Evolutionary Algorithms for One-sided Bipartite Crossing Minimisation

Jakob Baumann • Ignaz Rutter • Dirk Sudholt

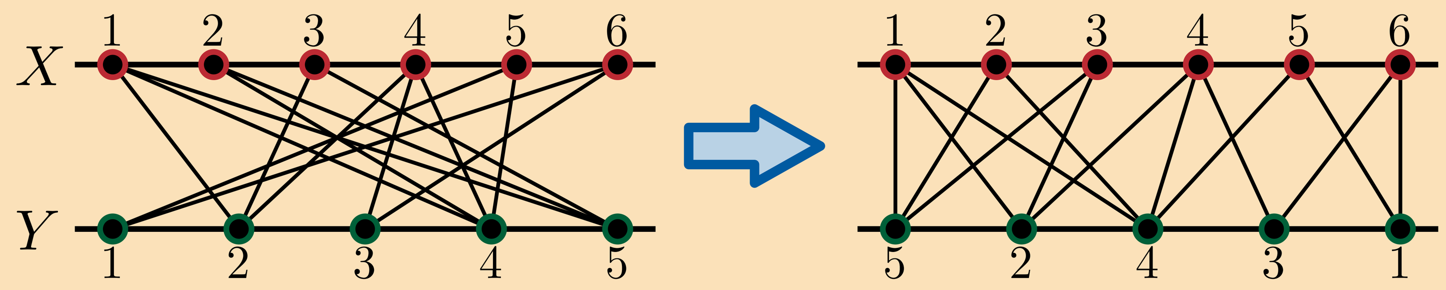
UNIVERSITÄT PASSAU
Fakultät für Informatik und Mathematik

Goals

- Understand EAs in the context of graph drawing by exploring a simple GD problem
- Complement theoretical findings in [1]
- Design specialised EA-based algorithm and compare it to the state-of-the-art

One-Sided Bipartite Crossing Minimization (OBCM)

Input: Bipartite Graph $G = (X, Y, E)$
Permutation π_X of X



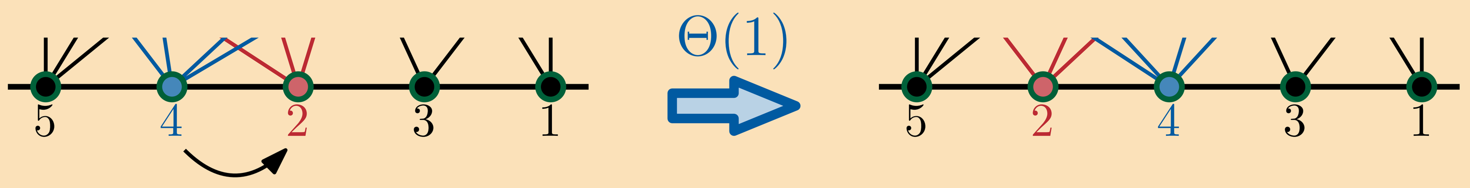
Problem: Find a permutation π_Y minimising the number of edge crossings

State-of-the-art algorithms

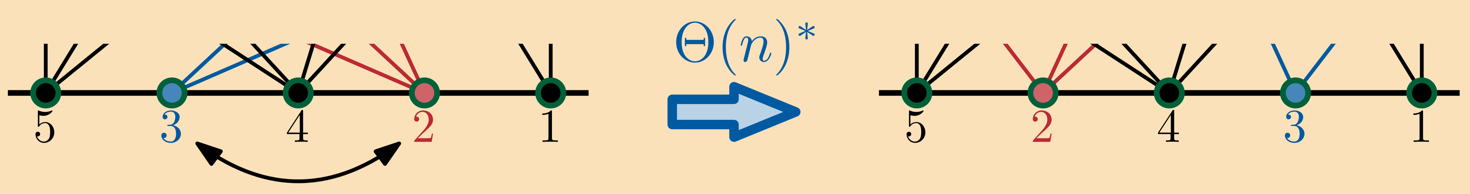
- **Median/Barycenter** $\Theta(n \log n + m)$: Place vertices at the average of their neighbors [2]
- **Nagamochi's algorithm** $\Theta(n \log n)$: Place vertices by some specific scheme; best theoretical bound [3]
- **Sifting** $\Theta(n^2 + nm)$: Sequentially place vertices at locally optimal positions [4]

