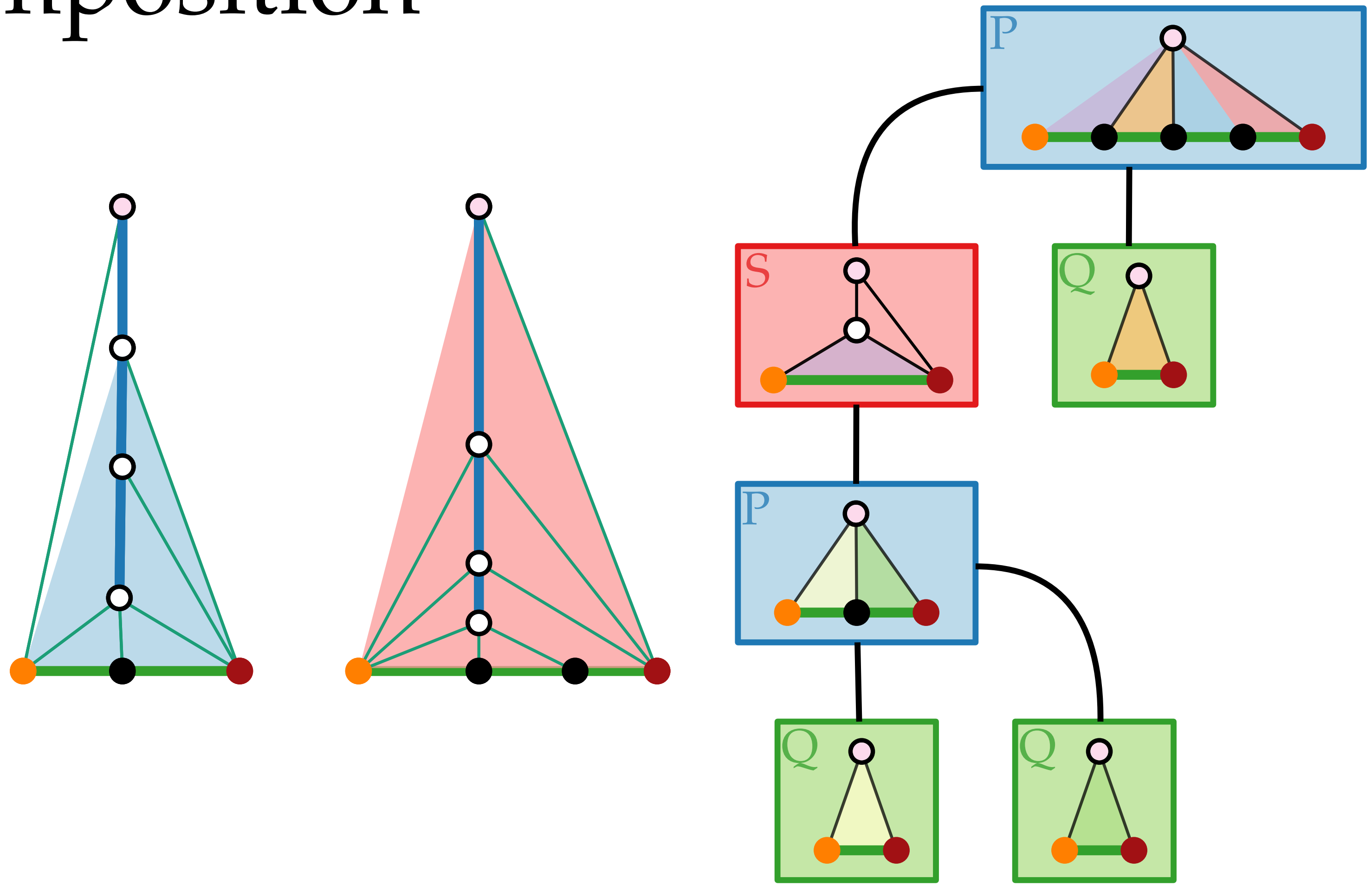

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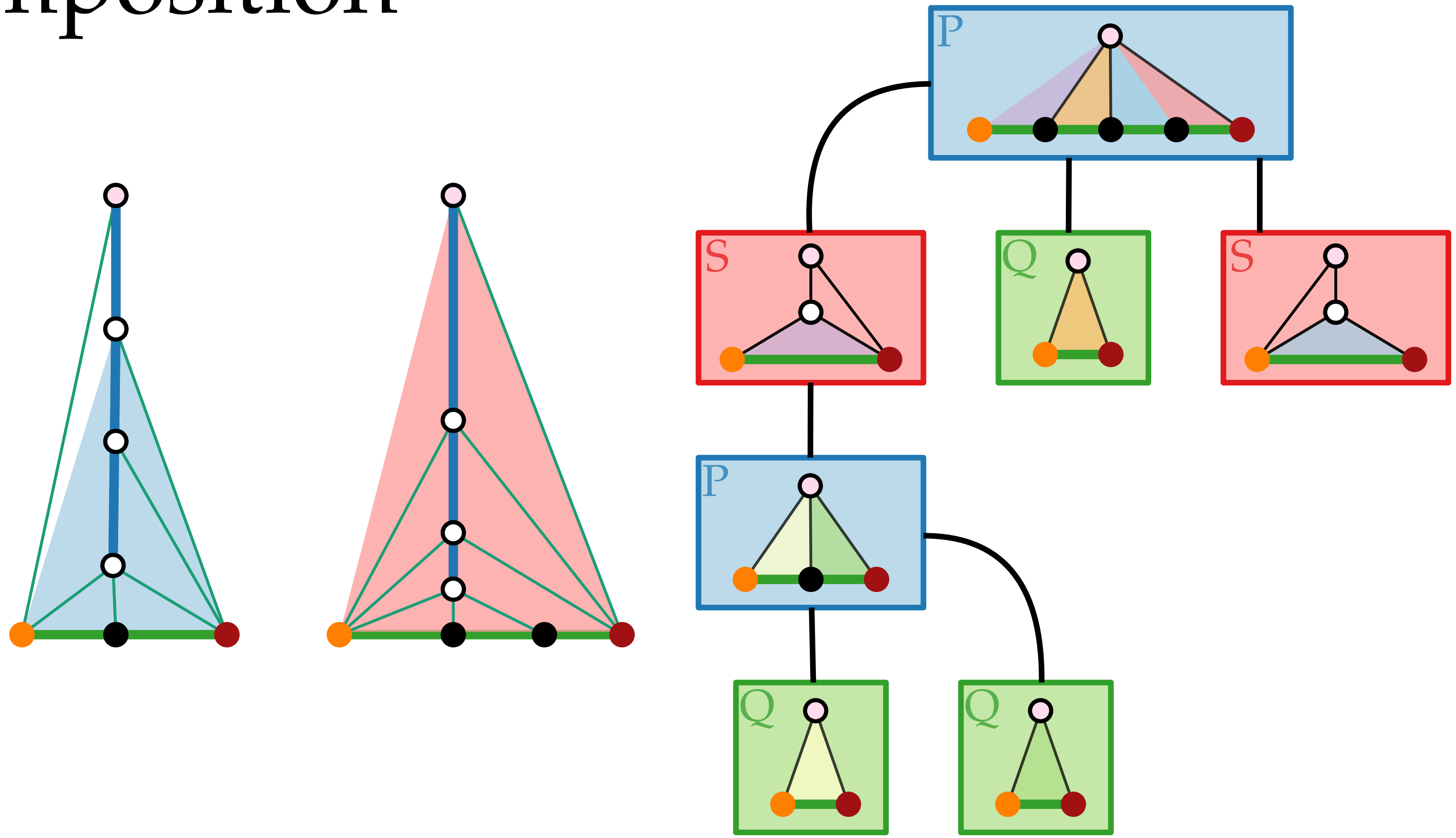

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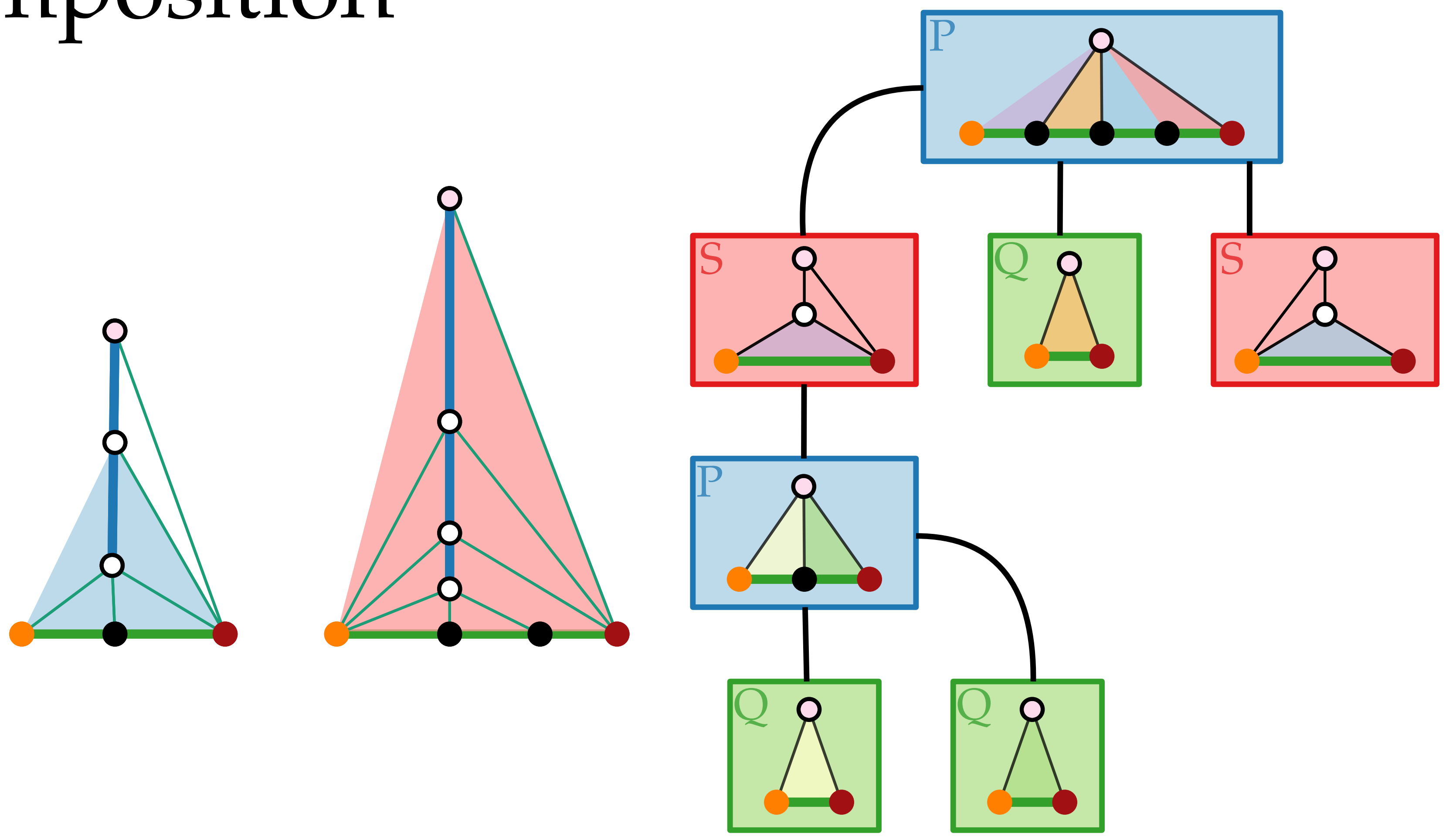

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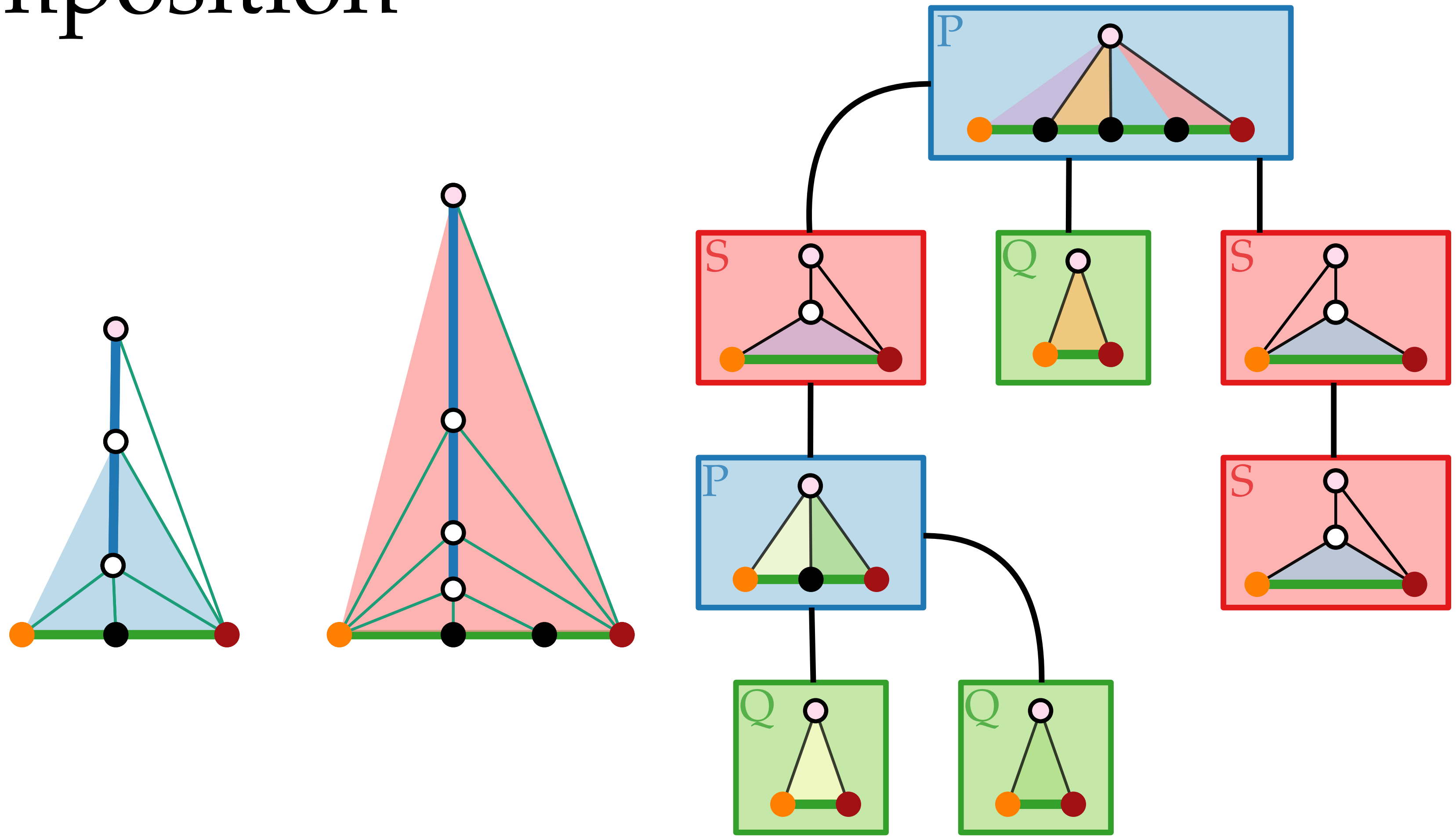

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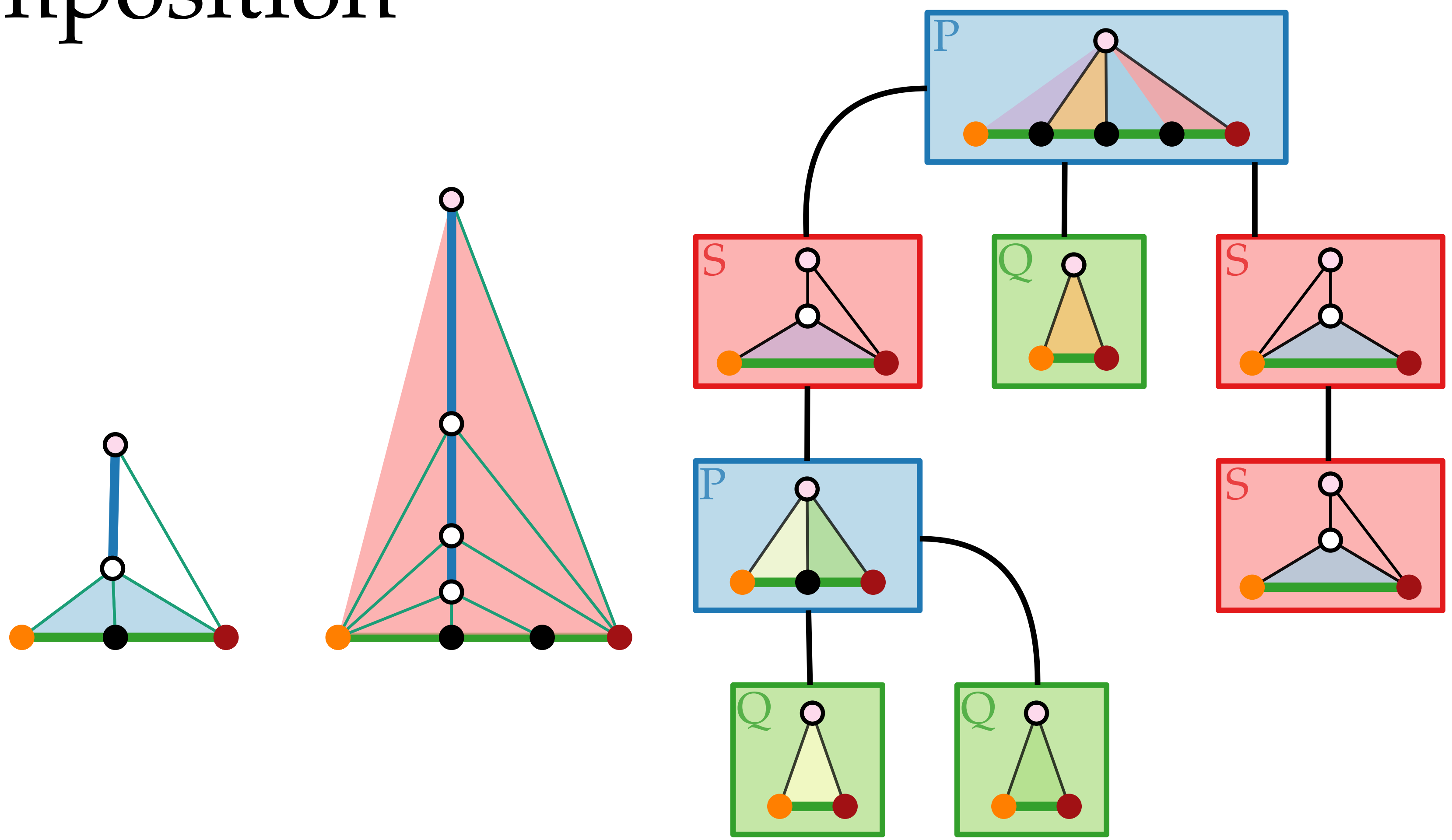

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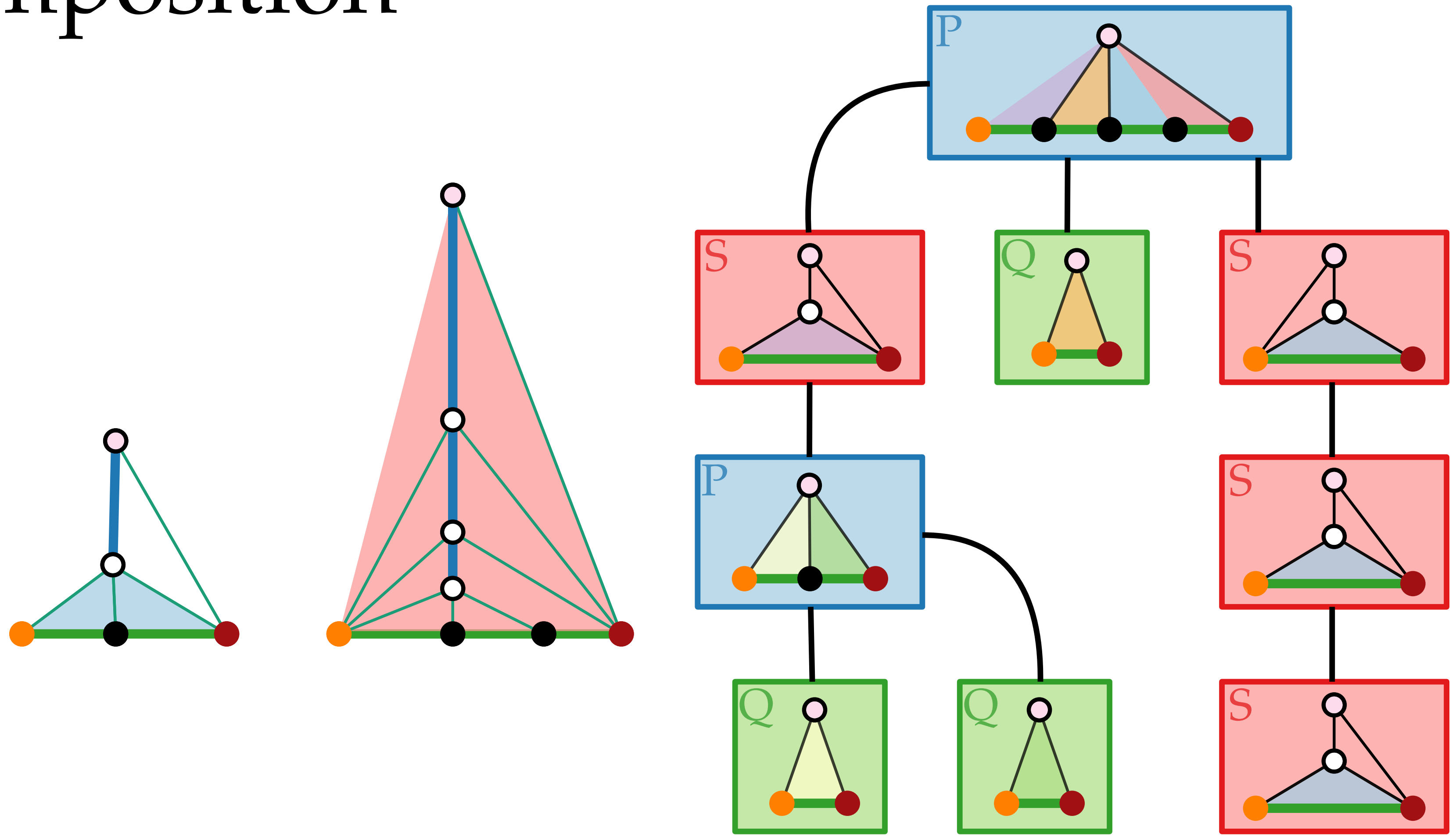
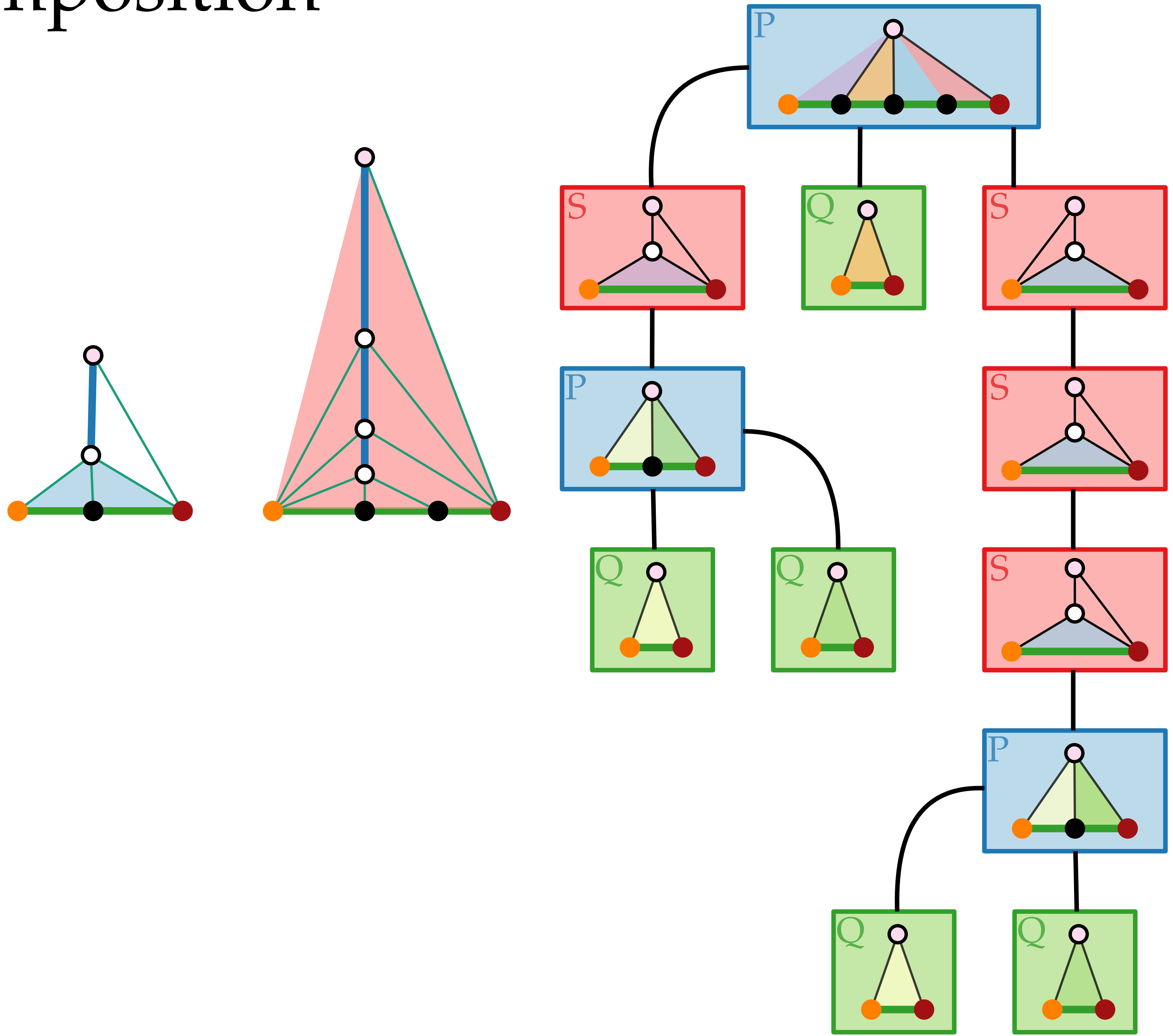
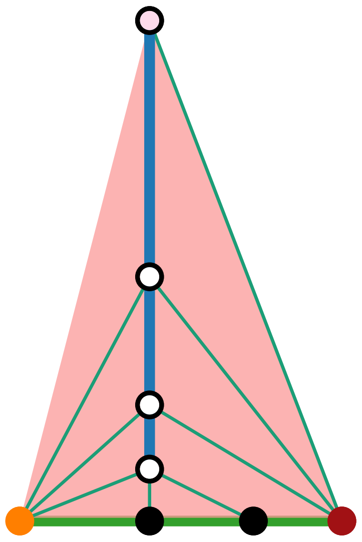
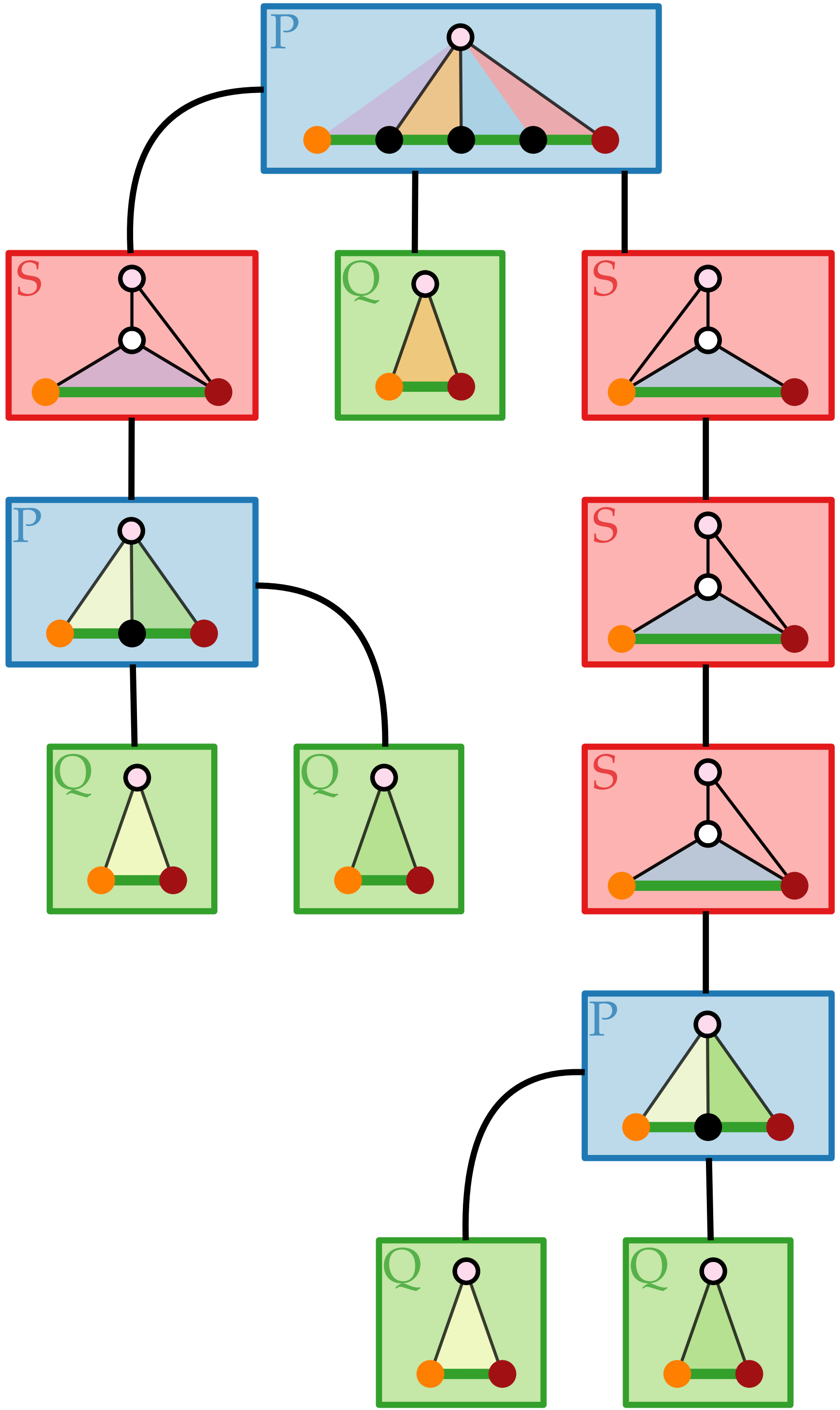


