Particle Therapy Patient Scheduling: First Heuristic Approaches

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1 Test Instances

We created artificial benchmark instances related to the expected situation at MedAustron and real particle treatments. The main characteristic of an instance is its number of therapies n_T. We consider 5 instances for 10, 20, 50, 70, 100, 150, 200, and 300 therapies. In the used naming schema we encode first the number of therapies followed by a consecutive number.

The results provided in Section 4 are achieved on transformed instances and applying the preprocessing described in Section 3 prior to executing the algorithm proposed in the paper. In the original instances the unavailability periods modeling the lunch break have been added in such a way that each activity requires exactly one resource with the unavailability period representing the lunch break. To simplify the detection of unavailability periods affecting all activities we introduced in the instance format the concept of global unavailability periods, which represent unavailability periods on all resources. Section 2 shows the updated input format.

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2 Input Format

The instances are encoded in JSON. Note that the instance format described below allows to state instances for a more general problem, hence it contains elements that are not relevant for the Particle Therapy Patient Scheduling Problem (PTPSP). An instance is represented by the following JSON-object:

- `GENERAL`: object, contains globally relevant problem information
  - `beam-resource-id`: integer, ID of the Beam resource
  - `proton-resource-id`: not relevant for PTPSP
  - `carbon-resource-id`: not relevant for PTPSP
  - `IR-rooms`: array of integers, IDs of the irradiation room resources, not relevant for PTPSP
  - `anesthetist-id`: not relevant for PTPSP
  - `working-days`: array of arrays of objects, working days partitioned into weeks sorted in increasing order of the day index
    - `d`: unique positive integer, index of day $d \in D'$
    - `start`: integer, opening time $\widetilde{W}_d^{\text{start}}$ in minutes
    - `end`: integer, closing time $\widetilde{W}_d^{\text{end}}$ in minutes
    - `unavailable`: array of objects (optional), global unavailability periods, i.e., unavailability periods for all resources $r \in R$
      - `start`: start time in minutes (is interpreted as $\overline{W}_{r,d,w}^{\text{start}}$ for all $r \in R$)
      - `end`: end time in minutes (is interpreted as $\overline{W}_{r,d,w}^{\text{end}}$ for all $r \in R$)
- `RESOURCES`: array of objects, all resources and their availabilities
  - `id`: unique positive integer, resource ID
  - `name`: string (optional), name for resource
  - `W`: array of objects
    - `d`: unique integer, day d
    - `start`: integer (optional, default: $\widetilde{W}_d^{\text{start}}$), start time $W_{r,d}^{\text{start}}$ in minutes
    - `end`: integer (optional, default: $\widetilde{W}_d^{\text{end}}$), end time $W_{r,d}^{\text{end}}$ in minutes
    - `unavailable`: array of objects (optional), unavailability periods
      - `start`: start time $\overline{W}_{r,d,w}^{\text{start}}$ in minutes