Common permutation-based mutation operators

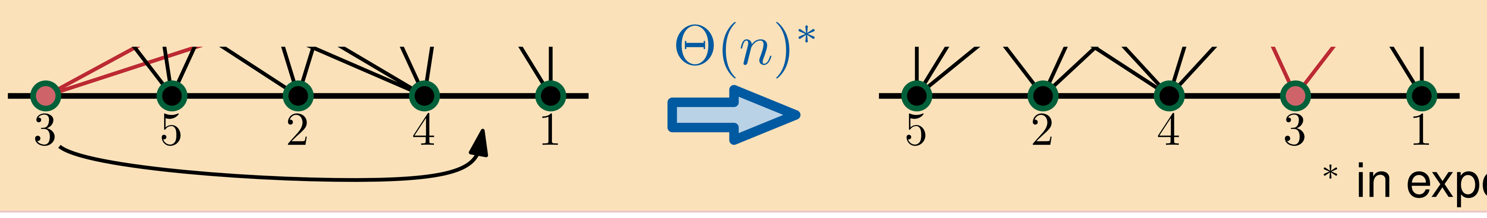
Swap: Pick vertex (u.a.r.) and swap with right neighbor



Exchange: Pick two vertices (u.a.r.) and exchange their positions



Jump: Pick a vertex (u.a.r.) and jump it to a new position (u.a.r.)



* in expectation

Evolutionary algorithms

- The **X-EA** is a typical (1+1)-EA with X being one of the mutation operators swap, exchange, and jump

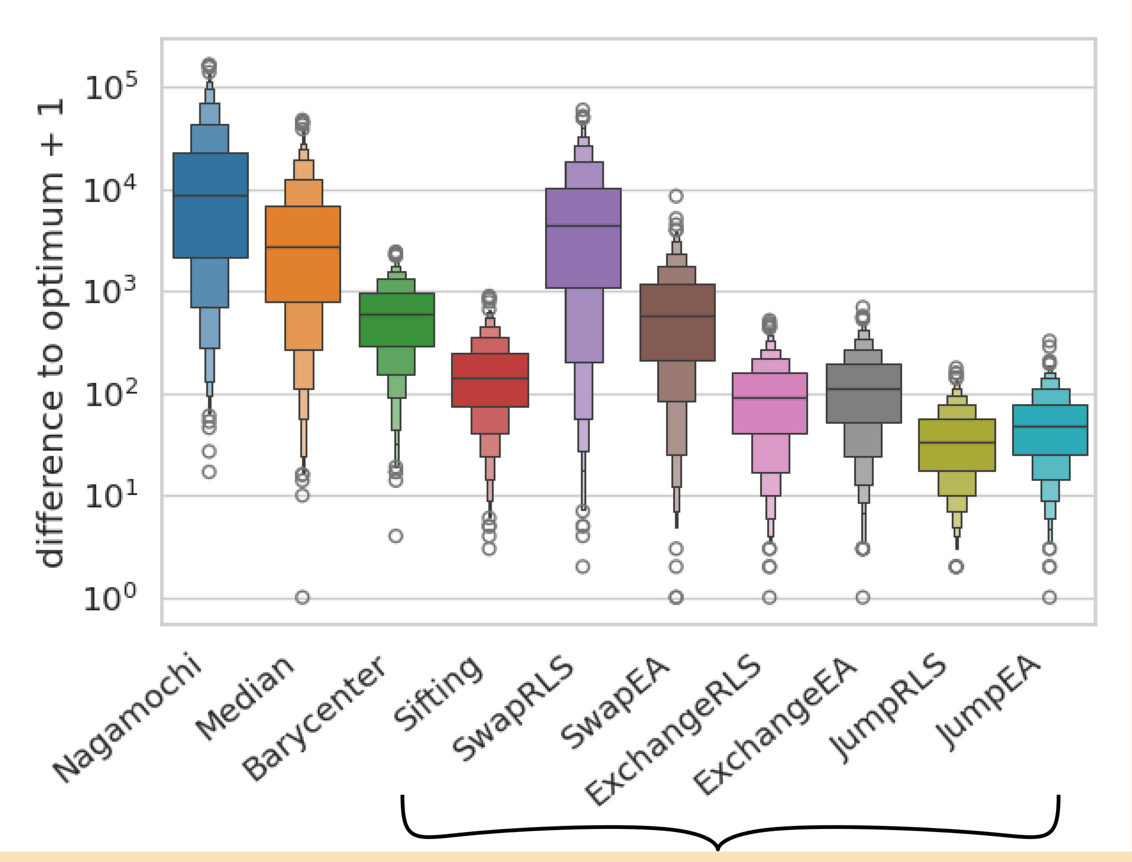
Algorithm 1: (1+1)-EA for permutation-based optimisation.