SPQ-Decomposition

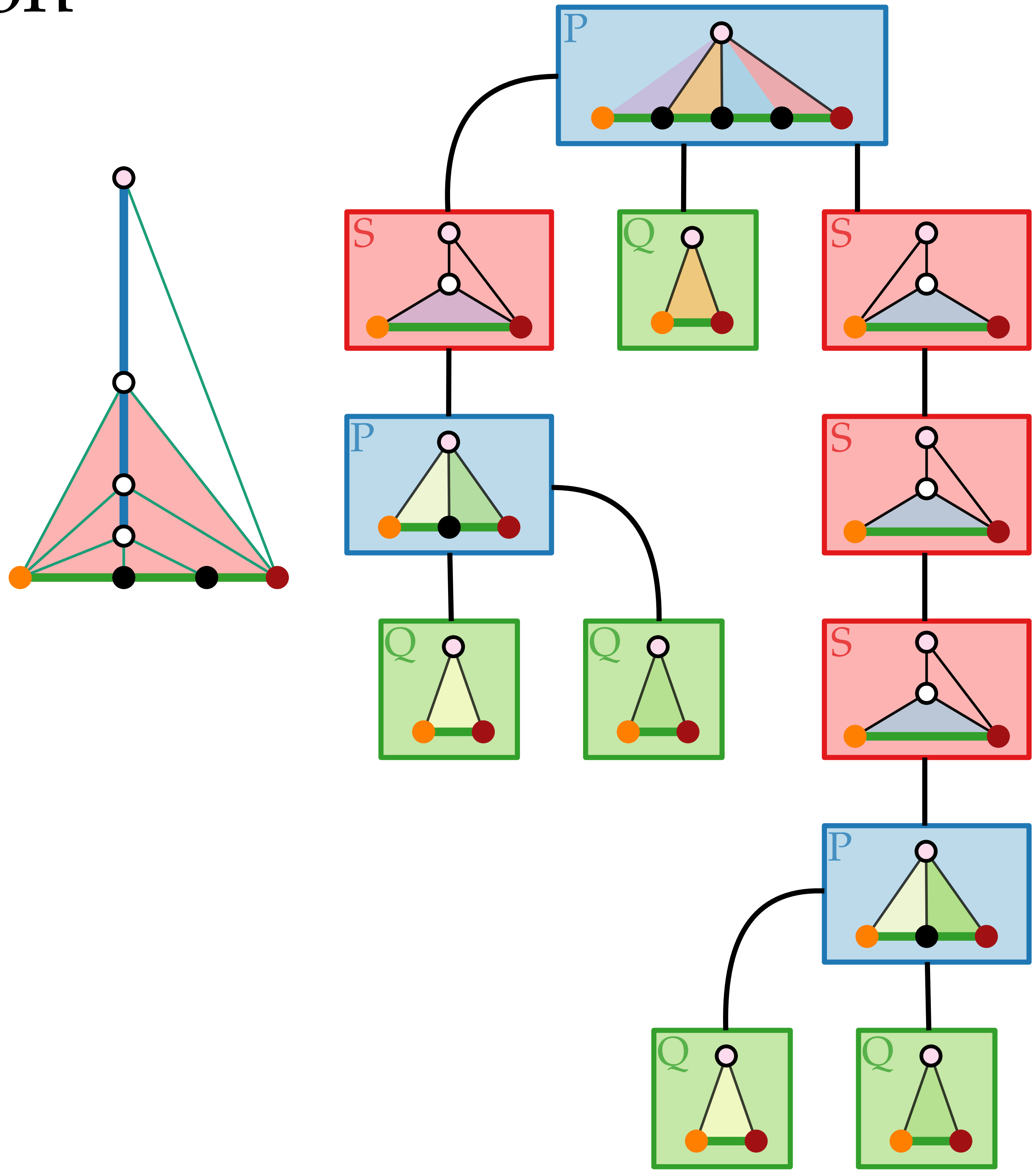


SPQ-Decomposition





SPQ-Decomposition



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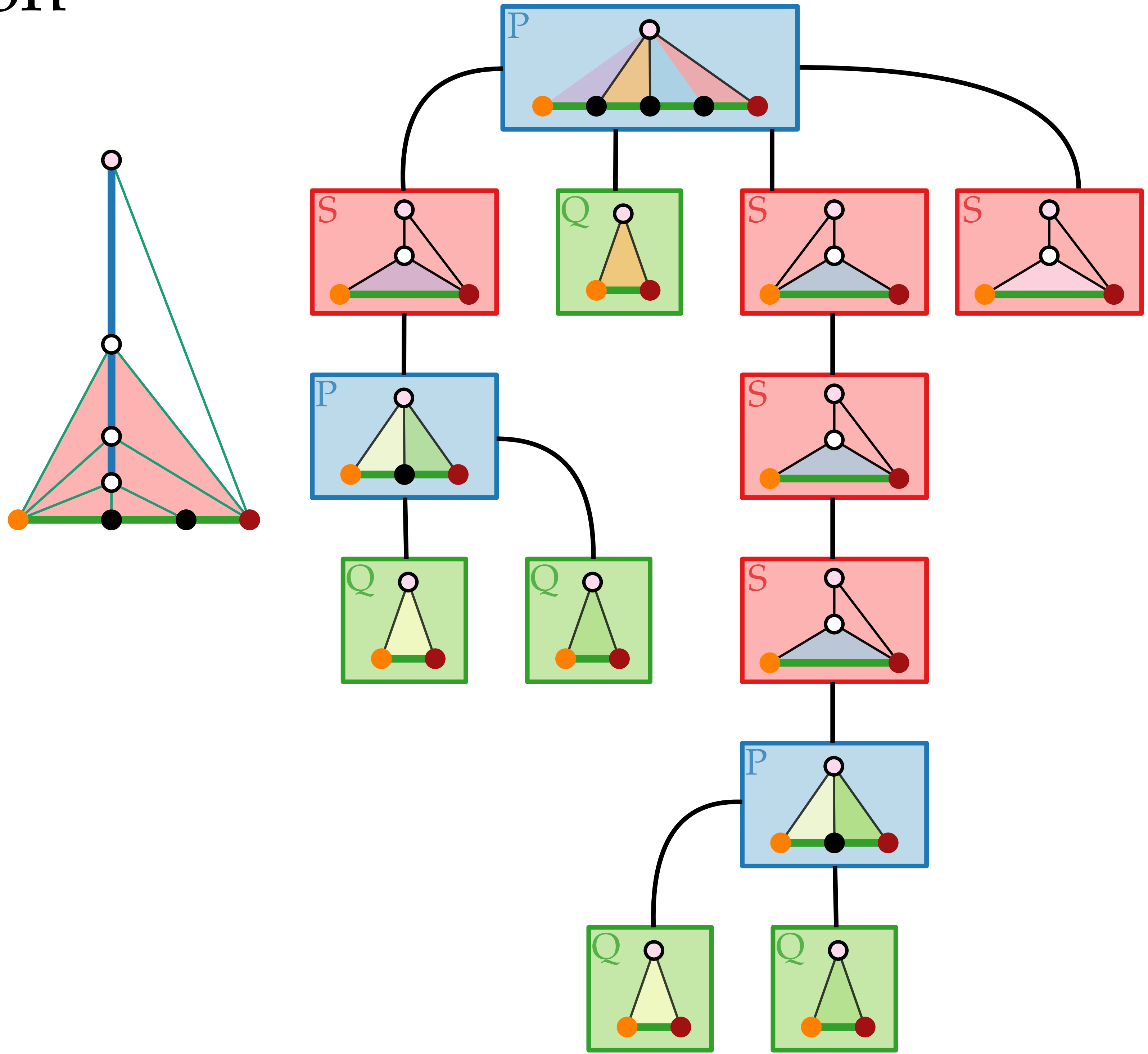
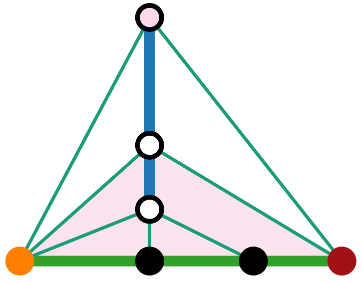
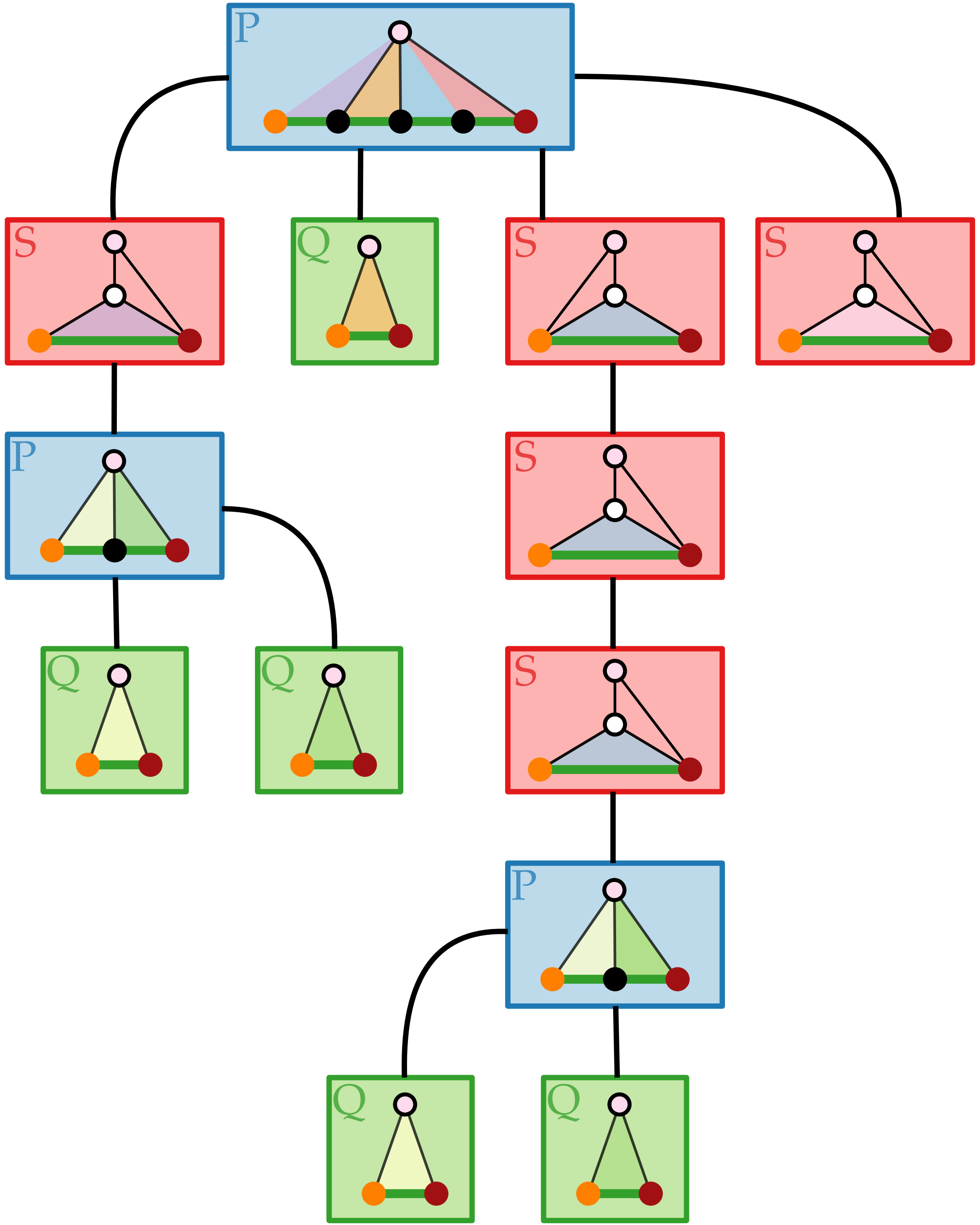
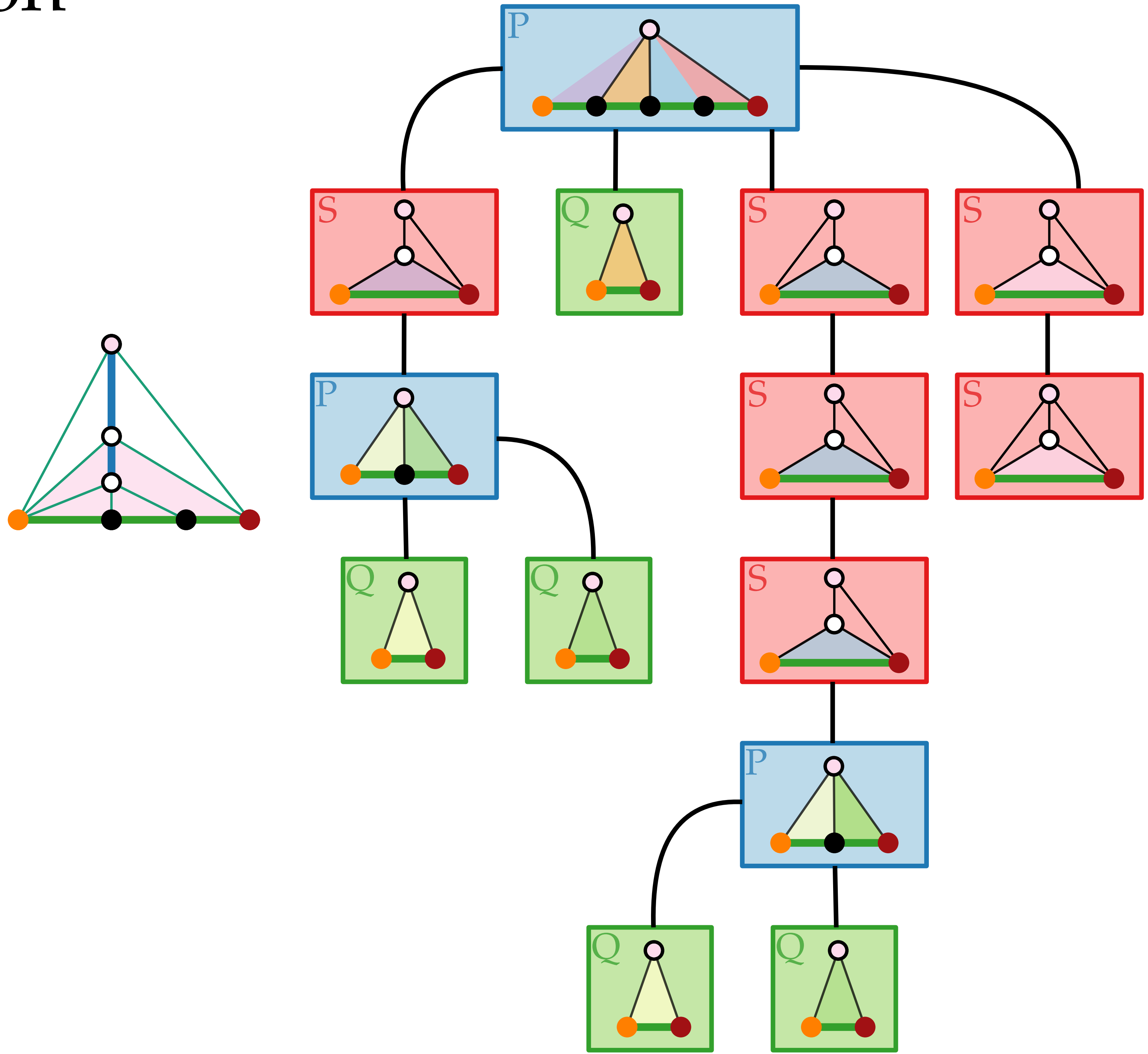


