Supplementary material

This document contains supplementary material for the following paper:

[1] Iurlano, E., Raidl, G. R.: SAT-Based Search for Minwise Independent Families. (submitted 2024)

The following are the certifying instances for [1, Propositions 1, 3]. Readers interested in inspecting these objects can find them below as `Julia` lists together with a standalone feasibility checker in Listing 1.2—the latter can be used as follows:

```
F = [apply_perm(offset,g) for offset in theta for g in G]
k=5; is_mw_indep(F, k)
```

Listing 1.1. Certifiers

```
#d,n,k;|G|=60,6,6;6
θ = [[2,4,5,3,6,1],[2,5,1,4,6,3],[4,6,1,3,2,5],[2,6,1,5,3,4],[3,2,1,4,5,6],
     [5,4,1,2,3,6],[4,3,1,5,6,2],[6,5,2,3,4,1],[4,3,2,5,1,6],[6,3,2,1,5,4]] # |θ|=10
G = [[1,2,3,4,5,6],[5,4,2,3,6,1],[6,3,4,2,1,5],[4,5,6,1,2,3],[2,1,5,6,3,4],[3,6,1,5,4,2]] # |G|=6

#d,n,k;|G|=60,6,5;6
θ = [[4,5,6,1,2,3],[3,4,5,2,6,1],[1,3,6,2,5,4],[1,3,6,4,2,5],[2,4,6,3,1,5],
     [1,2,6,5,4,3],[2,5,3,6,1,4],[3,6,4,5,1,2],[2,3,1,6,4,5],[2,5,3,1,6,4]] # |θ|=10
G = [[1,2,3,4,5,6],[5,4,2,3,6,1],[6,3,4,2,1,5],[4,5,6,1,2,3],[2,1,5,6,3,4],[3,6,1,5,4,2]] # |G|=6

#d,n,k;|G|=60,5,5;60
θ = [[1,4,3,2,5]] # |θ|=1
G = [[1,2,3,4,5],[1,4,2,3,5],[1,3,4,2,5],[1,4,3,5,2],[1,5,4,3,2],[1,3,5,4,2],
     [1,5,3,2,4],[1,2,5,3,4],[1,3,2,5,4],[1,5,2,4,3],[1,4,5,2,3],[1,2,4,5,3],
     [2,1,5,4,3],[2,4,1,5,3],[2,5,4,1,3],[2,4,5,3,1],[2,3,4,5,1],[2,5,3,4,1],
     [2,3,5,1,4],[2,1,3,5,4],[2,5,1,3,4],[2,3,1,4,5],[2,4,3,1,5],[2,1,4,3,5],
     [4,1,2,5,3],[4,5,1,2,3],[4,2,5,1,3],[4,5,2,3,1],[4,3,5,2,1],[4,2,3,5,1],
     [4,3,2,1,5],[4,1,3,2,5],[4,2,1,3,5],[4,3,1,5,2],[4,5,3,1,2],[4,1,5,3,2],
     [5,1,4,2,3],[5,2,1,4,3],[5,4,2,1,3],[5,2,4,3,1],[5,3,2,4,1],[5,4,3,2,1],
     [5,3,4,1,2],[5,1,3,4,2],[5,4,1,3,2],[5,3,1,2,4],[5,2,3,1,4],[5,1,2,3,4],
     [3,2,4,1,5],[3,1,2,4,5],[3,4,1,2,5],[3,1,4,5,2],[3,5,1,4,2],[3,4,5,1,2],
     [3,5,4,2,1],[3,2,5,4,1],[3,4,2,5,1],[3,5,2,1,4],[3,1,5,2,4],[3,2,1,5,4]] # |G|=60

 #d,n,k;|G|=36,8,4;2
G = [[1,2,3,4,5,6,7,8],[8,7,6,5,4,3,2,1]] # |G|=2, exceptionally here considered as right-coset
θ = [[1,2,3,4,5,6,7,8],[8,5,6,7,2,3,4,1],[1,6,7,3,2,4,8,5],
     [5,2,4,8,6,7,3,1],[1,4,6,5,8,3,2,7],[7,8,3,2,4,6,5,1],
     [5,1,8,7,3,4,2,6],[6,3,4,2,1,8,7,5],[5,1,8,3,6,4,7,2],
     [2,6,4,7,1,8,3,5],[2,6,1,5,8,4,7,3],[3,8,4,7,6,1,5,2],
     [6,2,1,5,4,3,8,7],[7,4,3,8,2,1,5,6],[5,7,1,3,6,4,2,8],
     [8,6,4,2,7,1,3,5],[4,3,8,1,7,6,2,5],[5,7,6,2,3,8,1,4]] # |θ|=18, inferred from Na et al. 2023, Proposition 4.6(iii)

#d,n,k;|G|=48,8,4;24
θ = [[5,8,7,2,3,6,1,4],[6,5,1,4,2,3,8,7]] # |θ|=2
G = [[1,2,3,4,5,6,7,8],[1,5,7,2,4,3,6,8],[1,4,6,5,2,7,3,8],[8,7,4,3,6,5,2,1],
     [8,6,2,7,3,4,5,1],[8,3,5,6,7,2,4,1],[3,6,8,1,2,7,5,4],[3,2,5,6,1,8,7,4],
     [3,1,7,2,6,5,8,4],[4,5,1,8,7,2,6,3],[4,7,6,5,8,1,2,3],[4,8,2,7,5,6,1,3],
     [6,4,2,7,1,8,3,5],[6,1,3,4,7,2,8,5],[6,7,8,1,4,3,2,5],[5,3,7,2,8,1,4,6],
     [5,8,4,3,2,7,1,6],[5,2,1,8,3,4,7,6],[2,8,5,6,4,3,1,7],[2,4,1,8,6,5,3,7],
     [2,6,3,4,8,1,5,7],[7,1,6,5,3,4,8,2],[7,3,8,1,5,6,4,2],[7,5,4,3,1,8,6,2]] # |G|=24

#d,n,k;|G|=24,7,4;12
θ = [[7,2,5,6,3,4,1],[7,6,4,3,1,5,2]] # |θ|=2
G = [[1,2,3,4,5,6,7],[1,2,3,5,4,7,6],[3,2,1,5,4,6,7],[3,2,1,4,5,7,6],
     [6,2,7,3,1,5,4],[6,2,7,1,3,4,5],[7,2,6,1,3,5,4],[7,2,6,3,1,4,5],
     [5,2,4,7,6,1,3],[5,2,4,6,7,3,1],[4,2,5,6,7,1,3],[4,2,5,7,6,3,1]] # |G|=12

#d,n,k;|G|=24,6,4;24
θ = [[2,3,5,1,6,4]] # |θ|=1
G = [[1,2,3,4,5,6],[2,1,3,4,6,5],[5,6,3,4,1,2],[6,5,3,4,2,1],[1,2,4,3,6,5],[2,1,4,3,5,6],
     [6,5,4,3,1,2],[5,6,4,3,2,1],[3,4,5,6,1,2],[4,3,5,6,2,1],[1,2,5,6,3,4],[2,1,5,6,4,3],
     [3,4,6,5,2,1],[4,3,6,5,1,2],[2,1,6,5,3,4],[1,2,6,5,4,3],[5,6,1,2,3,4],[6,5,1,2,4,3],
     [3,4,1,2,5,6],[4,3,1,2,6,5],[5,6,2,1,4,3],[6,5,2,1,3,4],[4,3,2,1,5,6],[3,4,2,1,6,5]] # |G|=24

#d,n,k;|G|=12,5,4;6
θ = [[1,4,2,3,5],[5,2,4,3,1]] # |θ|=2
G = [[1,2,3,4,5],[1,3,2,5,4],[2,3,1,4,5],[2,1,3,5,4],[3,1,2,4,5],[3,2,1,5,4]] # |G|=6

#d,n,k;|G|=12,4,4;12
θ = [[1,3,4,2]] # |θ|=1
G = [[1,2,3,4],[1,3,4,2],[1,4,2,3],[4,3,2,1],[4,2,1,3],[4,1,3,2],
     [3,4,1,2],[3,1,2,4],[3,2,4,1],[2,1,4,3],[2,4,3,1],[2,3,1,4]] # |G|=12
```

