Sequencing Jobs with One Common and Multiple Individual Resources with Multivalued Decision Diagrams

Dissertantenseminar

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Problem Definition: PC-JSOCMSR

We consider the Prize-Collecting Job Sequencing with One Common and Multiple Secondary Resources (PC-JSOCMSR) problem.

We are given

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Each job $j \in J$ has

- a processing time $p_j > 0$
  - during which it fully requires resource $q_j \in R$ and
  - the common resource 0 for a part of its execution
    - for p_j^0 time beginning at p_j^{pre} after the jobs' start
- a set of time windows $W_j = \bigcup_{w=0,\ldots,\omega_j} W_{j,w}$
  - with $W_{j,w} = [W_{j,w}^{\mathrm{start}}, W_{j,w}^{\mathrm{end}}]$
- a prize $z_j > 0$.

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- a prize $z_j > 0$.

We are looking for a subset of jobs $S \subseteq J$

- that can be feasibly scheduled and
- maximizes the total prize, i.e., $\sum_{j \in S} z_j$.

Solution Representation

Observe that each job requires resource 0.

- Hence, a schedule of the jobs S implies a total ordering of the jobs.

We represent a solution by a permutation $\pi = (\pi_i)_{i=1,\ldots,|S|}$.

A normalized schedule is obtained by scheduling each job from S in the order given by π at the earliest feasible time.

Obviously, any optimal solution either is

- a normalized schedule, or
- there exists a corresponding normalized schedule.

Closely Related Literature

Andre A. Cire and Willem-Jan Van Hoeve.
Multivalued decision diagrams for sequencing problems.
Operations Research, 61(6):1411–1428, 2013.

David Bergman, Andre A. Cire, Willem-Jan van Hoeve, and J. N. Hooker.
Discrete optimization with decision diagrams.
INFORMS Journal on Computing, 28(1):47–66, 2016.

David Bergman, Andre A. Cire, Willem-Jan van Hoeve, and John N. Hooker.
Decision Diagrams for Optimization.
Artificial Intelligence: Foundations, Theory, and Algorithms. Springer, 2016.

Recursive Model for PC-JSOCMSR

The control variables of the model are $\pi_1, \ldots, \pi_n \in J$.

A state (P, t) consists of

- the set $P \subseteq J$ of jobs that still can be scheduled, and
- the vector $t = (t_r)_{r \in R_0}$ of the times from which on each resource r is available for performing a next job.

The initial state is $\mathbf{r} = (J, (T^{\min}, \ldots, T^{\min}))$.

Recursive Model for PC-JSOCMSR (cont.)

The earliest feasible time for job $j \in J$ not smaller than t is given by

$$\mathrm{eft}(j,t) = \min\{\infty,\ t' \geq t \mid [t', t' + p_j] \subseteq W_j\}. \tag{1}$$

Let the starting time of a next job $j \in J$ w.r.t. a state (P, t) be

$$\mathrm{s}((P,t),j) = \begin{cases} \mathrm{eft}(j, \max(t_0 - p_j^{\mathrm{pre}}, t_{q_j})) & \text{if } j \in P \\ \infty & \text{else.} \end{cases} \tag{2}$$

Recursive Model for PC-JSOCMSR (cont.)

The transition function to obtain the successor (P', t') of state (P, t) when scheduling job $j \in J$ next is

$$\tau((P,t),j) = \begin{cases} (P \setminus \{j\}, t') & \text{if } \mathrm{s}((P,t),j) \neq \infty \\ \hat{0} & \text{else,} \end{cases} \quad (3)$$

with

$$t'_0 = \mathrm{s}((P,t),j) + p_j^{\mathrm{pre}} + p_j^0 \quad (4)$$

$$t'_r = \mathrm{s}((P,t),j) + p_j \qquad \text{for } r = q_j \quad (5)$$

$$t'_r = t_r \qquad \text{for } r \in R \setminus \{q_j\} \quad (6)$$

and $\hat{0}$ representing the infeasible state.

Recursive Model for PC-JSOCMSR (cont.)

All states except the infeasible state $\hat{0}$ are terminal states.

Any sequence of state transitions $\tau(\ldots\tau(\mathbf{r},\pi_1)\ldots,\pi_i)$ yielding a terminal state represents a solution $(\pi_1,\ldots,\pi_i)$.

The cost associated with a state transition are $h((P,t),j) = z_j$.