SPQ-Decomposition

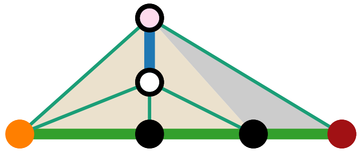


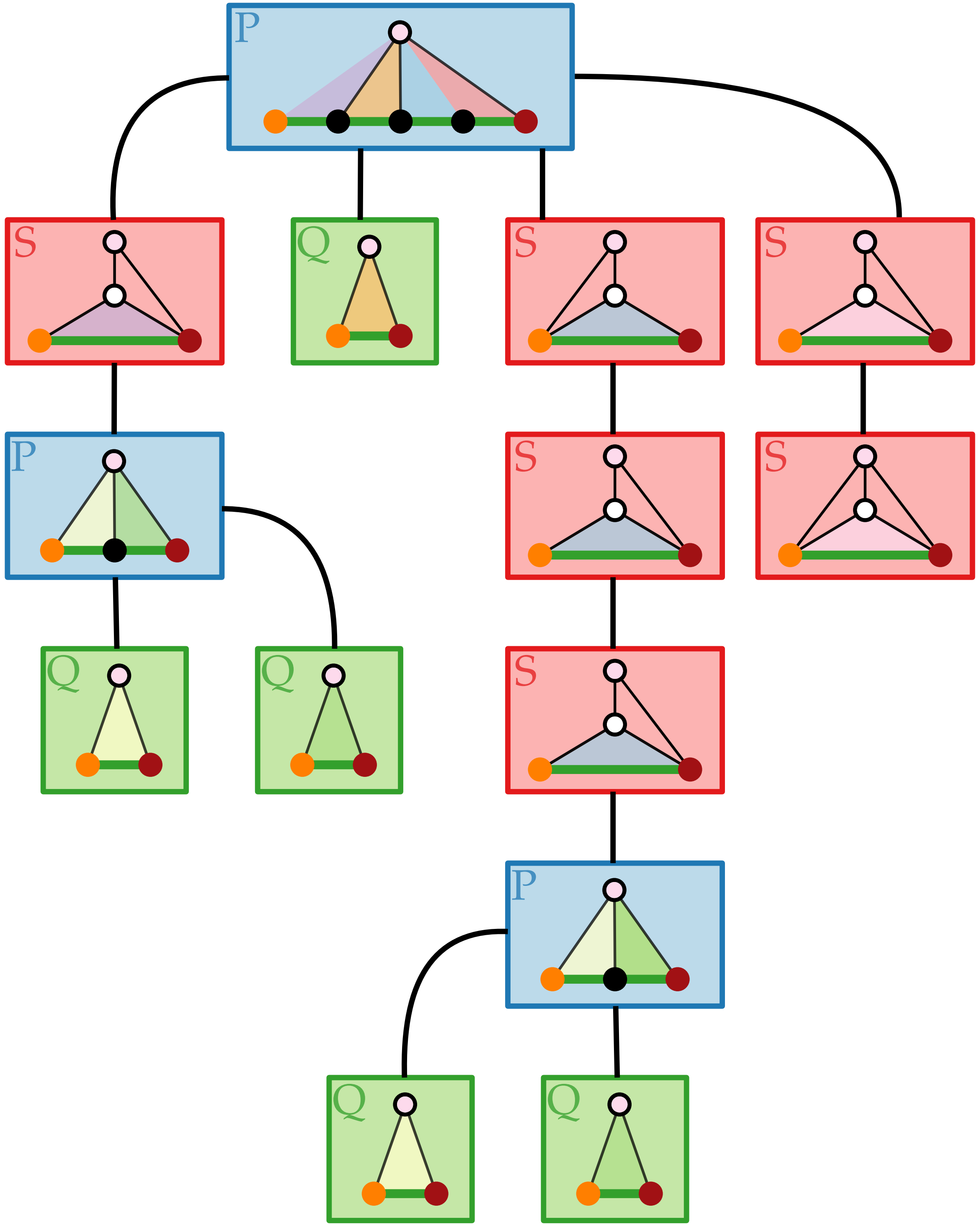


SPQ-Decomposition

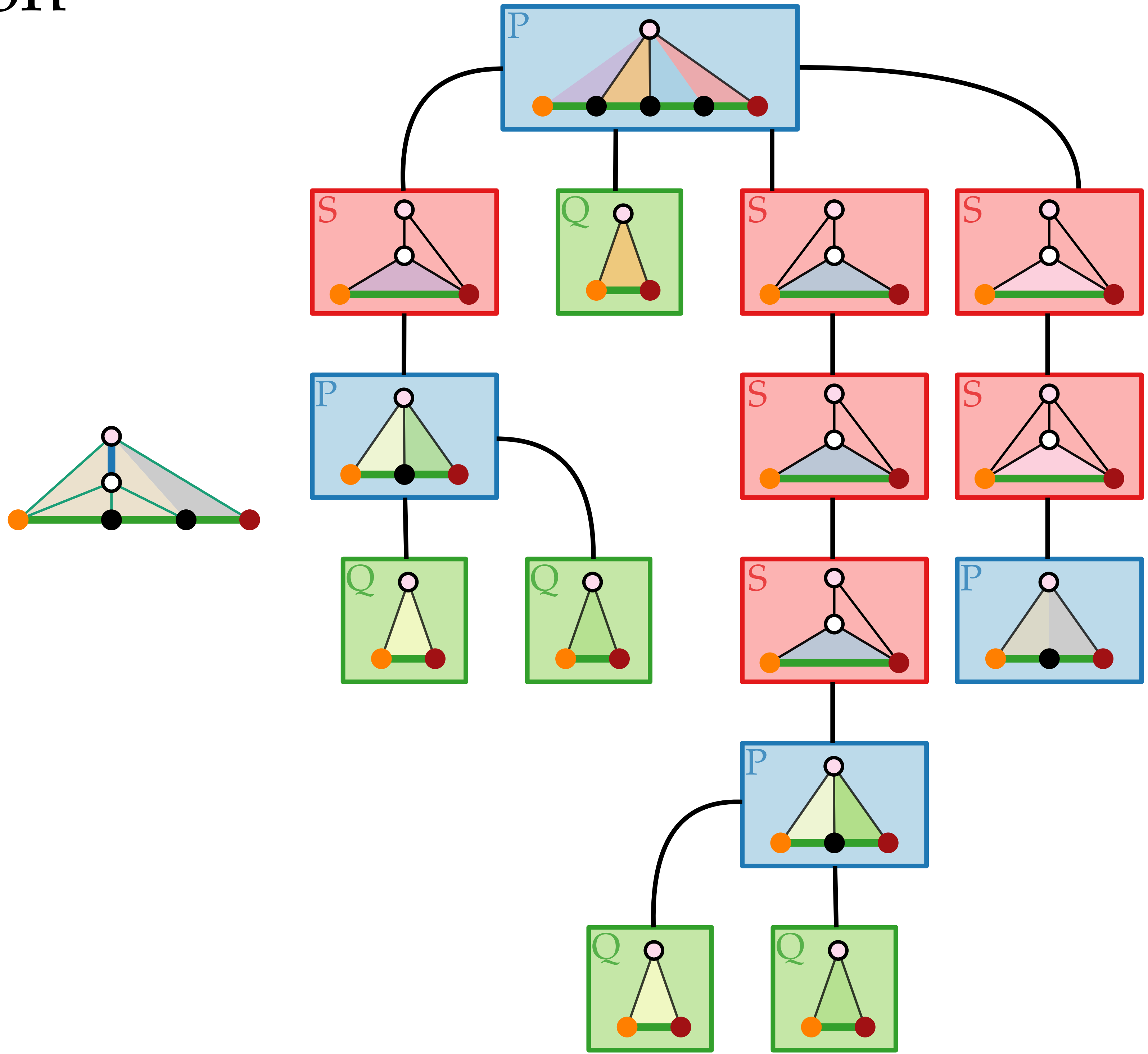


SPQ-Decomposition

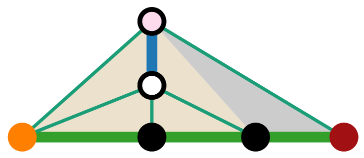


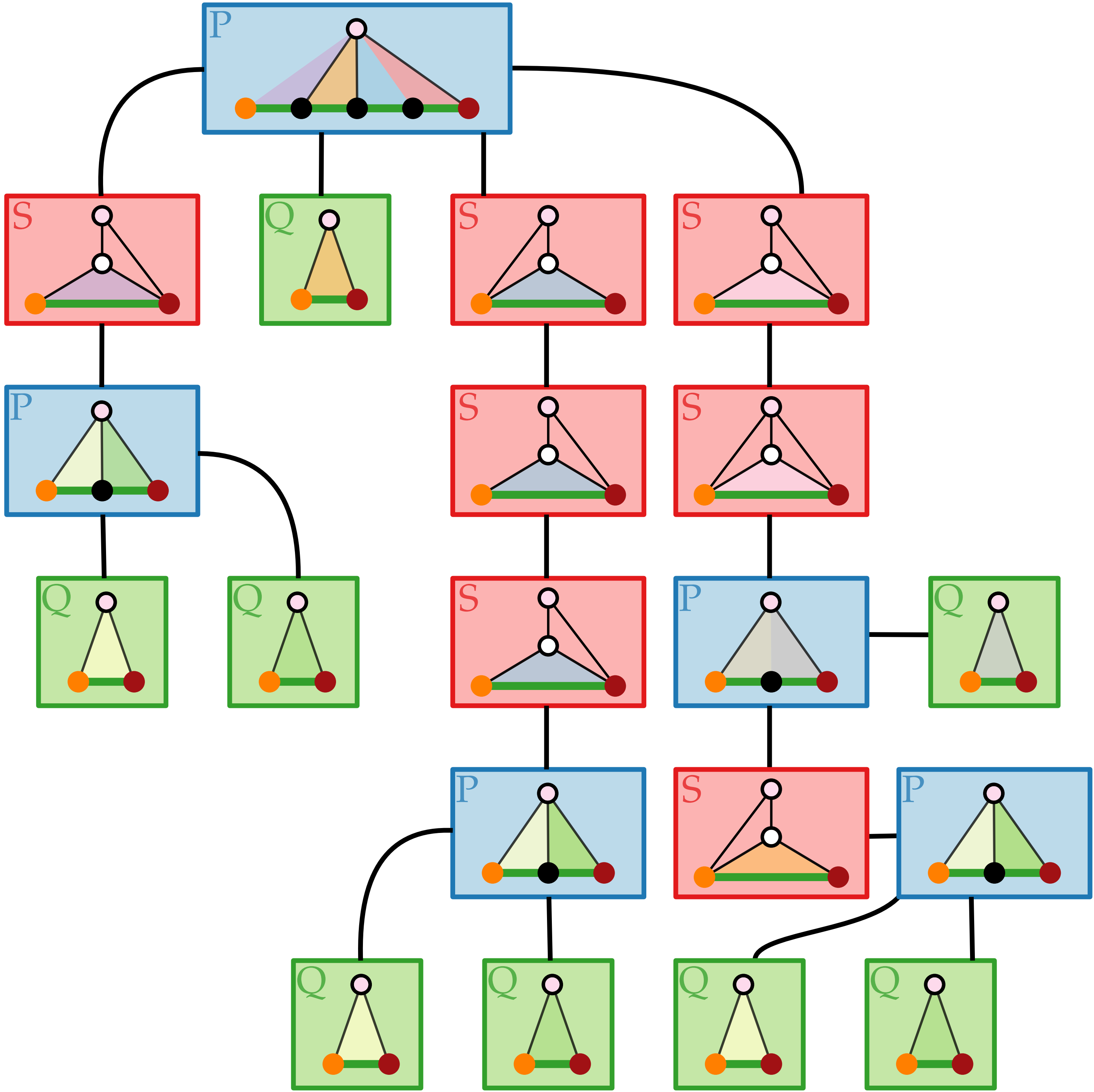


SPQ-Decomposition

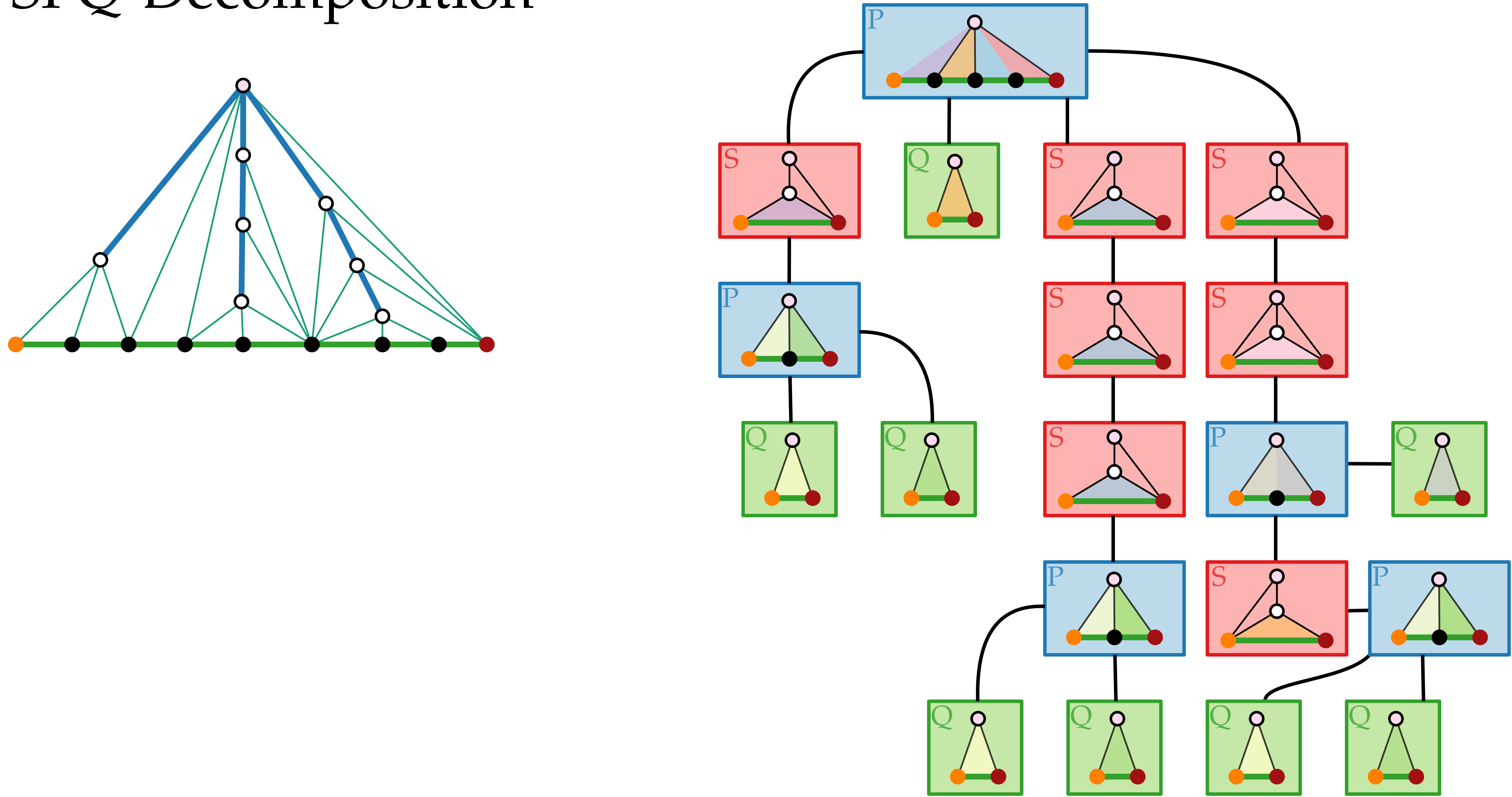


SPQ-Decomposition

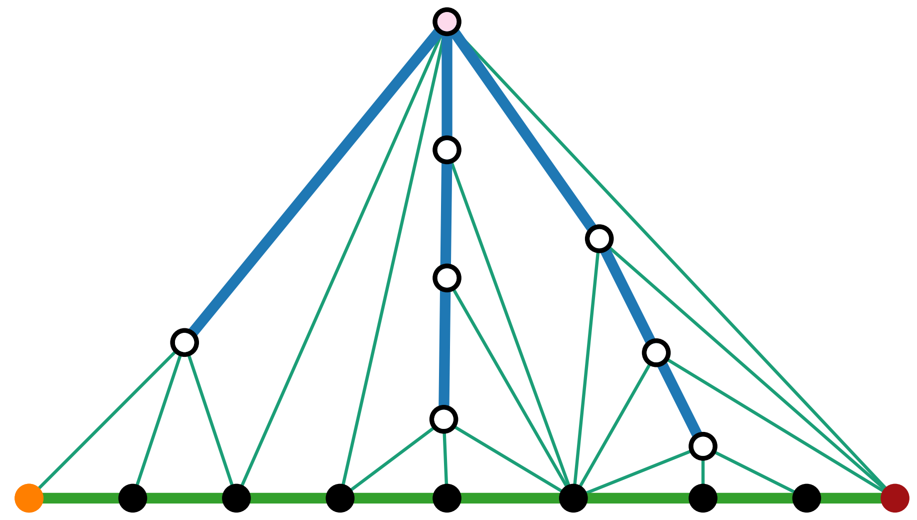


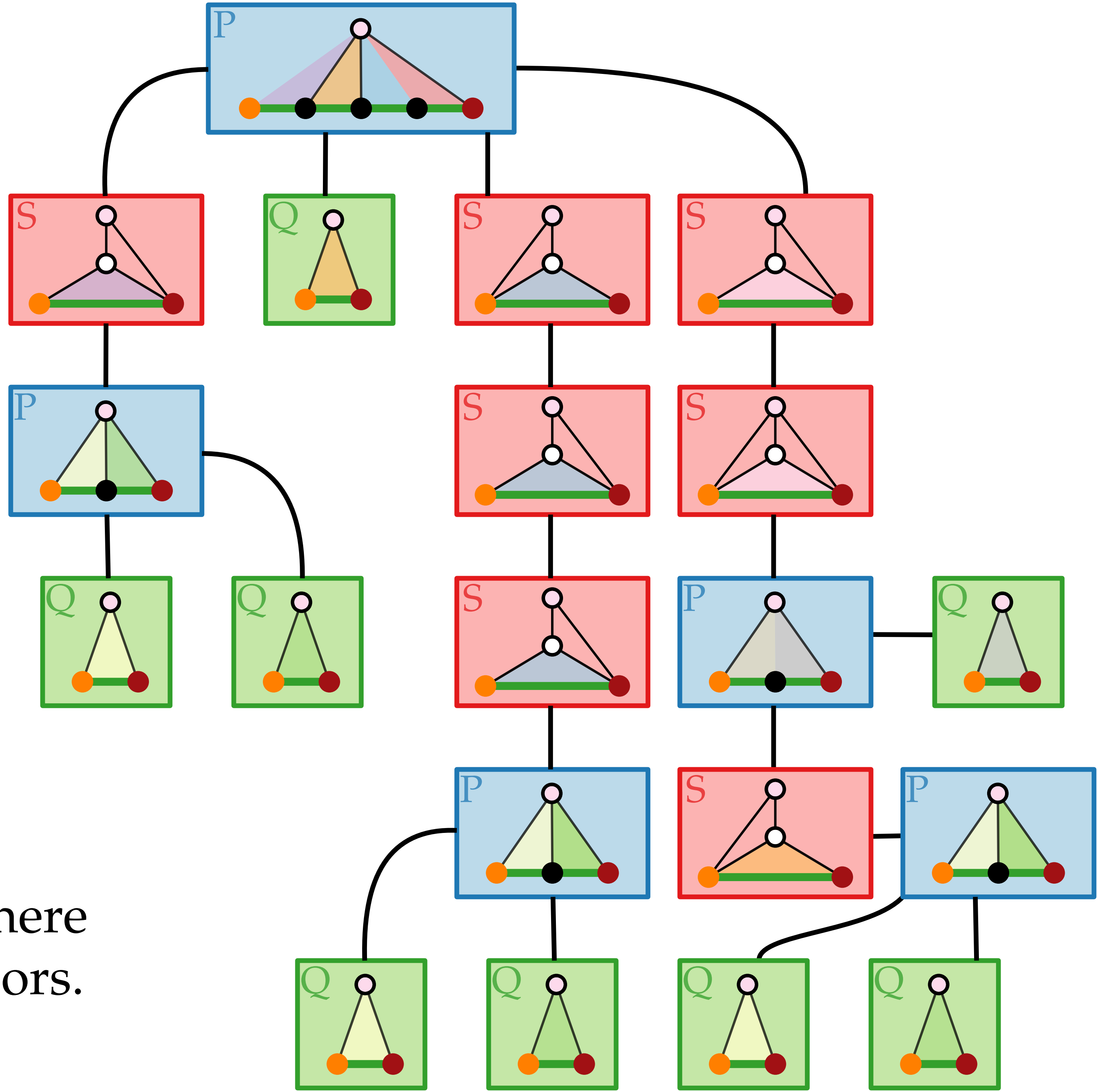


SPQ-Decomposition



SPQ-Decomposition





There is always an SPQ-decomposition where no two nodes of the same type are neighbors.

Shapes

- Always use one of 6 shapes

Shapes

- Always use one of 6 shapes

Shapes

- Always use one of 6 shapes

$(1,1)$-flat

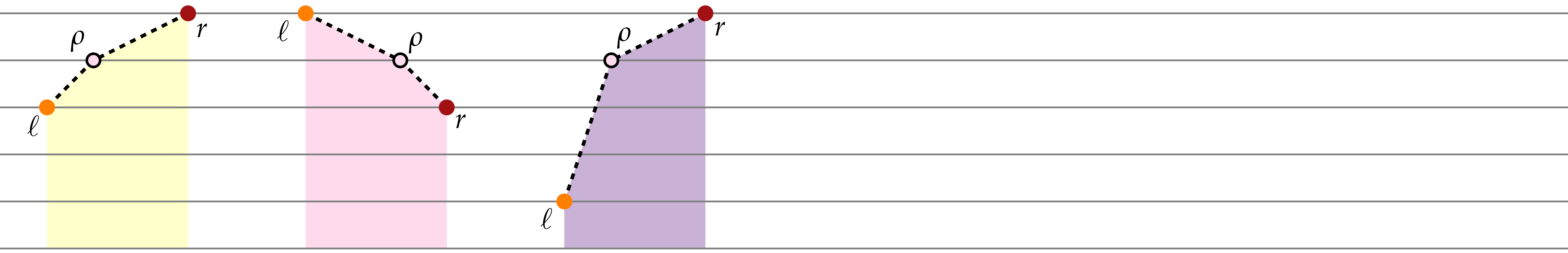
Shapes

- Always use one of 6 shapes

$(1, 1)$-flat $(-1, -1)$-flat

Shapes

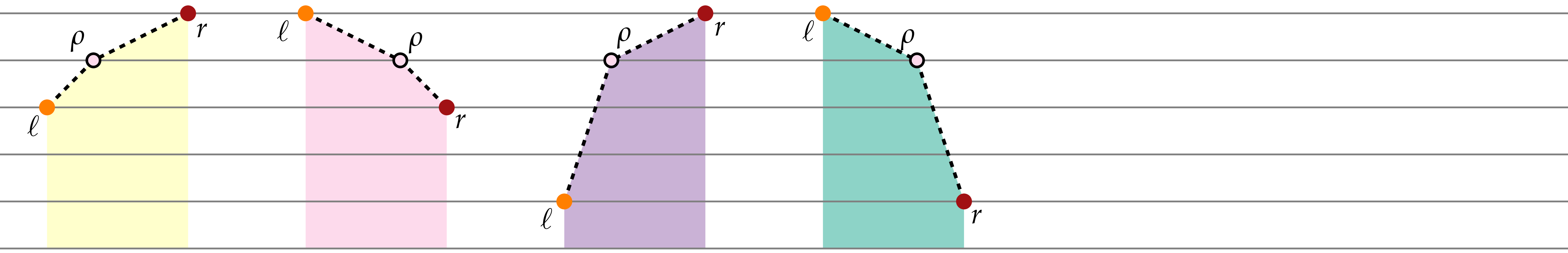
- Always use one of 6 shapes



$(1, 1)$-flat $(-1, -1)$-flat $(3, 1)$-flat

Shapes

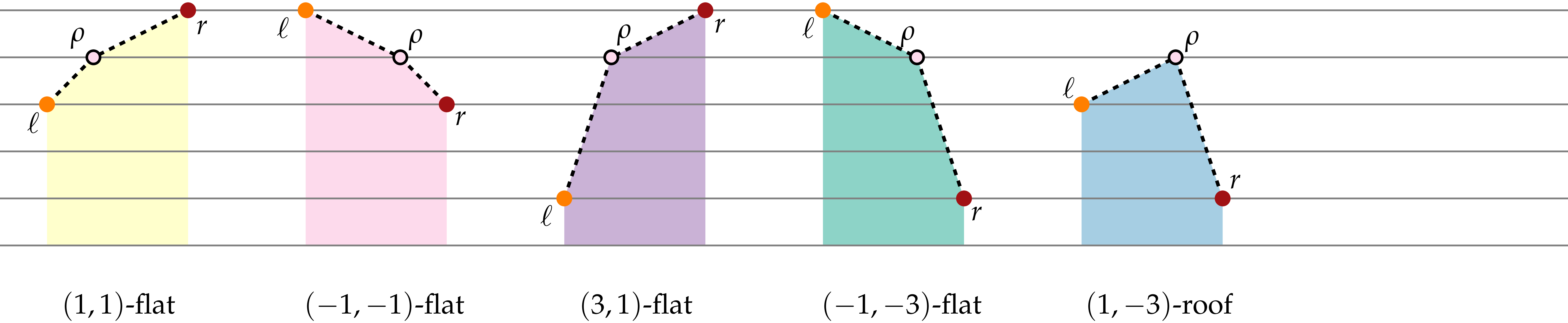
- Always use one of 6 shapes



$(1,1)$-flat $(-1,-1)$-flat $(3,1)$-flat $(-1,-3)$-flat

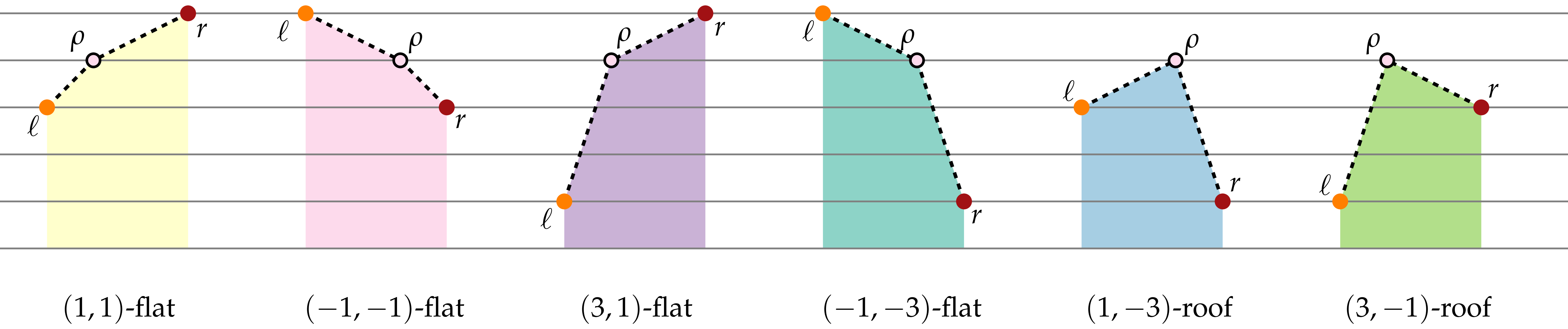
Shapes

- Always use one of 6 shapes



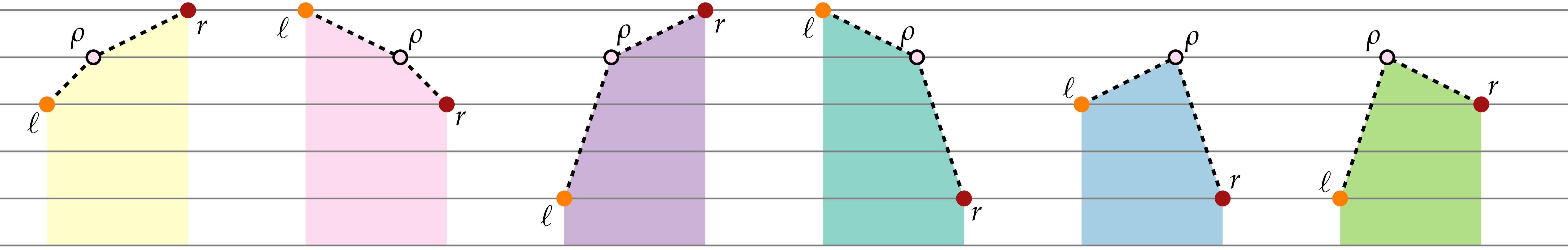
Shapes

- Always use one of 6 shapes



Shapes

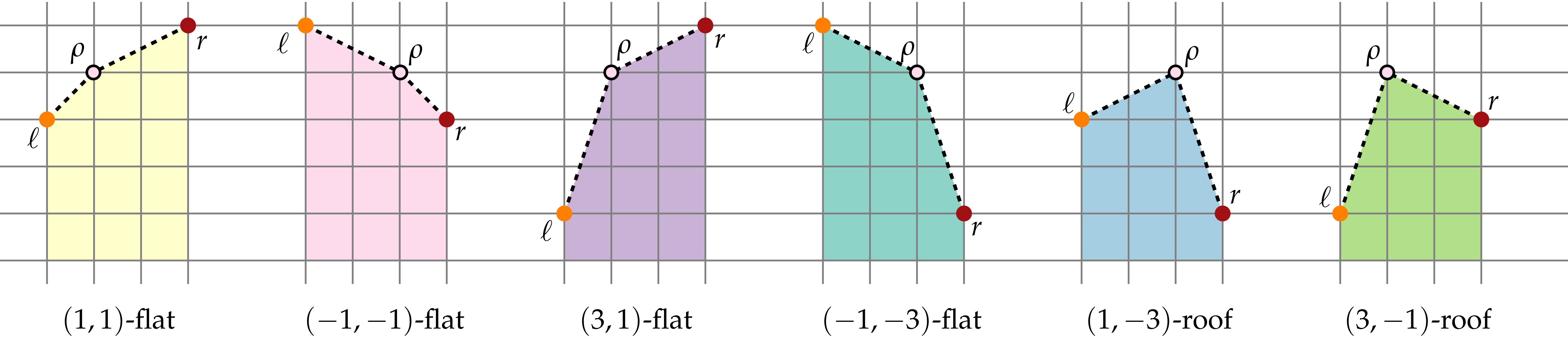
- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid



$(1, 1)$-flat $(-1, -1)$-flat $(3, 1)$-flat $(-1, -3)$-flat $(1, -3)$-roof $(3, -1)$-roof

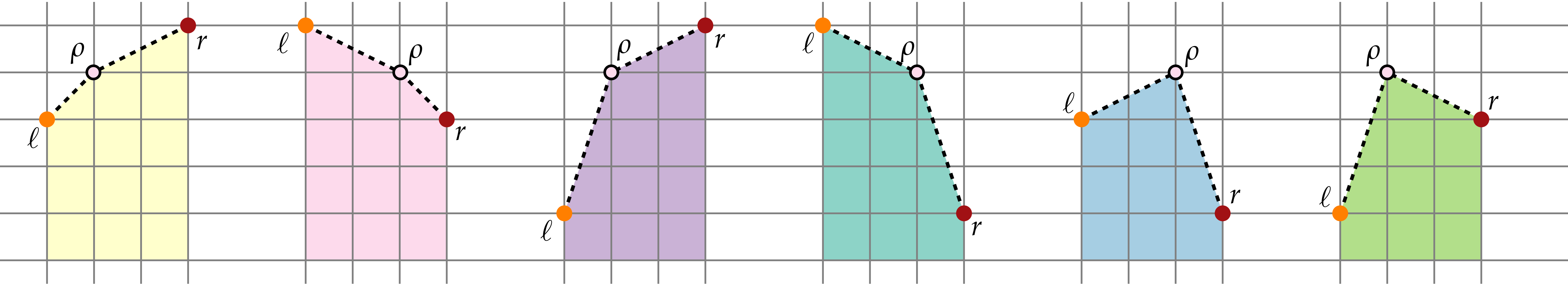
Shapes

- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid



Shapes

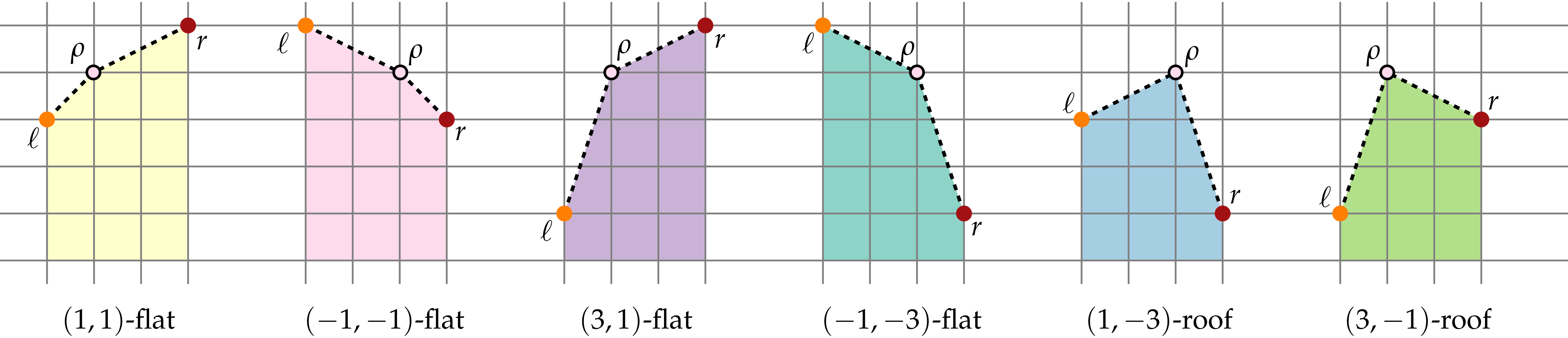
- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid
- P-nodes can only use flat shapes



$(1,1)$-flat $(-1,-1)$-flat $(3,1)$-flat $(-1,-3)$-flat $(1,-3)$-roof $(3,-1)$-roof

Shapes

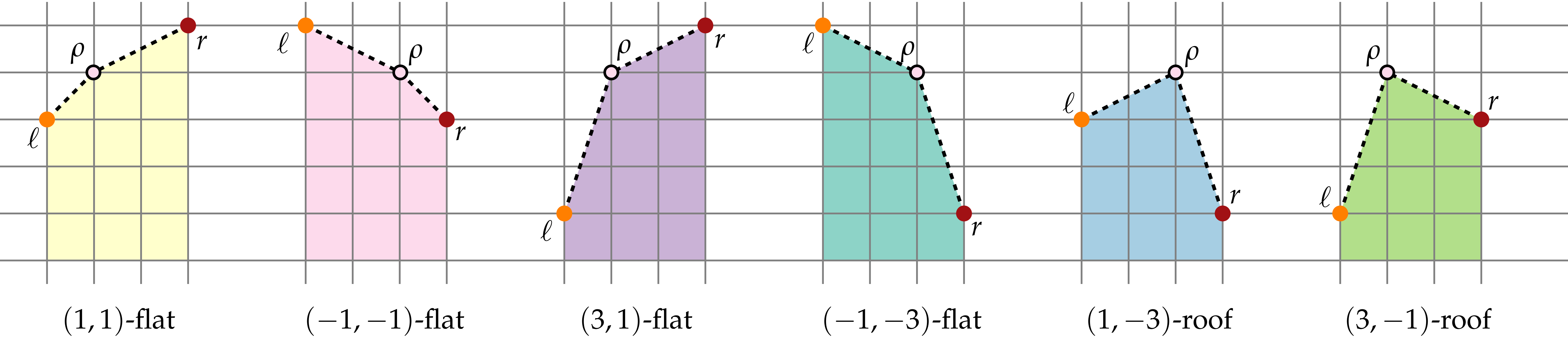
- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid
- P-nodes can only use flat shapes
- Create a drawing for every possible shape ➡ can combine what we need



Shapes

- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid
- P-nodes can only use flat shapes
- Create a drawing for every possible shape ➡ can combine what we need

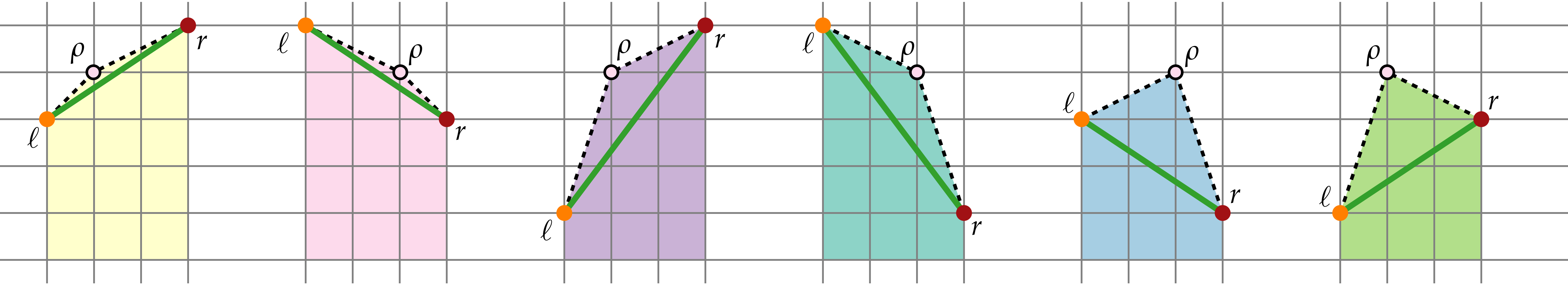
Q-nodes:



Shapes

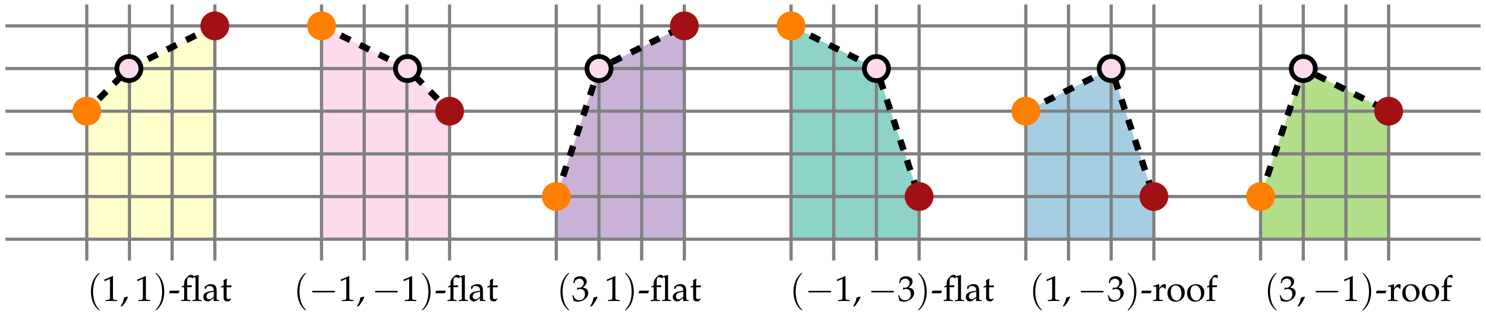
- Always use one of 6 shapes
- Draw with 1 bend per edge on the grid
- P-nodes can only use flat shapes
- Create a drawing for every possible shape ➡ can combine what we need

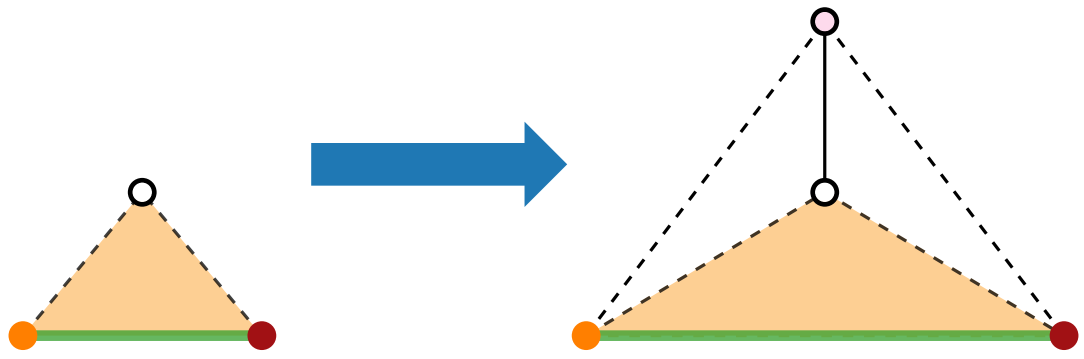
Q-nodes:



