Polygonally Anchored Graph Drawing

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Anchored Graph Drawing

We explore polygonally anchored graph drawing, where some nodes may have positional information in the form of a polygonal region. In particular, we use the standard force-directed graph layout algorithm by Fruchterman-Reingold, modified to restrict nodes to their associated "anchor" region (if specified). The low energy layouts produced by such algorithms may reveal geographic information about nodes with no such knowledge a priori.

Figure 1. The Vienna rail system [1].

Some applications of graph drawing with partial positional information include location-based social networks and rail networks. Work by social scientists supports the idea that one's social network (of family, friends, coworkers, etc.) is one of the best predictors for the region they identify with [2]. And in rail networks, stations and connections are often associated with the area they bring service to, so their particular placement in a map may be ambiguous [3, 8].

Methodology

In the standard Fruchterman-Reingold algorithm, repulsive forces between nodes and attractive forces between adjacent nodes are applied iteratively until the global "temperature" of the system decays to 0, a quantity that controls the amount of displacement.

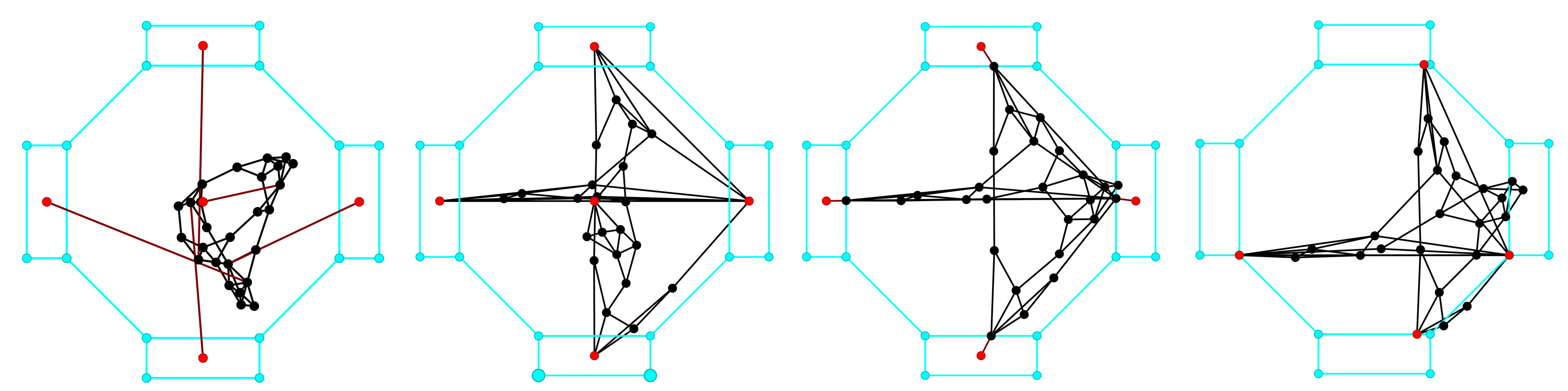
Our modification is to introduce an additional anchoring force from nodes to their associated region (if given) that is applied in each of these iterations. The exact direction of the anchoring force is chosen via one of our three metrics.

In order to ensure that such a node remains tethered to its the associated region, we apply a displacement force towards the region that is multiplied by a sufficiently large constant to act as if it were an "infinite force".

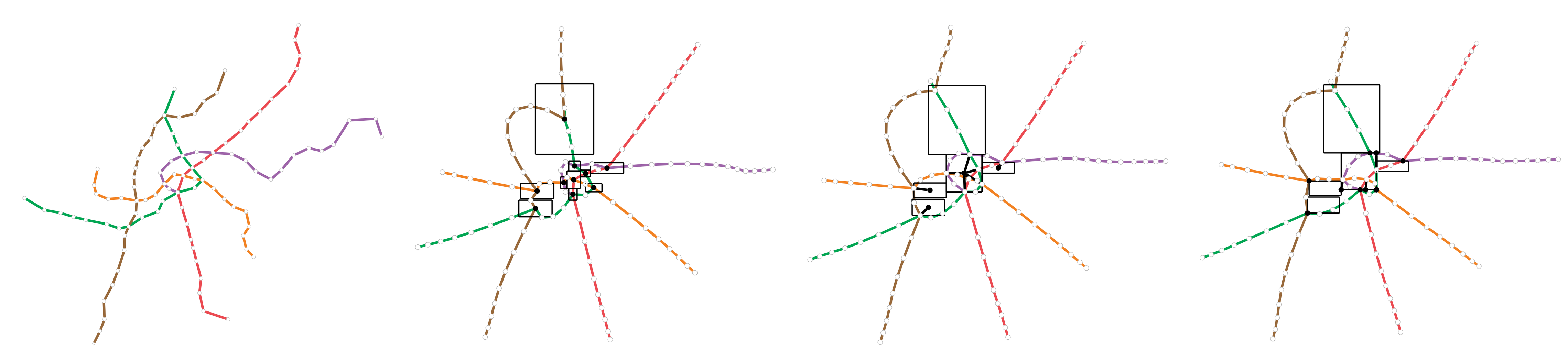
Graphs with Polygonal Anchoring Forces

Table 1. For (a) and (c), the black nodes and edges denote the original input graph, while the teal denotes the anchor regions. The anchoring forces are shown with a red edge connecting the vessel to the last point to pull it (the red node may overlap the vessel). The graphs shown from left-to-right are the input graph, graph with centroidal forces, graph with inside-out forces, and graph with closest point forces.