        - `end`: end time $\overline{W}^{\text{end}}_{r,d,w}$ in minutes

- `THERAPIES`: array of objects, all therapies with their data
  - `id`: unique positive integer, therapy ID
  - `name`: string (optional), name of the therapy
  - `n-twmin`: integer (optional, default: 4), minimum number of treatments per week n_t^{twmin}
  - `n-twmax`: integer (optional, default: 5), maximum number of treatments per week n_t^{twmax}
  - `delta-min`: integer (optional, default: 1), min. number of days between two consecutive DTs $\delta_t^{\min}$
  - `delta-max`: integer (optional, default: 5), max. number of days between two consecutive DTs $\delta_t^{\max}$
  - `daily-treatments`: array of objects, all DTs are given in the required order
    - `id`: unique positive integer, DT ID
    - `name`: string (optional), name of DT
    - `d-min`: integer (optional, default: 0), earliest possible day $d_{t,u}^{\min}$
    - `d-max`: integer (optional), latest possible day $d_{t,u}^{\max}$; if not specified or -1 no bound is assumed (an implicit limit is given through the number of considered days)
    - `activities`: array of objects, all activities that must be scheduled in this order at a single day
      - `id`: unique positive integer, activity ID
      - `name`: string (optional), name of activity
      - `p`: positive integer, processing time $p_{t,u,a}$ in minutes
      - `resources`: array of integers (optional), ID's of required resources
    - `min-lag`: array of objects (optional), minimum EtS time lag constraints
      - `a1`: integer, ID of first activity a
      - `a2`: integer, ID of second activity a'
      - `time`: integer, minimum EtS time lag $L^{\min}_{t,u,a,a'}$ in minutes
    - `max-lag`: array of objects (optional), maximum EtS time lag constraints
      - `a1`: integer, ID of first activity a
      - `a2`: integer, ID of second activity a'
      - `time`: integer, maximum EtS time lag $L^{\max}_{t,u,a,a'}$ in minutes

3 Preprocessing

The applied preprocessing technique utilizes the observation that, due to the structure of the DTs, some resources cannot be used by activities close to resource availability changes. Thus, some resource availabilities can be pruned.

For example, if the preparation steps that need to be done before the beam take for every DT at least 20 minutes then the beam resource cannot be used during the first 20 minutes of the day.

Let q_r^{pre} be a lower bound on the earliest time a resource $r \in R$ might be used by an activity iff all resources become available at the same time.

$$q_r^{\text{pre}} = \min_{t \in T} \min_{u \in U_t} \min_{a' \in A_{t,u} \; | r \in Q_{t,u,a}} \left(\sum_{a=1}^{a'-1} p_{t,u,a} + \sum_{(a,a+1) \in P_{t,u}^{\min} | 0 < a < a'} L_{t,u,a,a+1}^{\min} \right)$$

Let q_r^{post} be a lower bound on the latest time a resource $r \in R$ might be used by an activity iff all resources become available at the same time.

$$q_r^{\text{post}} = \min_{t \in T} \min_{u \in U_t} \min_{a' \in A_{t,u} | r \in Q_{t,u,a}} \left(\sum_{a=a'+1}^{\alpha_{t,u}} p_{t,u,a} + \sum_{(a,a+1) \in P_{t,u}^{\min} | a' \leq a < \alpha_{t,u}} L_{t,u,a,a+1}^{\min} \right)$$

The resource availabilities can be pruned by setting

$$\begin{aligned} W_{r,d} &:= [\max(W_{r,d}^{\text{start}}, \widetilde{W}_d^{\text{start}} + q_r^{\text{pre}}), W_{r,d}^{\text{end}}] \qquad \forall r \in R, \forall d \in D' \\ \widehat{W}_{r,d} &:= [W_{r,d}^{\text{end}}, \widetilde{W}_d^{\text{end}} - q_r^{\text{post}}] \qquad \forall r \in R, \forall d \in D'. \end{aligned}$$

Moreover, we are able to increase the size of unavailability periods derived from global unavailability periods by setting them to

$$\overline{W}_{r,d,w} = [\overline{W}_{r,d,w}^{\text{start}} - q_r^{\text{post}}, \overline{W}_{r,d,w}^{\text{end}} + q_r^{\text{pre}}].$$