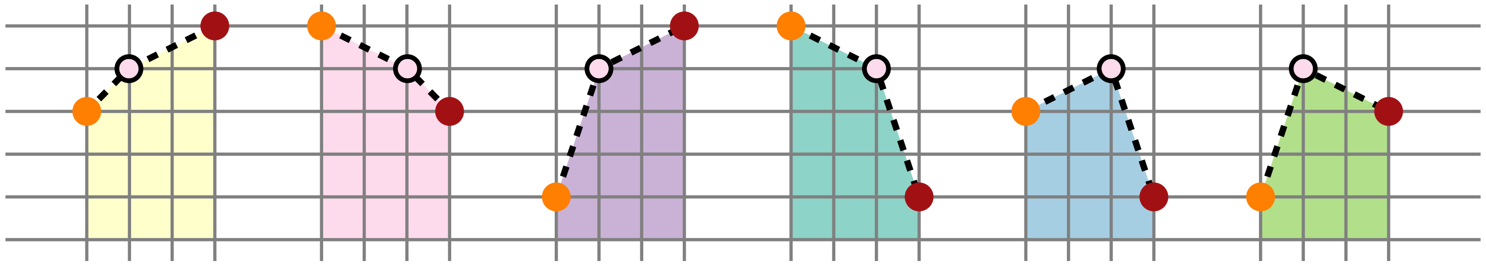
$(1,1)$-flat $(-1,-1)$-flat $(3,1)$-flat $(-1,-3)$-flat $(1,-3)$-roof $(3,-1)$-roof

S-nodes

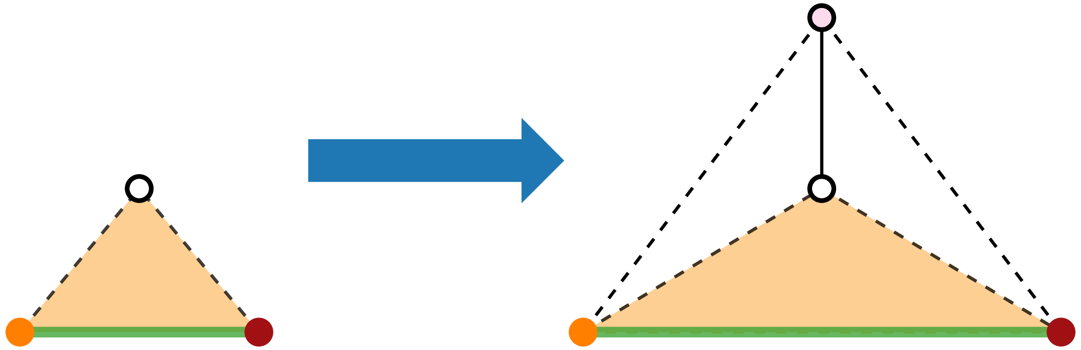




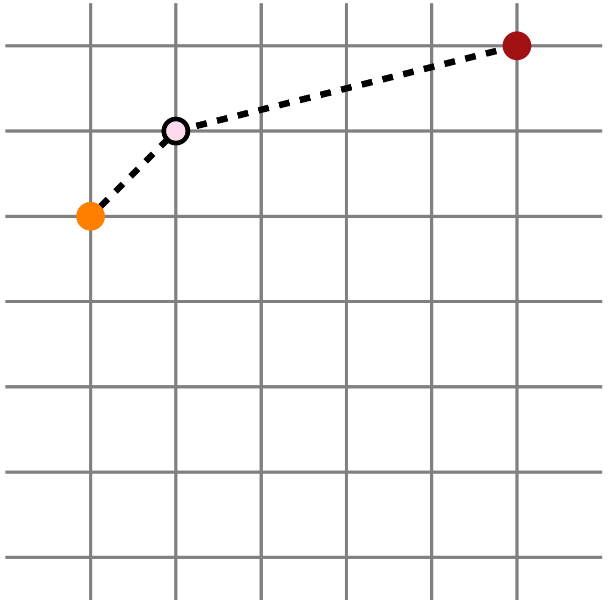
S-nodes



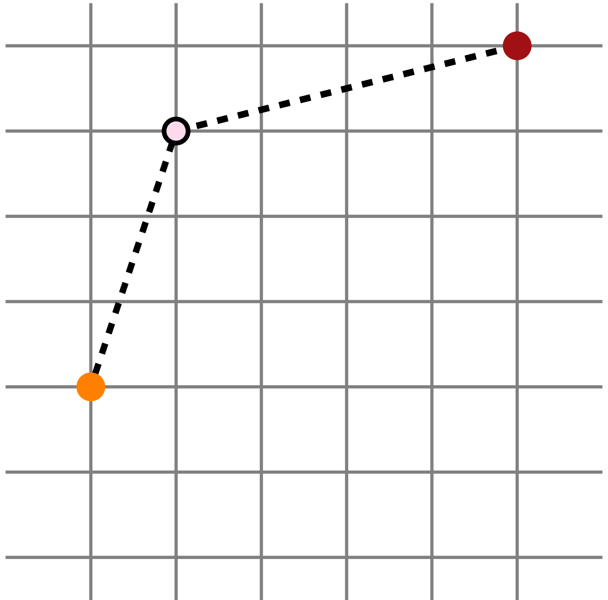
$(1,1)$-flat $(-1,-1)$-flat $(3,1)$-flat $(-1,-3)$-flat $(1,-3)$-roof $(3,-1)$-roof



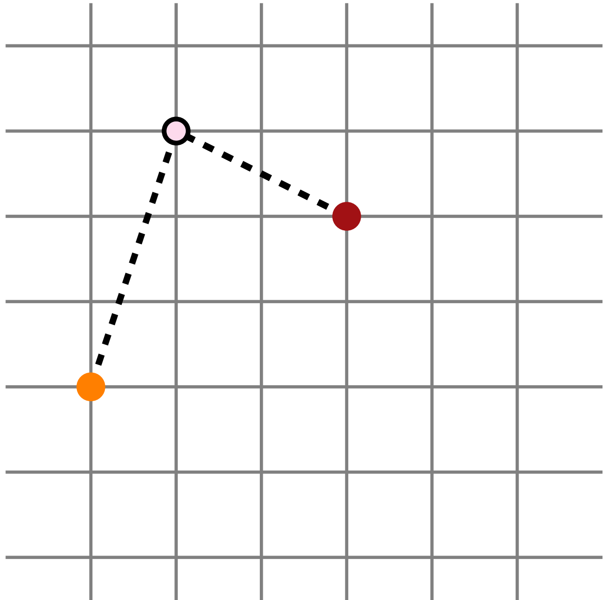
$(1,1)$-flat



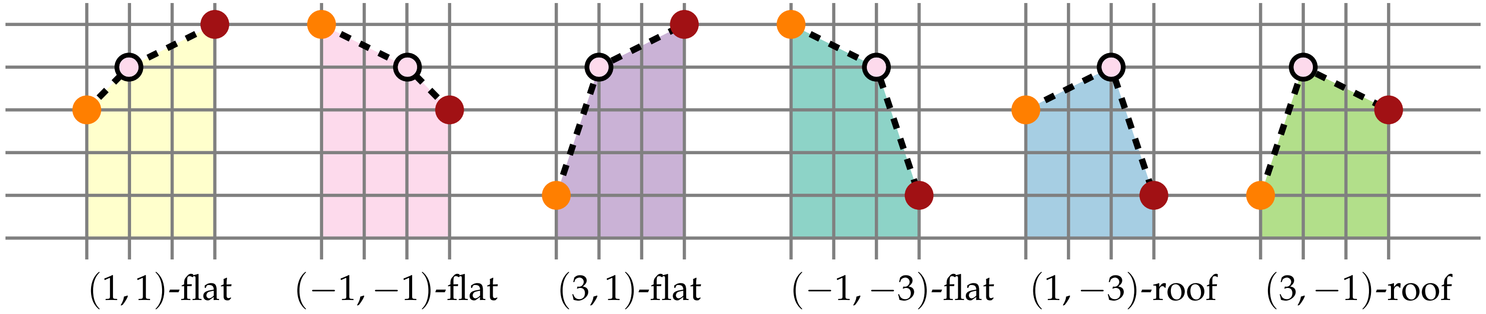
$(3,1)$-flat

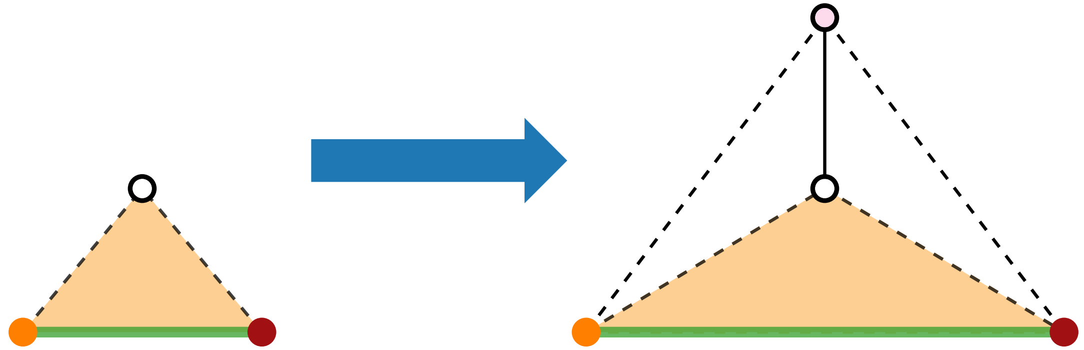


$(3,-1)$-roof

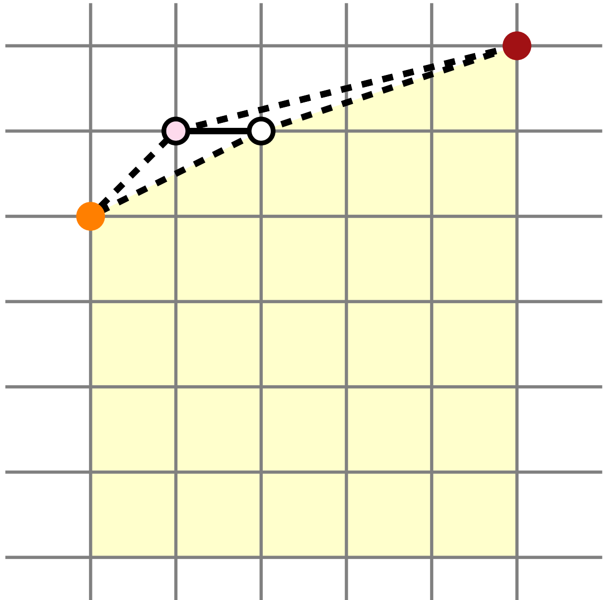


S-nodes

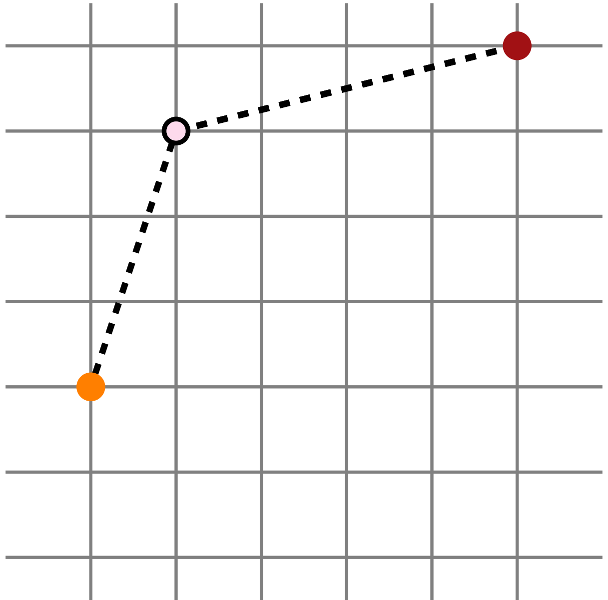




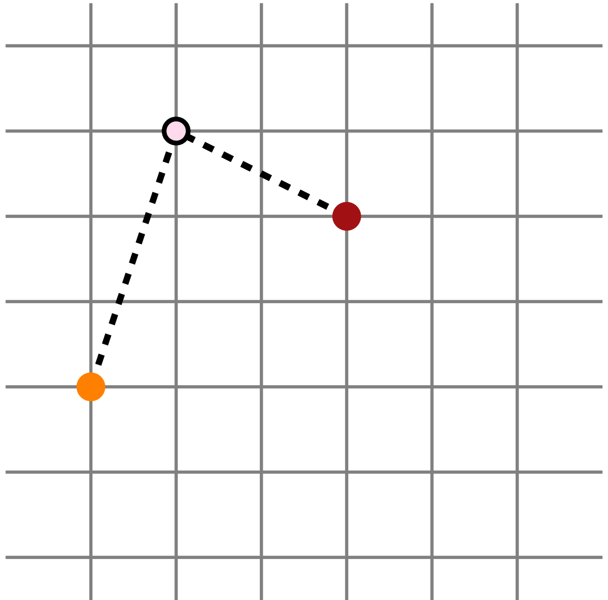
(1, 1)-flat



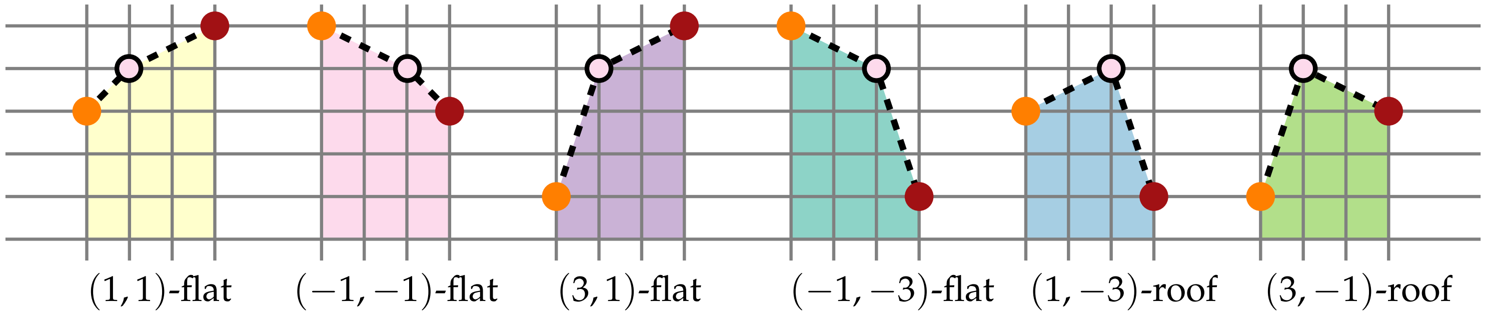
(3, 1)-flat



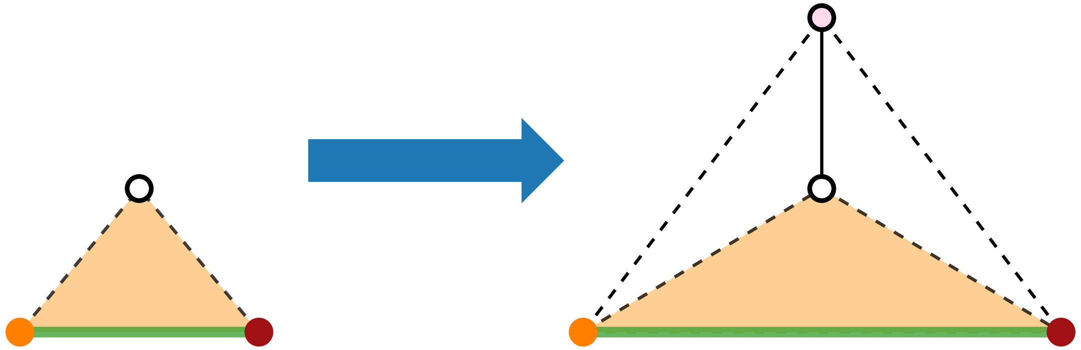
(3, −1)-roof

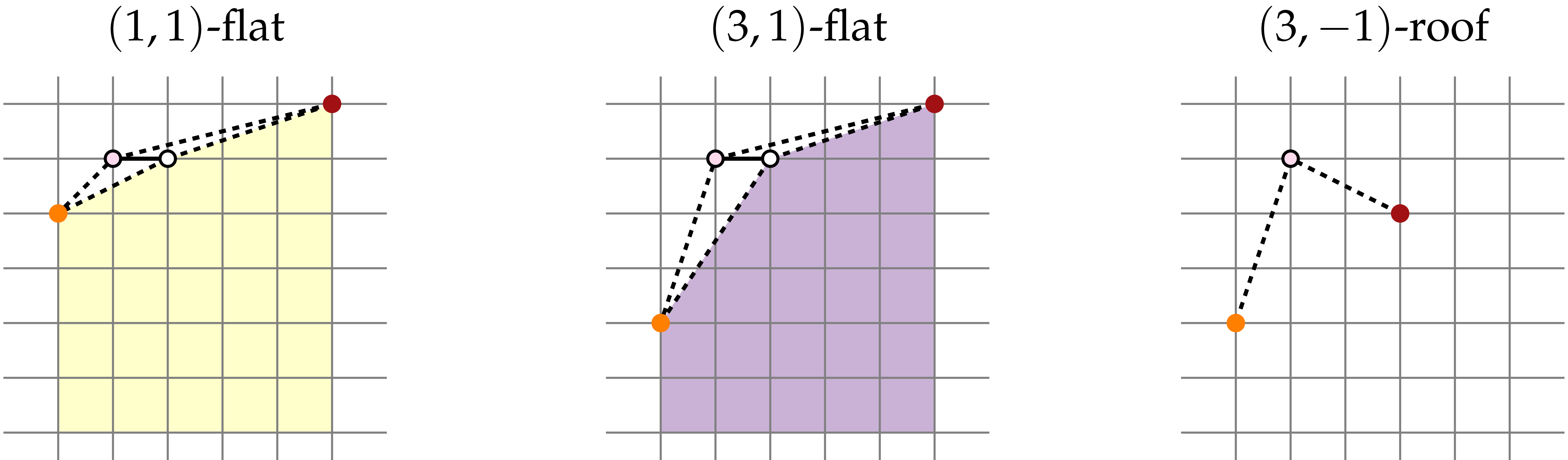


S-nodes



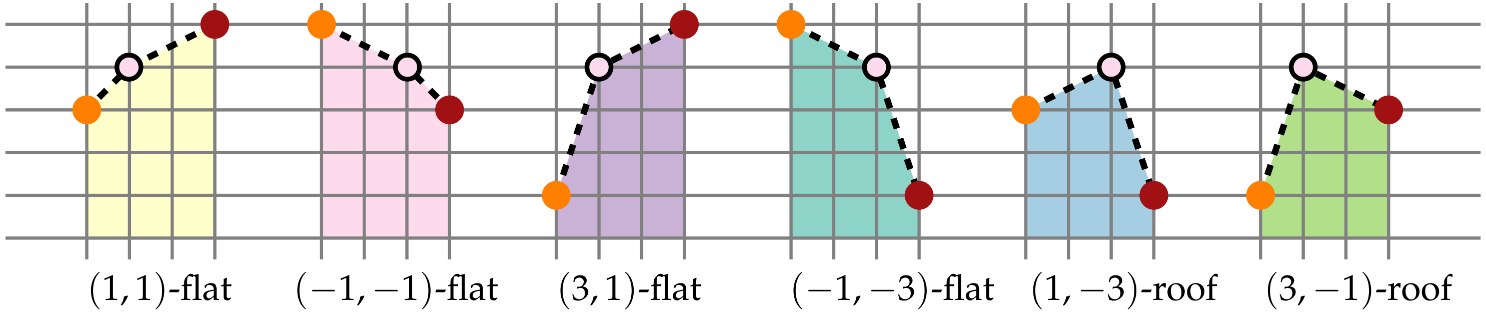
(1, 1)-flat
(−1, −1)-flat
(3, 1)-flat
(−1, −3)-flat
(1, −3)-roof
(3, −1)-roof