PC-JSOCMSR can be solved by calling the following function with $Z^*(\mathbf{r})$:

$$Z^*(P,t) = \max\{0, z_j + Z^*(\tau((P,t),j)) \mid j \in P \wedge \tau((P,t),j) \neq \hat{0}\}. \quad (7)$$

Multivalued Decision Diagrams for PC-JSOCMSR

An MDD is a directed acyclic multi-graph $G = (V, A)$.

MDD $G = (V, A)$ is obtained from the recursive model by creating

- nodes for the terminal (feasible) states,
- arcs for all state transitions between terminal states
  - of length $h((P, t), j) = z_j$,

Paths from **r** to some node $v \in V$ correspond to solutions.

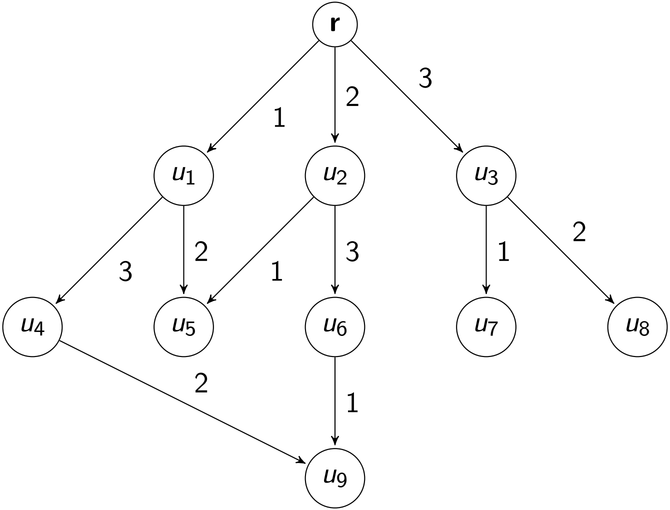
An optimal solution corresponds to a longest path in the MDD.

Such an MDD is called exact because we have

$$\mathrm{Sol}(\mathcal{P}) = \mathrm{Sol}(G), \tag{8}$$

$$Z(\pi) = Z^{\mathrm{lp}}. \tag{9}$$

Multivalued Decision Diagrams for PC-JSOCMSR



Relaxed MDDs for PC-JSOCMSR

A simple relaxation scheme merges a subset M of feasible states to obtain the state

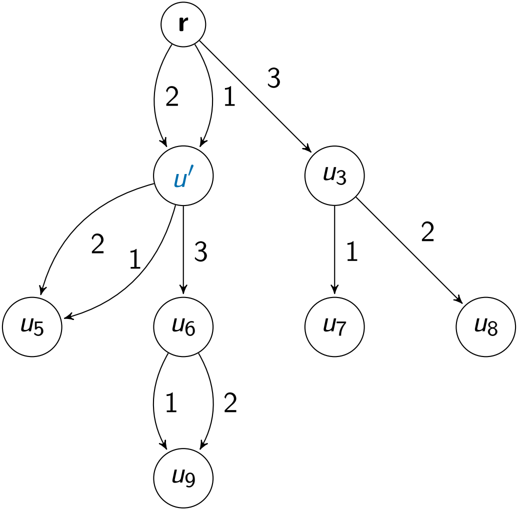
$$\oplus(M) = \left(\bigcup_{(P,t)\in M} P, \left(\min_{(P,t)\in M} t_r \right)_{r\in R_0} \right). \tag{10}$$

For a relaxed MDD we have

$$\mathrm{Sol}(\mathcal{P}) \subseteq \mathrm{Sol}(G), \tag{11}$$

$$Z(\pi) \leq Z^{\mathrm{lp}}. \tag{12}$$

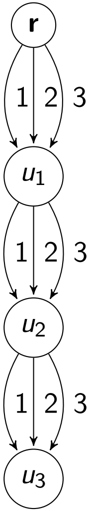
Relaxed MDDs for PC-JSOCMSR



Relaxed MDDs of Width One

A relaxed MDD of width one can be obtained by

- adding $\{0, \ldots, n+1\}$ nodes
  - where node 0 is associated with the initial state
- connect node $i = 0, \ldots, n$ with node $i+1$
  - by n arcs representing transitions for each job $j \in J$



An Incremental Refinement Algorithm for MDDs

Input: relaxed MDD $G = (V, A)$ with source node $\mathbf{r}$
Let $p = (a^{(1)}, \ldots, a^{(n)})$ be the longest path in G;
while *p is infeasible* **do**
    if *p contains a repetition of job j* **then**
        refine reptition of job j;
    else
        refine time window violation;
    end
    update longest path p;
end