Listing 1.2. Feasibility checker

```
using Combinatorics

"""
Returns the composition of two permutations ‘outer‘ and ‘inner‘ which both are passed as lists.
"""
function apply_perm(outer, inner)
    return [outer[inner[j]] for j in 1:length(inner)]
end

"""
Returns the union over all k in ‘k_range‘ of all sets of semiordered patterns SOP(n,k).
"""
function gen_SOP_n_k_range(k_range, n)
    SOP = []
    for j in k_range
        tmp_cll = []
        for cmb_j in combinations(1:n, j)
            for idx in 1:length(cmb_j)
                copy_cmb_j = copy(cmb_j)
                el = cmb_j[idx]
                deleteat!(copy_cmb_j, idx)
                push!(tmp_cll, tuple([el; sort(copy_cmb_j)]...))
            end
        end
        sort!(tmp_cll)
        append!(SOP, tmp_cll)
    end
    return SOP
end

"""
Checks if the family ‘F‘ is ‘k‘-restricted minwise independent.
"""
function is_mw_indep(F, k)
    SOP = gen_SOP_n_k_range(2:k, length(F[1]))
    unmet = []
    is_valid = true
    for sop in SOP
        counter = 0
        for l in 1:length(F)
            if all([F[l][sop[1]] < F[l][sop[j]] for j in 2:length(sop)])
                counter += 1
            end
        end
        if counter != div(length(F), length(sop))
            push!(unmet, (sop, counter))
            is_valid = false
        end
    end
    is_valid == false && println("unmet multiplicity: ", unmet)
    return is_valid
end
```