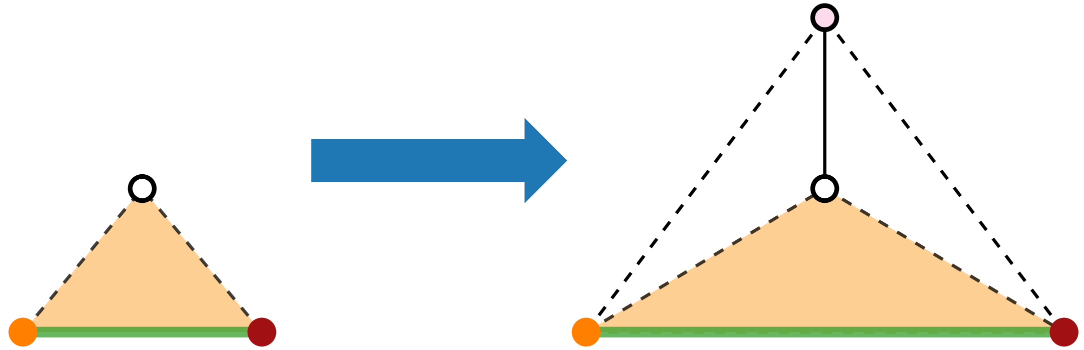




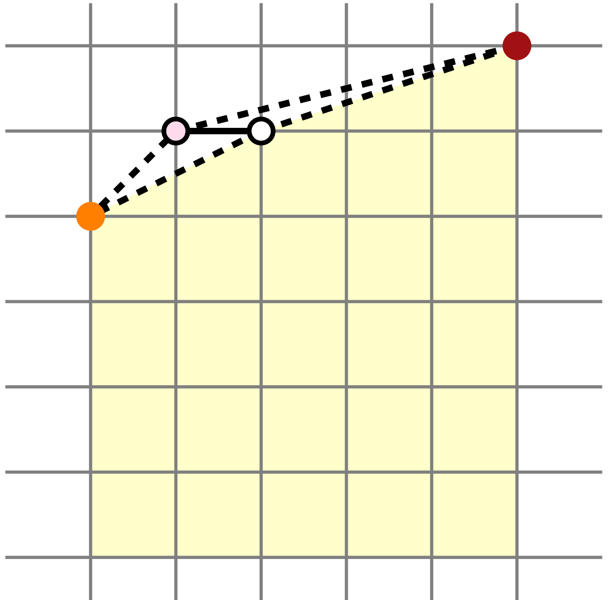
(1, 1)-flat
(3, 1)-flat
(3, −1)-roof

S-nodes

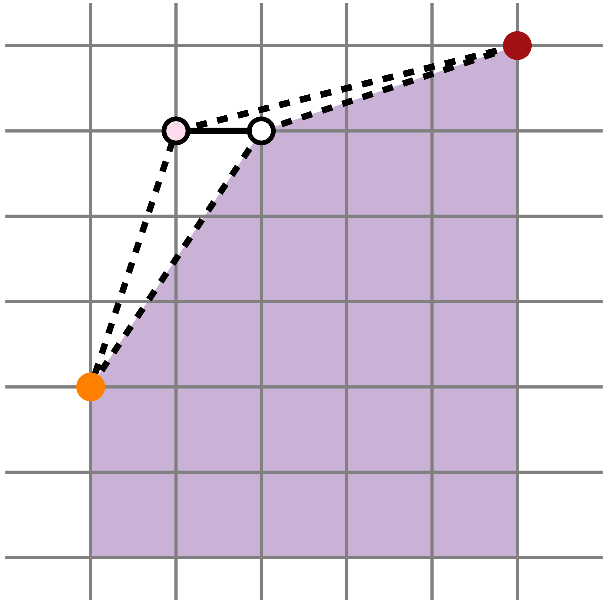




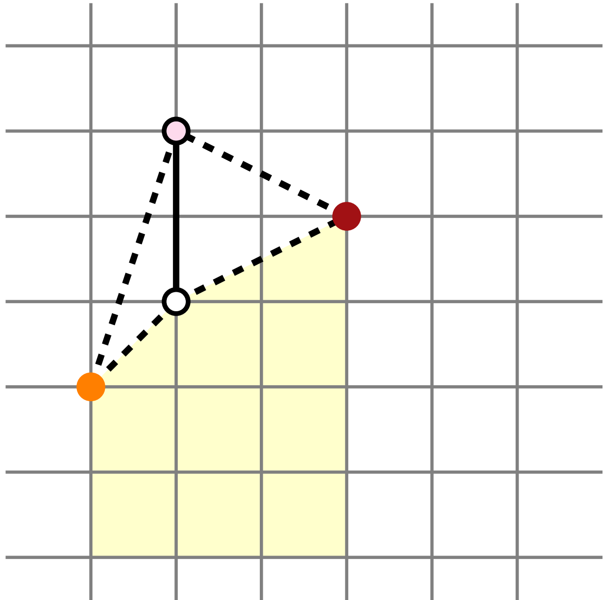
(1, 1)-flat



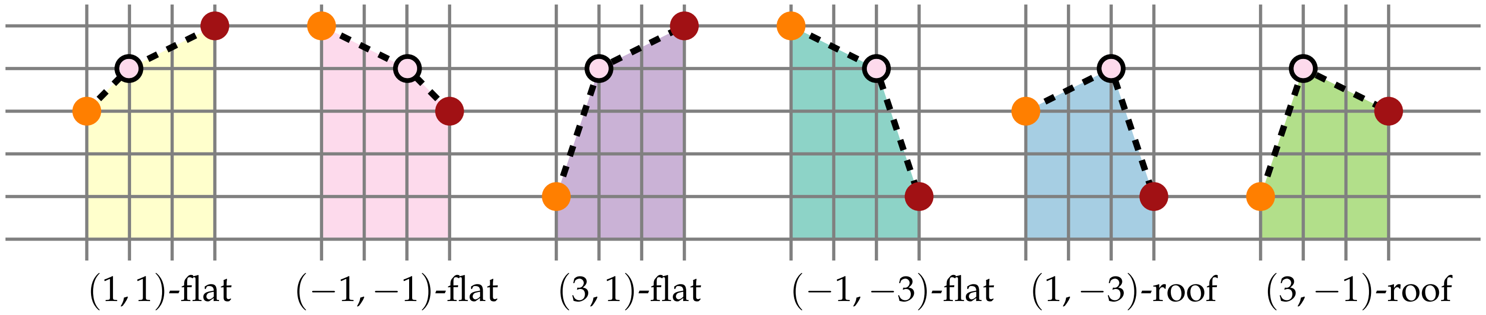
(3, 1)-flat

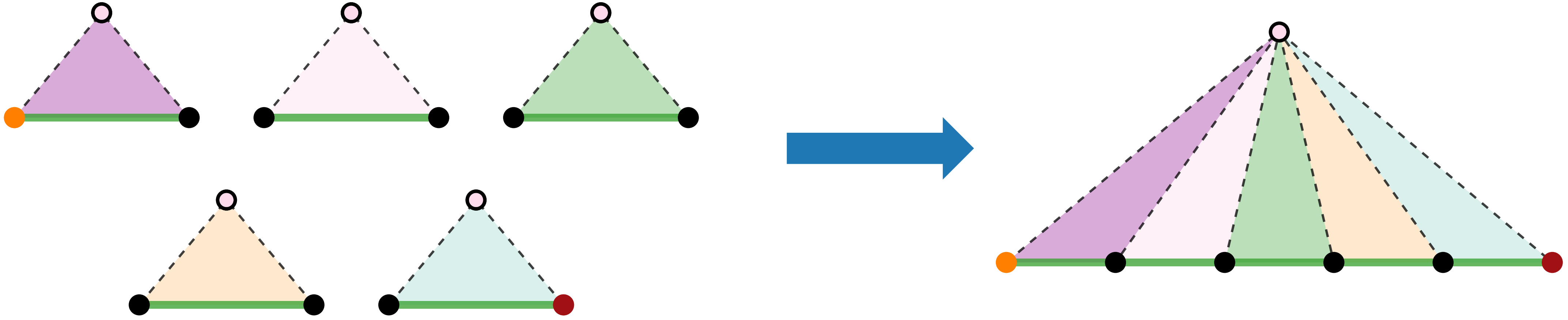


(3, −1)-roof

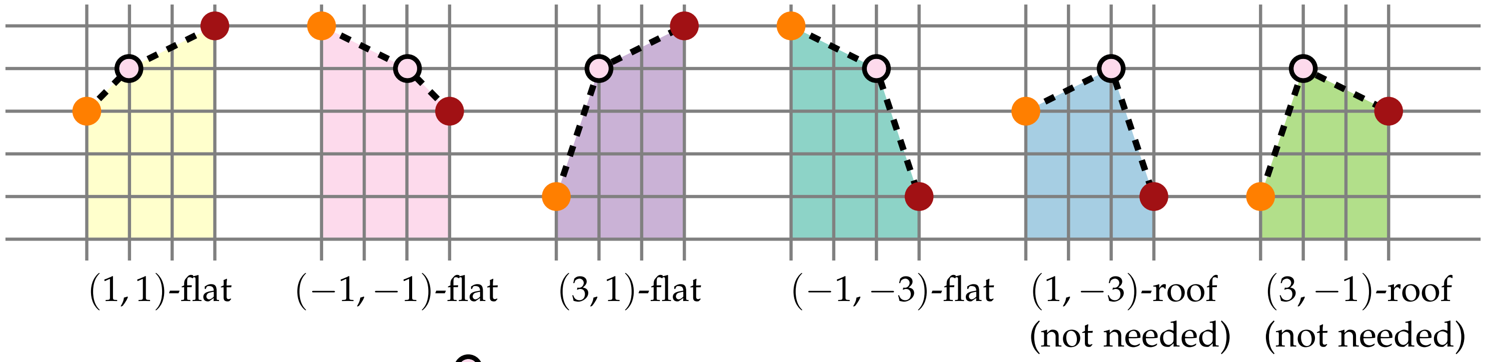


P-nodes

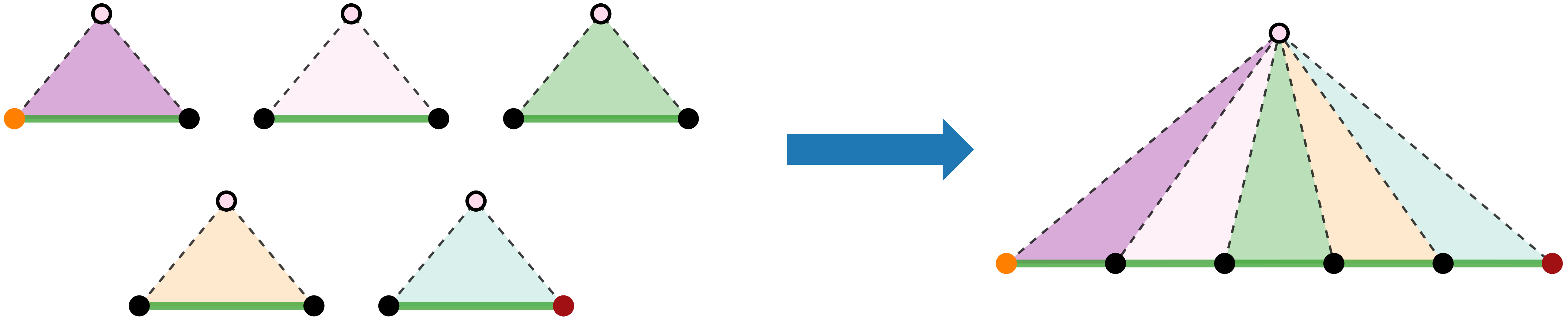




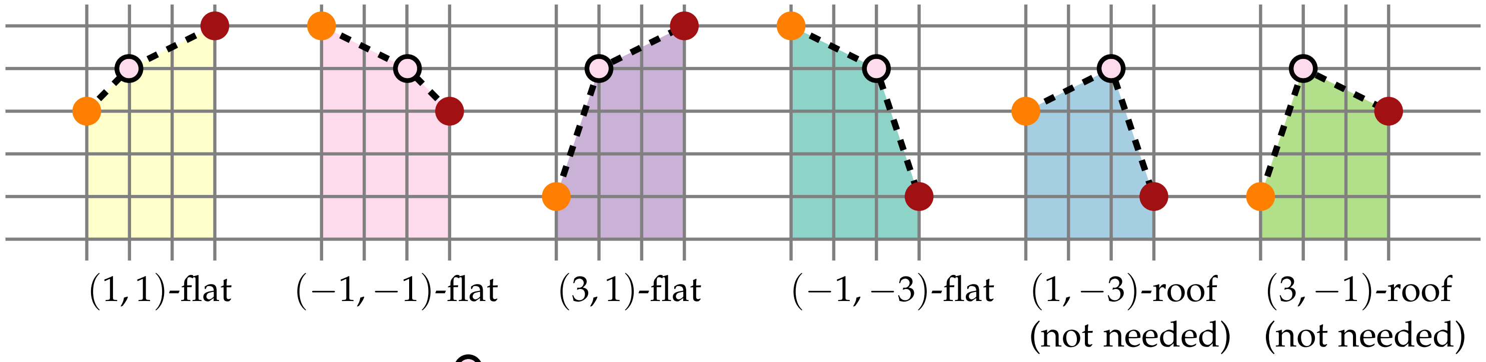
P-nodes

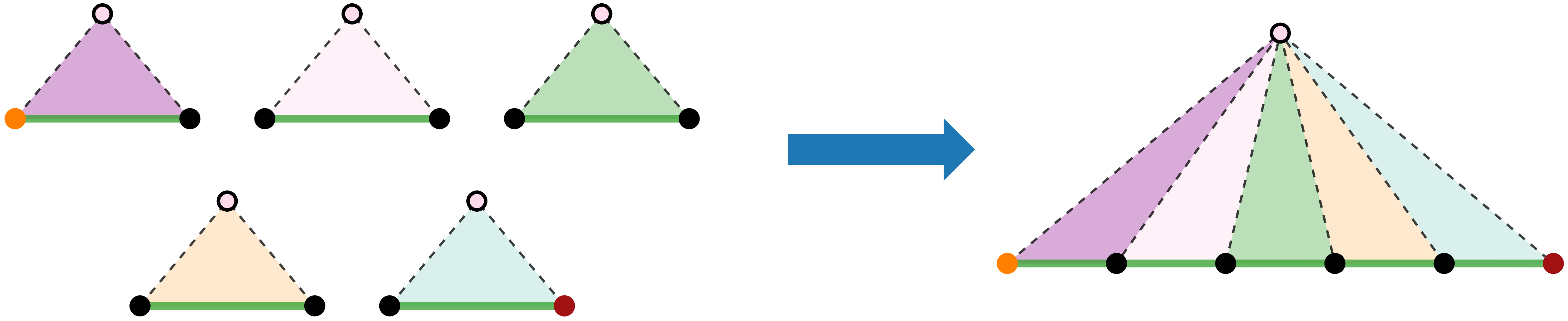


(1, 1)-flat
(−1, −1)-flat
(3, 1)-flat
(−1, −3)-flat
(1, −3)-roof
(not needed)
(3, −1)-roof
(not needed)