1. Choose permutation π u.a.r.;
2. **while** *stopping criterion not met* **do**
3. Choose k by Poisson distribution with $\lambda = 1$;
4. $\pi' \leftarrow$ apply mutation X $(k+1)$-fold to π;
5. **if** $c(\pi') \leq c(\pi)$ **then** $\pi \leftarrow \pi'$;

- **X-RLS** is a randomised local search variant, where $k = 1$ is constant

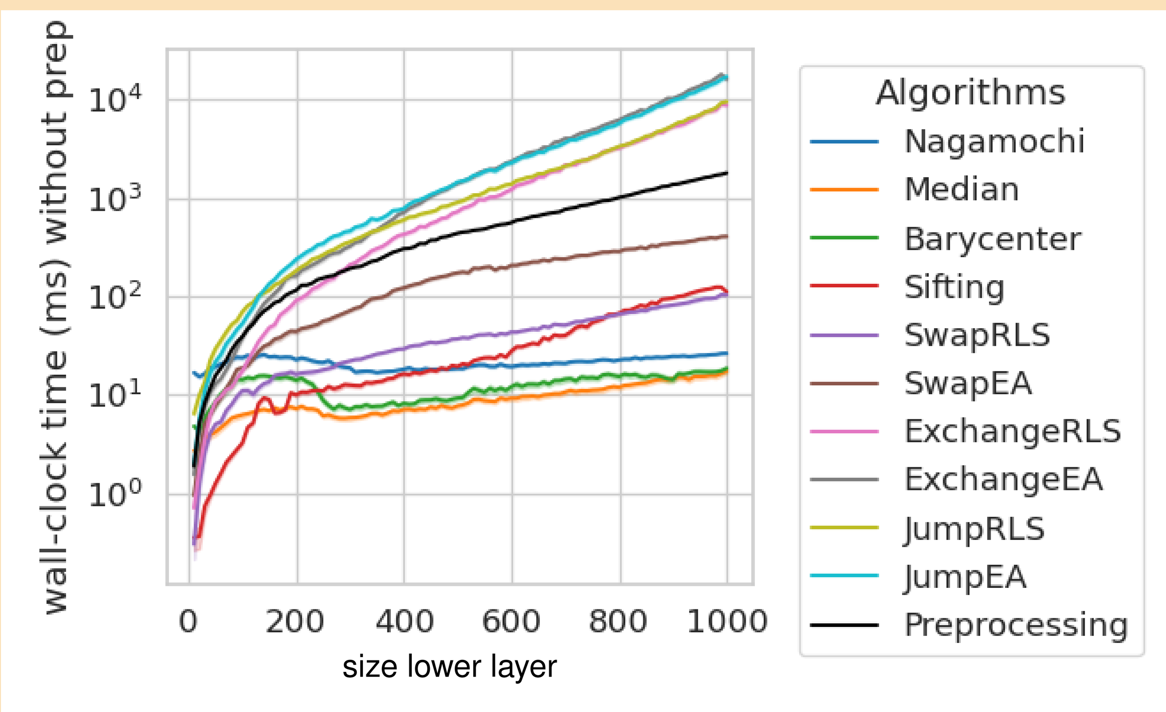
Experimental Comparison

- Instances of size $100 + 100$, random edges (with density 0.02 to 0.08)
- Stopping criterion: no improvement for $n^{1.5}$ generations