4 Updated Computational Results

Instance	TWCH's DayAssignment			Relaxed MILP		
	$\overline{da\text{-}obj}$	$\sigma(da\text{-}obj)$	$time[s]$	$da\text{-}obj$	LB	$time[s]$
ptpsp_010-01	**0.220**	0.000	0.029	**0.220**	0.220	3.3
ptpsp_010-02	**0.160**	0.000	0.031	**0.160**	0.160	5.0
ptpsp_010-03	**0.160**	0.000	0.026	**0.160**	0.160	3.3
ptpsp_010-04	**0.180**	0.000	0.028	**0.180**	0.180	4.2
ptpsp_010-05	**0.180**	0.000	0.032	**0.180**	0.180	5.7
ptpsp_020-01	**0.400**	0.000	0.045	**0.400**	0.400	6.9
ptpsp_020-02	**0.450**	0.000	0.050	**0.450**	0.450	10.1
ptpsp_020-03	**0.460**	0.000	0.054	**0.460**	0.460	9.6
ptpsp_020-04	**0.320**	0.000	0.051	**0.320**	0.320	10.6
ptpsp_020-05	**0.320**	0.000	0.048	**0.320**	0.320	7.7
ptpsp_050-01	15.394	0.181	0.116	**2.430**	2.143	7200.0
ptpsp_050-02	7.709	0.152	0.107	**2.200**	2.057	7200.0
ptpsp_050-03	20.147	0.000	0.127	**3.960**	3.675	7200.0
ptpsp_050-04	7.355	0.224	0.117	**2.610**	1.989	7200.0
ptpsp_050-05	19.835	0.069	0.114	**3.533**	3.165	7200.0
ptpsp_070-01	**55.388**	0.795	0.167	NA	13.877	7200.0
ptpsp_070-02	**67.584**	0.440	0.165	NA	23.165	7200.0
ptpsp_070-03	7.079	0.210	0.158	**3.843**	3.093	7200.0
ptpsp_070-04	25.268	0.243	0.162	**7.500**	6.697	7200.0
ptpsp_070-05	15.836	0.126	0.157	**4.727**	4.270	7200.0
ptpsp_100-01	18.916	0.964	0.221	**4.620**	4.339	7200.0
ptpsp_100-02	12.893	0.906	0.226	**4.590**	4.160	7200.0
ptpsp_100-03	28.005	0.886	0.230	**5.530**	5.125	7200.0
ptpsp_100-04	**26.822**	0.209	0.235	NA	6.171	7200.0
ptpsp_100-05	16.829	0.569	0.231	**4.250**	3.996	7200.0
ptpsp_150-01	**83.526**	0.732	0.377	NA	35.853	7200.0
ptpsp_150-02	**59.069**	0.737	0.362	NA	12.420	7200.0
ptpsp_150-03	**32.405**	0.274	0.343	NA	11.504	7200.0
ptpsp_150-04	24.303	0.459	0.334	**9.310**	8.355	7200.0
ptpsp_150-05	**60.536**	0.206	0.359	NA	15.056	7200.0
ptpsp_200-01	54.054	0.740	0.488	**13.343**	12.566	7200.0
ptpsp_200-02	105.074	0.533	0.470	**30.680**	28.607	7200.0
ptpsp_200-03	138.127	1.239	0.505	**50.627**	49.948	7200.0
ptpsp_200-04	**156.511**	1.967	0.511	NA	52.011	7200.0
ptpsp_200-05	68.851	0.317	0.491	**32.120**	31.411	7200.0
ptpsp_300-01	**142.361**	1.046	0.752	NA	-15.135	7200.0
ptpsp_300-02	**173.495**	2.466	0.744	NA	-15.649	7200.0
ptpsp_300-03	**153.488**	2.185	0.782	NA	-15.711	7200.0
ptpsp_300-04	**34.127**	1.157	0.733	NA	-15.272	7200.0
ptpsp_300-05	**163.548**	1.202	0.746	NA	-16.101	7200.0

Table 1: Results of TWCH's DayAssignment and the relaxed MILP, which also considers only day assignments.