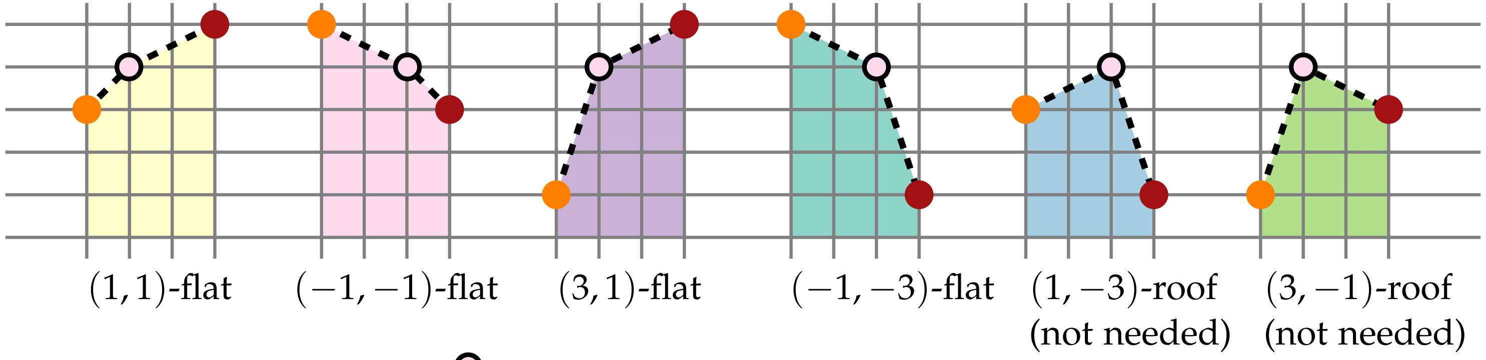
P-nodes

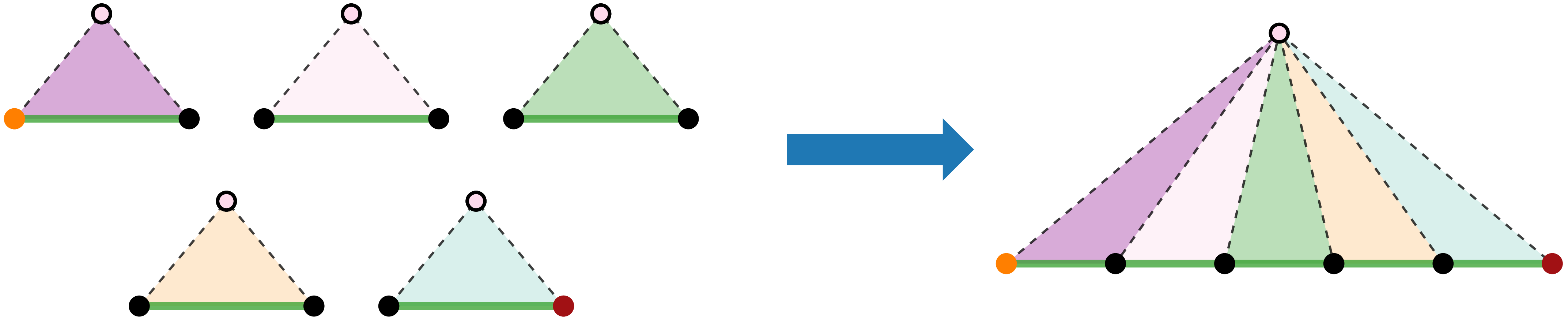




(1,1)-flat

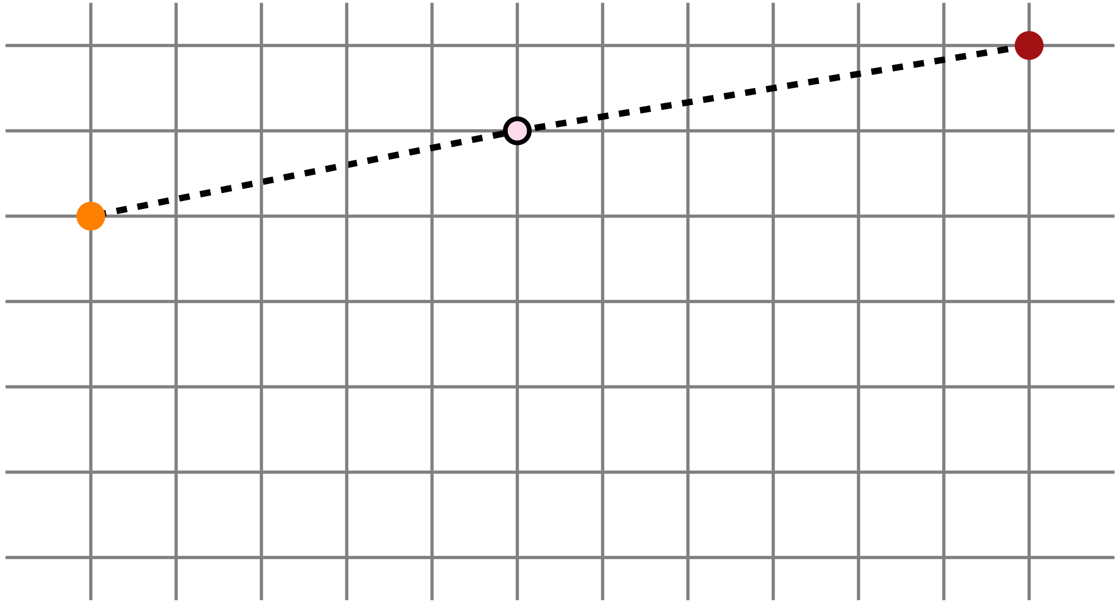
P-nodes



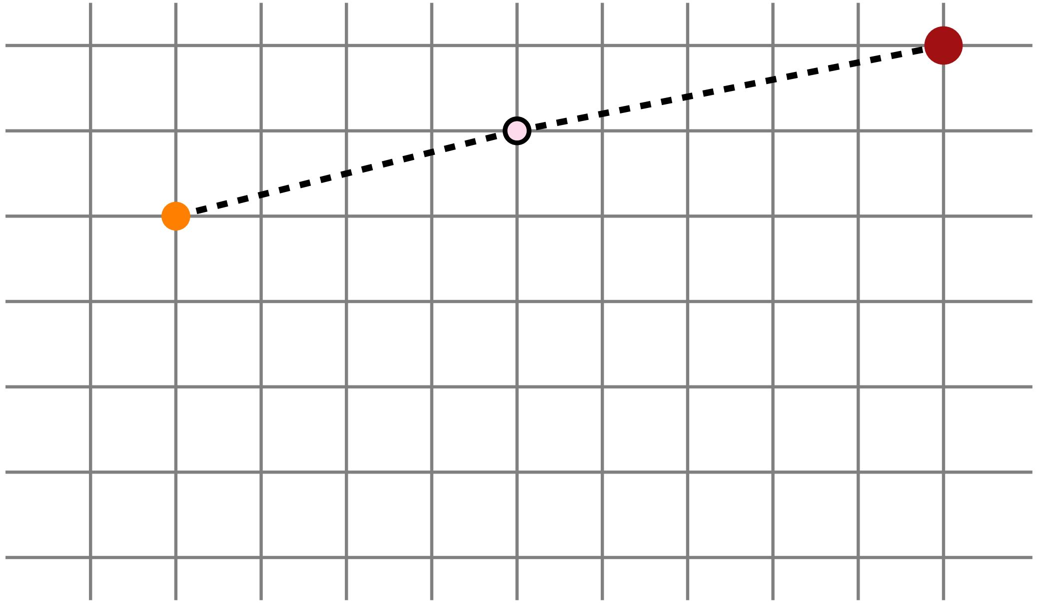


(1,1)-flat

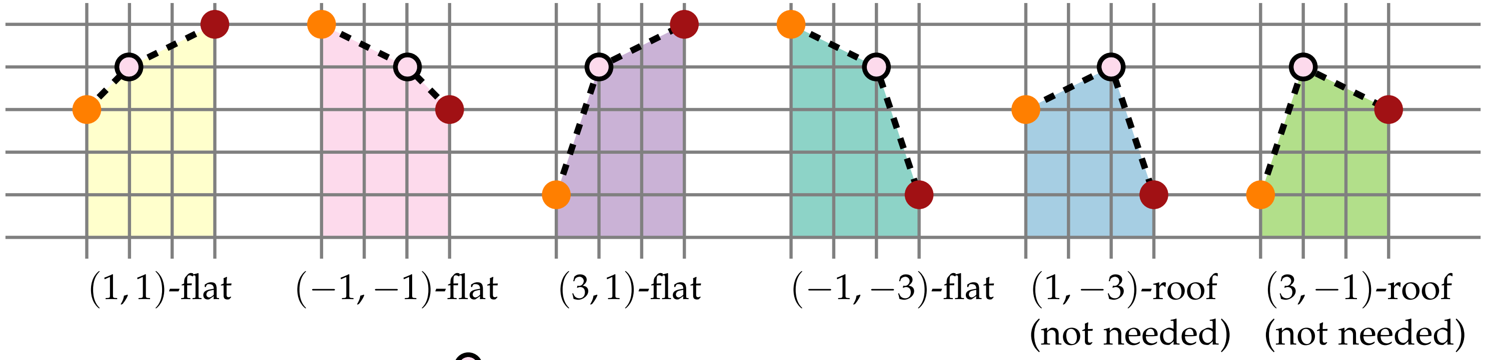
odd degree

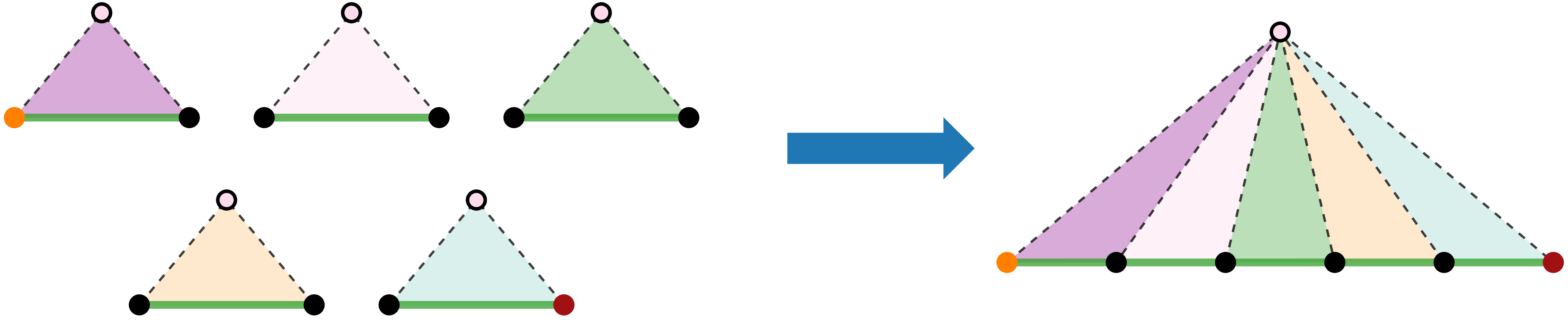


even degree



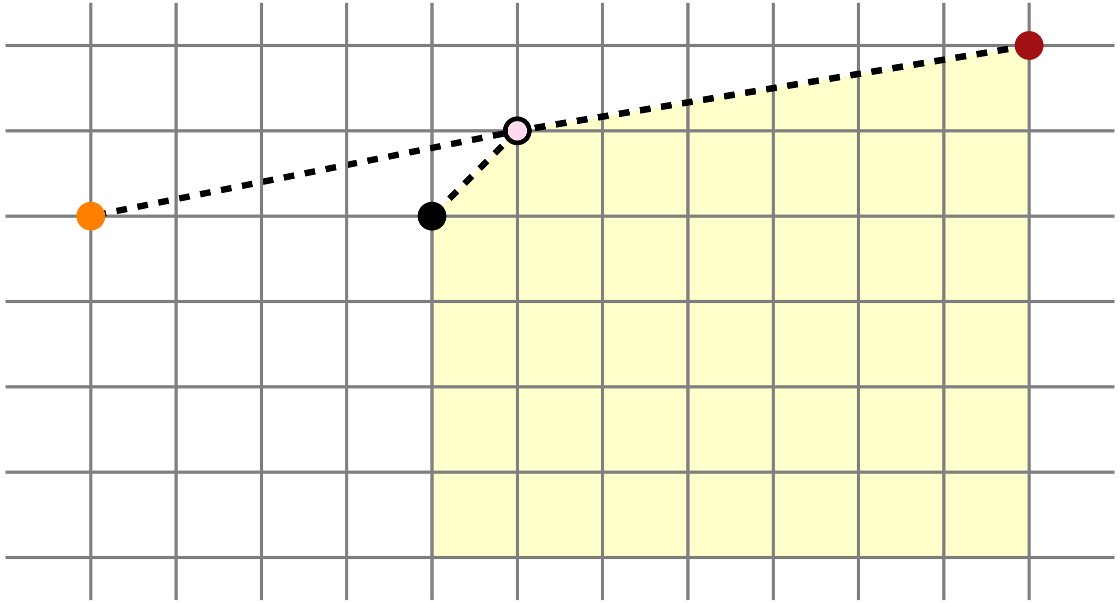
P-nodes



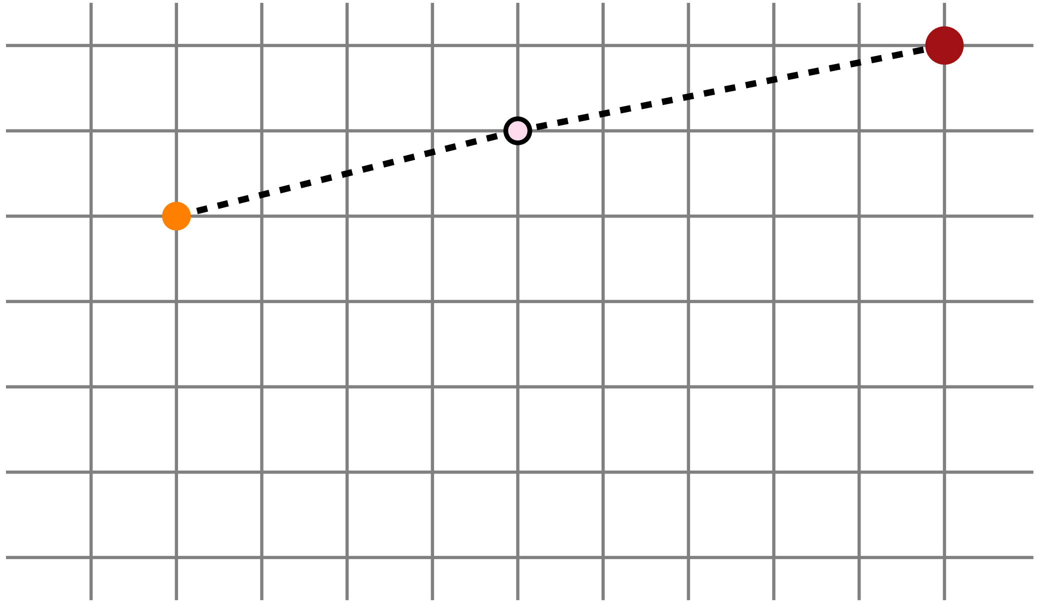


(1,1)-flat

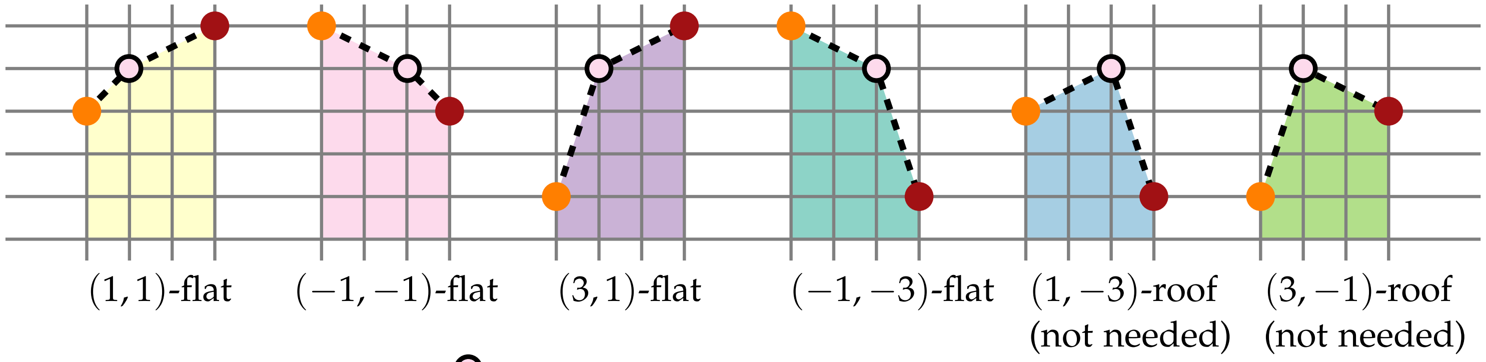
odd degree

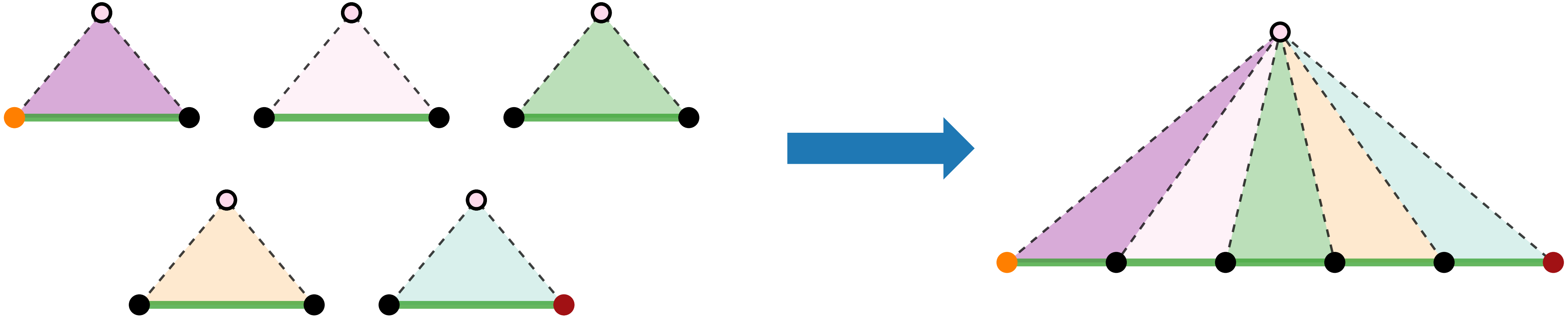


even degree



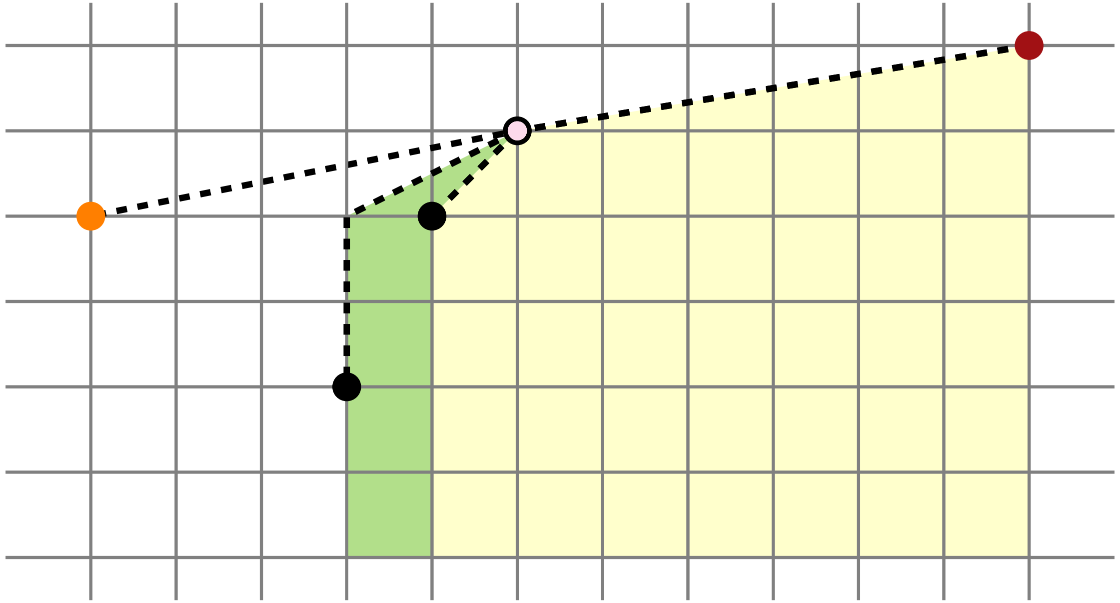
P-nodes



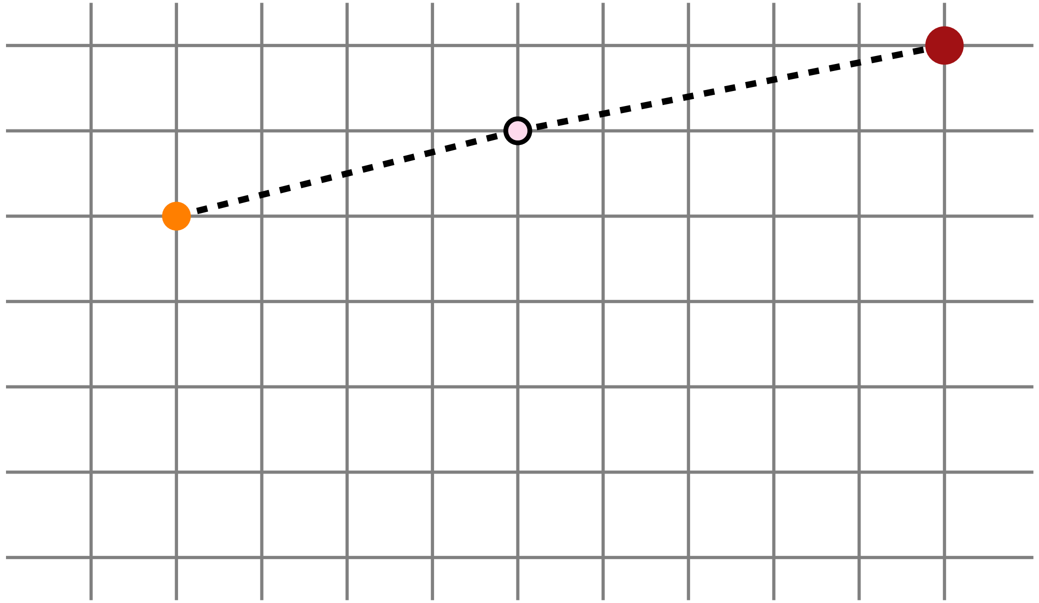


(1,1)-flat

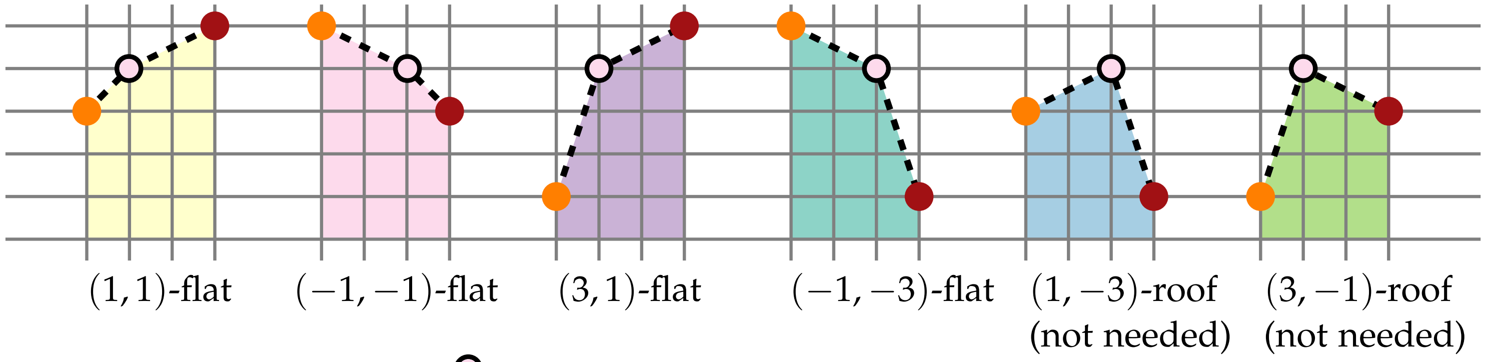
odd degree

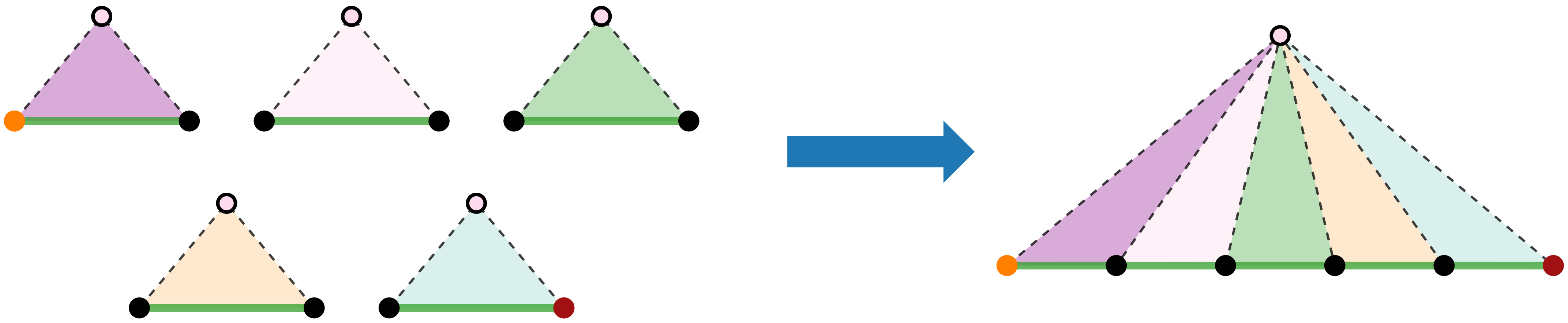


even degree



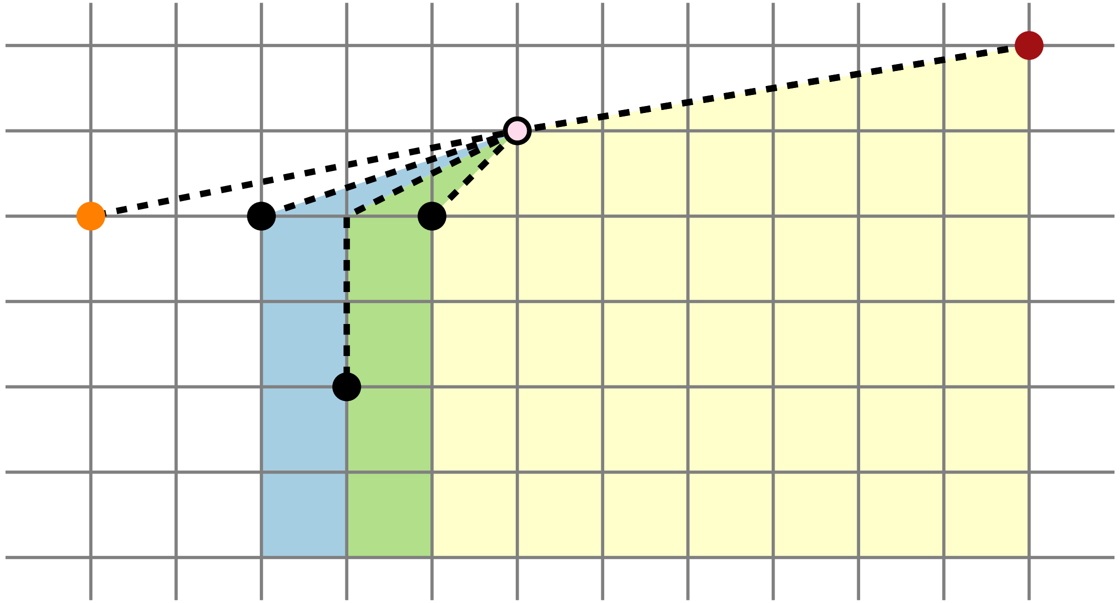
P-nodes



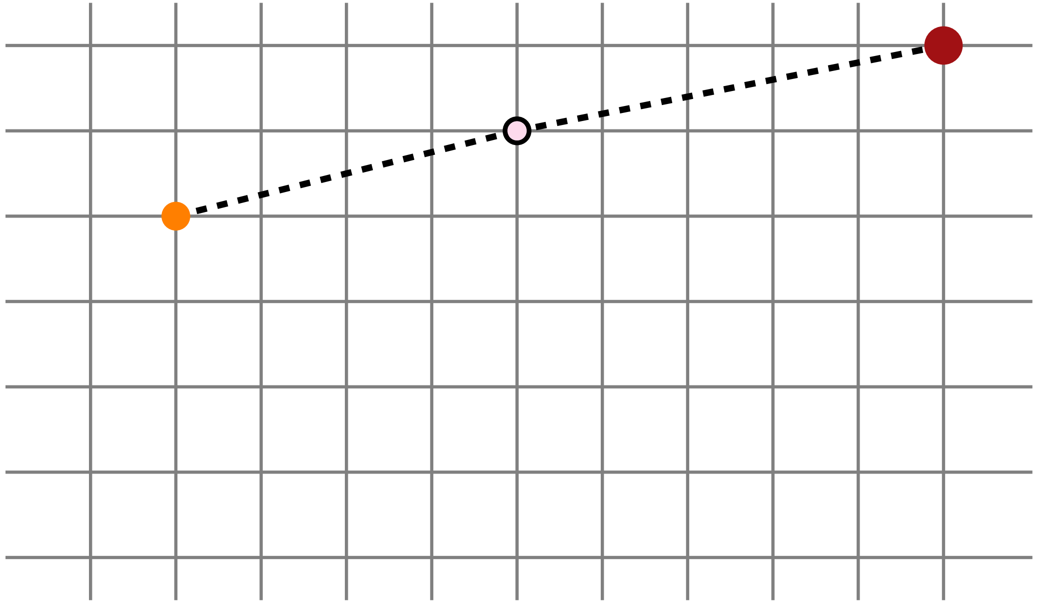


(1,1)-flat

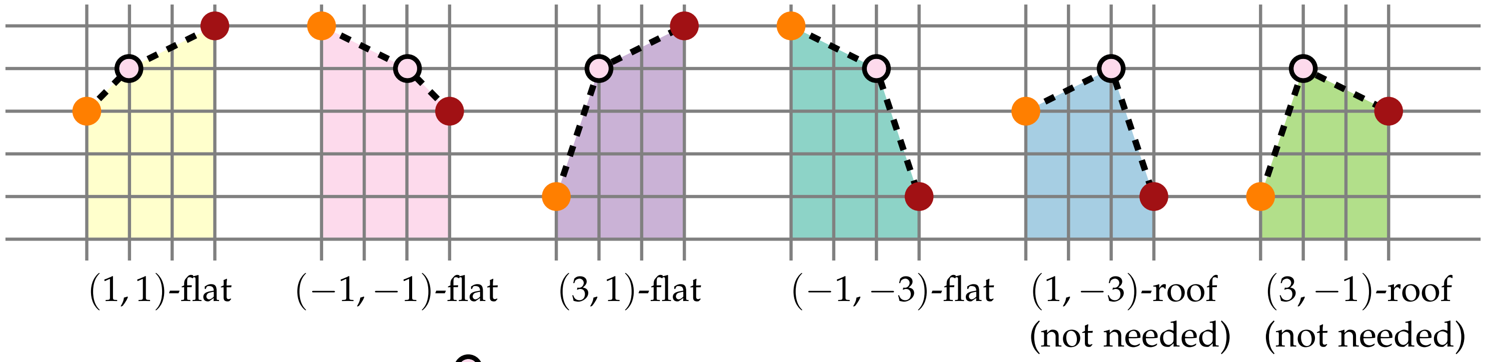
odd degree

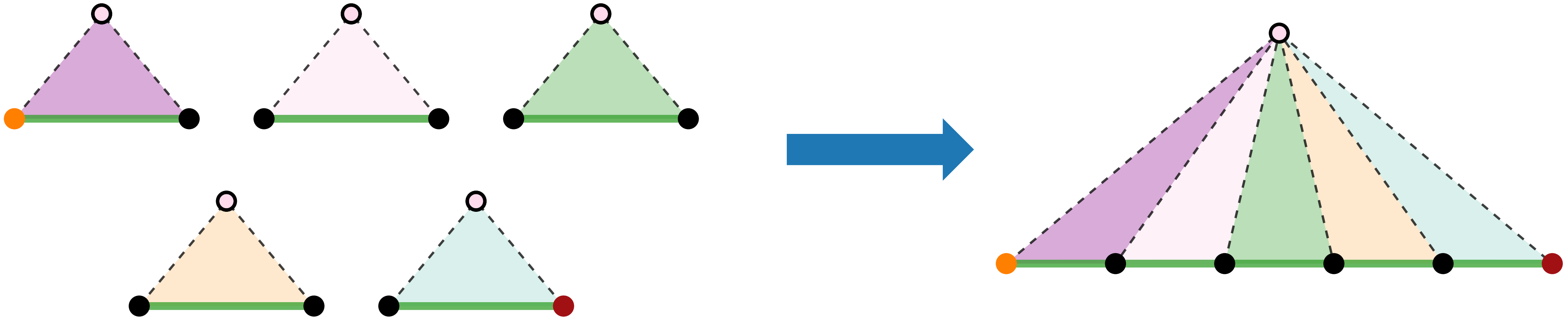


even degree



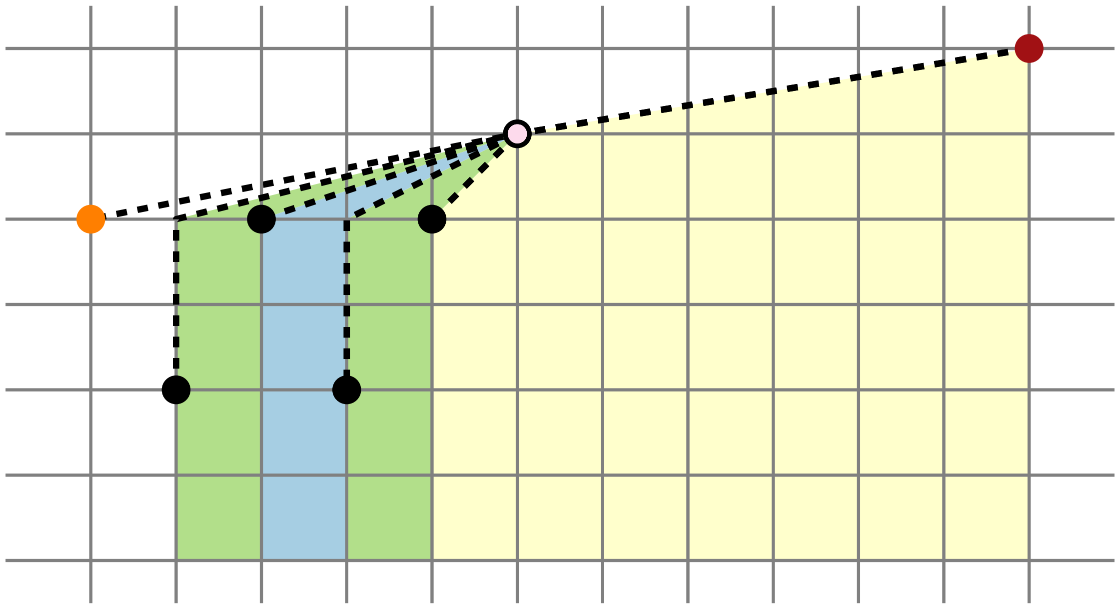
P-nodes





(1,1)-flat

odd degree



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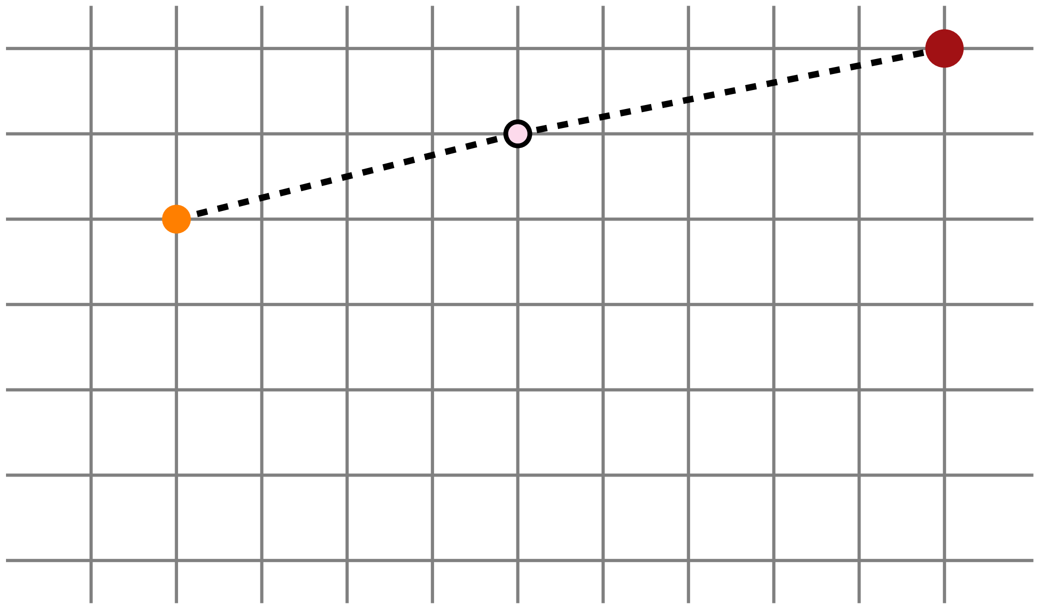

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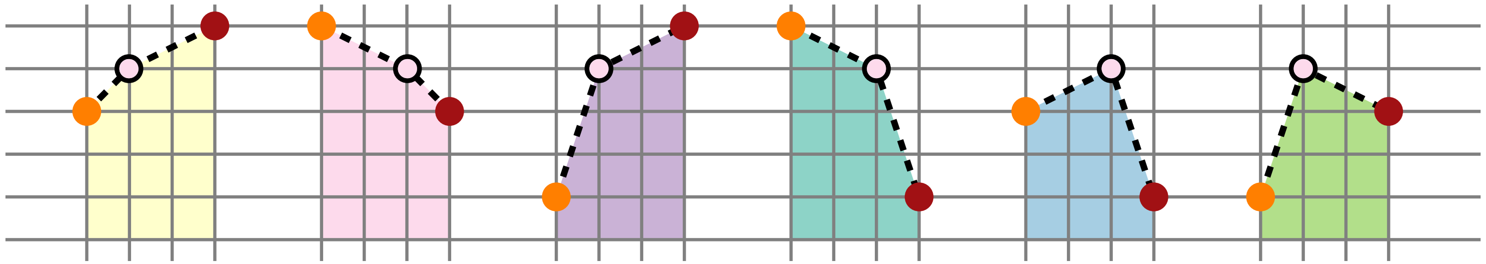

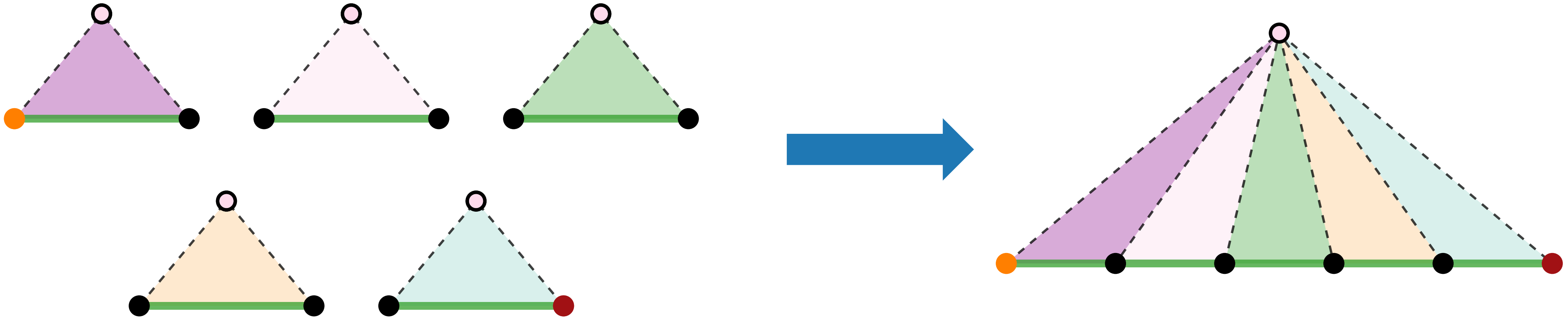

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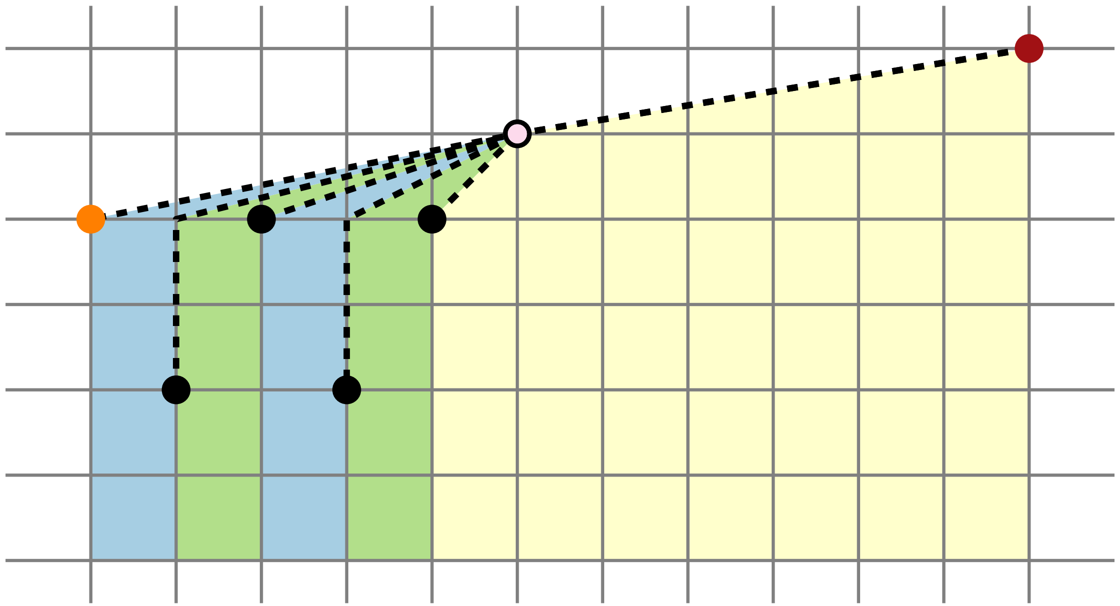
odd degree



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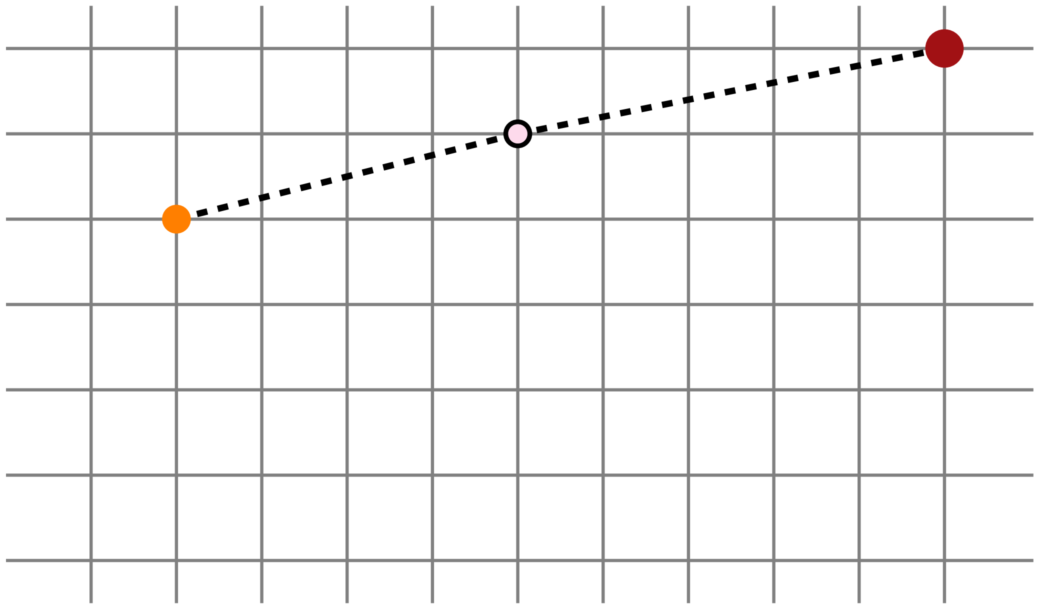

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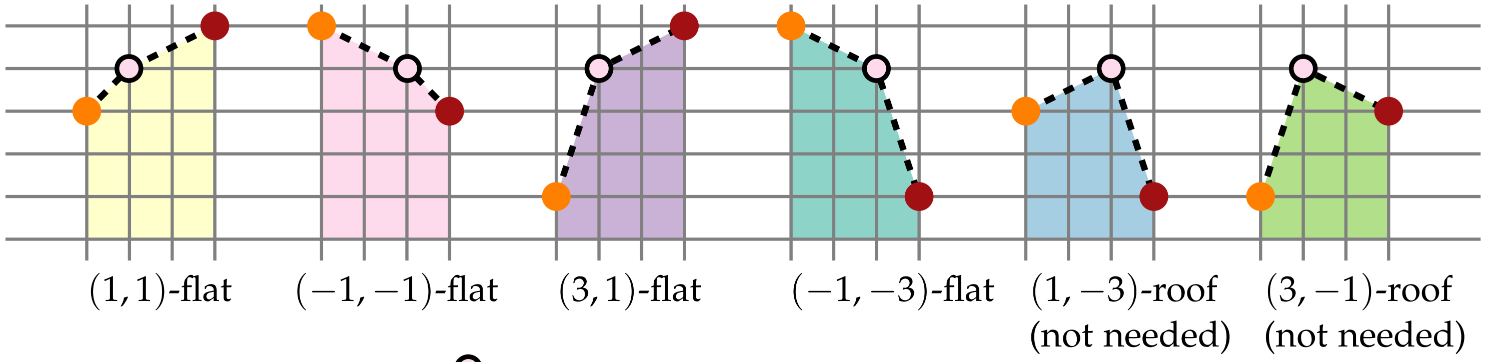

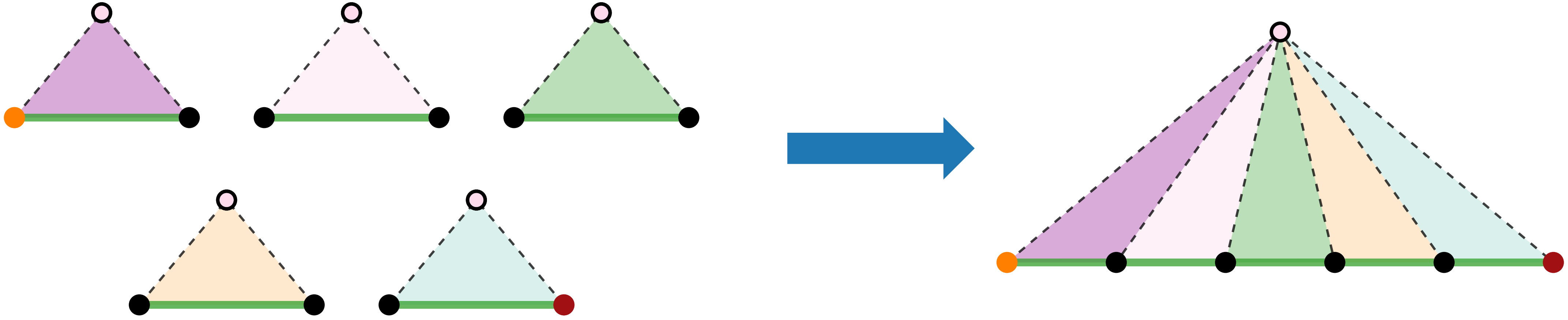

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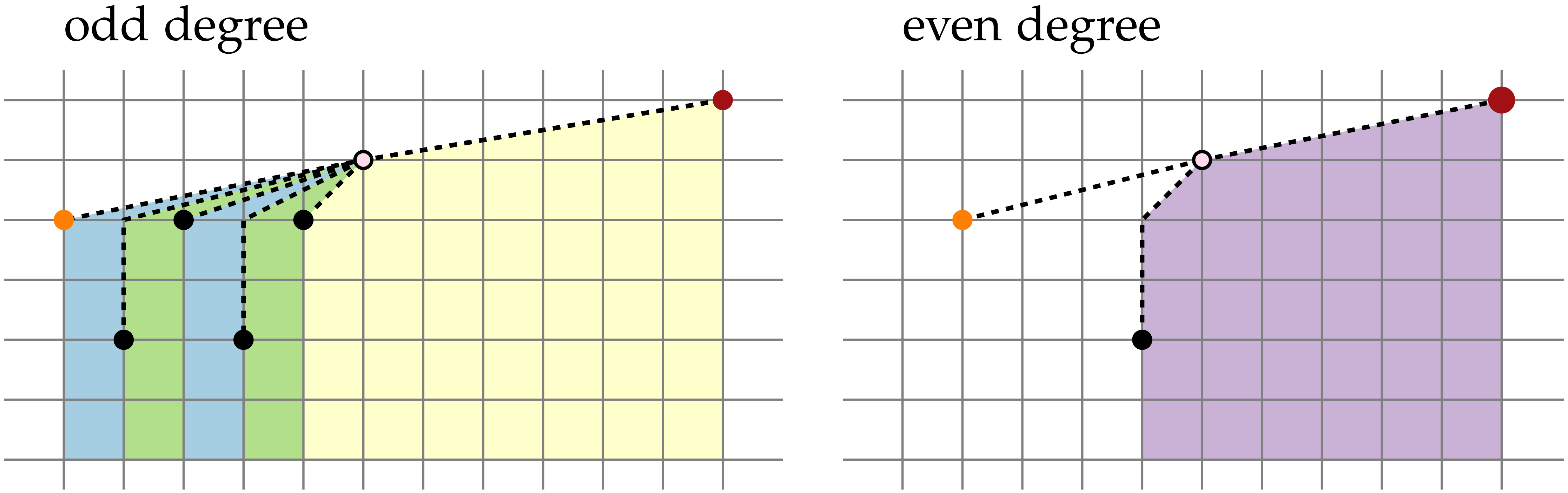

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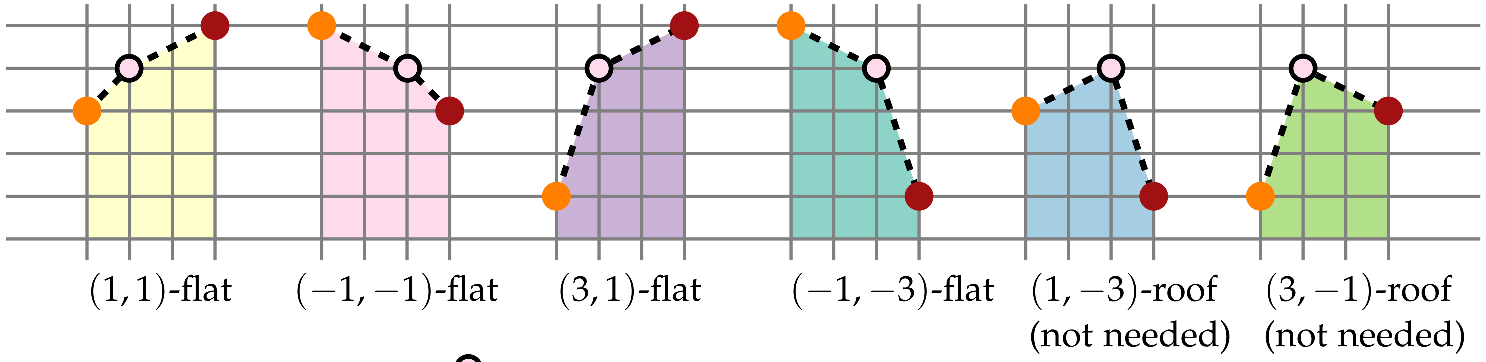