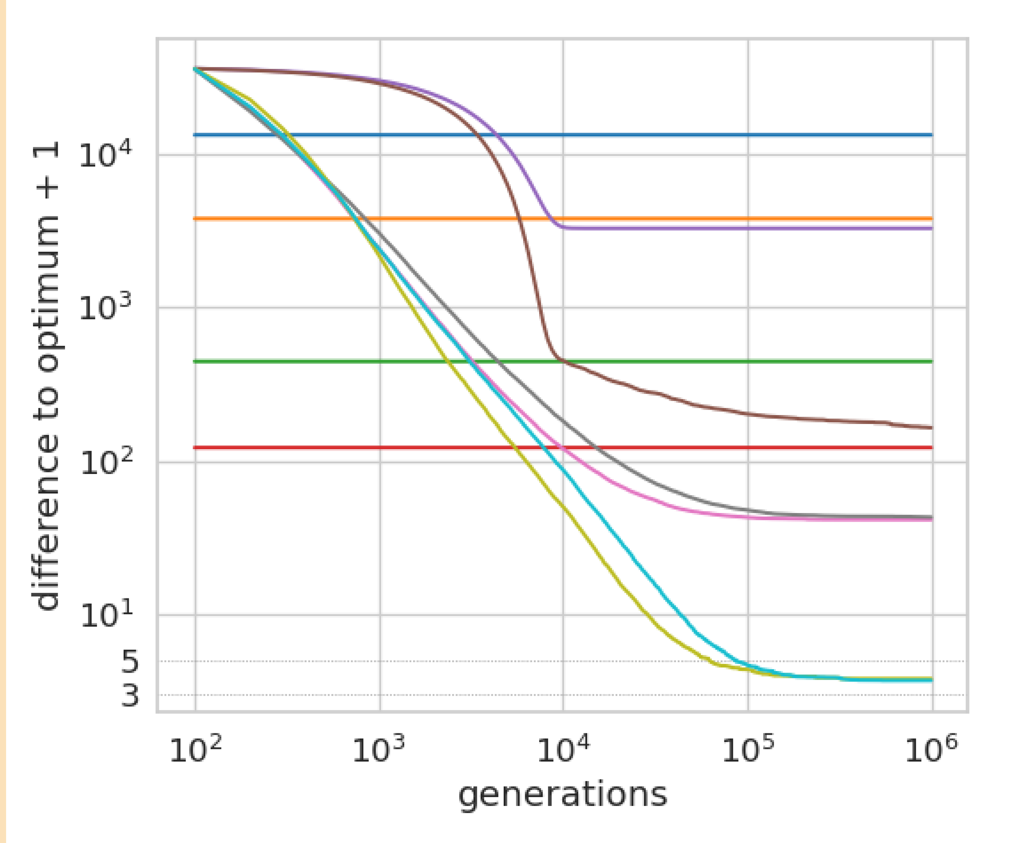


Preprocessing step $\Theta(nm)$

- Preprocessing usually takes a significant amount of time
- Jump- and Exchange-EAs need most time but get best results
- Tested on different classes, results look alike