Instance	TWCH			GRASP		IG	
	obj	$\sigma(obj)$	*time*[s]	*obj*	$\sigma(obj)$	*obj*	$\sigma(obj)$
ptpsp_010-01	**0.220**	0.000	0.029	**0.220**	0.000	**0.220**	0.000
ptpsp_010-02	**0.160**	0.000	0.033	**0.160**	0.000	**0.160**	0.000
ptpsp_010-03	**0.160**	0.000	0.026	**0.160**	0.000	**0.160**	0.000
ptpsp_010-04	**0.180**	0.000	0.028	**0.180**	0.000	**0.180**	0.000
ptpsp_010-05	**0.180**	0.000	0.033	**0.180**	0.000	**0.180**	0.000
ptpsp_020-01	**0.400**	0.000	0.048	**0.400**	0.000	**0.400**	0.000
ptpsp_020-02	**0.450**	0.000	0.053	**0.450**	0.000	**0.450**	0.000
ptpsp_020-03	**0.460**	0.000	0.059	**0.460**	0.000	**0.460**	0.000
ptpsp_020-04	**0.320**	0.000	0.056	**0.320**	0.000	**0.320**	0.000
ptpsp_020-05	**0.320**	0.000	0.052	**0.320**	0.000	**0.320**	0.000
ptpsp_050-01	137.633	3.006	0.125	90.729	1.674	**90.076**	1.362
ptpsp_050-02	110.162	3.328	0.116	**59.732**	0.899	60.062	1.037
ptpsp_050-03	141.862	0.030	0.138	99.357	1.010	**99.211**	0.978
ptpsp_050-04	130.468	3.719	0.129	80.334	3.137	**78.582**	2.356
ptpsp_050-05	152.183	0.524	0.127	106.755	2.428	**106.710**	2.037
ptpsp_070-01	285.514	3.376	0.180	196.236	1.096	**196.128**	1.006
ptpsp_070-02	273.576	2.338	0.176	**187.880**	2.579	189.072	2.755
ptpsp_070-03	133.315	2.884	0.167	81.386	1.781	**80.523**	1.653
ptpsp_070-04	183.234	1.712	0.171	**113.476**	0.990	113.683	1.251
ptpsp_070-05	117.064	1.453	0.162	**74.049**	1.406	74.333	1.245
ptpsp_100-01	158.296	5.158	0.248	97.666	3.514	**97.108**	2.944
ptpsp_100-02	101.672	3.062	0.255	56.287	1.996	**55.352**	2.128
ptpsp_100-03	217.635	3.827	0.259	**138.745**	2.881	138.851	2.147
ptpsp_100-04	136.362	2.451	0.263	92.526	0.941	**92.248**	1.177
ptpsp_100-05	218.756	8.927	0.256	**124.722**	1.728	124.906	1.779
ptpsp_150-01	308.486	1.940	0.422	242.174	1.814	**241.886**	1.538
ptpsp_150-02	337.925	4.270	0.417	246.376	1.585	**246.113**	1.360
ptpsp_150-03	245.445	4.273	0.386	161.461	1.801	**161.259**	1.601
ptpsp_150-04	130.472	5.718	0.378	77.765	2.086	**77.645**	2.250
ptpsp_150-05	254.204	3.411	0.402	175.912	1.112	**175.904**	1.461
ptpsp_200-01	326.001	4.473	0.533	213.734	2.545	**213.602**	2.058
ptpsp_200-02	405.628	4.338	0.530	310.743	1.670	**309.888**	2.269
ptpsp_200-03	477.678	9.252	0.568	**370.290**	1.894	370.686	2.094
ptpsp_200-04	544.991	7.560	0.568	432.290	5.469	**431.960**	4.799
ptpsp_200-05	303.526	4.011	0.536	226.527	1.830	**226.175**	2.119
ptpsp_300-01	693.159	7.051	0.848	**524.418**	2.961	524.564	3.256
ptpsp_300-02	776.983	8.528	0.841	**566.499**	4.183	566.849	4.607
ptpsp_300-03	727.553	10.955	0.860	521.020	5.366	**520.778**	5.285
ptpsp_300-04	432.027	12.356	0.831	240.094	7.798	**238.451**	6.455
ptpsp_300-05	696.374	11.895	0.849	**528.881**	2.621	530.379	2.565

Table 2: Average results of TWCH, GRASP and IG over 30 runs.