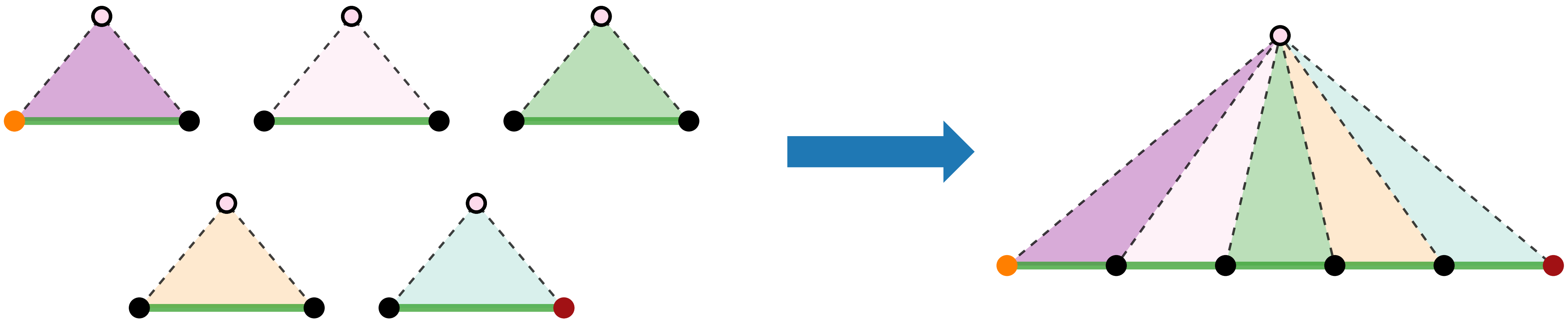

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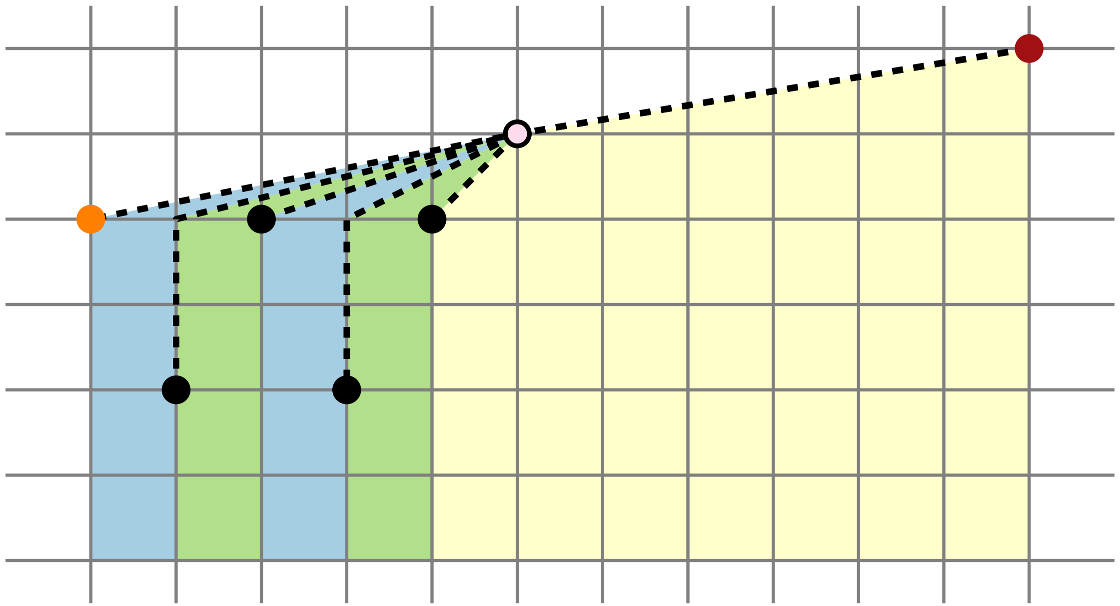

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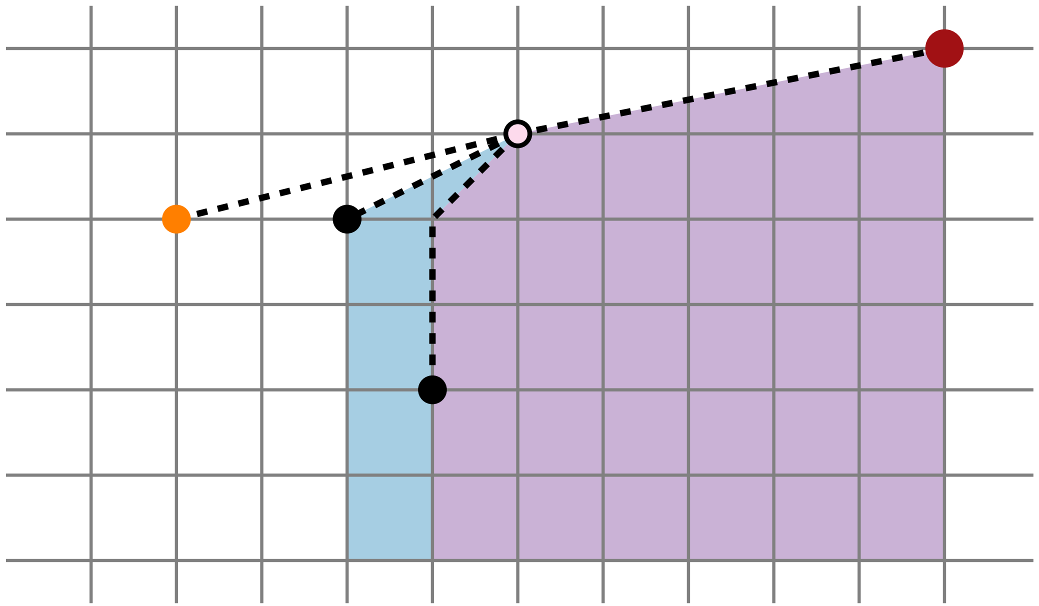

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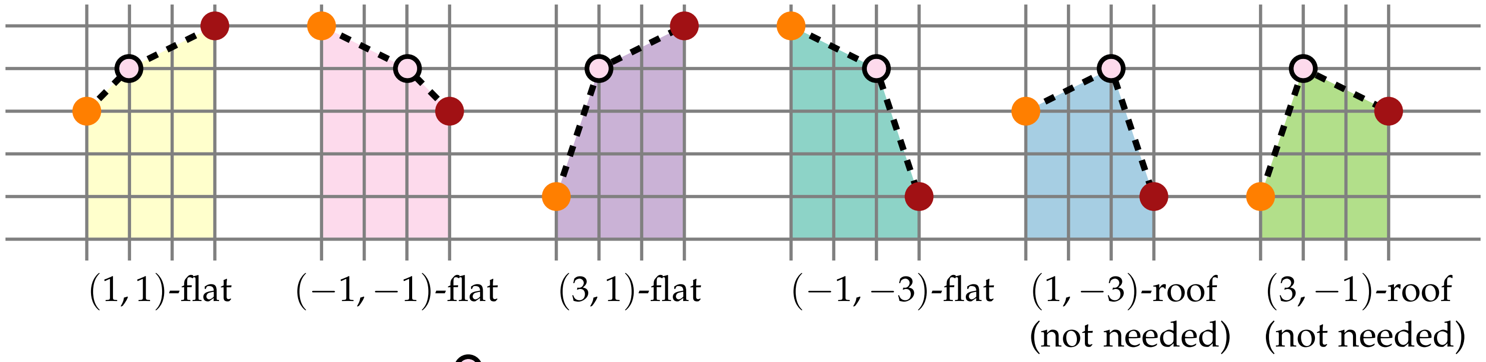

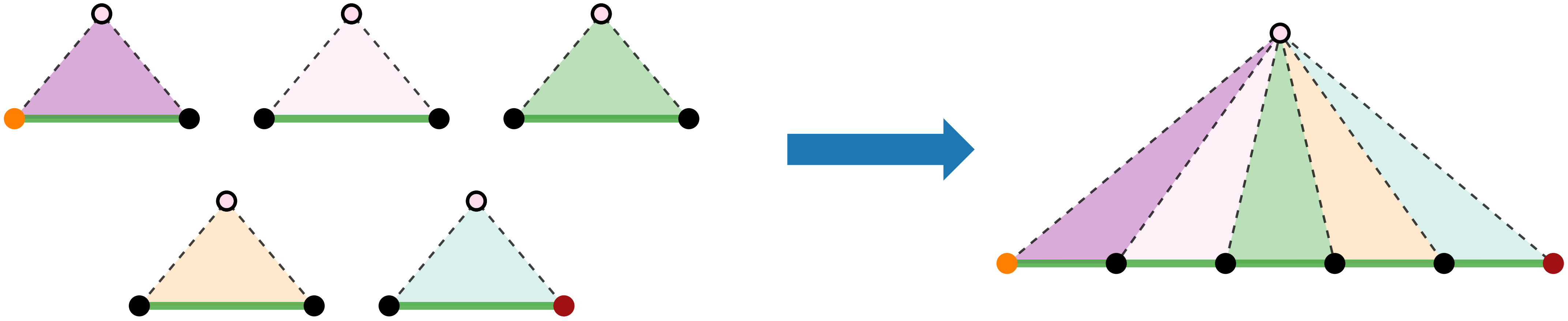

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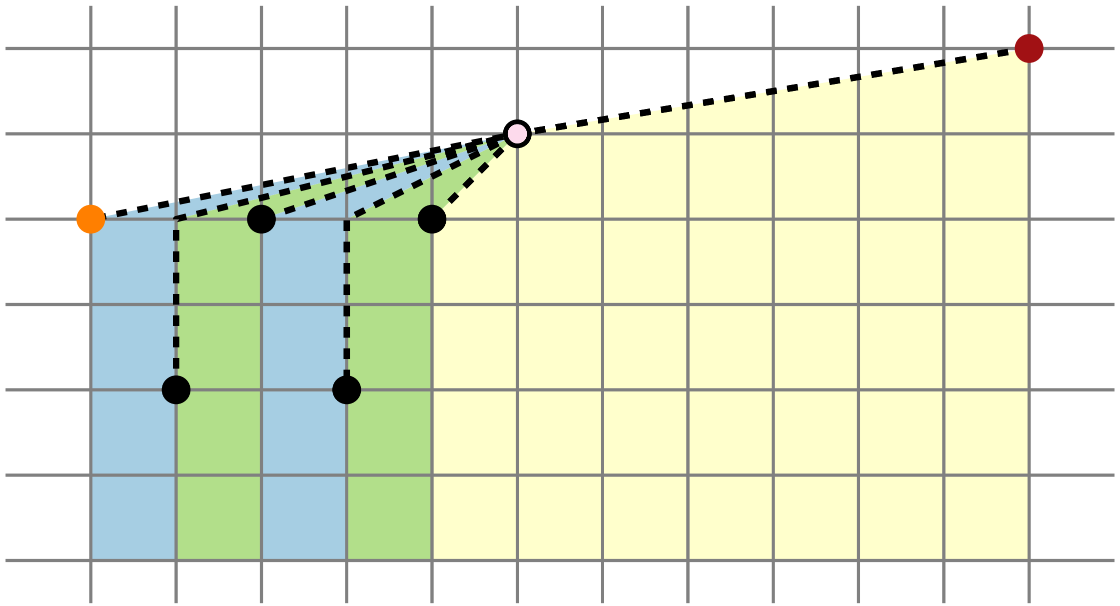

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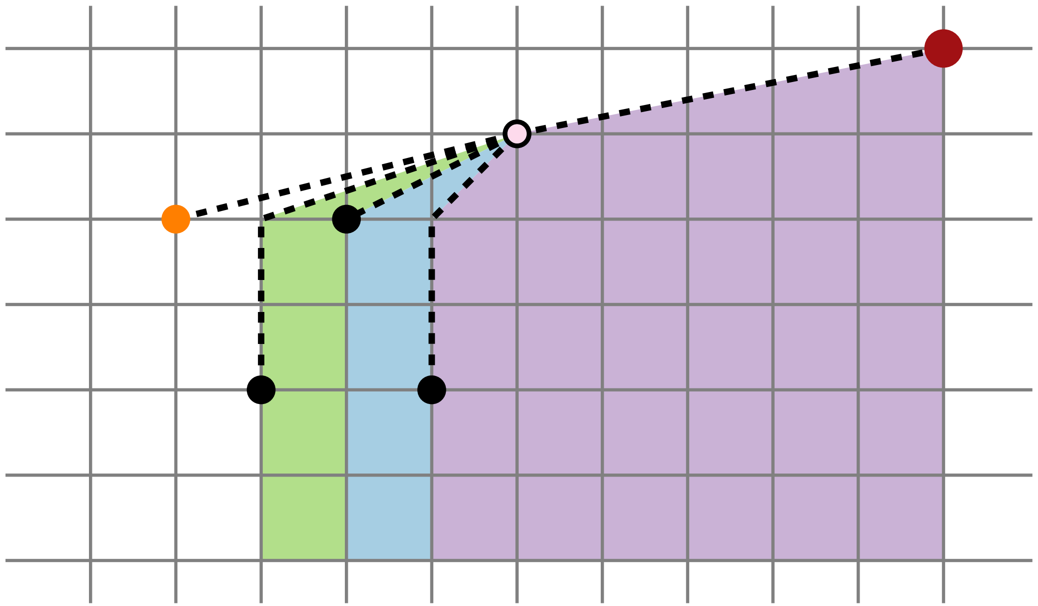

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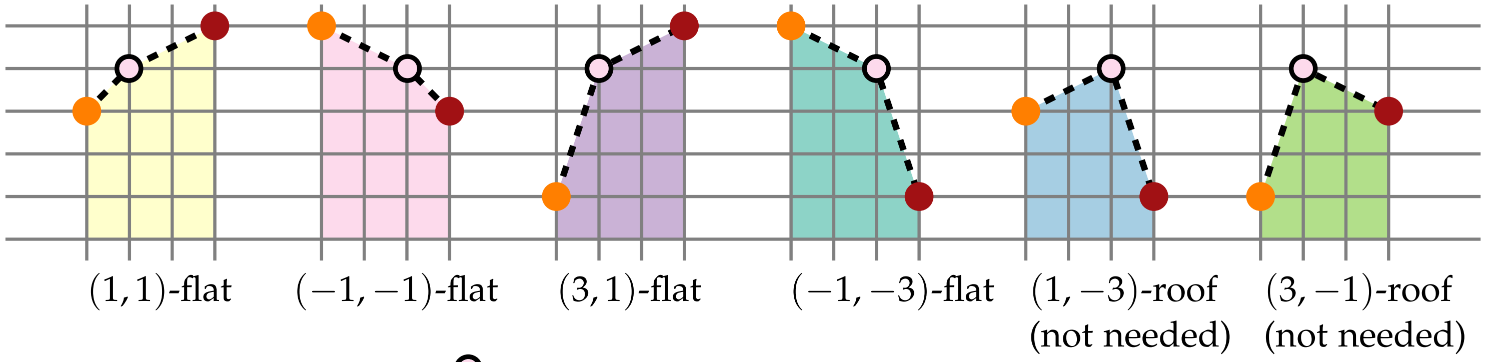

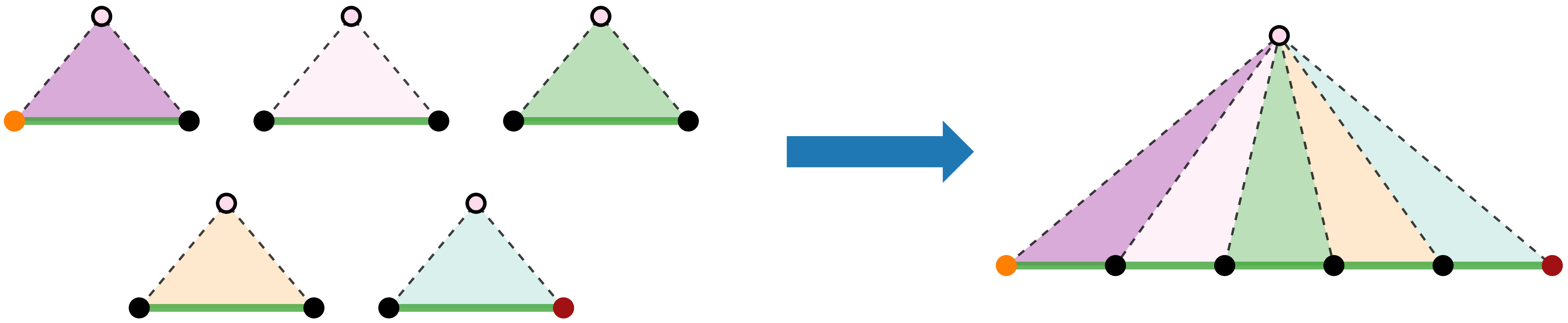

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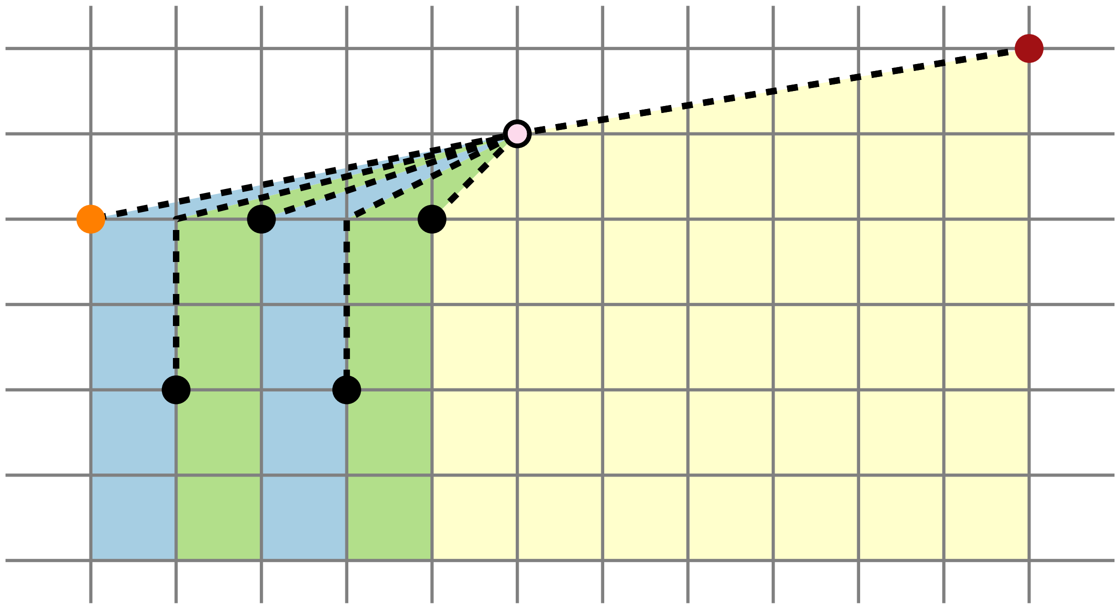

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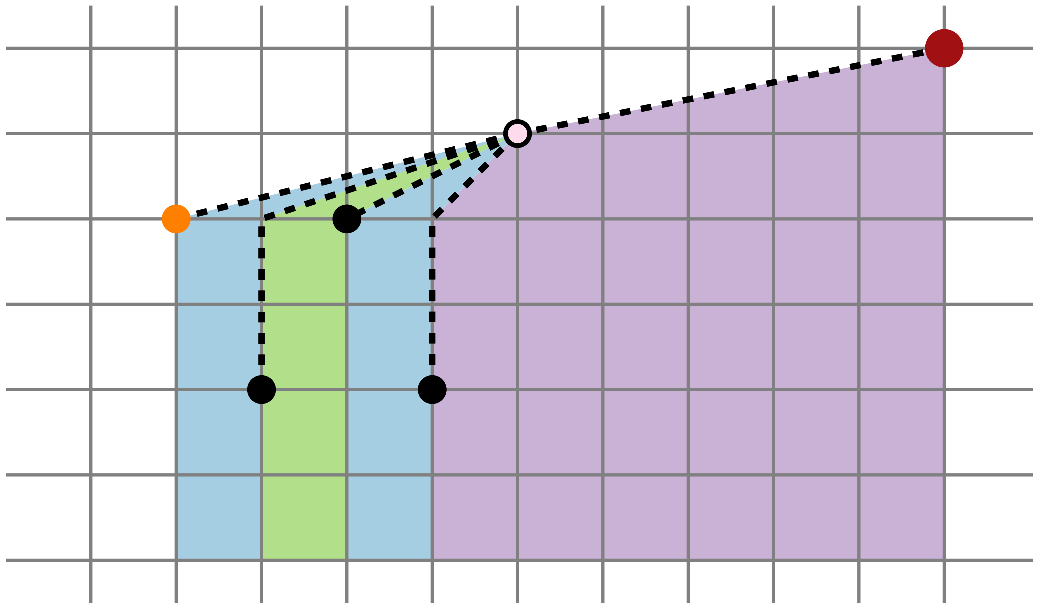

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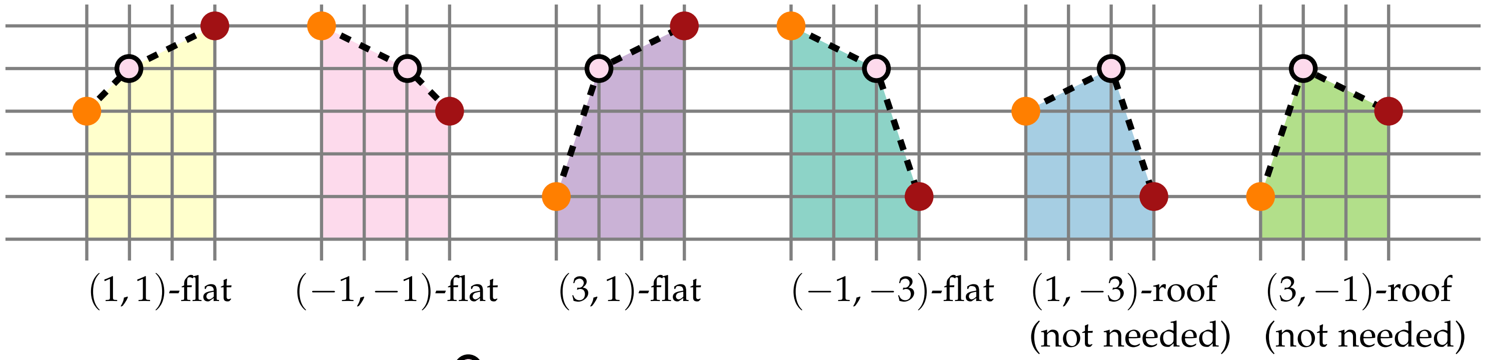
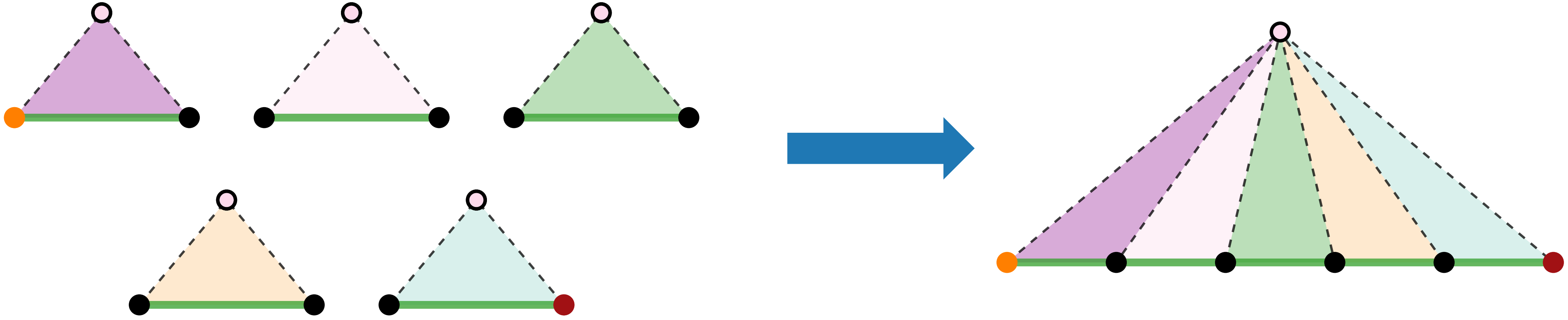
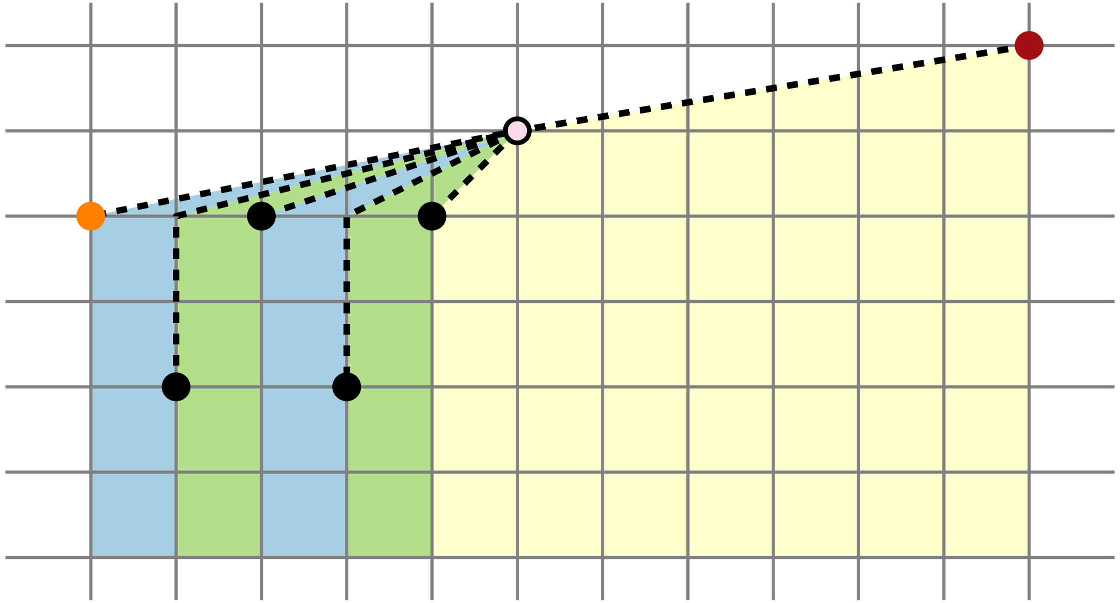


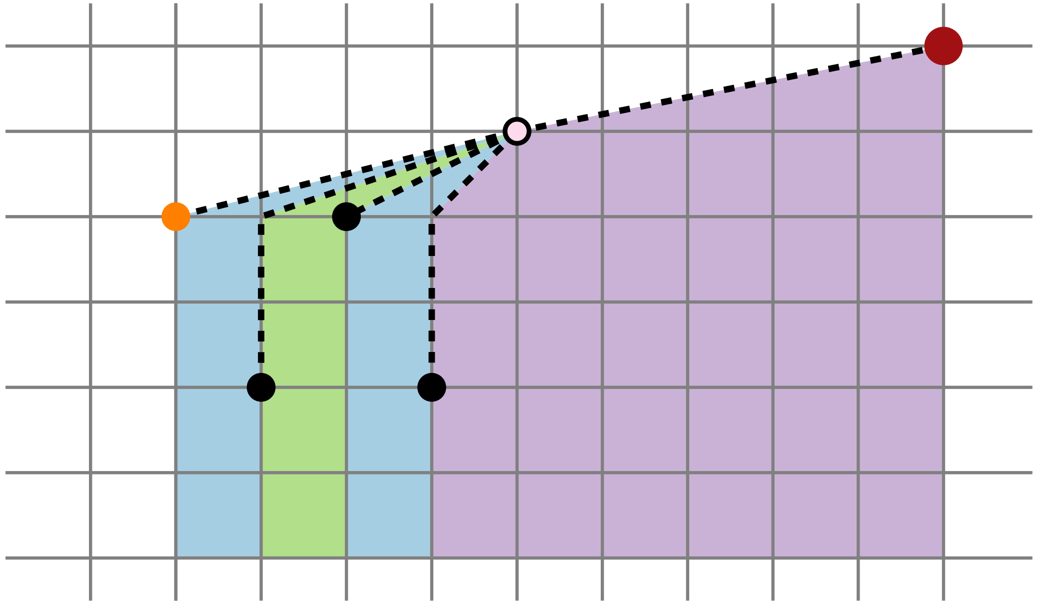


(3,1)-flat

odd degree



even degree



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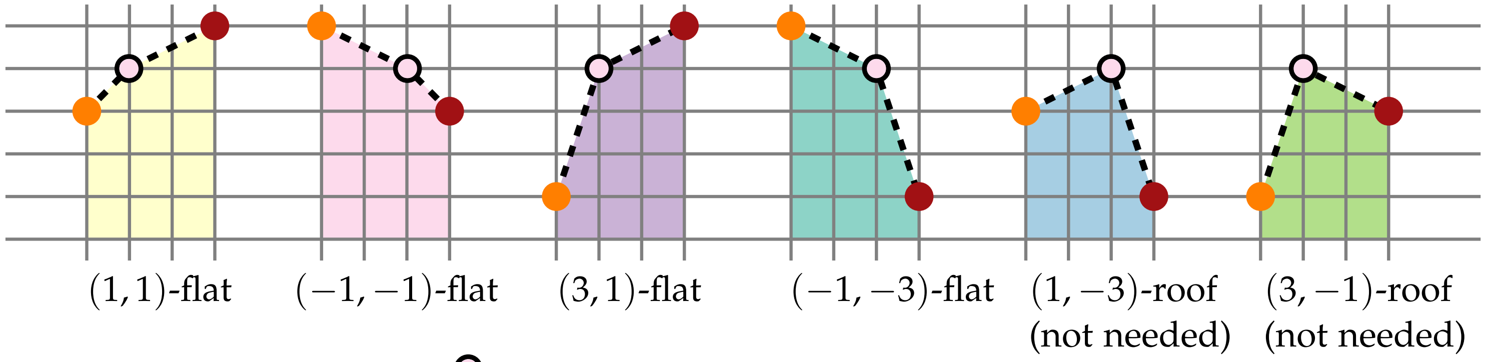