An Incremental Refinement Algorithm for MDDs

Let $\mathrm{All}_u^{\downarrow} \subseteq J$ be the jobs on all paths from $\mathbf{r}$ to $u \in V$, i.e.,

$$\mathrm{All}_u^{\downarrow} = \bigcap_{a=(v,u)\in A^+(u)} \left(\mathrm{All}_v^{\downarrow} \cup \{\mathrm{job}(a)\}\right) \tag{13}$$

Let $\mathrm{Some}_u^{\downarrow} \subseteq J$ be the jobs on some path from $\mathbf{r}$ to $u \in V$, i.e.,

$$\mathrm{Some}_u^{\downarrow} = \bigcup_{a=(v,u)\in A^+(u)} \left(\mathrm{Some}_v^{\downarrow} \cup \{\mathrm{job}(a)\}\right) \tag{14}$$

Let $\mathrm{Some}_u^{\uparrow} \subseteq J$ be the jobs on some path from $u \in V$ to any reachable node, i.e.,

$$\mathrm{Some}_u^{\uparrow} = \bigcup_{a=(v,u)\in A^-(u)} \left(\mathrm{Some}_v^{\uparrow} \cup \{\mathrm{job}(a)\}\right) \tag{15}$$

Filtering a Relaxed MDD

We remove arcs for which all paths that cross them violate a constraint.

We can remove any arc $a = (u, v)$ if

- $\mathrm{s}(u, \mathrm{job}(a)) = \infty$
- $\mathrm{job}(a) \in \mathrm{All}_u^{\downarrow}$
- $|\mathrm{Some}_u^{\downarrow}| = \mathrm{Hops}_u^{\min}$ and $\mathrm{job}(a) \in \mathrm{Some}_u^{\downarrow}$
- $Z^{\mathrm{lp}}(v) + Z^{\mathrm{ub}}(v) < Z^{\mathrm{lb}}$

Nodes without an ingoing arc except **r** can be removed together with all its outgoing arcs.

Refinement of Job Repetitions

Lemma

A job j is assigned on each path starting from $\mathbf{r}$ *at most once if and only if* $j \notin \mathrm{Some}_u^{\downarrow} \cap \mathrm{Some}_u^{\uparrow} \setminus \mathrm{All}_u^{\downarrow}$ *for all nodes* $u \in V$*.*

Given job j. For all nodes $u \in V$ with $j \in \mathrm{Some}_u^{\downarrow} \cap \mathrm{Some}_u^{\uparrow} \setminus \mathrm{All}_u^{\downarrow}$:

- Replace u by two nodes u_1 and u_2
  - redirect all incoming arcs $a = (v, u)$ to u_1 if $j \in \mathrm{All}_v^{\downarrow} \cup \{\mathrm{job}(a)\}$ and
  - to u_2 otherwise,
  - replicate all outgoing arcs for both nodes.

Refinement of Time Window Violations

Let $(a^{(1)}, \dots, a^{(k)})$ be a path in our MDD starting at root $\mathbf{r}$, where

- $(a^{(1)}, \dots, a^{(k-1)})$ is a feasible solution and
- the last job violates its time windows.

Let $(a^{(i)}, \dots, a^{(k)})$ be the smallest subpath s.t.
$\tau(\dots \tau((P^{(i)}, t^{(i)}), j^{(i)}) \dots, j^{(k)})$ violates the last job's time window.

For node $(P^{(i)}, t^{(i)})$ to $(P^{(k)}, t^{(k)})$ do:

- replace current node $u = (P^{(l)}, t^{(l)})$ by nodes u_1 and u_2.
- Let $(P', t') = \tau((P^{(l-1)}, t^{(l-1)}), j^{(l-1)})$.
- Redirect all incoming arcs $a = (v, u)$ to u_1 if $(P'', t'') = \tau(v, \mathrm{job}(a))$ and $t''_r \geq t'_r$ for all $r \in R$.
- All other incoming arcs are redirected to u_2.
- The outgoing arcs are replicated for u_1 and u_2.

Next Steps

- Implementation of the algorithm
- Alternative initial (relaxed) MDDs
- Preprocessing of initial (relaxed) MDDs
- Identifying supplementary filtering rules
- Combination with A* approach