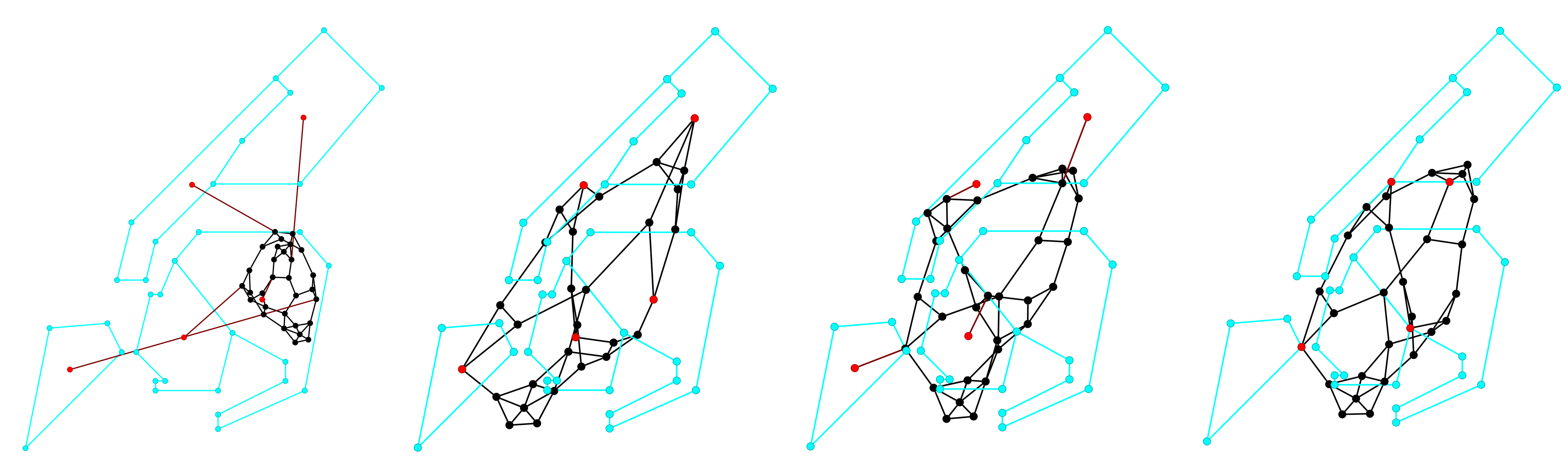
Input Graph	Centroidal Force	Inside-Out Force	Closest Point Force



(a) The input graph is a force-directed graph, where 5 nodes are anchored to one of the 5 regions, the octagon and the 4 rectangles.



(b) The input graph shows the Vienna subway map with its real geographical locations. The 10 stations with intersections/transfers are anchored (6 of them share the same anchor for the non-centroidal forces).



(c) The input graph shows a social network in Queens, one of the 5 boroughs of NYC. 5 nodes are then anchored one of the 5 boroughs.

Three Metrics for Anchoring Forces

We denote the region associated with a point to be its *anchor*, and call that point a *vessel*. We consider three types of anchoring forces:

1. **Centroidal Force:** Every point from the anchor region applies a displacement force (the displacement vector from the vessel to the point).
2. **Inside-Out Force:** The same force as in the first metric, but it is only applied if the vessel point is outside the anchor region.
3. **Closest Point Force:** Pull the vessel towards the closest point in the anchor region, but only if the vessel is outside that region.

Centroidal Force Equivalence

We show that the centroidal force is indeed equivalent to applying one singular force from the region's centroid, scaled by the region's area. Let $v = (v_x, v_y)$ be a vessel point attached to an anchor point (x, y), and let P be the region defined by an anchor with area A. Its centroid will be given by the point $c = (c_x, c_y)$, where

$$c_x = \frac{\iint_P x\,dx\,dy}{\iint_P dx\,dy} = \frac{\iint_P x\,dx\,dy}{A}, c_y = \frac{\iint_P y\,dx\,dy}{\iint_P dx\,dy} = \frac{\iint_P y\,dx\,dy}{A}$$

If every point (x, y) in an anchor region P applies a force vector $C \cdot \langle x - v_x, y - v_y \rangle$ on the vessel point, then the total force applied will be

$$F_x = \iint_P C \cdot (x - v_x)\,dx\,dy = CAc_x - CAv_x = CA(c_x - v_x)$$

$$F_y = \iint_P C \cdot (y - v_y)\,dx\,dy = CAc_y - CAv_y = CA(c_y - v_y)$$

Notice that the total force $\langle F_x, F_y \rangle$ is equal to just one force being applied from the centroid c, scaled up by the anchor's area A.

References

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