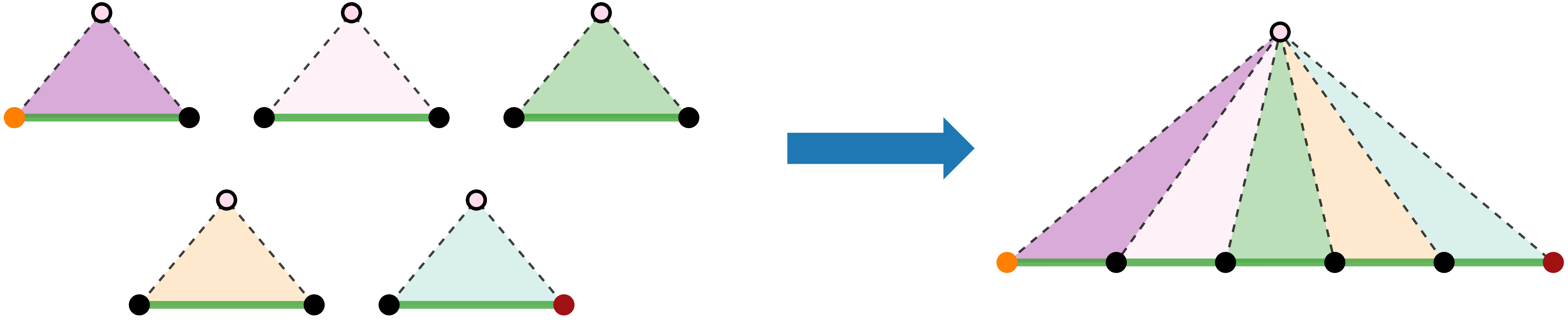

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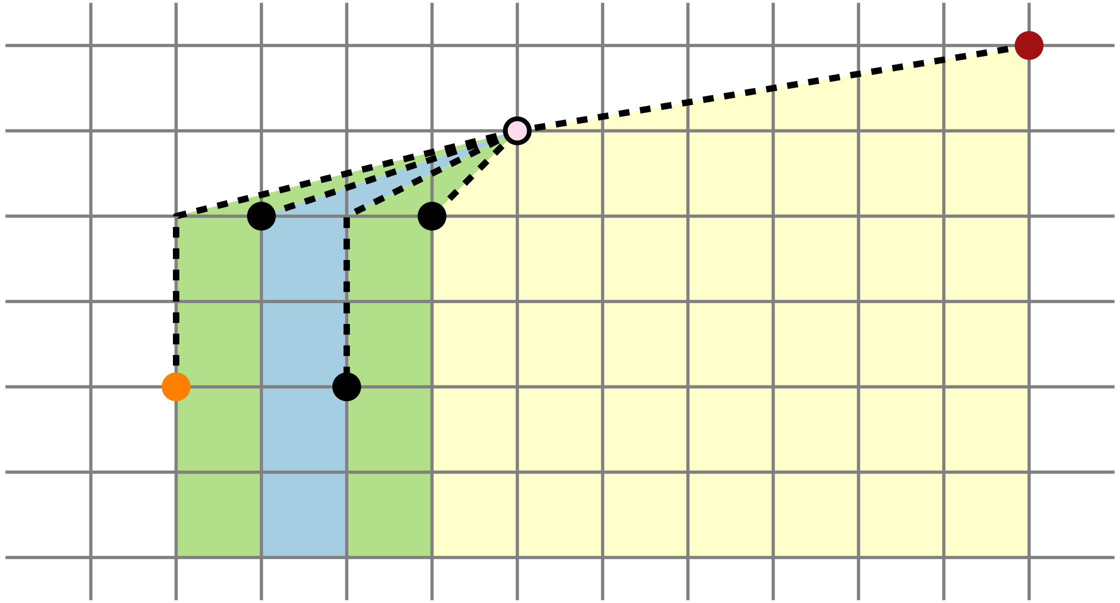

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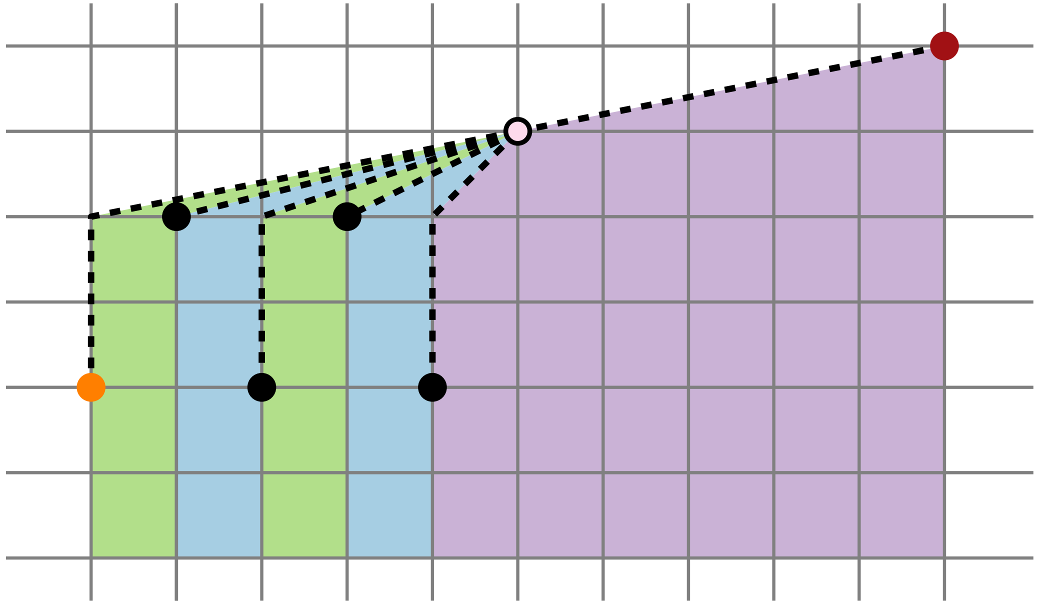

Results: Cycle-trees

Theorem.
Every almost-3-connected path-tree admits a 4-span weakly leveled planar drawing

Results: Cycle-trees

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- remove edge (ℓ, r)

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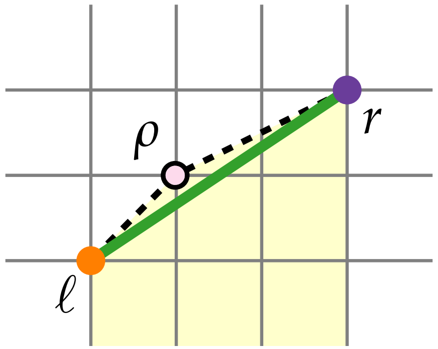
Cycle-trees?

- remove edge (ℓ, r)
- Create (1,1)-flat drawing



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Cycle-trees?

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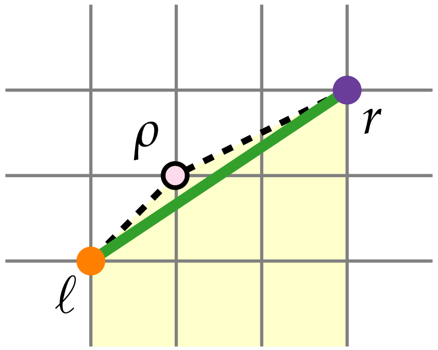
- Create (1,1)-flat drawing
- Reinsert (ℓ, r) with span 2



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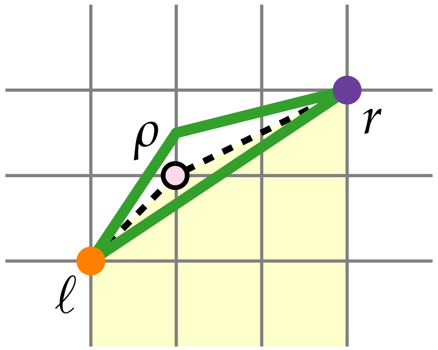

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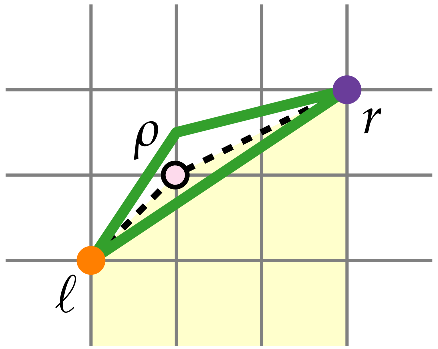

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Every 3-connected cycle-tree admits a 4-span weakly leveled planar drawing.

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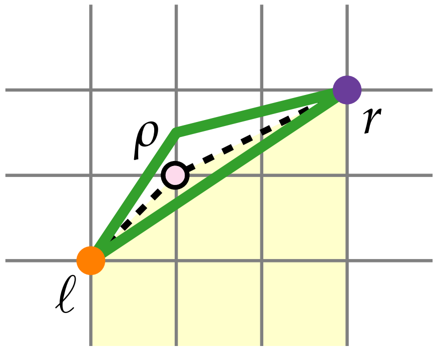

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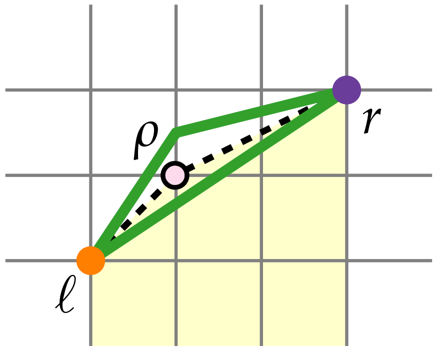

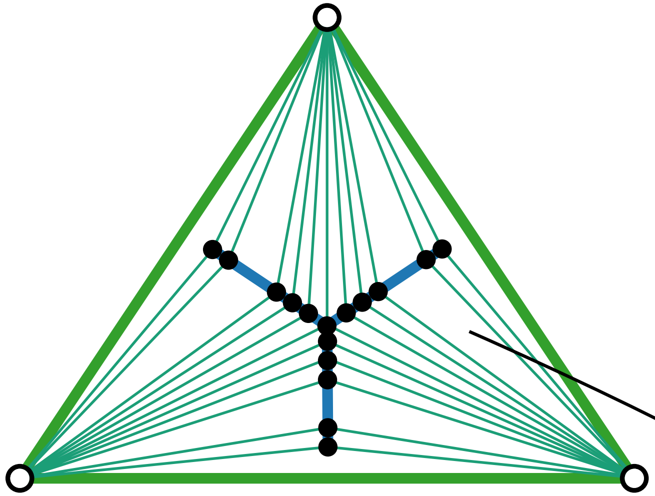

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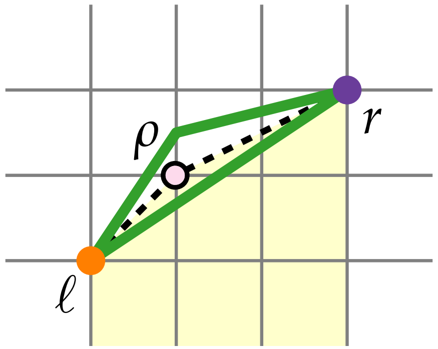

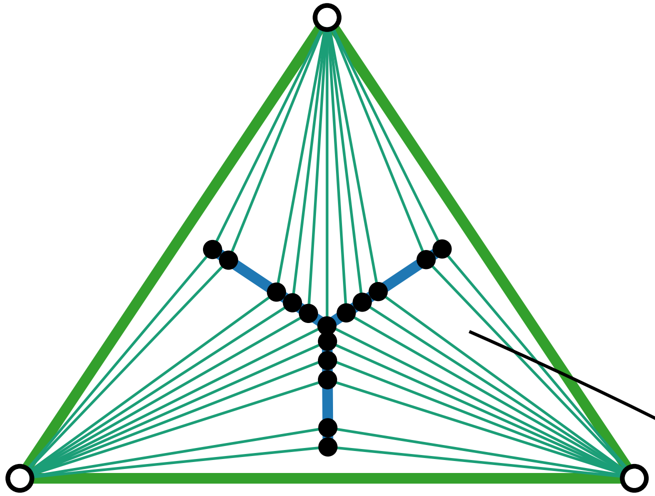

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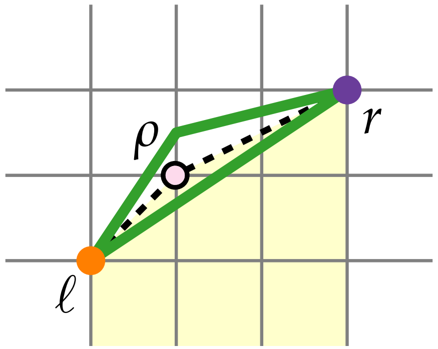

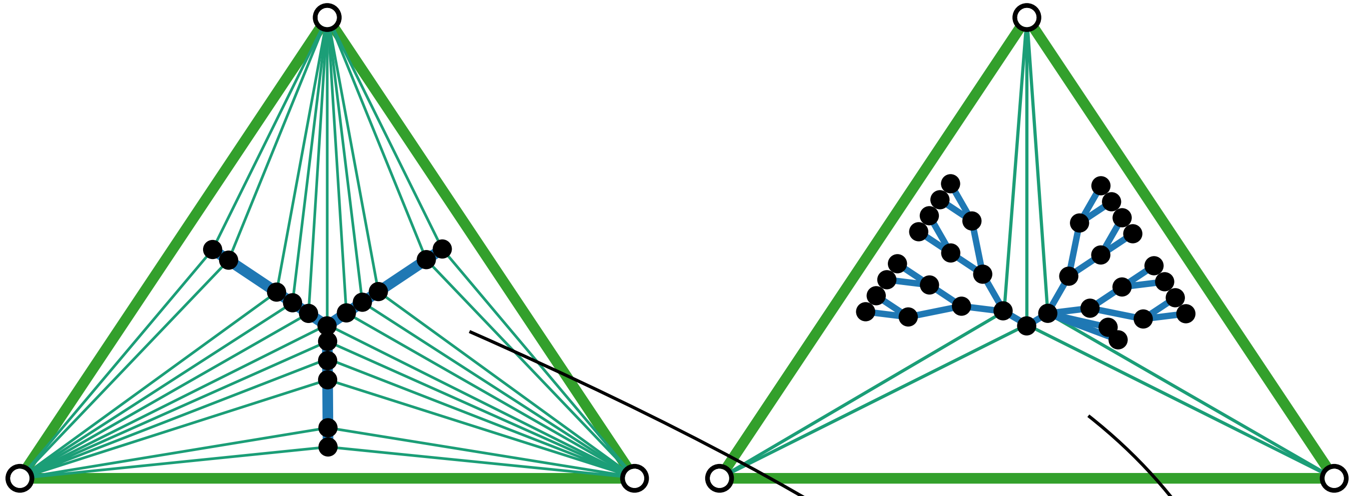

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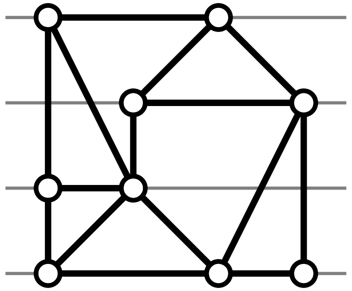

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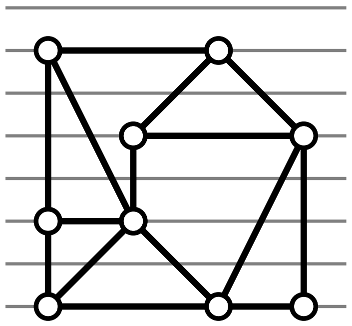

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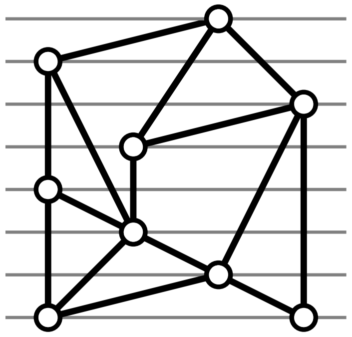

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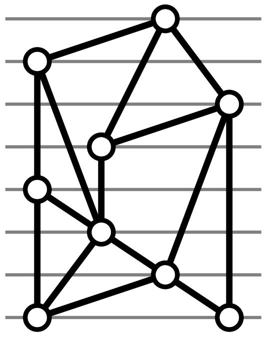

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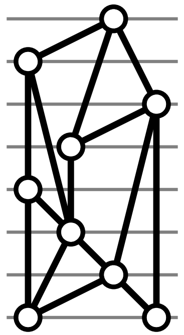

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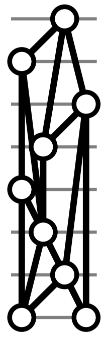

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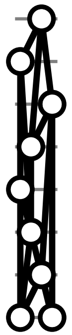

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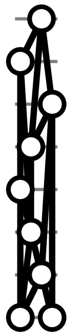

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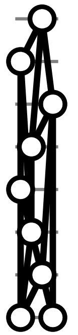
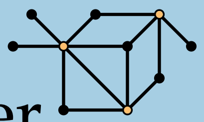
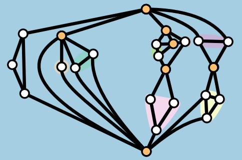


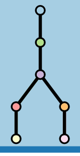
The previously best-known bound was $\mathcal{O}(n^{0.695})$.

Overview

FPT algorithms

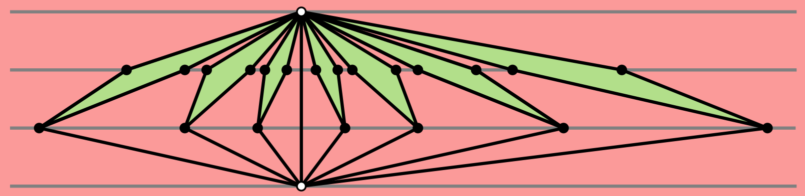
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

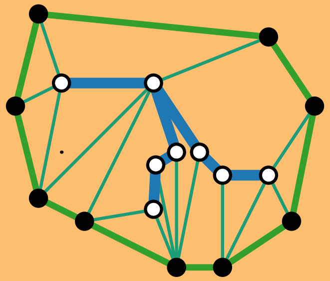
Reduction from leveled planar



Combinatorial results

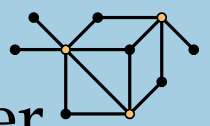
Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

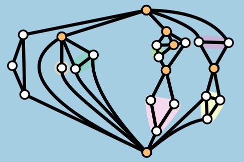
Implications

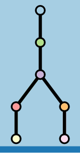


Overview

FPT algorithms

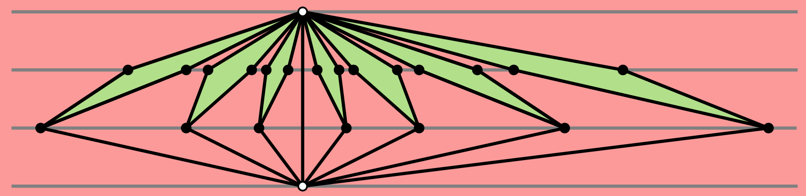
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

Reduction from leveled planar



Combinatorial results

Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
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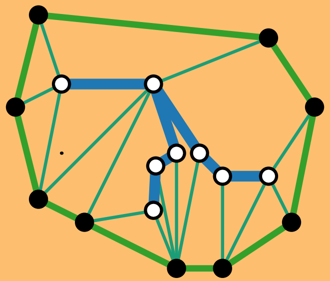
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

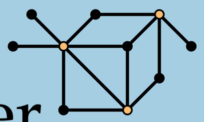
Implications

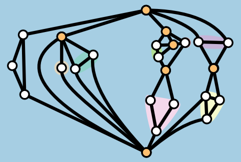


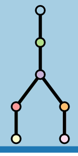
Treewidth-2 graphs have edge-length ratio $\mathcal{O}(\sqrt{n})$

Overview

FPT algorithms

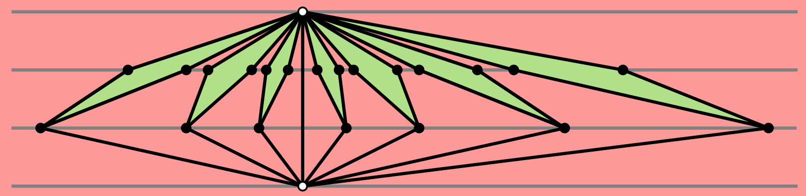
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth 

NP-hardness

Reduction from leveled planar

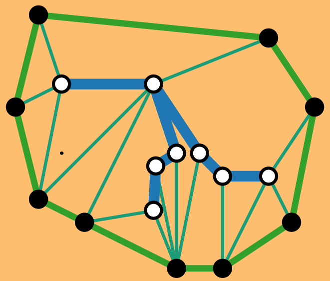


Combinatorial results

Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

Implications

3-connected cycle-trees have edge-length ratio ≤ 9

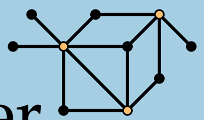


Treewidth-2 graphs have edge-length ratio $\mathcal{O}(\sqrt{n})$

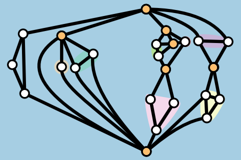
Overview

FPT algorithms

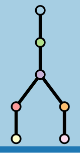
Linear kernel in size of vertex cover



FPT in size of b-modulator ($+b$)

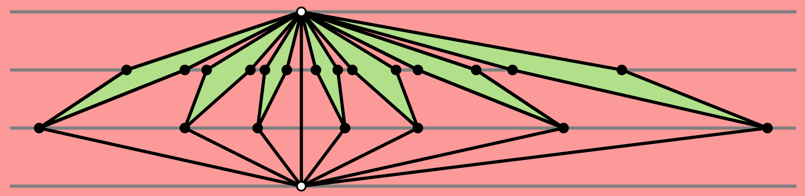


FPT in treedepth



NP-hardness

Reduction from leveled planar

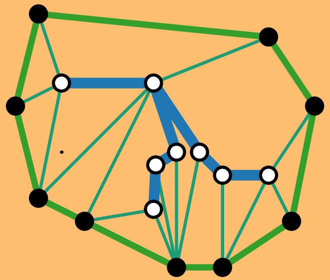


Combinatorial results

Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

Implications

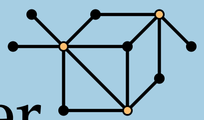
3-connected cycle-trees have
edge-length ratio ≤ 9
queue number ≤ 5

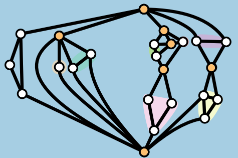


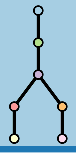
Treewidth-2 graphs have
edge-length ratio $\mathcal{O}(\sqrt{n})$

Overview

FPT algorithms

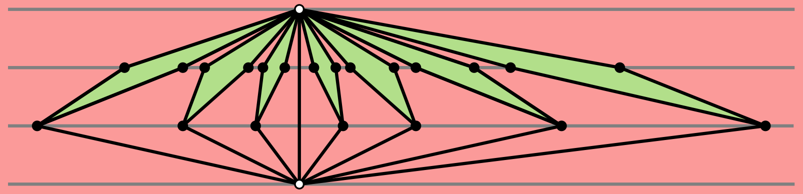
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth  Poly. kernel?

NP-hardness

Reduction from leveled planar

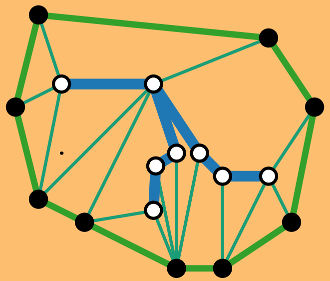


Combinatorial results

Graph class	LB	UB
2-outerplanar	$\Omega(n)$	n
3-connected cycle-tree	4	4
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

Implications

3-connected cycle-trees have



edge-length ratio ≤ 9

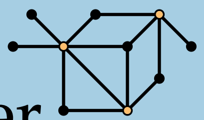
queue number ≤ 5

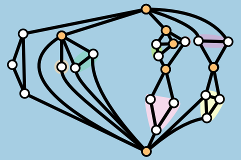
Treewidth-2 graphs have

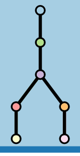
edge-length ratio $\mathcal{O}(\sqrt{n})$

Overview

FPT algorithms

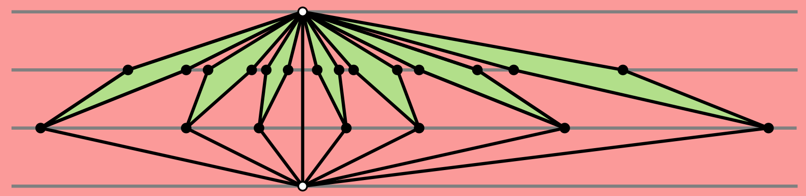
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth  Poly. kernel? Treewidth?

NP-hardness

Reduction from leveled planar



Combinatorial results

Graph class	LB	UB
2-outerplanar	$\Omega(n)$	n
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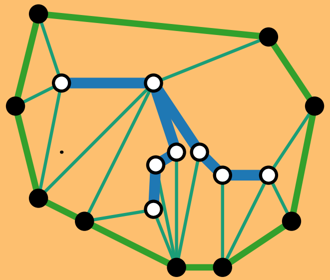
Implications

3-connected cycle-trees have



edge-length ratio ≤ 9

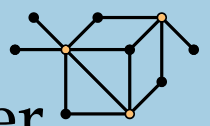
queue number ≤ 5

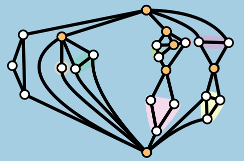
Treewidth-2 graphs have

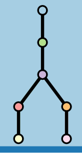
edge-length ratio $\mathcal{O}(\sqrt{n})$

Overview

FPT algorithms

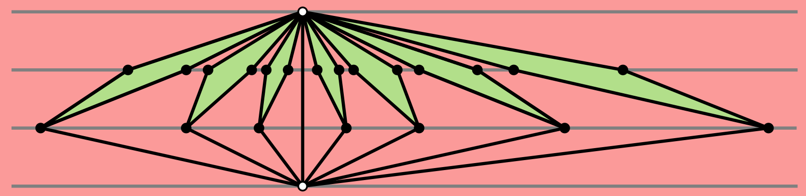
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth  Poly. kernel? Treewidth?

NP-hardness

Reduction from leveled planar



Combinatorial results

Graph class	**LB**	**UB**
2-outerplanar	$\Omega(n)$	n
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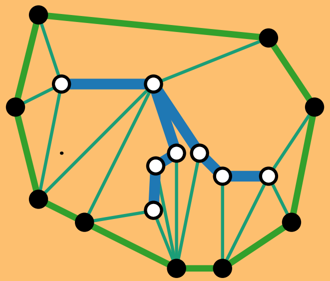
cycle-tree	$\Omega(\log n)$	$\mathcal{O}(\log n)$
treewidth 2	$2^{\Omega(\sqrt{\log n})}$	$\mathcal{O}(\sqrt{n})$

?

Implications

3-connected cycle-trees have



edge-length ratio ≤ 9

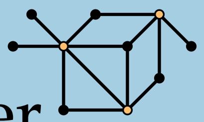
queue number ≤ 5

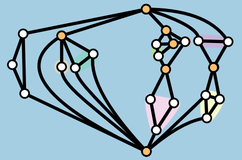
Treewidth-2 graphs have

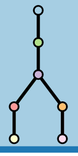
edge-length ratio $\mathcal{O}(\sqrt{n})$

Overview

FPT algorithms

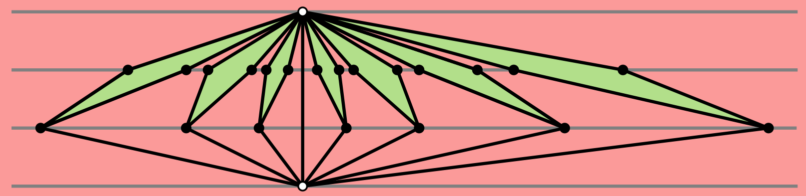
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth  Poly. kernel? Treewidth?

NP-hardness

Reduction from leveled planar



Combinatorial results

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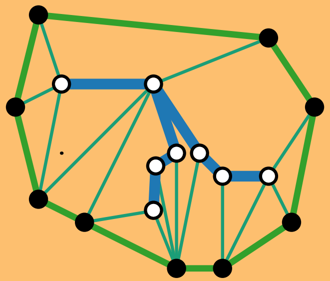
Implications

3-connected cycle-trees have



edge-length ratio ≤ 9

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Treewidth-2 graphs have

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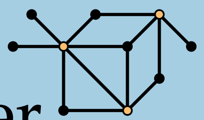
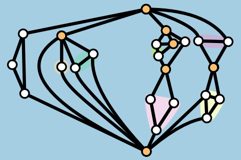
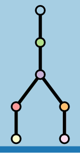
$\Omega(\log n)$

Overview

FPT algorithms

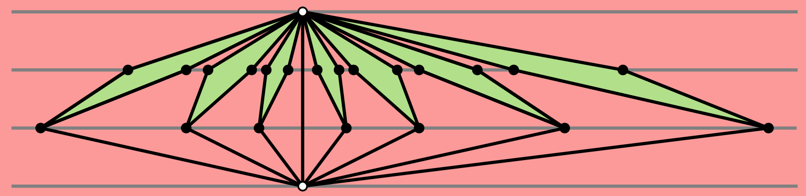
Linear kernel in size of vertex cover 

FPT in size of b-modulator $(+b)$ 

FPT in treedepth  Poly. kernel? Treewidth?

NP-hardness

Reduction from leveled planar



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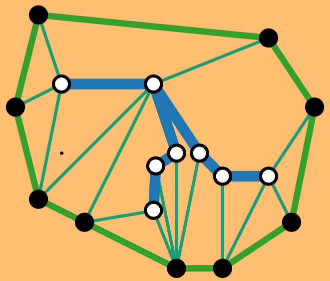
Implications

3-connected cycle-trees have



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