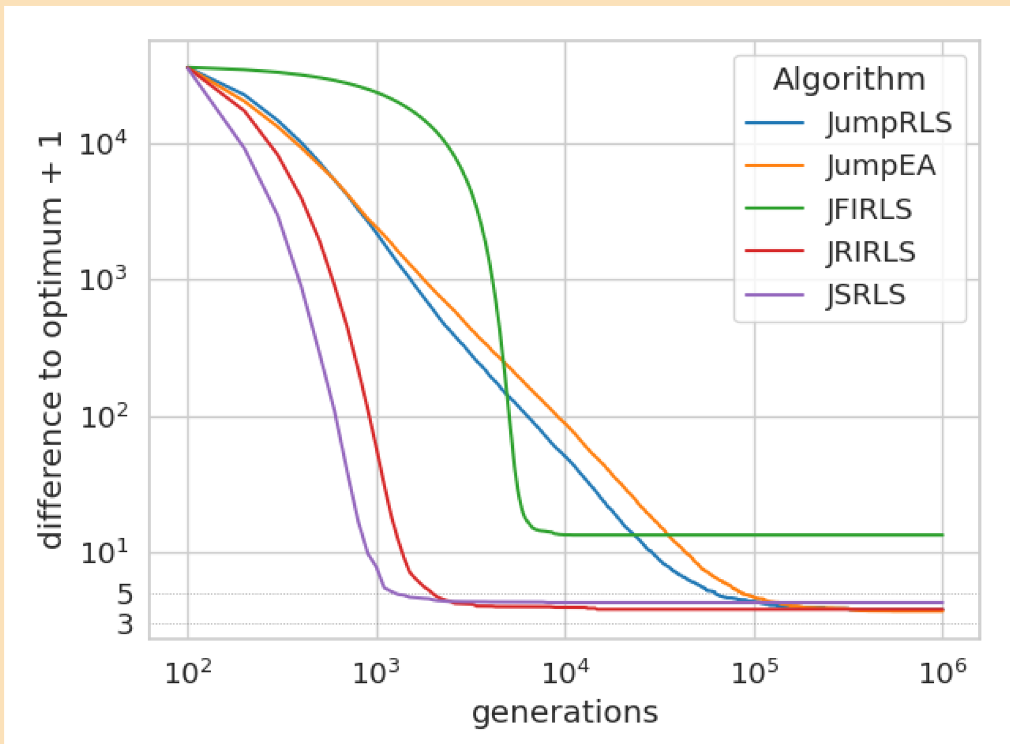


- Fitness over time plot reveals jump is by far the best! Taking roughly $n^{2.5}$ generations
- Performance differences are validated by Wilcoxon rank-sum test



Faster Convergence through Improved Algorithm Designs

- No overhead: instead of random jumps, scan for a fitness-improving jump
- Tested three variants: Select vertex u.a.r. then
  - **JFI-RLS**: Choose the first improving jump
  - **JRI-RLS**: Scan all jumps and choose an improving one u.a.r.
  - **JS-RLS**: scan all jumps and take the best (*sift*)
- Much better convergence rate for JRI- and JS-RLS, factor 100 $(= n?)$



- JFI-RLS statistically significantly worse, JS- and JRI-RLS roughly equal
- Very close approximation to optimum. JRI-RLS can better explore the plateau

Conclusion and Future Work

- Even simple "(1+1)"-type EA can beat best approx. algorithms (in reasonable time)
- Solutions become astonishingly close to the optimum
- Improvement with problem-specific knowledge yields very practical algorithm
- Can we prove an approximation ratio for jumps in poly time?
- Can the algorithm be improved using populations and crossover?
- Extend analysis to multiple layers, and see how EAs can actually be used to draw aesthetically pleasing graphs

1. J. Baumann, I. Rutter, D. Sudholt: Evolutionary computation meets drawing: Runtime analysis for crossing minimisation on layered graph drawings. *In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '24). ACM Press, 2024.481* (2024).
2. P. Eades and N. C. Wormald. Edge crossings in drawings of bipartite graphs. *Algorithmica, 11(4):379-403* (1994).
3. H. Nagamochi. An improved bound on the one-sided minimum crossing number in two-layered drawings. *Discrete and Computational Geometry, 33:569-591* (2005).
4. C. Matuszewski, R. Schönfeld, and P. Molitor. Using sifting for k-layer straightline crossing minimization. *In Graph Drawing: 7th International Symposium, Proceedings 7: 217-224* (1999)